A dynamic logic of knowledge, graded beliefs and graded goals and its application to emotion modelling

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Abstract. The paper introduces a logic which allows to represent different kinds of mental states of an agent such as knowledge, graded belief, and graded goal, and the notion of epistemic action (as the action of learning that a certain fact φ is true.) The logic is applied to the formalization of expectation-based emotions such as hope, fear, disappointment and relief, and of their intensity.

1 Introduction

In this paper, I will present a logic called DL-KGBG (*Dynamic Logic of Knowledge, Graded Beliefs and Goals*) which allows: (1) to express that a given epistemic action *is going to* occur; (2) to describe the effects of an epistemic action on an agent's mental state; (3) to represent different kinds of mental attitudes including knowledge, graded beliefs (*i.e.*, believing with a certain strength that a given proposition is true) and graded goals (*i.e.*, wishing with a certain strength a given proposition to be true.) An epistemic action is nothing but the mental action (or process) of learning that a given proposition is true, of changing the agent's beliefs in the light of a new incoming evidence. I here follow the tradition of Dynamic Epistemic Logic (DEL) [10] in modelling epistemic actions as basic operations of model transformation in the semantics.

In the second part of the paper, the logic DL-KGBG will be applied to the formalization of expectation-based emotions such as hope, fear, disappointment and relief. It will be shown that this logic allows to represent the notion of emotion intensity (*e.g.*, how much an agent feels happy or sad, disappointed or relieved, *etc.*), which is ignored by most of current logical models of emotions [19, 1, 18].[1] Following previous works on the cognitive theory of expectations [8, 20], an expectation-based emotion is here conceived as an emotion that an agent experiences when having an expectation about a certain fact φ, that is when: (1) *believing that φ is true with a certain strength, but envisaging the possibility that φ could be false, and (2) either*, *having the goal that φ is true (positive expectation) or*, *having the goal that φ is false (negative expectation)*.

The rest of the paper is organized as follows. Section 2 will be devoted to present the logic DL-KGBG. A complete axiomatization as well as a decidability result for this logic will be given in Section 3. In Section 4, the logic DL-KGBG will be applied to the formalization of expectation-based emotions and of their intensity.

[1] The only exception is [26]. However, different from the approach presented here, Meyer and coll. do not study the cognitive variables determining intensities (*e.g.*, the *strength* of an agent's beliefs and the *importance* of his goals) but instead propose a function describing how the intensity of an emotion decreases over time.

2 A dynamic logic of knowledge, graded beliefs and graded goals

This section presents the syntax and the semantics of the logic DL-KGBG, as well as a complete axiomatization and a decidability result for this logic.

2.1 Syntax

Assume a countable set of atomic propositions $Prop = \{p, q, \ldots\}$ and a finite set of natural numbers $Num = \{x \in \mathbb{N} : 0 \leq x \leq \mathsf{max}\}$ with $\mathsf{max} \in \mathbb{N}$. The language $\mathcal{L}$ of DL-KGBG is defined by the following grammar in Backus-Naur Form (BNF):

$$\begin{aligned} Act &: \alpha ::= *\varphi \\ Atm &: \chi ::= p \mid \mathsf{after}_{\epsilon|\alpha} \mid \mathsf{exc}_{\mathsf{h}} \mid \mathsf{des}_{\mathsf{h}} \\ Fml &: \varphi ::= \chi \mid \neg\varphi \mid \varphi \wedge \varphi \mid [\mathsf{K}]\varphi \mid [\alpha]\varphi \end{aligned}$$

where p ranges over $Prop$, h ranges over Num, α ranges over Act, and ϵ ranges over the set of epistemic action sequences Act^*. The other Boolean constructions $\top$, $\bot$, $\vee$, $\rightarrow$ and $\leftrightarrow$ are defined from p, $\neg$ and $\wedge$ in the standard way. I define the set of objective facts Obj as the set of all Boolean combinations of atomic propositions.

An epistemic action $*\varphi$ in Act is the mental action (or process) of learning that φ is true. As I will show in Section 2.2, technically this amounts to an operation of beliefs' conditionalization in Spohn's sense [24]. Act^* is the set of epistemic action sequences. An epistemic action sequence is of the form $\alpha_1; \ldots; \alpha_n$ where ";" stands for concatenation. I note *nil* the empty sequence of epistemic actions.

Atm contains special constructions of different types which are used to represent both the objective world as well as the mental state of an agent. The formula $\mathsf{after}_{\epsilon|\alpha}$ is a special atom that is read "after the sequence of epistemic actions ϵ, the epistemic action α will occur". The atomic formula $\mathsf{after}_{nil|\alpha}$ specifies the *actual* epistemic action that is going to occur next. Therefore, I define:

$$\mathsf{occ}_{\alpha} \stackrel{\mathtt{def}}{=} \mathsf{after}_{nil|\alpha}$$

where occ_{α} has to be read "the epistemic action α *is going to* occur".

The special atoms $\mathsf{exc}_{\mathsf{h}}$ are used to identify the degree of *plausibility* of a given world for the agent. Starting from [13], ranking among possible worlds have been extensively used in belief revision theory. I here use the notion of plausibility first introduced by Spohn [24]. Following Spohn's theory, the worlds that are assigned the smallest numbers are the most plausible, according to the beliefs of the individual. That is, the number h assigned to a given world rather captures the degree of *exceptionality* of this world, where the exceptionality degree of a world is nothing but the opposite of its plausibility degree (*i.e.*, the exceptionality degree of a world decreases when its plausibility degree increases.) Therefore, formula $\mathsf{exc}_{\mathsf{h}}$ can be read alternatively as "the current world has a degree of exceptionality h" or "the current world has a degree of plausibility $\mathsf{max} - \mathsf{h}$". The special atoms $\mathsf{des}_{\mathsf{h}}$ are used to identify the degree of *desirability* (or the degree of *goodness*) of a given world for the agent. Contrary to plausibility, the worlds that are assigned the biggest numbers are the most desirable for the agent. The degree of undesirability (or degree of a badness) of a given world is the opposite of its desirability degree. Therefore, formula des_h can be read alternatively as "the current world has a degree of desirability h" or "the current world has a degree of undesirability $\mathsf{max} - \mathsf{h}$".

The formula $[\alpha]\varphi$ has to be read "after the occurrence of epistemic action α, φ will be true", while $[\mathsf{K}]\varphi$ has to be read "the agent knows that φ is true". This concept of knowledge is the standard S5-notion, partition-based and fully introspective, that is commonly used both in computer science [11] and economics [4]. As I will show in the next Section 2.2, the operator $[\mathsf{K}]$ captures a form of 'absolutely unrevisable belief', that is, a form of belief which is stable under belief revision with any new *evidence*. A similar property for the notion of knowledge has been advanced by the so-called *defeasibility* (or *stability*) *theory of knowledge* [14, 25, 23]. According to this theory, a given piece of information φ is part of the agent's knowledge only if the agent's justification to believe that φ is true is sufficiently strong that it is not capable of being defeated by evidence that the agent does not possess. As pointed out by [5], two different interpretations of the term 'evidence' have been given in the context of this theory, each giving a different interpretation of what *knowledge* is. The first one defines knowledge as a form of belief which is stable under belief revision with 'any piece of *true* information', while the second one gives a stronger definition of knowledge as a form of belief which is stable under belief revision with 'any piece of information'. The concept formalized by the operator $[\mathsf{K}]$ captures this latter form of knowledge in a stronger sense.

