E-MARKET MECHANISM DESIGN FOR SUPPLY CHAIN MANAGEMENT

BY

MASABUMI FURUHATA

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The undersigned hereby certify that they have read and recommended for acceptance a thesis entitled "**E-Market Mechanism Design for Supply Chain Management**" by **Masabumi Furuhata** for the degree of **Doctor of Philosophy**.

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Supervisors:

_____

Dr. Dongmo Zhang

_____

Dr. Laurent Perrussel

_____

Prof. Yan Zhang

_____

Research Director Andreas Herzig

UNIVERSITY OF WESTERN SYDNEY

AND

UNIVERSITÉ DE TOULOUSE

Date: **31 March 2009**

Author: **Masabumi Furuhata**

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Signature of Author

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Abstract

This thesis investigates the problem of market mechanism design for supply chain management and e-marketplace development. Instead of viewing a market as an isolated entity, we consider each market to be a part of a supply chain and study the effects of interactions between different markets in the supply chain. We focus on three important issues in relation to market mechanism design and investigate them using a variety of approaches, such as game-theoretic, strategic and experimental methods.

Capacity allocation has been a research topic in management science and operations research for several years, dealing with imbalance between supply and demand in a market. In recent years, this topic, known as resource allocation, has become an active research area in computer science. The first issue we consider in this thesis is how to design a market mechanism design for capacity allocation. We extend the capacity allocation game to a supply chain model. Based on this model we examine the game-theoretic properties of allocation mechanisms, such as efficiency, profit maximising, and truth-telling, with respect to quantity competition and price competition in a related market of the supply chain. We find that a few typical allocation

mechanisms that have been generally used in industry are sensitive to supply chain settings. We design a new allocation mechanism for supply chains with quantity competition and prove that it is a truth-telling mechanism. We provide a comprehensive analysis of the relation between the properties of allocation mechanisms and pricing strategies of supply chain members.

The second issue we investigate is the design of market mechanisms for online markets. We introduce an approach to the modelling of online markets as supply chains, in which a coordination mechanism is applied to the market between the online market owner and the sellers. We examine a set of coordination mechanisms based on fixed-fee contracts, revenue-sharing contracts, and profit-sharing contracts in relation to different marketing strategies, such as advertisement. We prove that the fixed-fee contract achieves coordination while the revenue-sharing contract does not achieve coordination if we do not consider the effect of advertisement. However, fixed-fee contracts no longer achieve coordination if the effect of advertisement is taken into account. We design a new online market contact based on the idea of profit-sharing and prove that it achieves coordination with advertisement.

Finally, we explore the problem of market mechanism design using autonomous trading agents. We introduce a formal representation of market policies, such as accepting policies, charging policies, pricing policies and matching policies, based on double auction mechanisms. By utilising the Market Design Game platform for the Trading Agent Competition (TAC), we analyse how specific market policies influence market behaviour of specific types of autonomous trading agents using experimental methods. We design a range of market policies and test them with three types

of trading agents: random, learning and human-like bidding agents. We find that market performance, such as the profit of market makers, allocation efficiency and transaction volume, improves with time. The experiments also show that the periodic clearing policy gives market makers greater profits than the continuous clearing for all three types of trading agents we use. However, for learning and human-like bidding agents, the continuous clearing policy is preferable to the periodic clearing policy because it can improve allocation efficiency and transaction volume.

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Chapter 1

Introduction

Electronic business (e-business) has been embraced by many as the new frontier of the information revolution. In its early years the e-business revolution introduced the concept of sharing of information through open media, so that anyone with access to the Internet had an enormous amount of information available to them instantaneously. The next phase of the e-business revolution was led by companies who used the power of the Internet to conduct business-to-consumer (B2C) transactions which enabled them to drive up their market share through using this alternative channel. The current phase of the e-business revolution is driven by enterprises which have the vision and desire to surpass the B2C phase by way of enabling business-to-business (B2B) transactions over the Internet. Among many other benefits, these enterprises look to the integration of B2B and B2C solutions to boost efficiency, cut costs and facilitate supply chain processes. e-marketplaces are the vehicles that provide such a desired integration [83, 103].

An e-marketplace is an electronic gathering place that accommodates multiple

markets running under pre-defined and explicit market rules. An e-marketplace provides to its customers a unified view of sets of goods and services and enables them to engage in market transactions under a variety of different market mechanisms. However, the success of e-marketplaces relies on the ability to provide cutting-edge functionalities and a variety of trading mechanisms in order to attract and retain multiple participants: *buyers*, *sellers* and *other supply chain partners*. Nevertheless, the existing e-marketplaces are mostly running under relatively simple and fixed market mechanisms, such as catalog sale or types of auction, while an e-marketplace with B2C and B2B integration requires highly *dynamic*, *efficient* and *reliable* market mechanisms. The fact that, today, after a few years of the practice, a very small fraction of these businesses exist, tells us that there still is a lot to be learned before a new phase of e-business revolution becomes a reality [63, 36].

This thesis investigates the problem of market mechanism design from supply chain management and e-marketplace development point of view. Instead of viewing a market as an isolated entity, we consider each market in an e-marketplace as a component of a supply chain and examine the properties of a market mechanism in relation to the mechanisms applied in other markets in the same supply chain. We choose a few typical issues in market mechanism design and investigate them using theoretical, strategic and experimental methods. In this chapter, we briefly introduce these issues and summarise our major findings from this research.

1.1 Background

1.1.1 Supply Chain Management

The term *Supply Chain Management* has risen to prominence in industry and academia since 1980s. Generally speaking, a *supply chain* is the network of organisations that are involved in the processes and activities of manufacture, redistribution, retailing and delivery of certain products or services [17]. A supply chain encompasses all the physical, information and financial flows between suppliers and end-users through intermediaries, such as manufacturers, wholesalers, retailers, and distributors.

Figure 1.1 depicts a typical three-tier supply chain where a set of suppliers and a set of end-users are bridged by a single layer of intermediaries[1]. The supply chain members are linked with two markets: the *upstream market* and the *downstream market*. Products and services are distributed from the suppliers via the intermediaries to the end-users (product flows). Customers' orders to the intermediaries and the intermediaries' orders to the suppliers are examples of information flows.

Typical examples of the three-tier supply chains are the industries where end-products are redistributed to the end-users via wholesalers and retailers. For instance, we can view the pharmaceutical industry as a three-tier supply chain. The drug manufacturers are suppliers and the consumers of the medicines are end-users. The intermediaries are pharmacies. These supply chain members are linked by two types of markets: wholesale markets and retail markets. The medicine retailers purchase medicines from the manufacturers via wholesale markets and sell the medicines to

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[1]To simplify presentation, we ignore financial flow in the model.
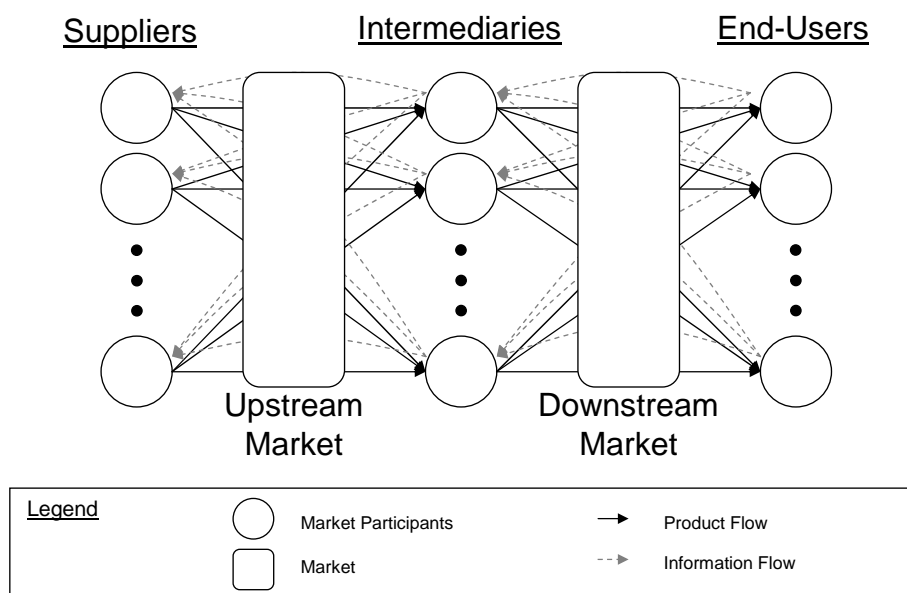
Figure 1.1: A Typical Three-Tier Supply Chain Network.

end-users via retail markets. A product flow represents a distribution of medicines from manufactures via pharmacies to end-users. A typical information flow is the order stream that comes from purchase orders of the end-users and goes up to the suppliers via the retailers.

Obviously, this three-tier supply chain model can be further extended and refined to cover a whole supply chain, that is, the system of organisations, facilities and activities from raw material procurement to end-product delivery. The idea of *supply chain management* is to view the chain as an integrated system, and to fine-tune the decisions about how to operate the various components in ways that can produce the

most desirable overall system performance in the long run [55].

1.1.2 Market Mechanism Design

In a supply chain, the markets play the central role in binding supply chain members together and adjusting their benefits and demands. Market behaviours are determined by the mechanisms that are applied to the market. The study of market mechanisms has been one of the central research tasks in economics and finance for several decades [72, 86, 119]. In general, a market mechanism specifies the set of actions that market participants can perform and the outcomes associated to every set of possible actions. One postulate behind the design of market mechanisms is that market participants have incentives to behave according to the market mechanism. Mechanism design is a field of economics that focuses on creating incentives and rules for strategic interactions such that the desired outcome or some desirable properties are achieved.

Existing research on market mechanism design is mostly focus on single market mechanism design [12, 60, 65, 86, 114]. The design and analysis of mechanisms for multiple markets in a supply chain is much more complicated. A well-known example in industry is the *bullwhip effect* in supply chains, which describes the phenomenon of market information being amplified through supply chains [16]. The main challenge of market mechanism design for supply chains is how to guarantee a mechanism to achieve desired properties when situations in other markets in the supply chain are changed.

To illustrate the complexity of market mechanism design for supply chains, let us consider a simple three-tier supply chain in which a single supplier supplies a non-substitutable product, such as a patented medicine, to a retail network. There are two ways for the supplier to wholesale its product: *exclusive distribution* and *selective distribution*. With exclusive distribution, end-users are only able to purchase the product from a single retailer, called *monopoly*. With selective distribution, however, end-users can have choice of a range of retailers. Such a retail market is normally referred to as an *oligopoly*. Obviously, the mechanism that is applied in the wholesale market (upstream market) depends heavily on the mechanisms applied in the retailer market (downstream market). This is because the effects of the market mechanism in one market can propagate to the other market through product flows and information flows. For example, a shortage of the product in downstream market could give retailers an incentive to exaggerate their demands, which would cause phantom demand in the upstream market.