2.2 Semantics

Definition 1 (Model). *DL-KGBG-models are tuples* $M = \langle W, \sim, \kappa_{\mathsf{exc}}, \kappa_{\mathsf{des}}, \mathcal{P}, \mathcal{V}\rangle$ *where:*

- *W is a nonempty set of possible worlds or states;*
- *$\sim$ is an equivalence relation between worlds in W;*
- *$\kappa_{\mathsf{exc}} : W \longrightarrow Num$ and $\kappa_{\mathsf{des}} : W \longrightarrow Num$ are functions from the set of possible worlds into the finite set of natural numbers Num;*
- *$\mathcal{P} : W \times Act^* \longrightarrow Act$ is a partial function called* protocol*, mapping worlds and action sequences to actions;*
- *$\mathcal{V} : W \longrightarrow 2^{Prop}$ is a valuation function.*

As usual, $p \in \mathcal{V}(w)$ means that proposition p is true at world w. The equivalence relation $\sim$, which is used to interpret the epistemic operator $[\mathsf{K}]$, can be viewed as a function from W to 2^W. Therefore, we can write $\sim(w) = \{v \in W : w \sim v\}$. The set $\sim(w)$ is the agent's *information state* at world w: the set of worlds that the agent considers possible at world w or, the set of worlds that the agent cannot distinguish from world w. As $\sim$ is an equivalence relation, if $w \sim v$ then the agent has the same information state at w and v (*i.e.*, the agent has the same knowledge at w and v.)

The function κ_{exc} represents a plausibility grading of the possible worlds and is used to interpret the atomic formulas exc_h. $\kappa_{\mathsf{exc}}(w) = \mathsf{h}$ means that, according to the agent the world w has a degree of exceptionality h or, alternatively, according to the agent the world w has a degree of plausibility $\mathsf{max} - \mathsf{h}$. (Remember that the degree of plausibility of a world is the opposite of its exceptionality degree.) The function κ_{exc}

allows to model the notion of belief: among the worlds the agent cannot distinguish from a given world w (*i.e.*, the agent's information state at w), there are worlds that the agent considers more plausible than others. For example, suppose that $\sim(w) = \{w, v, u\}$, $\kappa_{\mathsf{exc}}(w) = 2$, $\kappa_{\mathsf{exc}}(u) = 1$ and $\kappa_{\mathsf{exc}}(v) = 0$. This means that at world w the agent cannot distinguish the three worlds w, v and u, that is, $\{w, v, u\}$ is the set of worlds that the agent considers possible at world w. Moreover, according to the agent, the world v is strictly more plausible than the world u and the world u is strictly more plausible than the world w (as $\mathsf{max} - 0 > \mathsf{max} - 1 > \mathsf{max} - 2$.)

The function κ_{des} is used to interpret the atomic formulas $\mathsf{des}_{\mathsf{h}}$. $\kappa_{\mathsf{des}}(w) = \mathsf{h}$ means that, according to the agent the world w has a degree of goodness (or desirability) h or, alternatively, according to the agent the world w has a degree of badness (or undesirability) $\mathsf{max} - \mathsf{h}$. (Remember that the degree of undesirability of a world is the opposite of its desirability degree.)

Finally, the protocol function $\mathcal{P}$ is used to interpret the atomic formulas $\mathsf{after}_{\epsilon|\alpha}$. A similar notion of protocol has been studied by [6]. For every world $w \in W$, $\mathcal{P}(w, \epsilon) = \alpha$ means that, at world w, the epistemic action α will occur after the sequence of epistemic actions ϵ. For example, imagine that the agent wakes up in the morning. Then $\mathcal{P}(w, *8pm; *rain) = *earthquake$ means that, at world w, after learning in sequence that 'it is 8 p.m.' (by looking at the alarm) and that 'it is raining outside' (by opening the window), the agent (will switch on the radio) and will learn that 'during the night an earthquake of magnitude 5 has occurred'. $\mathcal{P}(w, nil) = \alpha$ means that, at world w, the epistemic action α will occur next. $\mathcal{P}$ is supposed to be a partial function as I want to allow states in which no action occurs.

DL-KGBG-models are supposed to satisfy the following additional *normality* constraint for the plausibility grading.

($NORM_{\kappa_{\mathsf{exc}}}$) for every $w \in W$, there is v such that $w \sim v$ and $\kappa_{\mathsf{exc}}(v) = 0$.

Definition 2 (Truth conditions). *Given a DL-KGBG-model M, a world w and a formula φ, $M, w \models \varphi$ means that φ is true at world w in M. The rules defining the truth conditions of formulas are:*

- $M, w \models p$ *iff* $p \in \mathcal{V}(w)$
- $M, w \models \mathsf{after}_{\epsilon|\alpha}$ *iff* $\mathcal{P}(w, \epsilon)$ *is defined and* $\mathcal{P}(w, \epsilon) = \alpha$
- $M, w \models \mathsf{exc}_{\mathsf{h}}$ *iff* $\kappa_{\mathsf{exc}}(w) = \mathsf{h}$
- $M, w \models \mathsf{des}_{\mathsf{h}}$ *iff* $\kappa_{\mathsf{des}}(w) = \mathsf{h}$
- $M, w \models \neg\varphi$ *iff not* $M, w \models \varphi$
- $M, w \models \varphi \wedge \psi$ *iff* $M, w \models \varphi$ *and* $M, w \models \psi$
- $M, w \models [\mathsf{K}]\varphi$ *iff* $M, v \models \varphi$ *for all* v *with* $w \sim v$
- $M, w \models [*\varphi]\psi$ *iff* $M^{*\varphi}, w \models \psi$

where the updated model $M^{*\varphi}$ is defined according to the Definition 9 below.

Following [24], I extend the exceptionality degree of a possible world to a plausibility/exceptionality degree of a formula viewed as a set of worlds.

Definition 3 (Exceptionality degree of a formula). *The exceptionality degree of a formula φ at world w, noted $\kappa^{w}_{\mathsf{exc}}(\varphi)$, is defined as follows:*

$$\kappa^{w}_{\mathsf{exc}}(\varphi) \stackrel{\mathsf{def}}{=} \min\{\kappa_{\mathsf{exc}}(v) : M, v \models \varphi \text{ and } w \sim v\}.$$

As expected, the *plausibility* degree of a formula φ, noted $\kappa^w_{\mathsf{plaus}}(\varphi)$, is defined as $\mathsf{max} - \kappa^w_{\mathsf{exc}}(\varphi)$. I do a similar manipulation for the desirability degree of a formula.

Definition 4 (Desirability degree of a formula). *The desirability degree of a formula φ at world w, noted $\kappa^w_{\mathsf{des}}(\varphi)$, is defined as follows:*

$$\kappa^w_{\mathsf{des}}(\varphi) \stackrel{\text{def}}{=} \min\{\kappa_{\mathsf{des}}(v) : M, v \models \varphi \text{ and } w \sim v\}.$$

As expected, the *undesirability* degree of a formula φ, noted $\kappa^w_{\mathsf{undes}}(\varphi)$, is defined as $\mathsf{max} - \kappa^w_{\mathsf{des}}(\varphi)$. Again following [24], I define the concept of belief as a formula which is true in all worlds that are maximally plausible (or minimally exceptional).

Definition 5 (Belief). *At world w the agent believes that φ is true if and only if, for every v such that $w \sim v$, if $\kappa_{\mathsf{exc}}(v) = 0$ then $M, v \models \varphi$.*

The following concept of graded belief is taken from [15]. I say that at world w the agent believes that φ with strength at least h if and only if, all possible worlds in which φ is false are exceptional at least degree h (or all possible worlds in which φ is false are plausible at most degree $\mathsf{max} - \mathsf{h}$.)

Definition 6 (Graded belief). *At world w the agent believes that φ with strength at least h if and only if, $\kappa^w_{\mathsf{exc}}(\neg\varphi) \geq \mathsf{h}$.*

I define the following concept of strong (or certain) belief: an agent has the strong belief that φ if and only if either he knows that φ is true (*i.e.*, he has an unrevisable belief that φ is true) or he believes that φ is true with maximal strength max.

Definition 7 (Strong belief). *At world w the agent strongly believes that φ (or at w the agent is certain that φ is true) if and only if, $M, v \models \varphi$ for all v with $w \sim v$ or $\kappa^w_{\mathsf{exc}}(\neg\varphi) = \mathsf{max}$.*

The following concept of graded goal is the motivational counterpart of the notion of graded belief. This concept has not been studied before in the logical literature. I say that at world w the agent wants (or wishes) φ to be true with strength at least h if and only if, all possible worlds in which φ is true are desirable at least degree h (or all possible worlds in which φ is true are undesirable at most degree $\mathsf{max} - \mathsf{h}$.) This implies that, all possible worlds which are desirable at most degree $\mathsf{h} - 1$ satisfy $\neg\varphi$ (or all possible worlds which are undesirable at least degree $\mathsf{max} - (\mathsf{h} - 1)$ satisfy $\neg\varphi$.)

Definition 8 (Graded goal). *At world w the agent wants/wishes φ to be true with strength at least h (or the agent has the goal that φ with strength at least h) if and only if, $\kappa^w_{\mathsf{des}}(\varphi) \geq \mathsf{h}$.*

The reason why the definition of graded goal is not symmetric to the definition of graded belief is that these two concepts satisfy different logical properties. As I will show below in Section 2.3, Definition 6 and Definition 8 allow to capture interesting differences between graded belief and graded goal, especially on the way they distribute over conjunction and over disjunction. As the following proposition highlights, the concepts of belief, graded belief, strong belief and graded goal semantically defined in Definitions 5-8 are all syntactically expressible in the logic DL-KGBG.

Proposition 1. *For every DL-KGBG-model M and world w:*

1. *at w the agent believes that φ is true if and only if*
   $M, w \models [\mathsf{K}](\mathsf{exc}_0 \to \varphi)$
2. *at w the agent believes that φ is true with strength at least* h *if and only if*
   $M, w \models \bigvee_{\mathsf{l} \in Num : \mathsf{l} \geq \mathsf{h}} (\langle \mathsf{K} \rangle (\mathsf{exc}_\mathsf{l} \wedge \neg\varphi) \wedge \bigwedge_{\mathsf{k} \in Num : \mathsf{k} < \mathsf{l}} [\mathsf{K}](\mathsf{exc}_\mathsf{k} \to \varphi))$
3. *at w the agent strongly believes that φ is true (or the agent is certain that φ is true) if and only if*
   $M, w \models \bigwedge_{\mathsf{k} \in Num : \mathsf{k} < \mathsf{max}} [\mathsf{K}](\mathsf{exc}_\mathsf{k} \to \varphi)$
4. *at w the agent wants φ to be true with strength at least* h *if and only if*
   $M, w \models \bigvee_{\mathsf{l} \in Num : \mathsf{l} \geq \mathsf{h}} (\langle \mathsf{K} \rangle (\mathsf{des}_\mathsf{l} \wedge \varphi) \wedge \bigwedge_{\mathsf{k} \in Num : \mathsf{k} < \mathsf{l}} [\mathsf{K}](\mathsf{des}_\mathsf{k} \to \neg\varphi))$

In the light of Proposition 1, I provide the following four syntactic abbreviations:

$$
\begin{aligned}
\mathsf{Bel}\varphi &\stackrel{\text{def}}{=} [\mathsf{K}](\mathsf{exc}_0 \to \varphi) \\
\mathsf{Bel}^{\geq \mathsf{h}}\varphi &\stackrel{\text{def}}{=} \bigvee_{\mathsf{l} \in Num : \mathsf{l} \geq \mathsf{h}} (\langle \mathsf{K} \rangle (\mathsf{exc}_\mathsf{l} \wedge \neg\varphi) \wedge \bigwedge_{\mathsf{k} \in Num : \mathsf{k} < \mathsf{l}} [\mathsf{K}](\mathsf{exc}_\mathsf{k} \to \varphi)) \\
\mathsf{SBel}\varphi &\stackrel{\text{def}}{=} \bigwedge_{\mathsf{k} \in Num : \mathsf{k} < \mathsf{max}} [\mathsf{K}](\mathsf{exc}_\mathsf{k} \to \varphi) \\
\mathsf{Goal}^{\geq \mathsf{h}}\varphi &\stackrel{\text{def}}{=} \bigvee_{\mathsf{l} \in Num : \mathsf{l} \geq \mathsf{h}} (\langle \mathsf{K} \rangle (\mathsf{des}_\mathsf{l} \wedge \varphi) \wedge \bigwedge_{\mathsf{k} \in Num : \mathsf{k} < \mathsf{l}} [\mathsf{K}](\mathsf{des}_\mathsf{k} \to \neg\varphi))
\end{aligned}
$$

where $\mathsf{Bel}\varphi$, $\mathsf{Bel}^{\geq \mathsf{h}}\varphi$, $\mathsf{SBel}\varphi$ and $\mathsf{Goal}^{\geq \mathsf{h}}\varphi$ respectively mean: "the agent believes that φ", "the agent believes that φ with strength *at least* h",[2] "the agent strongly believes that φ" and "the agent wants/wishes φ to be true with strength at least h".