A market run electronically could add more complexity to its mechanism design. E-markets diverge from traditional markets in several aspects. Firstly, the partnership between traders (sellers or buyers) and the market owners in e-markets is loosely tied and mostly temporal. A trader can enter or leave a market any time even during a transaction. The market owner often has poor information about the traders. Legal restrictions usually used to regulate market behaviours are hard to apply to online markets in many cases. Therefore it is important to devise proper market mechanisms that can give traders and market makers an incentive to behave well. Secondly, due to easy access and information availability, the competition between e-markets of the

same kind is much more severe than in traditional trading models. An e-market owner should be able to diversify trading models as well as business strategies in order to meet different needs of traders and avoid severe competition. Thirdly, the integration of B2B and B2C models requires highly dynamic, contract-based and scalable mechanisms [28, 83, 103].

1.2 Issues and Methods

This thesis investigates the problem of market mechanism design for supply chain management and e-marketplace development. Since it is hard to study the whole problem in the thesis, we focus on three important aspects of the problem and investigate them using a variety of methods, including: *game-theoretic*, *strategic* and *experimental*.

1.2.1 Mechanism Design for Capacity Allocation

A fundamental problem in market mechanism design is imbalance between supply and demand. In many situations, suppliers are not able to satisfy the customer demand over a certain period due, perhaps because of lack of raw materials, a shortage of key parts, shortage of available stock, limited production or delivery capacity, and so on. The ability to fill orders is normally referred to as the supplier's capacity. When orders from customers exceed the capacity of the supplier, a certain mechanism that maps the supplier's capacity to each customer is required. Such mechanisms are called *capacity allocation mechanisms*. These mechanisms are commonly used in

industries to deal with the gap between market supply and demand (compared to price leverage) [11, 36].

Capacity allocation mechanisms are normally quantity based, which means that these do not use price-leveraging to adjust demand imbalance, unlike most economists would suggest. There are many reasons for a supplier to prefer a quantity-based mechanism to a price-based mechanism. Firstly, business transactions in business-to-business models are generally governed by pre-existing contracts with negotiated prices [36, 63]. Secondly, buyers are usually more sensitive to price than quantity. In the situation when capacity is extremely scarce, price-based mechanisms tend to cause dramatic increases in prices. Thirdly, there are many situations where price is not a major concern, such as printer jobs, IP addresses, or network bandwidth. A large range of industries use allocation mechanisms, for example, the automobile [10], semiconductor [38, 74, 73], and food industries [11]. Capacity allocation mechanisms are also applicable to many computing-related resource allocation problems [19, 10, 35, 39, 57, 80].

The problem of capacity allocation can be more complicated and import in supply chain management because information about supply shortages could be exaggerated through the supply chain as has been illustrated with the pharmacy example in Section 1.1. To explain the problem, let us consider a three-tier supply chain model in which there is only one supplier (a special case of the general model shown in Figure 1.1). Suppose that this supplier distributes products to a set of retailers via wholesale markets (upstream market). The retailers sell the products to end-users

via retail markets (downstream market) as shown in Figure 1.2. Obviously, the upstream market is a monopoly and the downstream market is an oligopoly, which is competitive.
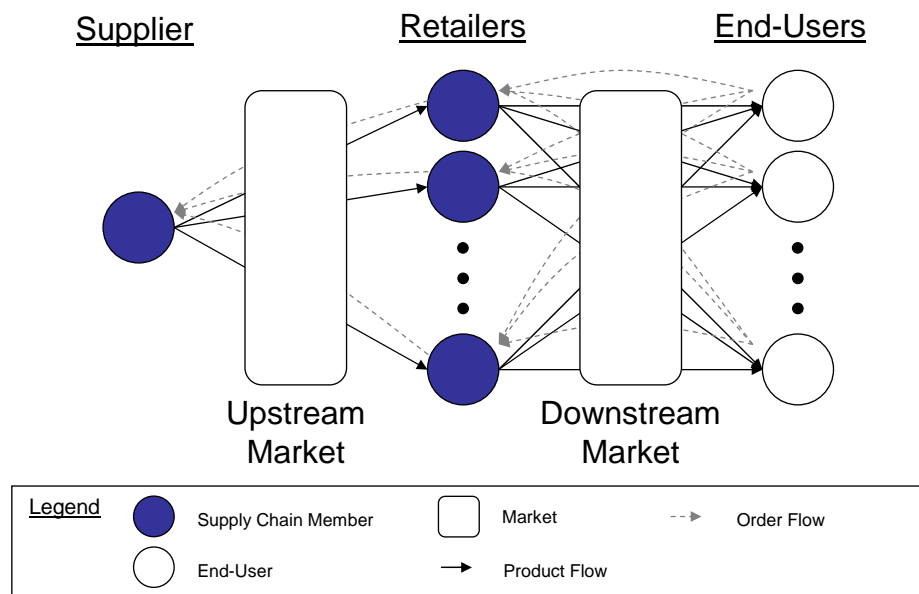
Figure 1.2: A Supply Chain Model for Capacity Allocations.

Assume that a capacity allocation mechanism is applied to the upstream market. It is not hard to imagine that the behaviours of the capacity allocation mechanism in the upstream market heavily relies on the mechanisms in the downstream market. Let us take the automobile industry as an example. Consider a car manufacturer, say Toyota. Assume that there is only a single Toyota dealer in each region. Local

dealers observe demand in their area and report their needs to the manufacturer. The manufacturer distributes cars according to the dealers' orders. Whenever the market demand exceeds the market supply, the available products are allocated in the proportion to the sizes of the orders. Such an allocation mechanism is called *proportional allocation.* This is common in some industries [10, 11]. A few questions arise: (i) Under such an allocation mechanism, will the local dealers report their demand truthfully? (ii) Can the mechanism efficiently allocate all products the manufacturer produces? (iii) Can the mechanism maximize the profit of the supplier or the profit of all supply chain members (social welfare).

In [10, 11], Cachon and Lariviere considered the problem of capacity allocation in supply chains in which the downstream market is a monopoly, which means that there is no competition in the downstream market. In this thesis, we consider the more interesting situation in which the downstream market is competitive. We examine two typical forms of market competition: *quantity competition* and *price competition.* In quantity competition, sellers determine supply quantities; the market price is induced by the demand function. At the opposite end is price competition, in which sellers determine their selling prices and buyers choose sellers based on their prices. We investigate how the competitive mechanisms applied in the downstream market influence the properties of the allocation mechanisms in the upstream market. Based on the supply chain model introduced in Chapter 2, we examine the game-theoretic properties of allocation mechanisms, such as incentive compatibility, efficiency and social welfare, with regard to quantity competition (Chapter 3) and price competition (Chapter 4).

1.2.2 Market Mechanism Design for Online Markets

When we consider the problem of capacity allocation, we normally assume that the product flow and information flow are symmetric, which means that the product flow goes from the supplier to the end-user and the order flow goes in the reverse direction. The second issue we are interested in is how market mechanisms effect each other in a supply chain when the product flow and information flow are asymmetric. One of the representative situations is online markets, where information flow goes through all supply chain members but product flow bypasses market makers (see Figure 1.3).

Consider an online hotel booking service provider. End-users book hotel rooms via the website of the service provider. These orders are collected by the online market owner and are forwarded to each hotel. This forms an information flow. However, the product flow does not go through the online market owner because the hotel services are directly delivered to the end-users.

This example illustrates a generic characteristic of online markets: product flows are separate from information flow and financial flow. We view an online market as a three-tier supply chain as shown in Figure 1.3. The orders for products or services are conveyed from buyers via the online market owner to the sellers while the products or services are directly shipped or provided from the sellers to the buyers. The online market owner operates two markets: an upstream market for sellers and a downstream market for buyers. In most existing e-marketplaces, the upstream market is contract-based, say revenue-sharing, like eBay, and the downstream market uses the mechanisms such as fixed-price or English auction, like Amazon and eBay,
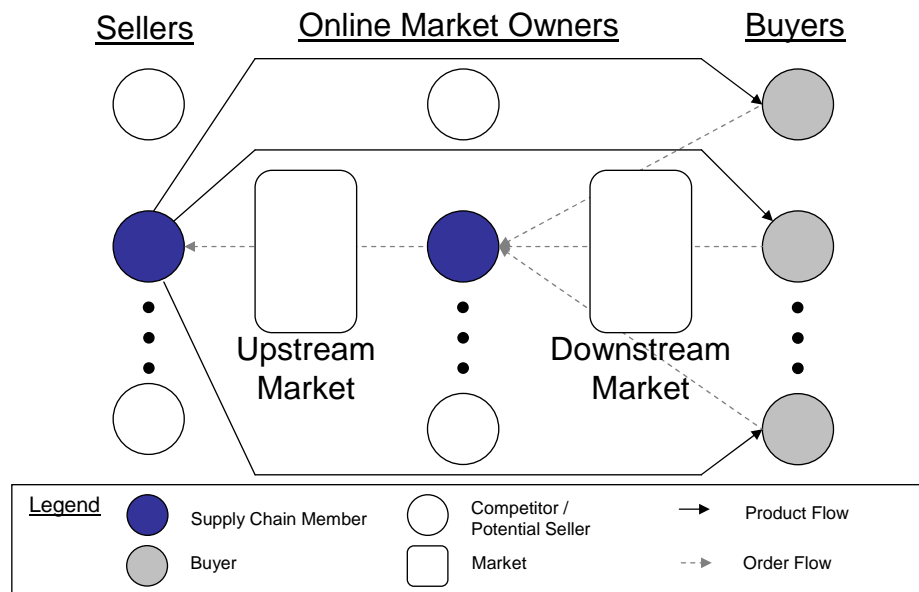
Figure 1.3: A Supply Chain Network of Online Markets.

respectively[2].

Due to the separation of product flows, information flows and financial flows in online markets, many issues arise: (i) How does the benefit of end-users bind with the suppliers of products and services? (ii) How does the dynamically changing market situation in the downstream market affect relatively static contract-based mechanisms in the upstream market? (iii) How do market mechanisms in both markets influence

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[2]A contact-based market mechanism means that the market rules are explicitly described in a contract, which consists of a set of promises to undertake certain business transactions.

the behaviour of customers?

In this thesis, we focus on the design of the upstream market. We examine a set of coordination mechanisms based on fixed-fee contracts, revenue-sharing contracts, and profit-sharing contracts in relation to different marketing strategies, such as advertisement (Chapter 5).

1.2.3 Market Mechanism Design for Autonomous Trading Agents

In the capacity allocation and online market design issues, we focus on the discussion of game-theoretic or strategic properties of a market mechanism. The last issue of concern in this thesis is how a market mechanism behaves under autonomous trading agents. Our investigation of the issue is also based on a supply chain framework as shown in Figure 1.4.

In an online market, the relation between sellers and the market owner is static and the online market owner operates two separate markets. In order to test autonomous trading agents, we consider market situations in which the market owners' relations with both sellers and buyers are dynamic. The most typical example of such a mechanism is double auction. In a double auction, the sellers and buyers submit asks and bids to the market owner (also called the *market maker* in auction theory). The market maker tries to match these asks and bids. Each matched ask and bid pair will lead to a transaction. Due to the tight link between the upstream market and downstream market, a small change of policy in one market could affect the behaviour
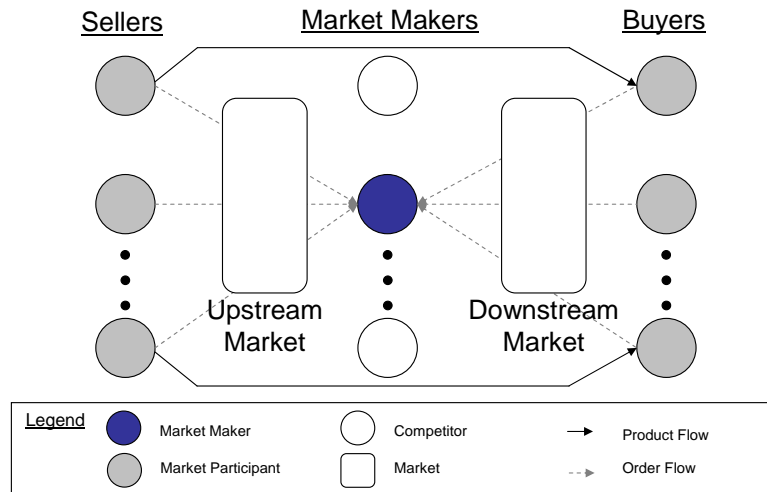
Figure 1.4: A Supply Chain Network in Double Auction Market.

of the other market.

By utilizing the platform of Trading Agent Competition in Market Design (TAC-MD or CAT), we explore an experimental method of market mechanism design. We devise a set of different market policies, such as accepting policies, charging policies, pricing policies and matching policies, and investigate how these policies affect market share, transaction rate and market maker's profit. The framework and experimental results are presented in Chapters 6 and 7.

1.3   List of Contributions of the Thesis

This thesis has made the following contributions to the research of market mechanism design for supply chain management and e-market development:

1. *Introduces a methodology of market mechanism design that views the mechanism design problem as a supply chain management problem.*

2. *Extends the capacity allocation game to competitive supply chain models.*

3. *Introduces an approach to the modelling of online markets as supply chains.*

4. *Introduces a formal description of market mechanisms with double auction for autonomous trading agents.*

5. *Conducts a set of experiments to test the relationship between market mechanisms and trading strategies.*

1.4   Thesis Layout

This thesis is organised as follows: Chapter 1 describes the background, motivation and methods of the research.

Chapter 2 introduces formal representations of supply chain models for capacity allocation and desirable properties of allocation mechanisms. We introduce several allocation mechanisms and describe the properties of these allocation mechanisms.

Chapter 3 focuses on designing capacity allocation mechanisms for quantity competition based on the capacity allocation game presented in Chapter 2. In our model,

the desirable outcome of the mechanisms in a market depends on the market behaviour in the connected market. We investigate properties of several allocation mechanisms with respect to efficiency and stability. An important finding is that proportional allocation, one of the most commonly used allocation techniques, is no longer efficient in our supply chain model in contrast to the monopolistic downstream markets model. We also compare how a cost structure of retailers influences the market behaviour and the properties of allocation mechanisms.

Chapter 4 considers allocation mechanisms for price competition in the downstream market. We compare properties of allocation mechanisms and market behaviours between two major competitive market mechanisms in the downstream market: quantity competition and price competition.

Chapter 5 deals with market mechanism design for online markets that have asymmetric relations between product flow and information flow. We present a formal framework of online markets and examine the properties of coordination mechanisms for online markets. We analyse how coordinating mechanisms are affected by market owner's marketing strategies.

Chapter 6 presents a formal representation of market mechanisms with double auction. We decompose market mechanisms into several components. We propose a learning-based accepting policy which decides whether the market institution accepts incoming orders based on past experiences. We present four market indices to evaluate market performance.

Chapter 7 presents experimental design and results for double auction with autonomous trading agents. We use three types of agents: random, learning and human-like bidding agents. In the experimental results, we show that the proposed accepting policy is robust with respect to fluctuating clearing prices.

Chapter 8 summarises the thesis and suggests directions for future work.

Chapter 2

Mechanism Design for Capacity Allocation

From this chapter to Chapter 4, we consider a specific scenario of supply chain formation where the product flow starts from a single source in the supply chain. As we have mentioned in Section 1.2.1, such a scenario can be abstracted as a three-tier supply chain with a single supplier who produces a certain kind of product (or provides a certain resource) and a set of intermediaries who purchase the product from the supplier and resell it to the end-users (see Figure 1.2)[1]. In other words, the upstream market is a typical monopoly and the downstream market is a typical oligopoly. Our concern is what kind of mechanism is applicable to each market and how these markets affect each other. To facilitate the analysis, we introduce a general model for this supply chain management problem in this chapter and discuss the properties of mechanisms in different settings of markets in Chapter 3 and 4.

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[1]To simplify analysis, we focus on the situation where only one product is distributed.

2.1 Capacity Allocation Problem

Imbalance between supply and demand frequently occurs. A key idea in economics is that prices adjust the supply and demand gap: prices rise if there is excess demand; prices fall otherwise. Managers in practice often think differently. For them, excess demand means loss of opportunity and excess supply means waste of resources. They require other approaches to handle the problem rather than simply applying price adjustments.

The reasons that cause the supply and demand gap are two folds: (i) demand changes quickly or demand is unpredictable, (ii) supply is inflexible. Therefore, a key issue is how to manage these two causes. Firms prepare resources, such as people, products, parts, raw materials, tools, machines, or facilities, to meet changing demand, however it may not always be enough. These suppliers' abilities are frequently referred to as their capacities. Thus, this issue is called as *capacity allocation problem* in management science or operations research.

Capacity allocation problem is observed in most industries. However, an impact of capacity allocation is significant if a supplier is the only supplier that has an essential resource for the production of goods. This situation is notable in capital-intensive industry, such as steel, chemical, paper and semiconductor where suppliers are required to invest substantial amount of capital on the production of goods such as capacity [11, 63, 36, 38, 74, 73]. In such industries, the suppliers may not be able to quickly adjust their capacities to correspond to demand.

According to Cachon and Lariviere [10], capacity allocation can be abstracted in

the context of supply chains as follows:

> *A single company, referred to as a supplier, produces a certain product with a limited capacity. All the products are exclusively allocated to a number of intermediaries, referred to as retailers, who order the products from the supplier and sell them to end-users.*

A single supplier means there are no alternative suppliers for the retailers. Each retailer is a local monopolist means that application is directly relevant to its payoff. This type of supply chains is known as an exclusive distribution model or a franchise network.

In [11, 10], Cachon and Lariviere deal with the capacity allocation problem with allocation mechanisms in the above abstraction of supply chains. Erkoc and Wu [38] extend this problem in production department and sales department which is fundamentally similar to Cachon and Lariviere's model.

In more general and complex situations such as ones where the retailers have direct competition in a retail market, the market behaviours and the properties of allocation mechanisms may differ in a significant way. In this thesis, we will relax Cachon and Lariviere's assumption and investigate the properties of capacity allocation mechanisms in the situations where competition exists in the retail market as illustrated in Figure 1.2 in Section 1.2.1. Unlike the Cachon and Lariviere's model, the output of allocation mechanisms is not directly relevant to payoff of the retailers. It requires how the retailers behave in the retail market. Hence, our aim is to present

how properties of allocation mechanism is influenced by market rules in the downstream market. Furthermore, our concern is how allocation mechanisms influence market behaviour in the downstream market.

In order to design mechanisms for such competitive markets, we address three issues related to capacity allocation. First, allocation mechanisms should guarantee that all the available capacity is fully allocated to the retailers as long as there are demands. Second, how does the allocation mechanism maximise profits of all supply chain members given that each member is individually rational? Third, how can allocation mechanisms induce truthful demand from the retailers? Retailers frequently request the supplier to deliver the products more than their real demand in order to gain advantage from allocation.

In the retail market, we assume that all retailers compete with each other for customers in an oligopolistic market. As we will see, the allocation mechanisms used in the upstream market are highly related to the mechanisms used in the downstream market. We consider two major competition rules in the downstream market: *quantity competition* and *price competition*. We discuss the two market rules in Chapter 3 and Chapter 4, respectively.

In this chapter, we formalize the problem of capacity allocation. In Section 2.2, we present our capacity allocation game as a non-cooperative game. In Section 2.3, we present typical allocation mechanisms. In Section 2.4, we introduce a set of criteria that are used as a guideline for the design and analysis of allocation mechanisms. In Section 2.5, we describe summary and discussion of this chapter.

2.2 The Model

This section formally represents our capacity allocation game in the three-tier supply chain model that consists of a supplier, numbers of retailers and numbers of end-users as mentioned in Section 1.2.1. First, we define allocation mechanism in the upstream market. Then, we present competition rules in the downstream market. Finally, we present the overall capacity game.

2.2.1 Allocation Mechanism in the Upstream Market

Let s be a supplier who is capable of producing K units of a certain product within a certain period of time. We call K the capacity of the supplier. c is the unit cost of the supplier to construct the capacity.

Let $N = \{1, \cdots, n\}$ be the set of retailers. For each $i \in N$, let m_i be the order (in quantity) of retailer i to purchase products from the supplier produced in the underlying period. We use the subscript $j \neq i$ to refer to the set of all the retailers except i, e.g. $m_{j \neq i} = (m_1, \ldots, m_{i-1}, m_{i+1}, \ldots, m_n)$. Let $m_{-i} \equiv \sum_{j \in N \setminus \{i\}} m_j$. We assume that all retailers submit their orders simultaneously and independently. We also assume that the supplier sells products at the fixed unit price w.

If the sum of orders exceeds the capacity of the supplier, the supplier is not able to make promises of the delivery for all orders. Hence, the supplier allocates its capacity to some orders. Let $\mathcal{A} = \{a \in \Re^n : a \geq 0 \ \& \ \sum_{i=1}^n a_i \leq K\}$, where $a \geq 0$ means that for any i, $a_i \geq 0$ (a_i is the i-th component of a). We call each $a \in \mathcal{A}$ a *feasible allocation*.