Definition 9 (Model update). *Given a DL-KGBG-model $M = \langle W, \sim, \kappa_{\mathsf{exc}}, \kappa_{\mathsf{des}}, \mathcal{P}, \mathcal{V} \rangle$, $M^{*\varphi}$ is the model such that:*

$$
\begin{aligned}
W^{*\varphi} &= W \\
\sim^{*\varphi} &= \sim \\
\text{for all } w,\ \kappa^{*\varphi}_{\mathsf{exc}}(w) &= \begin{cases} \kappa_{\mathsf{exc}}(w) - \kappa^{w}_{\mathsf{exc}}(\varphi) & \text{if } M, w \models \varphi \\ Cut_{\mathsf{max}}(\kappa_{\mathsf{exc}}(w) + \Delta) & \text{if } M, w \models \neg\varphi \end{cases} \\
\text{for all } w,\ \kappa^{*\varphi}_{\mathsf{des}}(w) &= \kappa_{\mathsf{des}}(w) \\
\text{for all } w,\ \mathcal{P}^{*\varphi}(w, \epsilon) &= \begin{cases} \alpha & \text{if } \mathcal{P}(w, *\varphi;\epsilon) = \alpha \\ \text{undefined} & \text{if } \mathcal{P}(w, *\varphi;\epsilon) \text{ is undefined} \end{cases} \\
\mathcal{V}^{*\varphi} &= \mathcal{V}
\end{aligned}
$$

where $\Delta \in Num \setminus \{0\}$ and

$$
Cut_{\mathsf{max}}(x) = \begin{cases} x & \text{if } 0 \leq x \leq \mathsf{max} \\ \mathsf{max} & \text{if } x > \mathsf{max} \end{cases}
$$

An epistemic action $*\varphi$ makes the protocol $\mathcal{P}$ to advance one step forward (see the definition of $\mathcal{P}^{*\varphi}$.) That is, if α will occur after the sequence of events $*\varphi;\epsilon$ then, after the occurrence of the epistemic action $*\varphi$, it is the case that α will occur after the sequence

[2] Similar operators for graded belief are studied in [15, 3, 9].

of events ϵ. As epistemic actions only affect the agent's beliefs and do not affect the objective world, the valuation function $\mathcal{V}$ is not altered by them (see the definition of $\mathcal{V}^{*\varphi}$.) Moreover, the epistemic action $*\varphi$ modifies the plausibility ordering (see the definition of $\kappa_{\mathsf{exc}}^{*\varphi}$) but does not modify the agent's information state (see the definition of $\sim^{*\varphi}$) nor the desirability ordering (see the definition of $\kappa_{\mathsf{des}}^{*\varphi}$.) In particular, the epistemic action of learning that φ is true induces a kind of belief conditionalization in Spohn's sense [24]. The plausibility ranking over possible worlds is updated as follows.

- For every world w in which φ is true, the degree of exceptionality of w decreases from $\kappa_{\mathsf{exc}}(w)$ to $\kappa_{\mathsf{exc}}(w) - \kappa_{\mathsf{exc}}^{w}(\varphi)$, which is the same thing as saying that, degree of plausibility of w increases from $\mathsf{max} - \kappa_{\mathsf{exc}}(w)$ to $\mathsf{max} - (\kappa_{\mathsf{exc}}(w) - \kappa_{\mathsf{exc}}^{w}(\varphi))$. (Note that, by Definition 3, we have $\kappa_{\mathsf{exc}}(w) - \kappa_{\mathsf{exc}}^{w}(\varphi) \leq \kappa_{\mathsf{exc}}(w)$.)
- For every world w in which φ is false, the degree of exceptionality of w increases from $\kappa_{\mathsf{exc}}(w)$ to $Cut_{\mathsf{max}}(\kappa_{\mathsf{exc}}(w) + \Delta)$, which is the same thing as saying that, degree of plausibility of w decreases from $\mathsf{max} - \kappa_{\mathsf{exc}}(w)$ to $\mathsf{max} - Cut_{\mathsf{max}}(\kappa_{\mathsf{exc}}(w) + \Delta)$.

Cut_{max} is a minor technical device, taken from [3], which ensures that the new plausibility assignment fits into the finite set of natural numbers *Num*. The parameter Δ is a *conservativeness* index which captures the agent's disposition to radically change his beliefs in the light of a new evidence. More precisely, the higher is the index Δ, and the higher is the agent's disposition to decrease the plausibility degree of those worlds in which the learnt fact φ is false. (When $\Delta = \mathsf{max}$, the agent is minimally conservative.) I assume that Δ is different from 0 in order to ensure that, after learning that p is true, the agent will believe p. That is, from $\Delta \in Num \setminus \{0\}$ it follows that $[*p]\mathsf{Bel}p$ is valid for every proposition $p \in Prop$. Note that the operation $*\varphi$ preserves the constraints on DL-KGBG-models: if M is a DL-KGBG-model then $M^{*\varphi}$ is a DL-KGBG-model too.

In the sequel I write $\models_{\mathsf{DL\text{-}KGBG}} \varphi$ to mean that φ is *valid* in DL-KGBG (φ is true in all DL-KGBG-models.)

2.3 Some properties of mental attitudes

The following are some interesting examples of validity. For every $\mathsf{h}, \mathsf{k} \in Num$ we have:

$$\models_{\mathsf{DL\text{-}KGBG}} \mathsf{Bel}^{\geq \mathsf{h}}\varphi \rightarrow \mathsf{Bel}^{\geq \mathsf{k}}\varphi \text{ if } \mathsf{h} \geq \mathsf{k} \tag{1}$$

$$\models_{\mathsf{DL\text{-}KGBG}} \mathsf{Goal}^{\geq \mathsf{h}}\varphi \rightarrow \mathsf{Goal}^{\geq \mathsf{k}}\varphi \text{ if } \mathsf{h} \geq \mathsf{k} \tag{2}$$

$$\models_{\mathsf{DL\text{-}KGBG}} [\mathsf{K}]\varphi \rightarrow \neg\mathsf{Bel}^{\geq \mathsf{h}}\varphi \tag{3}$$

$$\models_{\mathsf{DL\text{-}KGBG}} \mathsf{SBel}\varphi \leftrightarrow ([\mathsf{K}]\varphi \vee \mathsf{Bel}^{\geq \mathsf{max}}\varphi) \tag{4}$$

$$\models_{\mathsf{DL\text{-}KGBG}} \neg(\mathsf{Bel}\varphi \wedge \mathsf{Bel}\neg\varphi) \tag{5}$$

$$\models_{\mathsf{DL\text{-}KGBG}} [*\varphi]\mathsf{Bel}\varphi \text{ if } \varphi \in Obj \tag{6}$$

$$\models_{\mathsf{DL\text{-}KGBG}} [\mathsf{K}]\varphi \rightarrow [*\psi][\mathsf{K}]\varphi \text{ if } \varphi \in Obj \tag{7}$$

$$\models_{\mathsf{DL\text{-}KGBG}} (\mathsf{Bel}^{\geq \mathsf{h}}\varphi \wedge \mathsf{Bel}^{\geq \mathsf{k}}\psi) \rightarrow \mathsf{Bel}^{\geq \min\{\mathsf{h},\mathsf{k}\}}(\varphi \wedge \psi) \tag{8}$$

$$\models_{\mathsf{DL\text{-}KGBG}} (\mathsf{Goal}^{\geq \mathsf{h}}\varphi \wedge \mathsf{Goal}^{\geq \mathsf{k}}\psi) \rightarrow (\mathsf{Goal}^{\geq \max\{\mathsf{h},\mathsf{k}\}}(\varphi \wedge \psi) \vee [\mathsf{K}](\neg\varphi \vee \neg\psi)) \tag{9}$$

$$\models_{\mathsf{DL\text{-}KGBG}} (\mathsf{Bel}^{\geq \mathsf{h}}\varphi \wedge \mathsf{Bel}^{\geq \mathsf{k}}\psi) \rightarrow (\mathsf{Bel}^{\geq \max\{\mathsf{h},\mathsf{k}\}}(\varphi \vee \psi) \vee [\mathsf{K}](\varphi \vee \psi)) \tag{10}$$

$$\models_{\mathsf{DL\text{-}KGBG}} (\mathsf{Goal}^{\geq \mathsf{h}}\varphi \wedge \mathsf{Goal}^{\geq \mathsf{k}}\psi) \rightarrow \mathsf{Goal}^{\geq \min\{\mathsf{h},\mathsf{k}\}}(\varphi \vee \psi) \tag{11}$$