Definition 2.2.1. An *allocation mechanism* $g : \Re^n \to \mathcal{A}$ is a function which assigns a feasible allocation to each vector of orders such that for any retailers' order vector m, $g_i(m) \leq m_i$ for each $i = 1, \cdots, n$.

In other words, an allocation mechanism assigns a feasible allocation of the suppliers' capacity to each retailer so that the allocated quantity does not exceed the retailer's order quantity.

In general, the allocation that fully uses the capacity is a desired property of capacity allocations for the supplier. An allocation g is said to be *efficient* if

$$\sum_{i=1}^{n} g_i(m) = K \tag{2.2.1}$$

whenever $\sum_{i=1}^{n} m_i \geq K$.

In contrast to typical mechanism design literatures, the output of the allocation mechanisms does not have a straightforward relation to payoff of the corresponding retailers in our model. In other words, the retailer takes further actions, in the downstream market, corresponding to the output of the allocation mechanisms. Therefore, it is significant to consider the market rules of the adjacent market to analyse the properties of the allocation mechanisms.

2.2.2 Competition Rules in the Downstream Market

In this section, we present how retailers behave in the downstream market. We assume the following demand function for the end-users. Let $D(p)$ be the demand of the end-users at price p and $P(q)$ be its inverse function, where q stands for the supply quantity. We assume that the function $P(q)$ is strictly positive on some bounded

interval $(0, \hat{q})$, on which it is twice-continuously differentiable, strictly decreasing and concave. That is, as the seller increase its supplies, the market price strictly decreases. For $q \geq \hat{q}$, we assume $P(q) = 0$. Hence, there is an upper limit for the demand. We assume that the cost of retailers is the purchase cost w.

Consider two major competition rules in oligopoly: *quantity competition* and *price competition*. First, we describe the case of quantity competition based on the Cournot model shown in [26, 27] or Chapter 12 in [76]. Note that the Cournot model, each retailer (seller in his model) determines its supply quantity; the selling price is determined by the total supply quantity. However, in the capacity allocation setting, the retailers set order quantities rather than supply quantities. The supply quantities then correspond to the allocation quantities. Based on this setting, we have the following profit function: for retailer i,

$$\pi_i(a_i, a_{j \neq i}) = a_i \left(P \left(\sum_{i=1}^{n} a_i \right) - w \right) . \tag{2.2.2}$$

In other word, the total allocation quantity $\sum_{i=1}^{n} a_i$ determines the market price $P \left(\sum_{i=1}^{n} a_i \right)$, and the profit π_i for retailer i is determined by the market price, allocated quantity a_i and wholesale price w.

Next, we consider the case of price competition. Using a classic price competition model, the Bertrand model in [3] or Chapter 12 in [76], the strategic decision of each retailer (seller in his model) is pricing. Retailer i sets selling price $p_i \in \Re^+$ independently and simultaneously. End-users choose from the lowest price offering

retailers. The consequence of Bertrand model is that the retailers decrease their selling prices to the marginal costs because of the competition and the unlimited supply limits. In contrast to Bertrand model, our model has a supply limit. As a result, retailers may not always fulfil all the demand and some residual demands may exist. In such a case, we present how end-users behave to describe the residual demand, called *rationing rule*. One of the most common rationing rules is *surplus maximising rule* [69], where the end-users choose a retailer that offers the lowest price among the retailers. If there is a residual demand, the second lowest price offering retailers fulfil it and so on. Let d_i be the demand of firm i. This demand depends on how the retailers set their prices. Let N_l be the set of retailers that offer $(l+1)$-st lowest price. Therefore N_0 represents the set of the retailers who offer the lowest price. Under the surplus maximising rule, the demand for retailer i is

$$
d_i =
\begin{cases}
D(p_i)\dfrac{a_i}{\sum\limits_{i \in N_0} a_i}, & \text{if } i \in N_0 \\[2em]
\max\left\{0, \left(D(p_i) - \sum\limits_{j \in \bigcup\limits_{\eta < l} N_\eta} a_j\right)\dfrac{a_i}{\sum\limits_{i \in N_l} a_i}\right\}, & \text{if } i \in N_l \text{ where } l > 0.
\end{cases}
$$

In this rationing rule, each retailer obtain residual demand that is proportional to its upper supply limit (allocation) among the retailers that set the same price. In other words, all retailers have the same sales opportunity if they set the same price. This rationing rule is suitable for the online markets that list all the sellers on the site.

To show how the retailers map allocations to pricing decisions, let us assume that x_i is the sales quantity of retailer i. Obviously, $x_i \leq \min\{a_i, d_i\}$. Let

$$
H_i = \{h_i \mid h_i : \mathcal{A} \to [0, P(\hat{q})]\}. \tag{2.2.3}
$$

Each $h_i \in H_i$ represents a pricing rule of retailer i, i.e., $h_i(a)$ gives the retailer i's price when the allocation is a. H_i then represents the pricing space of retailer i.

Based on the above setting, we obtain the profit function of retailer i as follows: given a $a \in \mathcal{A}$ and an $h_i \in H_i$,

$$\pi_i(a) = h_i(a)x_i - wa_i$$

In other words, in the case of price competition in the downstream market, the profit of a retailer is determined by its pricing rule, capacity allocation, and its sales quantity.

2.2.3   Capacity Allocation Game

Now we are ready to define capacity allocation games.

Definition 2.2.2. A capacity allocation game $\mathcal{G}$ is a triplet $\langle N, \mathcal{S}, \{\pi_{i \in I}\} \rangle$, where

1. $I = \{s\} \cup N$, where s is the supplier and N is the set of retailers.

2. $\mathcal{S} = \prod_{i \in I} S_i$, where S_i is the strategy space of retailer i.

3. π_i is the profit function of retailer i.

We call each $\sigma = (\sigma_s, \sigma_1, \cdots, \sigma_n) \in \mathcal{S}$ a *strategy profile*. For the supplier, the strategy space S_s is actually the choices of feasible allocation mechanisms. For each retailer i, the strategy space is determined by the market mechanisms applied in both upstream market and downstream market.

In the case where the downstream market is under quantity competition, $S_i = \Re^+$, where a strategy of retailer i is its order quantity. The profit function π_i is:

$$\pi_i(\sigma) = \begin{cases} w \sum_{i \in N} g_i(m) - cK, & \text{if } i = s \\ P(g_i(m) + g_{-i}(m))g_i(m) - wg_i(m), & \text{if } i \in N \end{cases} \quad (2.2.4)$$

In the case where the downstream market is under price competition, $S_i = \Re^+ \times H_i$, where the strategy of retailer i determines its order quantities and pricing rules (note that a retailer's price rule is related to the allocation mechanism applied in the upstream market). The profit function is then

$$\pi_i(\sigma) = \begin{cases} w \sum_{i \in N} g_i(m) - cK, & \text{if } i = s \\ h_i(g(m))x_i - wg_i(m), & \text{if } i \in N \end{cases} \quad (2.2.5)$$

2.3 Typical Allocation Mechanisms

There are many allocation mechanisms that have be used in industry or proposed in the literature. In this section we list a few typical ones.

2.3.1 Proportional Allocation

The most commonly used allocation mechanism in industry is *proportional allocation.* An allocation mechanism g is called proportional if for each retailers' order vector m,

$$g_i(m) = min\left\{m_i, \frac{m_i}{\sum_{j=1}^n m_j}K\right\}. \quad (2.3.1)$$

If the demand is greater than the capacity, each allocation is the same fraction of its order. Otherwise, each allocation is equals to its order quantity. This mechanism has been analysed by Cachon and Lariviere [12]. They conjecture the prevalence of

this mechanism due to its intuitiveness and equitable possibility of allocations, since it never allocates zero. We will discuss the properties of proportional allocation in Section 2.4.1.

2.3.2 Uniform Allocation

Uniform allocation mechanism is introduced by Sprumont in [114], also known as *max-min fair allocation* in resource allocation problems [51].

For each m of order quantity vector, we index the retailers according to the ascending order of their order quantity, i.e., $m_1 \leq m_2 \leq \ldots \leq m_n$. An allocation mechanism g is called uniform allocation if

$$
g_i(m) = \begin{cases} K/n, & \text{if } nm_1 > K, \\ m_i, & \text{if } i \leq \lambda, \\ m_\lambda + \left(K - (n - \lambda + 1)m_\lambda - \sum\limits_{j=1}^{\lambda-1} m_j \right) /(n - \lambda), & \text{otherwise.} \end{cases}
$$

where $\lambda = \max \left\{ i : K - nm_1 - \sum\limits_{j=2}^{i} (n - l)(m_j - m_{j-1}) > 0 \right\}$.

Under the uniform allocation mechanism, the retailers with orders less than a threshold m_λ receive the same quantities as their respective orders. For those retailers who send orders greater than m_λ, the rest of capacity is equally divided by the number of these retailers and allocate each retailer an amount of m_λ (see Figure 2.1).

The threshold of m_λ is led by the following procedure. If $m_1 \times n$ is greater than the capacity, all retailers receive $\frac{K}{n}$. otherwise, there is a threshold m_λ where λ is greater than or equal to 1. In case of $m_1 \times n \leq K$, the supplier counts up the number of retailers, while $\sum\limits_{j=1}^{i} m_j + m_j \times (n - i) \leq K$ (the sum of i-th smallest orders and
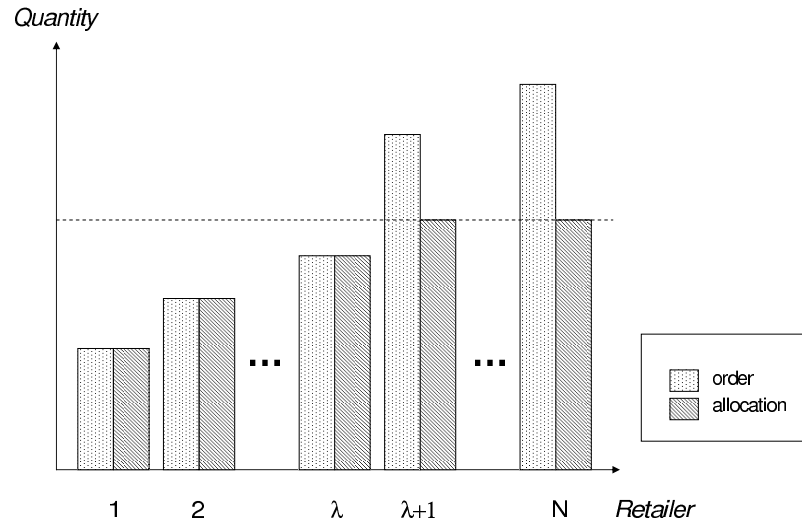
Figure 2.1: Uniform Allocation Mechanism

the quantity of the i-th order times the number of the rest of retailers is less than the capacity size of the supplier). If the sum of orders is greater than or equal to the capacity, the allocation quantity is equal to the capacity, which is the area below the horizontal dashed line in Figure 2.1.

Since uniform allocation prioritises smaller orders, the retailers are not able to increase their allocations by inflating orders. Therefore, under uniform allocation, the retailers do not have any incentives to place orders more than their desired allocations. This property is known as *truth-inducing* which is described in Section 2.4.3.

2.3.3 Capped Allocation

In [42], we proposed an allocation mechanism, named *capped allocation.* Assume that the supplier sets up a maximum allocation to each retailer. If a retailer sends in an order less than this maximal allocation quantity, the retailer will receive the full amount of its order. Otherwise, the retailer will receive the maximal quantity. Let $\gamma_i \in \Re^+$ be the maximal allocation quantity to retailer i such that $\sum_{i=1}^{n} \gamma_i \leq K$. This condition guarantees the feasibility of the capped allocation. Under capped allocation, retailer i is allocated $g_i(m)$, where

$$g_i(m) = min\left\{m_i, \gamma_i\right\}, \quad i = 1, \cdots, n.$$

Allocation is either the order quantity or the capped quantity. If $\gamma_1 = \cdots = \gamma_n$, we call such an allocation mechanism as *uniformly capped allocation.*

2.3.4 Max-max allocation

We propose an allocation mechanism which strictly allocates from the maximum orders. Let m be a order vector such that $m_1 \geq \ldots \geq m_n$. Under max-max allocation, retailer i is allocated $g_i(m)$, where

$$g_i(m) = \begin{cases} min\left\{m_i, K\right\}, & \text{for } i = 1, \\ min\left\{m_i, max\left\{0, K - \sum_{j=1}^{i-1} m_j\right\}\right\}, & \text{otherwise.} \end{cases} \tag{2.3.2}$$

Max-max allocation mechanism aims to allocate the capacity fully from the largest orders to the smaller orders sequentially. This procedure is similar to a winner determination in auctions. They normally determine the winner according to the best

price. Contrary, this allocation determines the winner according to quantities rather than prices.

2.4 Evaluation Criteria of Allocation Mechanisms

One of the most important issues in mechanism design is to determine how to evaluate mechanisms. This section presents three properties of allocation mechanisms for evaluation of mechanisms.

2.4.1 Individually Responsive Allocations

A widely acceptable allocation is to prioritise the important customers. The principle of individually responsive in the setting of capacity allocation can be described as follows:

Definition 2.4.1. Let m be a vector of retailers' orders, m_i be the i's component of m, $m_{j\neq i}$ be a vector of the other retailers' orders. An allocation mechanism g is called to be *individually responsive* (IR) if for any i and any order vector m_i',

$$m_i' > m_i \text{ implies } g_i(m_i', m_{j\neq i}) > g_i(m_i, m_{j\neq i})$$

$$\text{unless } g_i(m) = K. \tag{2.4.1}$$

An allocation mechanism g is called to be *weakly individually responsive* (WIR) if for any i,

$$m_i' > m_i \text{ implies } g_i(m_i', m_{j\neq i}) \geq g_i(m_i, m_{j\neq i}), \tag{2.4.2}$$

Under an IR mechanism, greater orders are allocated greater. In other words, a retailer receives more allocation if it places more orders. A representative example of this property is proportional allocation in Equation 2.3.1. If the retailer is a local monopolist like [11], the profit is increased with respect to the allocated quantity as long as this quantity is less than the monopoly quantity. This implies that the retailer inflates its order rather than submitting its optimal allocation quantity, as long as it expects that the supplier does not have enough capacity. Thus, the supplier may not obtain truthful order infromation from the retailers.

2.4.2 Pareto Optimal Allocation

The main goal of supply chain management is to maximise the supply chain profit. The following allocation mechanisms achieve such a goal.

Definition 2.4.2. [11] Let x^* be the optimal sales quantity of the retailers. An allocation mechanism g is *Pareto allocation mechanism* if $g(x^*)$ maximises the sum of retailer profits, that is,

$$\forall m. \sum_{i=1}^{n} \pi_i(g(x^*)) \geq \sum_{i=1}^{n} \pi_i(g(m))$$

In other words, the allocation mechanism maximizes the sum of retailers' profits if all retailers truthfully submit their optimal orders.

In the definition of Pareto optimal allocation mechanism, Cachon and Lariviere assume that all retailers truthfully submit their orders. This assumption is necessary if decisions of the retailers are based on the complex model like our two-connected

markets. By splitting the markets into individual markets, we obtain the simplified market models. In the isolated downstream market, we obtain the truthful demand according to the classic economic analysis shown in Section 2.2.2. Therefore, this definition implies that the property of market mechanisms is analysed by fixing market behaviours of a connected market.

In [11], Cachon and Lariviere show that the proportional allocation is actually a Pareto allocation mechanism in their model. Because Pareto allocation mechanisms are efficient and stable, the authors mention that proportional allocation is popular in many different industries. However, we prove that proportional allocation is not always Pareto allocation mechanism in our competitive settings. We present cost symmetric case in Section 3.3 and asymmetric case in Section 3.5.1.

2.4.3 Truth-Inducing Mechanisms

The truthful order information is a significant for the suppliers. The retailers tend to inflate orders under IR mechanisms as we have shown in Section 2.4.1. Moreover, in order to check Pareto optimality of allocation mechanisms, we use truthful orders as presented in Definition 2.4.2. An interesting property of the mechanisms is to tell the truth instead of inflating orders.

Definition 2.4.3. Let x^* be the optimal sales quantity of the retailers. An allocation mechanism g is called to be *truth-inducing* if x^* is a Nash equilibrium of g, that is,

$$\forall m. \forall i. \ \pi_i(g(x_i^*, x_{j \neq i}^*) \geq \pi_i(g(m_i, x_{j \neq i}^*)$$

In other words, the strategy profile that all retailers place orders truthfully at their

optimal sales quantities is a Nash equilibrium of g.

Under truth-inducing mechanisms, the supplier is able to know the truthful demand through the orders from the retailers. Several truth-inducing mechanisms, such as uniform allocation, lexicographic allocation, and relaxed linear allocation are proposed in the literature [12, 11, 114]. The advantage of the truth-inducing mechanism is that the supplier is able to evaluate the size of the constructed capacities. Subsequently, the supplier is able to review the capacity investment plan for the future. This criterion corresponds to the retailers' profits that rely on the behaviours in the downstream market.

2.5 Discussion and Related Work

In this chapter, we have presented a supply chain model of the capacity allocation problem and defined a few typical capacity allocation mechanisms. Capacity allocation is a standard solution for long-term and large-volume trades in industries, typically in the upper stream of supply chains. In contrast, auction mechanisms are popular for short-term and small-volume trades [36]. Queuing models are alternative solutions for scarce capacity such as [78, 68]. Our model differs from queuing models, since our capacity allocation solution does not find solutions with delay of delivery. This is a significant difference for perishable items, new products, or seasonable products. For those items, the late deliveries would mean loss of opportunities.

In this chapter, we formally represent the capacity allocation game where the supply chain members make strategic decisions in the two-connected markets. In contrast

to the existing capacity allocation literature such as single market models [12, 114], single supplier and monopolistic downstream markets models [11, 38], or multiple suppliers and monopolistic downstream markets [74, 73], our model has a competitive market as a connected market to a market uses allocation mechanisms. We investigate how two-connected markets are influenced by their market mechanisms each other. Furthermore, our model is more complex to analyse the properties of allocation mechanisms compared to the existing works, since individual allocation for each retailer does not simply deduce a profit function of the corresponding retailer. In our model, the profit functions are obtained by using the strategic interactions in the competitive downstream market.

We have assumed allocation mechanisms with a fixed-wholesale price contract as market mechanisms in the upstream market. This is a typical contract in industries seek for long-term relationships [63, 36]. Anupindi and Bassok [1] identify delivery commitments as key parameters in contracts between buyers and suppliers in industry rather than prices. Furthermore, in our setting, the supplier is not able to determine the profit maximal wholesale price, since the wholesale price is set prior to obtain demand information.

For the competitive market rules, we consider two typical rules, the quantity competition and the price competition. In Chapter 3 and 4 we investigate the properties of allocation mechanisms for the competitive markets under the quantity competition and the price competition, respectively.

The two-connected market model is a common model in industrial organization which is a subfield of microeconomics. The major works are vertical integration and

multilateral vertical contracting in [52, 77, 105]. Aims of these works are closely related to ours. They show how market rules in the downstream market affects the strategic choice in the upstream market and how strategic choices influence the market behaviour in the downstream market. Those studies consider problems of mergers and acquisitions hinging on full demand information. In our model, the supplier does not have demand information. In addition, those works do not consider allocation mechanisms.

The presented model is not restricted to product distribution in supply chains. Our approach is effective to analyse cases where strategic interactions are sequentially connected environments.

Chapter 3

Allocation Mechanisms with Quantity Competition

In the previous chapter, we have presented our supply chain model for the capacity allocation problems. A characteristic of this model is the introduction of competitive market in the connected market to capacity allocation problems. In this model, analysis of mechanism properties requires analysis of market behaviour in the downstream market. In this chapter, we consider the case of quantity competition in the downstream market.

3.1   Background

Most of research on allocation mechanisms ignores strategic interactions once buyers are allocated items. In the single market model [114], utilities of buyers are

directly mapped from allocation quantities. In the monopolistic downstream markets model [11, 12] or the exclusive distribution model of the internal product managers [38], a payoff function of each agent is based on its own allocation quantity. In this chapter, we extend the analysis to a competitive market, namely the oligopolistic market where the sellers directly compete each other. Similarly to the standard quantity competition, the payoff of the agents is influenced by the strategic interactions. Hence, it is important to understand how agents behave in the market based on the market rule. As a market rule, we consider a case of quantity competition which suits for substitute items or services such as agricultural products and natural resources. Our goal is to demonstrate how market rules influence the properties of allocation mechanisms that are not directly obtained from the individual outcome for each agent.

We have presented our supply chain model in Section 2.1 that consists of two connected markets. That is, in the upstream market, the supplier allocates capacities to the retailers based on the allocation mechanism. The retailers then sell the allocated products in the downstream market based on the quantity competition. In contrast to the case of the monopolistic downstream market markets, the outcome of the mechanisms depends on the strategic interactions of the retailers. At first, we temporary split the model into two markets. In such a case, our downstream market is the classical quantity competition, known as Cournot competition [26, 27]. We use the Cournot quantity, which is the equilibrium outcome of the classical quantity competition, as the truthful order quantity in our model. Next, by assuming the retailers submit the truthful orders, we analyse the properties of allocation mechanisms.