According to the validity (1), if the agent believes that φ with strength at least h and $\mathsf{h} \geq \mathsf{k}$, then he believes that φ is true with strength at least k. Validity (2) is the corresponding property for graded goals. The validity (3) highlights that knowledge and graded belief are distinct concepts. According to the validity (4), the agent has the strong belief that φ (*i.e.*, the agent is certain that φ) if and only if, either he knows that φ is true (*i.e.*, he has an unrevisable belief that φ is true) or he believes that φ with maximal strength max. According to the validity (5) (which follows from the normality constraint $NORM_{\kappa_{exc}}$ given in Section 2.2), an agent cannot have inconsistent beliefs.

The validity (6) highlights a basic property of belief revision in the sense of AGM theory [2]: if φ is an objective fact then, after learning that φ is true, the agent believes that φ is true. The validity (7) highlights a basic property of knowledge as *unrevisable belief* (see Section 2.1 for a discussion): if φ is an objective fact and the agent knows that φ is true then, after learning a new fact ψ, he will continue to know that φ is true. In this sense, knowledge is stable under belief revision. According to the validity (8), if the agent believes that φ with strength at least h and believes that ψ with strength at least k, then the strength of the belief that $\varphi \wedge \psi$ is at least $\min\{\mathsf{h}, \mathsf{k}\}$. According to the validity (10), if the agent believes that φ with strength h and believes that ψ with strength k, then either he believes $\varphi \vee \psi$ with strength at least $\max\{\mathsf{h}, \mathsf{k}\}$ or he knows that $\varphi \vee \psi$. (Remember that knowledge and graded belief are distinct concepts.)

The validities (9) and (11) are corresponding principles for goals. According to the validity (9), if the agent wishes φ to be true with strength at least h and wishes ψ to be true with strength at least k, then either he wishes $\varphi \wedge \psi$ to be true with strength at least $\max\{\mathsf{h}, \mathsf{k}\}$ or he knows that φ and ψ are inconsistent facts. According to the validity (11), if the agent wishes φ to be true with strength at least h and wishes ψ to be true with strength at least k, then he wishes $\varphi \vee \psi$ to be true with strength at least $\min\{\mathsf{h}, \mathsf{k}\}$.

The interesting aspect is that graded goals distribute over conjunction and over disjunction in the opposite way as graded beliefs. Consider for instance the validity (9) and compare it to the validity (8). The joint occurrence of two events φ and ψ is less plausible than the occurrence of a single event. This is the reason why in the right side of the validity (8) we have the *min*. On the contrary, the joint occurrence of two desired events φ and ψ is more desirable than the occurrence of a single event. This is the reason why in the right side of the validity (9) we have the *max*. For example, suppose Peter wishes to go to the cinema in the evening with strength at least h (*i.e.*, $\mathsf{Goal}^{\geq \mathsf{h}} goToCinema$) and, at the same time, he wishes to spend the evening with his girlfriend with strength at least k (*i.e.*, $\mathsf{Goal}^{\geq \mathsf{k}} stayWithGirlfriend$.) Then, according to the validity (9), either Peter wishes to to go the cinema with his girlfriend with strength at least $\max\{\mathsf{h}, \mathsf{k}\}$ (*i.e.*, $\mathsf{Goal}^{\geq \max\{\mathsf{h},\mathsf{k}\}}(goToCinema \wedge stayWithGirlfriend)$) or Peter knows that going to the cinema with his girlfriend is impossible (*i.e.*, $[\mathsf{K}](\neg goToCinema \vee \neg stayWithGirlfriend)$), perhaps because he knows that his girlfriend has the flu and cannot go out.

2.4 Exact strength of belief and exact strength of goal

The operators of type $\mathsf{Bel}^{\geq \mathsf{h}}$ and $\mathsf{Goal}^{\geq \mathsf{h}}$ introduced above enable to specify a concept of graded belief of the form "the agent believes that φ with strength h" in which the *exact* strength of the agent's belief is specified. I assume that "the agent believes that φ exactly with strength h", noted $\mathsf{Bel}^{\mathsf{h}}\varphi$ if and only if, the agent believes that φ with strength at least h and it is not the case that the agent believes that φ with strength at least $\mathsf{h}+1$. Formally, for every $\mathsf{h} < \mathsf{max}$, I define:

$$\mathsf{Bel}^{\mathsf{h}}\varphi \stackrel{\text{def}}{=} \mathsf{Bel}^{\geq \mathsf{h}}\varphi \wedge \neg \mathsf{Bel}^{\geq \mathsf{h}+1}\varphi$$

In order to have a uniform notation, I will use $\mathsf{Bel}^{\geq \mathsf{max}}\varphi$ and $\mathsf{Bel}^{\mathsf{max}}\varphi$ as interchangeable expressions. I do a similar manipulation for goals. For every $\mathsf{h} < \mathsf{max}$, I define:

$$\mathsf{Goal}^{\mathsf{h}}\varphi \stackrel{\text{def}}{=} \mathsf{Goal}^{\geq \mathsf{h}}\varphi \wedge \neg \mathsf{Goal}^{\geq \mathsf{h}+1}\varphi$$

where $\mathsf{Goal}^{\mathsf{h}}\varphi$ has to be read "the agent wants/wishes φ to be true exactly with strength h". As for beliefs, I will use $\mathsf{Goal}^{\geq \mathsf{max}}\varphi$ and $\mathsf{Goal}^{\mathsf{max}}\varphi$ as interchangeable expressions.

3 Decidability and axiomatization

Let L-KGBG be the fragment of the logic DL-KGBG without dynamic operators, that is, let the language of L-KGBG be the set of formulas defined by the following BNF:

$$\begin{aligned}
Act &: \alpha ::= *\varphi \\
Atm &: \chi ::= p \mid \mathsf{after}_{\epsilon|\alpha} \mid \mathsf{exc}_{\mathsf{h}} \mid \mathsf{des}_{\mathsf{h}} \\
Fml &: \varphi ::= \chi \mid \neg\varphi \mid \varphi \wedge \varphi \mid [\mathsf{K}]\varphi
\end{aligned}$$

where p ranges over Atm, h ranges over Num, α ranges over Act, and ϵ ranges over Act^*.

Proposition 2. *The following formulas are DL-KGBG valid for every $\mathsf{h}, \mathsf{k} \in Num$, $\alpha \in Act$, $\epsilon \in Act^*$.*

$$\begin{aligned}
&\mathsf{after}_{\epsilon|\alpha} \rightarrow \neg\mathsf{after}_{\epsilon|\beta} \textit{ if } \alpha \neq \beta \\
&\textstyle\bigvee_{\mathsf{h} \in Num} \mathsf{exc}_{\mathsf{h}} \\
&\textstyle\bigvee_{\mathsf{h} \in Num} \mathsf{des}_{\mathsf{h}} \\
&\langle \mathsf{K} \rangle \mathsf{exc}_0 \\
&\mathsf{exc}_{\mathsf{h}} \rightarrow \neg\mathsf{exc}_{\mathsf{k}} \textit{ if } \mathsf{h} \neq \mathsf{k} \\
&\mathsf{des}_{\mathsf{h}} \rightarrow \neg\mathsf{des}_{\mathsf{k}} \textit{ if } \mathsf{h} \neq \mathsf{k}
\end{aligned}$$

Proposition 3. *The following equivalences are DL-KGBG valid for every $p \in Atm$, $\mathsf{h} \in Num$, $\alpha \in Act$, $\epsilon \in Act^*$.*

(R.1) $[*\varphi]p \leftrightarrow p$
(R.2) $[*\varphi]\mathsf{after}_{\epsilon|\alpha} \leftrightarrow \mathsf{after}_{*\varphi;\epsilon|\alpha}$
(R.3) $[*\varphi]\mathsf{exc}_{\mathsf{h}} \leftrightarrow ((\varphi \wedge \bigvee_{\mathsf{k},\mathsf{l}:\mathsf{k}-\mathsf{l}=\mathsf{h}}(\mathsf{exc}_{\mathsf{k}} \wedge \mathsf{Bel}^{\mathsf{l}}\neg\varphi)) \vee (\neg\varphi \wedge \bigvee_{\mathsf{k}:Cut_{\mathsf{max}}(\mathsf{k}+\Delta)=\mathsf{h}} \mathsf{exc}_{\mathsf{k}}))$
(R.4) $[*\varphi]\mathsf{des}_{\mathsf{h}} \leftrightarrow \mathsf{des}_{\mathsf{h}}$
(R.5) $[*\varphi]\neg\psi \leftrightarrow \neg[*\varphi]\psi$
(R.6) $[*\varphi](\psi_1 \wedge \psi_2) \leftrightarrow ([*\varphi]\psi_1 \wedge [*\varphi]\psi_2)$
(R.7) $[*\varphi][\mathsf{K}]\psi \leftrightarrow [\mathsf{K}][*\varphi]\psi$

The equivalences of Proposition 3 provide a complete set of reduction axioms for DL-KGBG. Call *red* the mapping on DL-KGBG formulas which iteratively applies the above equivalences from the left to the right, starting from one of the innermost modal operators. *red* pushes the dynamic operators inside the formula, and finally eliminates them when facing an atomic formula.

Proposition 4. *Let φ be a formula in the language of DL-KGBG. Then*

1. *$red(\varphi)$ has no dynamic operators $[\alpha]$*
2. *$red(\varphi) \leftrightarrow \varphi$ is DL-KGBG valid*
3. *$red(\varphi)$ is DL-KGBG valid iff $red(\varphi)$ is L-KGBG valid.*

Sketch of Proof. The first item is clear. The second item is proved using Proposition 3 and the rule of replacement of equivalents. The last item follows from the second item and the fact that DL-KGBG is a conservative extension of L-KGBG. ■

Theorem 1. *Satisfiability in DL-KGBG is decidable.*

Sketch of Proof. We first show that the logic L-KGBG is decidable. Note that the problem of satisfiability in L-KGBG is reducible to the problem of *global* logical consequence in **S5**, where the set of global axioms Γ is the set of all formulas of Proposition 2. That is, we have $\models_{\text{L-KGBG}} \varphi$ if and only if $\Gamma \models_{\mathbf{S5}} \varphi$. We can show that $\Gamma \models_{\mathbf{S5}} \varphi$ if and only if $\Gamma_\varphi \models_{\mathbf{S5}} \varphi$, where

$$\Gamma_\varphi = \Gamma \setminus \{ \mathsf{after}_{\epsilon|\alpha} \rightarrow \neg \mathsf{after}_{\epsilon|\beta} : \mathsf{after}_{\epsilon|\alpha} \notin SUB(\varphi) \text{ or } \mathsf{after}_{\epsilon|\beta} \notin SUB(\varphi) \}$$

and $SUB(\varphi)$ is the set of all subformulas of φ. In other words, Γ_φ is the subset of Γ containing only formulas of the type $\mathsf{after}_{\epsilon|\alpha} \rightarrow \neg \mathsf{after}_{\epsilon|\beta}$ which are φ-relevant. Observe that every Γ_φ is finite. It is well-known that the problem of global logical consequence in **S5** with a finite number of global axioms is reducible to the problem of satisfiability in **S5**. The problem of satisfiability checking in **S5** is decidable [11, 7]. It follows that the problem of satisfiability checking in the logic L-KGBG is decidable too. *red* provides an effective procedure for reducing a DL-KGBG formula φ into an equivalent L-KGBG formula $red(\varphi)$. As L-KGBG is decidable, DL-KGBG is decidable too.

■

Theorem 2. *The validities of DL-KGBG are completely axiomatized by*

- *some axiomatization of* **S5** *for the epistemic operator* $[\mathsf{K}]$
- *the schemas of Proposition 2*
- *the reduction axioms of Proposition 3*
- *the inference rule* $\frac{\varphi \leftrightarrow \psi}{[\alpha]\varphi \leftrightarrow [\alpha]\psi}$

4 Formalization of expectation-based emotions and their intensity

The modal operators of graded belief and graded goal defined above are used here to provide a logical analysis of expectation-based emotions and their intensities.

4.1 Hope and fear

According to some psychological models [22, 16, 21] and computational models [12] of emotions, the intensity of hope with respect to a given event is a monotonically increasing function of: the degree to which the event is desirable and the likelihood of the event. That is, the higher is the desirability of the event φ, and the higher is the intensity of the agent's hope that this event will occur; the higher is the likelihood of the event φ, and the higher is the intensity of the agent's hope that this event will occur. Analogously, the intensity of fear with respect to a given event is a monotonically increasing function of: the degree to which the event is undesirable and the likelihood of the event. There are several possible merging functions which satisfy these properties. For example, I could define the merging function *merge* as an average function, according to which the intensity of hope about a certain event φ is the average of the strength of the belief that φ will occur and the strength of the goal that φ will occur. That is, for every $\mathsf{h}, \mathsf{k} \in \mathit{Num}$ representing respectively the strength of the belief and the strength of the goal, I could define *merge*(h,k) as $\frac{\mathsf{h}+\mathsf{k}}{2}$. Another possibility is to define *merge* as a product function $\mathsf{h} \times \mathsf{k}$ (also used in [12]), according to which the intensity of hope about a certain event φ is the product of the strength of the belief that φ will occur and the strength of the goal that φ will occur. Here I do not choose a specific merging function, as this would require a serious experimental validation and would much depend on the domain of application in which the formal model has to be used.