The rest of this chapter is organised as follows. In Section 3.2, we present the truthful order information under quantity competition without considering allocation mechanisms in the downstream market. We also present that the conditions of allocations to maximise the total retailer profits if the retailers submit the truthful orders. We consider two cases for the retailers costs: symmetric and asymmetric costs. In the case of symmetric costs, we investigate Pareto optimal allocations in Section 3.3 and truth-inducing mechanisms in Section 3.4. In Section 3.5, we investigate how asymmetric costs in the downstream market influence the properties of the allocation mechanisms. We summarise our results in Section 3.6.

3.2   Desired Market Behaviour under Quantity Competition

This section focuses on describing desired allocations in the downstream market as a whole. If we assume that the supplier does not have a capacity constraint, each retailer is able to set its supply quantity which is exactly the same case to the Cournot model shown in Section 2.2.2. First, we present the Cournot equilibrium in our capacity allocation model. Then, we present how desired allocations change over the given capacities of the supplier.

As presented in Section 2.2, the selling price is determined by the total supply quantity in the downstream market. If we assume that each retailer is allocated

as much as it requires, we have the following profit function according to Equation (2.2.2): $\pi_i(a_i, a_{j \neq i}) = P(a_i + a_{-i})a_i - wa_i$. Then, the desired allocation denoted by the best response function r is obtained,

$$r(a_{-i}) = \arg\max_{a_i}\{a_i P(a_i + a_{-i}) - wa_i\}. \tag{3.2.1}$$

We assume that $a_i P(a_i + a_{-i}) - wa_i$ is concave in a_i for all a_{-i}. Based on this assumption, $r(a_{-i})$ is a unique solution of $P(a_i + a_{-i}) + a_i \frac{\mathrm{d}P(a_i + a_{-i})}{\mathrm{d}a_i} - w = 0$. There is the unique intersection of the best response functions of all agents which is the Cournot-Nash equilibrium. Let us denote Cournot-Nash equilibrium as $q^{cw} = (q_1^{cw}, \ldots, q_n^{cw})$ and the total Cournot quantity as $Q^{cw} = \sum_{i=1}^n q_i^{cw}$, respectively. Note that $r(q_{-i}^{cw}) = q^{cw}$. At the Cournot-Nash equilibrium , each retailer's profit is maximized, i.e.,

$$\forall q_i \in \Re^+ \ (\pi_i(q^{cw}) \geq \pi_i(q_i, q_{j \neq i}^{cw})) \quad (i = 1, \cdots, n) \tag{3.2.2}$$

Suppose a linear demand $P(Q) = \max\{\alpha - \beta Q, 0\}$ where $\alpha > w \geq 0$ and $\beta > 0$ are constants. In such a case, Cournot-Nash equilibrium is

$$q_i^{cw} = \frac{\alpha - w}{(n+1)\beta} \quad (i = 1, 2, \ldots, n). \tag{3.2.3}$$

Therefore, the desired allocation for retailer i is $a_i^* = q_i^{cw}$

The desired allocation above is a case where there is no capacity limit for the supplier. If the capacity of the supplier is limited, the supplier is not able to allocate Cournot quantities to the retailers. In such cases, feasible allocations that maximise the total retailer profit are desirable for the retailers. In the following theorem,

we present the conditions of allocations that maximise the total retailer profit with assuming all retailers submit $m^* = q^{cw}$:

Theorem 3.2.1. *Suppose all retailers submit $m^* = q_i^{cw}$ under the linear demand $P(Q) = \max\{\alpha - \beta Q, 0\}$. An allocation a^* maximises the total profit of the retailers if it satisfies the following conditions:*

1. $0 \leq a_i^* \leq m_i^* (i = 1, \ldots, n)$.

2. $\sum_{i \in N} a_i^* = K$, *when* $K < \frac{\alpha - w}{2\beta}$.

3. $\sum_{i \in N} a_i^* = \frac{\alpha - w}{2\beta}$, *when* $K \geq \frac{\alpha - w}{2\beta}$.

Proof. Let $\Pi(a)$ be the total retailer profits with given allocation a such that

$$\Pi(a) = \sum_{i \in N} \pi_i(a)$$

$$= \sum_{i \in N} (\alpha - \beta(a_i + a_{-i}) - w)a_i.$$

The optimization problem of the total retailer profits is formulated as follows:

$$\max_{a \in \mathcal{A}} \Pi_i(a) \text{ subject to } \sum_{i \in N} a_i \leq K \text{ and } 0 \leq a_i \leq m_i^* \text{ for } i = 1, \ldots, n.$$

Since the above optimisation problem consists of an objective function which is concave and $n + 1$ inequality constraints, we apply Karush-Kuhn-Tucker (KKT) constraints [61, 66]. Let λ be KKT coefficients such that $\lambda = (\lambda_1, \ldots, \lambda_{n+1})$. For the above optimisation problem with nonnegativity constraints, we have the following modified Lagrangean $\mathcal{L}$,

$$\mathcal{L}(a, \lambda) = \Pi(a) - \lambda_1 \left(\sum_{i \in N} a_i - K \right) - \sum_{i=2}^{n+1} \lambda_i (a_i - m_i^*).$$

For $\mathcal{L}$, we have the following partial derivatives,

$$\frac{\partial \mathcal{L}(a, \lambda)}{\partial a_i} = \alpha - 2\beta(a_i + a_{-i}) - w - \lambda_1 - \lambda_{i+1} \quad \forall i = 1, \ldots, n.$$

Now the optimisation problem is to find (a^*, λ) that satisfies all the following KKT conditions for the modified Lagrangean,

$$\text{KKT}_1 : \frac{\partial \mathcal{L}(a, \lambda)}{\partial a_i} = 0, \quad a_i \geq 0, \quad a_i \frac{\partial \mathcal{L}(a, \lambda)}{\partial a_i} = 0 \quad \forall i = 1, \ldots, n$$

$$\text{KKT}_2 : \lambda_1 \geq 0, \quad \sum_{i=1}^{n} a_i - K \leq 0, \quad \lambda_1 \left(\sum_{i=1}^{n} a_i - K \right) = 0$$

$$\text{KKT}_3 : \lambda_i \geq 0, \quad a_i - m_i^* \leq 0, \quad \lambda_i(a_i - m_i^*) = 0, \quad \forall i = 1, \ldots, n$$

We check the existence of the optimal solutions in the following six cases.

Case 1: $\lambda_i = 0 \quad \forall i = 1, \ldots, n + 1$.

According to KKT_1, we obtain $\sum\limits_{i \in N} a_i^* = \frac{\alpha - w}{2\beta}$ and $a_i^* \geq 0 \ \forall i = 1, \ldots, n$. By putting this result into KKT_2, we have $K \geq \frac{\alpha - w}{2\beta}$. According to KKT_3, we have $a_i^* \leq m_i^*, \ \forall i = 1, \ldots, n$. Therefore, we have a solution a^* such that

$$\sum_{i \in N} a_i^* = \frac{\alpha - w}{2\beta} \text{ and } 0 \leq a_i^* \leq m_i^*, \forall i = 1, \ldots, n, \text{ if } K \geq \frac{\alpha - w}{2\beta}.$$

Case 2: $\lambda_1 = 0$, and $\lambda_i \neq 0 \quad \forall i = 2, \ldots, n + 1$.

This case does not hold, since, for all i, $a_i^* = m_i^*$ induced by KKT_3 do not satisfy KKT_1.

Case 3: $\lambda_1 = 0$ and there exists at least one but not for all $\tilde{i} \in \{2, \ldots, n + 1\}$ such that $\lambda_{\tilde{i}} \neq 0$, otherwise $\lambda_i = 0$.

According to KKT_3, we have $a_i^* \leq m_i^*$ for any i. We also obtain $a_{\tilde{i}}^* = m_{\tilde{i}}^*$. According to KKT_1, we obtain $\sum\limits_{i \in N} a_i^* \leq \frac{\alpha - w}{2\beta}$ and $a_i^* \geq 0 \ \forall i = 1, \ldots, n$. Furthermore, the following

conditions are obtained either $\sum\limits_{i\in N} a_i^* = \frac{\alpha-w}{2\beta}$ or $a_i = 0$ for all $i \neq \tilde{i}$. According to

KKT_2, we have $\sum\limits_{i\in N} a_i^* \leq K$. Therefore, we have a solution a^* such that

$$\sum_{i\in N} a_i^* \leq \frac{\alpha-w}{2\beta} \text{ and } 0 \leq a_i^* \leq m_i^*, \forall i = 1, \ldots, n, \text{ if } K \leq \frac{\alpha-w}{2\beta}.$$

There is at least one $\tilde{i}$ such that $a_{\tilde{i}}^* = m_{\tilde{i}}^*$. Furthermore, $\sum\limits_{i\in N} a_i^* = \frac{\alpha-w}{2\beta}$ or $a_i = 0$ for

all $i \neq \tilde{i}$.

Case 4: $\lambda_i \neq 0 \quad \forall i = 1, \ldots, n+1$.

This case does not hold, since, for all i, $a_i^* = m_i^*$ induced by KKT_3 does not satisfy

KKT_1.

Case 5: $\lambda_1 \neq 0$ and $\lambda_i = 0 \quad \forall i = 2, \ldots, n+1$.

According to KKT_2, we have $\sum\limits_{i\in N} a_i^* = K$. By putting this result into KKT_1, we obtain

$K \leq \frac{\alpha-w}{2\beta}$ and $a_i^* \geq 0 \ \forall i = 1, \ldots, n$. According to KKT_3, we have $a_i^* \leq m_i^*, \ \forall i = 1, \ldots, n$. Therefore, we have a solution a^* such that

$$\sum_{i\in N} a_i^* = K \text{ and } 0 \leq a_i^* \leq m_i^*, \forall i = 1, \ldots, n, \text{ if } K < \frac{\alpha-w}{2\beta}.$$

Case 6: $\lambda_1 \neq 0$, there exist at least one but not for all $\tilde{i} \in \{2, \ldots, n+1\}$, such that $\lambda_{\tilde{i}} \neq 0$ otherwise $\lambda_i = 0$.