Let me now define the notions of hope and fear with their corresponding intensities. An agent is experiencing a hope with with intensity i about φ if and only if there are $\mathsf{h}, \mathsf{k} \in \mathit{Num}$ such that such that $\mathsf{h} < \mathsf{max}$, h is the strength to which the agent believes that φ is true, k is the strength to which the agent wishes φ to be true and $\mathit{merge}(\mathsf{h},\mathsf{k}) = \mathsf{i}$:

$$\mathsf{Hope}^{\mathsf{i}}\varphi \stackrel{\mathsf{def}}{=} \bigvee_{\mathsf{h},\mathsf{k}\in \mathit{Num}:\mathsf{h}<\mathsf{max} \text{ and } \mathit{merge}(\mathsf{h},\mathsf{k})=\mathsf{i}} (\mathsf{Bel}^{\mathsf{h}}\varphi \wedge \mathsf{Goal}^{\mathsf{k}}\varphi)$$

The notion of fear can be defined in a similar way, after assuming that if an agent wishes φ to be true, then the situation in which φ is false is undesirable for him.[3] An agent is experiencing a fear with with intensity i about φ if and only if there are $\mathsf{h}, \mathsf{k} \in \mathit{Num}$ such that $\mathsf{h} < \mathsf{max}$, h is the strength to which the agent believes that φ is true, k is the strength to which the agent wishes φ to be false and $\mathit{merge}(\mathsf{h},\mathsf{k}) = \mathsf{i}$:

$$\mathsf{Fear}^{\mathsf{i}}\varphi \stackrel{\mathsf{def}}{=} \bigvee_{\mathsf{h},\mathsf{k}\in \mathit{Num}:\mathsf{h}<\mathsf{max} \text{ and } \mathit{merge}(\mathsf{h},\mathsf{k})=\mathsf{i}} (\mathsf{Bel}^{\mathsf{h}}\varphi \wedge \mathsf{Goal}^{\mathsf{k}}\neg\varphi)$$

In the preceding definitions of hope and fear, the strength of the belief is supposed to be less than max in order to distinguish hope and fear, which imply some form of uncertainty, from happiness and sadness (or unhappiness) which are based on certainty. In order to experience hope (resp. fear) about φ, the agent should have a minimal degree of uncertainty that φ might be false. Indeed, we have that:

$$\models_{\mathsf{DL\text{-}KGBG}} \mathsf{Hope}^{\mathsf{i}}\varphi \rightarrow \neg\mathsf{SBel}\varphi \tag{12}$$

$$\models_{\mathsf{DL\text{-}KGBG}} \mathsf{Fear}^{\mathsf{i}}\varphi \rightarrow \neg\mathsf{SBel}\varphi \tag{13}$$

This means that if an agent hopes (resp. fears) φ to be true, then he is not certain that φ (*i.e.*, the agent does not have the strong belief that φ.) For example, if I hope that

[3] If an agent wishes φ to be true, then in the situation in which φ is false he will be frustrated. This is the reason why the latter situation is undesirable for the agent.

my paper will be accepted at the next LORI III workshop, then it means that I am not certain that my paper will be accepted. The preceding two validities are consistent with Spinoza's quote "Fear cannot be without hope nor hope without fear". Indeed, if an agent hopes that φ will be true then, according to the validity (12), he envisages the possibility that φ will be false. Therefore, he experiences some fear that φ will be false. Conversely, if an agent fears that φ will be true then, according to the validity (13), he envisages the possibility that φ will be false. Therefore, he experiences some hope that φ will be false.

Remark 1. It has to be noted that hope and fear, and more generally expectations, are not necessarily about a *future* state of affairs, but they can also be about a *present* state of affairs or a *past* state of affairs. For example, I might say 'I hope that you feel better now!' or 'I fear that you did not enjoy the party yesterday night!'.

On the contrary, to feel happy (resp. sad) about φ, the agent should be *certain* that φ is true. For example, if I am happy that my paper has been accepted at the LORI III workshop, then it means that I am certain that my paper has been accepted. More precisely, an agent is experiencing happiness with intensity h about φ if and only if, the agent strong believes (is certain) that φ is true and h is the strength to which the agent wishes φ to be true:

$$\mathsf{Happiness}^{\mathsf{h}}\varphi \stackrel{\mathtt{def}}{=} \mathsf{SBel}\varphi \wedge \mathsf{Goal}^{\mathsf{h}}\varphi$$

Moreover, an agent is experiencing sadness with intensity h about φ if and only if, the agent strongly believes (is certain) that φ is true and h is the strength to which the agent wishes φ to be false:

$$\mathsf{Sadness}^{\mathsf{h}}\varphi \stackrel{\mathtt{def}}{=} \mathsf{SBel}\varphi \wedge \mathsf{Goal}^{\mathsf{h}}\neg\varphi$$

The following validities highlight some relationships between hope and fear, and happiness and sadness. Suppose $\Delta = \mathsf{max}$ and $\varphi \in Obj$, then:

$$\models_{\mathsf{DL\text{-}KGBG}} \mathsf{Hope}^{\mathsf{i}}\varphi \to [*\varphi] \bigvee_{\mathsf{k}\in Num} \mathsf{Happiness}^{\mathsf{k}}\varphi \tag{14}$$

$$\models_{\mathsf{DL\text{-}KGBG}} \mathsf{Fear}^{\mathsf{i}}\varphi \to [*\varphi] \bigvee_{\mathsf{k}\in Num} \mathsf{Sadness}^{\mathsf{k}}\varphi \tag{15}$$

This means that, if the agent is minimally conservative and hopes φ to be true then, after learning that φ, his sadness will transform into happiness about φ (with a certain strength.) On the contrary, if he is minimally conservative and fears φ to be true then, after learning that φ, his fear will transform into sadness about φ (with a certain strength.) As the following two validities highlight, if the agent is minimally conservative and learns that φ is false, then his hope about φ will transform into sadness about $\neg\varphi$ and his fear about φ will transform into happiness about $\neg\varphi$. That is, suppose $\Delta = \mathsf{max}$ and $\varphi \in Obj$, then:

$$\models_{\mathsf{DL\text{-}KGBG}} \mathsf{Hope}^{\mathsf{i}}\varphi \to [*\neg\varphi] \bigvee_{\mathsf{k}\in Num} \mathsf{Sadness}^{\mathsf{k}}\neg\varphi \tag{16}$$

$$\models_{\mathsf{DL\text{-}KGBG}} \mathsf{Fear}^{\mathsf{i}}\varphi \to [*\neg\varphi] \bigvee_{\mathsf{k}\in Num} \mathsf{Happiness}^{\mathsf{k}}\neg\varphi \tag{17}$$

4.2 Disappointment and relief

An interesting class of expectation-based emotions are those based on the invalidation of either a positive or a negative expectation. In agreement with Ortony et al.'s psychological model [21], I distinguish two types of these emotions: disappointment and relief. Disappointment is the emotion due to the invalidation of an agent's hope, while relief is the emotion due to the invalidation of an agent's fear. On the one hand, the higher is the intensity of the invalidated hope that φ is true, and the higher is the intensity of the agent's disappointment. On the other hand, the higher is the intensity of the invalidated fear that φ is true, and the higher is the intensity of the agent's relief.