According to KKT_3, we have $a_i^* \leq m_i^*$ for any i. We also obtain $a_{\tilde{i}}^* = m_{\tilde{i}}^*$. According

to KKT_2, we have $\sum\limits_{i\in N} a_i^* = K$. By putting this result into KKT_1, we obtain $K < \frac{\alpha-w}{2\beta}$

and $a_i^* \geq 0 \ \forall i = 1, \ldots, n$. Therefore, we have a solution a^* such that

$$\sum_{i\in N} a_i^* = K \text{ and } 0 \leq a_i^* \leq m_i^*, \forall i = 1, \ldots, n, \text{ if } K \leq \frac{\alpha-w}{2\beta}.$$

There is at least one $\tilde{i}$ such that $a_{\tilde{i}}^* = m_{\tilde{i}}^*$

According to the above six cases, a^* is obtained if the following optimality conditions are satisfied,

1. $0 \leq a_i^* \leq m_i^* (i = 1, \ldots, n)$.

2. $\sum_{i \in N} a_i^* = K$, when $K < \frac{\alpha - w}{2\beta}$.

3. $\sum_{i \in N} a_i^* = \frac{\alpha - w}{2\beta}$, when $K \geq \frac{\alpha - w}{2\beta}$.

$\square$

According to the above theorem, we obtain a condition of allocations that maximises the total retailer profit. Therefore, this condition can be used to evaluate Pareto optimality check of allocation mechanisms. The condition consists of two cases. The first case considers that the capacity is less than the joint monopoly quantity. In this case, allocations must be efficient. The second case considers that the capacity exceeds the joint monopoly quantity. In this case, the sum of allocation must be equal to the joint monopoly quantity. As long as allocation mechanisms satisfy this condition, the allocation mechanisms are Pareto optimal.

3.3   Pareto Optimal Allocation Mechanism

In this section, we explore how the market mechanism of the downstream market influences all of supply chain members' profits. In our supply chain model, it is necessary to consider both the supplier's perspective and the retailers' perspectives. As we have mentioned in Chapter 2, Cachon and Lariviere prove that the most popular

allocation mechanism, proportional allocation shown in Equation (2.3.1) maximises both the supplier's profit and the sum of retailers' profits in the monopolistic downstream market. In that sense, proportional allocation can be considered as the best capacity allocation mechanism in their model. However, this is no longer true in our oligopolistic downstream market model. To show this difference, we first prove that the proportional allocation mechanism does maximise the supplier's profit in Section 3.3.1. However, we prove in Section 3.3.2 that the proportional allocation mechanism does not always maximise the sum of the retailers' in our model.

3.3.1 Supplier's Profit Maximization

In general, profit maximisation is the fundamental goal for the suppliers. We present a condition to maximise a profit related to allocation mechanism in this section. Next, we show that proportional allocation is a profit maximising allocation mechanism for the supplier.

At the time of the capacity investment, the supplier does not exactly know either the demand of the end-users in the downstream market or the orders from the retailers. In such a case, the supplier is not able to set the capacity that maximises its profit shown in Equation (2.2.5). Hence, we focus on how allocation has an impact on the profit of the supplier. Let A be the total allocation. According to Equation 2.2.5, we have the following profit function, $\pi_s(A) = wA - cK$, where $A \leq K$. For the supplier, it is important to implement allocation mechanisms that maximise the supplier's profit whatever the given capacity size is. In the following lemma, we show that any

efficient allocation mechanisms maximises the profit of the supplier.

Lemma 3.3.1. *Let g be an efficient allocation mechanism. Given capacity K and order vector m, any allocation mechanism g maximises the supplier's profit π_s in the oligopolistic downstream market.*

Proof. According to the supplier's profit function shown in Equation (2.2.5), the supplier's profit π_s is linearly increased in the total supply A. Therefore, the profit of the supplier is maximised if the total supply A is maximised by mechanism g. According to the definition of efficient allocation shown in Equation (2.2.1), there is no other ways to allocate more quantities than efficient allocations. Hence, any efficient allocation g maximises the supplier's profit π_s. $\square$

According to this lemma, given capacity K and order m, any efficient allocation maximises the supplier's profit. Hence, this lemma provides a way to check preferable allocation mechanisms for the supplier. For instance, proportional allocation is efficient allocation according to the definition presented in Equation 2.3.1. Therefore, proportional allocation is a profit maximising mechanism for the supplier.

In the following section, we shift the view from the supplier to the retailers. Furthermore, we analyse how the retailers place orders according to the mechanism.

3.3.2   Retailers' Profit Maximization

To check the Pareto optimality of proportional allocation, we have shown that it is in favour of the supplier in the previous section. We now analyse its property from the retailers' point of view. As previously mentioned, we use the Cournot quantity as

the truthful order quantity. According to the definition of Pareto optimal allocations (Definition 2.4.2 in page 32), Pareto allocation mechanism maximises the total retailer profits if the retailers submit orders truthfully. In the following theorem, we show that proportional allocation is not always Pareto optimal in our supply chain model.

Theorem 3.3.2. *If all retailers order truthfully Cournot-Nash equilibrium quantity* $m_i^* = \frac{\alpha-w}{(n+1)\beta}$, *the proportional allocation mechanism is Pareto optimal if and only if* $K \leq \frac{\alpha-w}{2\beta}$.

Proof. According to the definition of Pareto optimal mechanism (Definition 2.4.2 in page 32), the total retailer profit is maximised under Pareto optimal mechanism if the retailers submit truthful orders. According to Equation (2.3.1), allocation of retailer i with orders m^* is,

$$g(m^*) = \begin{cases} \frac{K}{n} & \text{if } K < nm_i^*, \\[2ex] m_i^* & \text{otherwise .} \end{cases}$$

Hence, we obtain $\sum_{i=1}^{n} g_i(m^*) = K$ if $K \leq \frac{\alpha-w}{2\beta}$, which satisfies the conditions to maximise the total profit of the retailers presented in Theorem 3.2.1. At the opposite, we yield $\sum_{i=1}^{n} g_i(m^*) = K$ if $\frac{\alpha-w}{2\beta} < K \leq nm_i^*$ and we obtain $\sum_{i=1}^{n} g_i(m^*) = nm_i^*$ if $K > nm_i^*$. Hence g does not satisfy the conditions if $\frac{\alpha-w}{2\beta} < K$. Therefore, proportional allocation g is Pareto optimal iff $K \leq \frac{\alpha-w}{2\beta}$. $\qquad\square$

Theorem 3.3.2 indicates that proportional allocation maximises the total retailer profits if capacity K is less than the joint monopoly quantity $\frac{\alpha-w}{2\beta}$. This is a situation where the capacity is strictly restricted since the total market demand is $\frac{n(\alpha-w)}{(n+1)\beta}$. Therefore, proportional allocation is Pareto optimal if the capacity is strictly limited

compared to the market size of the downstream market. This is a significant difference compared to the monopolistic downstream market model in [11].

Pareto allocation mechanism is restricted to the case where the retailers submit truthful orders. A question arises whether this prerequisite holds in our model. In the next section, we check whether the retailers submit truthful orders under proportional allocation.

3.4 Truth-inducing Mechanisms

In the previous section, we focus on the Pareto optimal allocations. Now we switch to the other two typical evaluation criteria of allocation mechanism design: truth-inducing and individual responsiveness (IR). As Cachon and Lariviere mention in [11], the truth-inducing and the individual responsiveness are the properties that we cannot have both if the downstream markets are monopolistic. Similarly, we show that allocation mechanisms do not satisfy truth-inducing and individual responsiveness at the same time. Then we investigate specific truth-inducing mechanisms.

At first, we prove that the individually responsive mechanisms are not the truth-inducing mechanisms if the capacity binds.

Theorem 3.4.1. *Let g be an efficient individually responsive mechanism. Let m^* be the truthful order quantity (Cournot-Nash equilibrium quantity). Allocation mechanism g is not a truth-inducing mechanism if $K \leq \sum_{i=1}^{n} m_i^*$.*

Proof. In order that mechanism g to be a truth-inducing mechanism, the following condition must hold,

$$\pi_i(g(m^*)) \geq \pi_i(g(m_i, m^*_{-i})) \text{ for any } m_i. \tag{3.4.1}$$

Since $K \leq \sum\limits_{i=1}^{n} m_i^*$ and g is efficient, we have $\sum\limits_{i=1}^{n} g_i(m_i, m^*_{j \neq i}) = K$. The difference of the profits between the two cases, $(m_i, m^*_{j \neq i})$ and m^*, is

$$\pi_i(g(m_i, m^*_{j \neq i})) - \pi_i(g(m^*)) = (\alpha - \beta K - w)(g_i(m_i, m^*_{j \neq i}) - g_i(m^*)). \tag{3.4.2}$$

According to the supposition of the theorem, $K \leq \sum\limits_{i=1}^{n} m_i^*$, we obtain $K < \frac{\alpha - w}{\beta}$ which induces $\pi_i(g(m_i, m^*_{j \neq i})) - \pi_i(g(m^*)) > 0$ according to Equation 3.4.2. This inequation contradicts the truth-inducing condition presented in Equation 3.4.1. Therefore, the retailers do not submit the truthful orders under individual responsive mechanisms.

$\square$

According to Theorem 3.4.1, IR mechanisms are not truth-inducing mechanisms in our model. Since proportional allocation is a representative IR mechanism, it is not truth-inducing according to the result of Theorem 3.4.1. From now on, we investigate specific truth-inducing mechanisms.

A representative truth-inducing mechanism presented in [11, 12] is uniform allocation in the monopolistic downstream markets. We first check if uniform allocation is truth-inducing in the quantity competition in the following theorem.

Theorem 3.4.2. *Uniform allocation is truth-inducing.*

Proof. We show that it is not possible to increase the profit by changing the orders. If retailer i submits an order greater than the truthful order quantity, $m_i > m_i^*$, the

allocation is not changed, $g_i(m_i, m^*_{j \neq i}) = g_i(m^*)$ and $g_j(m_i, m^*_{j \neq i}) = g_j(m^*)$. Hence, the profit is not increased.

If $m_i < m^*_i$, we have to consider the following three cases, (i) $K \leq nm_i$, (ii) $nm_i < K \leq m_i + (n-1)m^*_i$, and (iii) $m_i + (n-1)m^*_i < K$. In case (i), we have $g_i(m_i, m^*_{j \neq i}) = \frac{K}{n}$ and $g_j(m_i, m^*_{j \neq i}) = \frac{K}{n}$. Therefore, $g_i(m_i, m^*_{j \neq i}) = g_i(m^*)$ implies $\pi_i(g(m_i, m^*_{j \neq i})) = \pi_i(g(m^*))$. Hence, the profit is not increased in case (i). In case (ii), we obtain $g_i(m_i, m^*_{j \neq i}) = m_i$ and $\sum_{i=1}^{n} g_i(m_i, m^*_{j \neq i}) = K$. Hence, we obtain the following profit function $\pi_i(g(m_i, m^*_{j \neq i})) = (\alpha - \beta K - w)g_i(m_i, m^*_{j \neq i}))$. Since this profit function linearly increases with respect to allocation quantity, the profit is not increased by decreasing m_i. In case (iii), we have $g_i(m_i, m^*_{j \neq i}) = m_i$ and $g_j(m_i, m^*_{j \neq i}) = m^*_i$. Hence, we obtain the following profit function $\pi_i(g(m_i, m^*_{j \neq i})) = (\alpha - \beta(m_i + m^*_{j \neq i}) - w)m_i$. Since this profit function is equivalent to the profit function under Cournot model, the profit is not increased by decreasing m_i as long as $m_i < m^*_i$.