More precisely, an agent is going to experience a disappointment with intensity i about φ if and only if the agent hopes φ to be true with intensity i and the agent is going to learn that φ if false:

$$\mathsf{Disappointment}^{\mathsf{i}}\varphi \stackrel{\mathsf{def}}{=} \mathsf{Hope}^{\mathsf{i}}\varphi \wedge \mathsf{occ}_{*\neg\varphi}$$

Moreover, an agent is going to experience a relief with intensity i about φ if and only if the agent fears φ to be true with intensity i and the agent is going to learn that φ if false:

$$\mathsf{Relief}^{\mathsf{i}}\varphi \stackrel{\mathsf{def}}{=} \mathsf{Fear}^{\mathsf{i}}\varphi \wedge \mathsf{occ}_{*\neg\varphi}$$

It has to be noted that disappointment and relief always imply some form of surprise, where surprise is defined as the invalidation of an agent's belief (see also [17]):

$$\mathsf{Surprise}^{\mathsf{h}}\varphi \stackrel{\mathsf{def}}{=} \mathsf{Bel}^{\mathsf{h}}\varphi \wedge \mathsf{occ}_{*\neg\varphi}$$

where $\mathsf{Surprise}^{\mathsf{h}}\varphi$ has to be read the agent feels surprised about φ with intensity h.

5 Conclusion

The analysis of expectation-based emotions presented in Section 4 is obviously very simplistic, as it misses a lot of important psychological aspects. For example, the fact that mental states on which emotions such as hope, fear, disappointment and relief are based are usually joined with bodily activation and components, and these components shape the whole subjective state of the agent and determine his behavior. I have only focused on the cognitive structure of emotions, without considering the *felt* aspect of emotions. This is of course a limitation of the model presented in this paper, as the intensity of emotion also depends on the presence of these somatic components (*e.g.*, the intensity of fear is amplified by the fact that, when experiencing this emotion I feel my stomach contracted, my throat dry, *etc*.) However, an analysis of the cognitive structure of emotions (*i.e.*, identifying the mental states which determine a given type of emotion), as the one presented above, is a necessary step for having an adequate understanding of affective phenomena. I postpone to future work an analysis of the relationships between emotion and action (*i.e.*, how an emotion with a given intensity determines the agent's future reactions), and of the relationships between cognitive structure and somatic aspects of emotions (*i.e.*, how somatic components affect emotion intensity.)

References

1. Adam, C., Herzig, A., Longin, D.: A logical formalization of the OCC theory of emotions. Synthese 168(2), 201–248 (2009)
2. Alchourrón, C.E., Gärdenfors, P., Makinson, D.: On the logic of theory change: partial meet contraction and revision functions. J. of Symbolic Logic 50(2), 510–530 (1985)
3. Aucher, G.: A combined system for update logic and belief revision. In: Proc. of PRIMA 2004. LNAI, vol. 3371, pp. 1–18. Springer-Verlag (2005)
4. Aumann, R.: Interactive epistemology I: Knowledge. International J. of Game Theory 28(3), 263–300 (1999)
5. Baltag, A., Smets, S.: A qualitative theory of dynamic interactive belief revision. In: Proc. of LOFT 7, Texts in Logic and Games, vol. 3, pp. 13–60. Amsterdam University Press (2008)
6. van Benthem, J., Gerbrandy, J., Hoshi, T., Pacuit, E.: Merging frameworks for interaction. J. of Philosophical Logic 38(5), 491–526 (2009)
7. Blackburn, P., de Rijke, M., Venema, Y.: Modal Logic. Cambridge Univ. Press (2001)
8. Castelfranchi, C., Lorini, E.: Cognitive anatomy and functions of expectations. In: Sun, R. (ed.) Proc. of IJCAI'03 Workshop on Cognitive modelling of Agents and Multi-Agent Interactions. pp. 29–36 (2003)
9. van Ditmarsch, H.: Prolegomena to dynamic logic for belief revision. Synthese 147(2), 229–275 (2005)
10. van Ditmarsch, H., van der Hoek, W., Kooi, B.: Dynamic Epistemic Logic, Synthese Library, vol. 337. Springer (2007)
11. Fagin, R., Halpern, J., Moses, Y., Vardi, M.: Reasoning about Knowledge. MIT Press, Cambridge (1995)
12. Gratch, J., Marsella, S.: A domain independent framework for modeling emotion. J. of Cognitive Systems Research 5(4), 269–306 (2004)
13. Grove, A.: Two modellings for theory change. J. of Philosophical Logic 17, 157–170 (1988)
14. Klein, P.D.: A proposed definition of propositional knowledge. J. of Philosophy 68, 471–482 (1971)
15. Laverny, N., Lang, J.: From knowledge-based programs to graded belief-based programs, part II: off-line reasoning. In: Proc. of IJCAI'05. pp. 497–502. Professional Book Center (2005)
16. Lazarus, R.S.: Emotion and adaptation. Oxford Univ. Press, New York (1991)
17. Lorini, E., Castelfranchi, C.: The cognitive structure of surprise: looking for basic principles. Topoi: an International Review of Philosophy 26(1), 133–149 (2007)
18. Lorini, E., Schwarzentruber, F.: A logic for reasoning about counterfactual emotions. Artificial Intelligence 175(3-4), 814–847 (2011)
19. Meyer, J.J.C.: Reasoning about emotional agents. International J. of Intelligent Systems 21(6), 601–619 (2006)
20. Miceli, M., Castelfranchi, C.: The mind and the future: the (negative) power of expectations. Theory and Psychology 12(3), 335–366 (2002)
21. Ortony, A., Clore, G.L., Collins, A.: The cognitive structure of emotions. Cambridge University Press, Cambridge, MA (1988)
22. Reisenzein, R.: Emotions as metarepresentational states of mind: naturalizing the belief-desire theory of emotion. Cognitive Systems Research 10, 6–20 (2009)
23. Rott, H.: Stability, strength and sensitivity: converting belief into knowledge. Erkenntnis 61, 469–493 (2004)
24. Spohn, W.: Ordinal conditional functions: a dynamic theory of epistemic states. In: Causation in decision, belief change and statistics, pp. 105–134. Kluwer (1998)
25. Stalnaker, R.: On logics of knowledge and belief. Philosophical Studies 128, 169–199 (2006)
26. Steunebrink, B.R., Dastani, M., Meyer, J.J.C.: A formal model of emotions: integrating qualitative and quantitative aspects. In: Proc. of ECAI 2008. pp. 256–260. IOS Press (2008)