Therefore, there are no ways to increase profit by changing order from the truthful quantity. $\qquad \square$

According to Theorem 3.4.2, we show that uniform allocation is a truth-inducing mechanism similarly to the cases of monopoly downstream market [11] and a market with a single peaked demand [114].

Several truth-inducing mechanisms, such as uniform allocation, lexicographic allocation, and relaxed linear allocation are proposed in the literature [12, 11, 114]. However, such allocation mechanisms are not so commonly used in industry, due to their non-intuitiveness. In the following theorem, we present that capped allocation

mechanism is a truth-inducing mechanism:

Theorem 3.4.3. *Any uniformly capped allocation mechanism is truth-inducing.*

Proof. Let $\gamma = \gamma_1 = \ldots = \gamma_n$ be a uniformly capped quantity under uniformlly capped allocation g. Since the truthful order quantity m_i^* is the Cournot quantity q^{cw}, it is sufficient to show that g satisfies the following condition,

$$\pi_i(g(m_i^*, m_{j\neq i}^*)) \geq \pi_i(g(m_i, m_{j\neq i}^*)) \text{ for any } m_i. \tag{3.4.3}$$

Since the profit function is dependent on output of the allocation mechanism, we obtain the following profit function according to the standard quantity competition presented in Equation (2.2.2),

$$\pi_i\left(g\left(m_i, m_{j\neq i}^*\right)\right) = \left(\alpha - \beta g_{-i}\left(m_i, m_{j\neq i}^*\right) - w\right) g_i\left(m_i, m_{j\neq i}^*\right) - \beta g_i^2\left(m_i, m_{j\neq i}^*\right), \tag{3.4.4}$$

and

$$\pi_i\left(g\left(m^*\right)\right) = (\alpha - (n-1)\beta g_i\left(m^*\right) - w) g_i\left(m^*\right) - \beta g_i^2\left(m^*\right). \tag{3.4.5}$$

We split into two cases at the capped quantity is the truthful order quantity.

Case 1: $\gamma \geq m_i^*$, we have $g_j(m_i, m_{j\neq i}^*) = m_j^*$, for any $j \neq i$, according to the definition of capped allocation. According to the condition of case 1, the capacity is greater than the total Cournot quantity, $K \geq Q^{cw}$. Since allocated quantity of retailer $j \neq i$ is Cournot quantity, we are able to apply the standard Cournot result that is the profit of retailer i is maximised at $g_i(m_i, m_{j\neq i}^*) = m_i^*$ which is a case $m_i = m_i^*$. Therefore, the condition (3.4.3) is satisfied in case 1.

Case 2: $\gamma < m_i^*$, we have $g_j(m_i, m_{j\neq i}^*) = \gamma$, for any m_i. Therefore, according to

Equation (3.4.4) and (3.4.5), we have

$$\pi_i \left(g\left(m_i, m^*_{j\neq i}\right)\right) = \left(\alpha - w - (n-1)\beta\gamma\right) g_i\left(m_i, m^*_{j\neq i}\right) - \beta g_i^2\left(m_i, m^*_{j\neq i}\right),$$

and

$$\pi_i \left(g\left(m^*_i, m^*_{j\neq i}\right)\right) = \left(\alpha - w - (n-1)\beta\gamma\right)\gamma - \beta\gamma^2.$$

If $m_i \geq \gamma$, then $g(m_i, m^*_{j\neq i}) = g(m^*)$. Therefore, it satisfies the condition (3.4.3). Otherwise, we have $\pi_i\left(g\left(m_i, m^*_{j\neq i}\right)\right) = \left(\alpha - w - (n-1)\beta\gamma\right)m_i - \beta m_i^2$. Hence, we obtain

$$\pi_i\left(g\left(m^*_i, m^*_{j\neq i}\right)\right) - \pi_i\left(g\left(m_i, m^*_{j\neq i}\right)\right)$$

$$= \left(\alpha - w - (n-1)\beta\gamma\right)\gamma - \beta\gamma^2 - \left(\alpha - w - (n-1)\beta\gamma\right)m_i + \beta m_i^2$$

$$= \left(\alpha - w - n\beta\gamma - \beta m_i\right)\left(\gamma - m_i\right),$$

$$> \left(\alpha - w - (n+1)\beta\gamma\right)\left(\gamma - m_i\right),$$

$$> 0.$$

The last inequation above is obtained, since we have $\alpha - w - (n+1)\beta\gamma > 0$ according to $\gamma < m^*_i = \frac{\alpha - w}{(n+1)\beta}$ and also $\gamma > m_i$. Therefore, case 2 satisfies the condition (3.4.3). $\square$

Under uniformly capped allocation mechanism, the supplier obtains truthful orders from the retailers. This leads the supplier's secure decision-makings on capacity planning and sales planning. However, for the supplier, truth-inducing mechanisms are less profitable mechanisms than IR mechanisms if capacity exceeds the total Cournot quantity. As we have shown in Section 3.3.1, the profit of the supplier is increased in the total allocation. Therefore, the inflated orders by IR mechanisms make the supplier obtaining greater profit than the truthful orders. It is a trade-off

for the supplier to choose either efficient and profitable mechanisms such as proportional allocation or its variations, or informative truth-telling mechanisms such as uniformly capped allocation.

3.5   Cost Asymmetric Case

In the previous sections, we assume that wholesale price is unique to all retailers. We call this case cost symmetric. In this section, we release this assumption and consider the more general case.

Under the symmetric cost structure, the retailers have the same competitive powers in the downstream market. However, in the cost asymmetric case, the retailers have different competitive powers to distribute items to the end-users. Let us denote ω_i the variable cost of retailer i which includes wholesale price w. Hence, the retailers have symmetric wholesale price w and each retailer has variable cost $\omega_i - w$. We assume strong asymmetric cost $\omega_1 < \ldots < \omega_n$. In that case, profit function of retailer i is $\pi_i(q_i, q_{j \neq i}) = (\alpha - \beta Q)q_i - \omega_i q_i$. Similarly to Equation (3.2.3), Cournot equilibrium quantity $q_i^{c\omega}$ is obtained, $q_i^{c\omega} = \frac{1}{\beta}\left(\frac{\alpha + \sum_{i=1}^{n}\omega_i}{n+1} - \omega_i\right)$. Therefore, Cournot equilibrium quantity is not symmetric. Notice that the retailer with the smallest cost has the greatest equilibrium quantity. At the Cournot equilibrium, we obtain that the profit of retailer i is: $\pi_i(q_i^{c\omega}, q_{-i}^{c\omega}) = \frac{1}{\beta}\left(\frac{\alpha + \sum_{i=1}^{n}\omega_i}{n+1} - \omega_i\right)^2$. Hence, the lower cost retailers have the greater profits. In the above settings, we investigate Pareto optimal mechanisms and truth-inducing mechanisms.

3.5.1 Pareto Optimality with the Cost Asymmetric Retailers

In this section, we investigate Pareto optimal allocation mechanisms. Similarly to the case of symmetric cost retailers, we first present the conditions of allocations that maximise the total profit of the retailers. Then, we check specific mechanisms if they satisfies the conditions.

To analyse the conditions, we assume that the retailers submit their orders at the Cournot quantity $q^{c\omega}$. The following theorem presents the conditions of allocations to maximise the total profit of the retailers if the retailers submit truthful orders under asymmetric case:

Theorem 3.5.1. *Let $m^* = q^{c\omega}$ be the truthful order vector (Cournot equilibrium quantities) with asymmetric cost retailers such that $m_1^* < \ldots < m_n^*$. Let $\hat{i}$ be the largest sequential number of retailers that satisfies $\sum_{j=1}^{\hat{i}} m_j^* \leq \frac{\alpha - \omega_{\hat{i}}}{2\beta}$. An allocation a^* maximises the total profit of the retailers if it satisfies the following conditions:*

(i) when $K \geq \sum_{i=1}^{\hat{i}} m_i^*$, $\sum_{i \in N} a_i^* = \frac{\alpha - \omega_{\hat{i}}}{2\beta}$ and

$$
a_i^* = \begin{cases}
m_i^* & \text{if } i \leq \hat{i}, \\
\max\left\{ \frac{\alpha - \omega_{\hat{i}}}{2\beta} - \sum_{j=1}^{\hat{i}} m_j^*, 0 \right\} & \text{if } i = \hat{i} + 1 \\
0 & \text{otherwise}
\end{cases}
$$

(ii) when $K < \sum_{i=1}^{\hat{i}} m_i^*$, $\sum_{i \in N} a_i^* = K$.

Proof. Let $\Pi(a)$ be the total retailer profits with given allocation a such that

$$\Pi(a) = \sum_{i \in N} \pi_i(a)$$

$$= \sum_{i \in N} (\alpha - \beta(a_i + a_{-i}) - \omega_i)a_i.$$

Similarly to Theorem 3.2.1, we have the following modified Lagrangean $\mathcal{L}$

$$\mathcal{L}(a, \lambda) = \Pi(a) - \lambda_1 \left(\sum_{i \in N} a_i - K \right) - \sum_{i=2}^{n+1} \lambda_i (a_i - m_i^*).$$

and the optimisation problem is to find (a^*, λ) that satisfies all the KKT conditions for the modified Lagrangean:

$$\text{KKT}_1 : \frac{\partial \mathcal{L}(a, \lambda)}{\partial a_i} = 0, \ \ a_i \geq 0, \ \ a_i \frac{\partial \mathcal{L}(a, \lambda)}{\partial a_i} = 0 \ \ \forall i = 1, \ldots, n$$

$$\text{KKT}_2 : \lambda_1 \geq 0, \ \ \sum_{i=1}^{n} a_i - K \leq 0, \ \ \lambda_1 \left(\sum_{i=1}^{n} a_i - K \right) = 0$$

$$\text{KKT}_3 : \lambda_i \geq 0, \ \ a_i - m_i^* \leq 0, \ \ \lambda_i(a_i - m_i^*) = 0, \ \ \forall i = 1, \ldots, n$$

We check the existence of the optimal solutions in the following six cases.

Case 1: $\lambda_i = 0 \ \ \forall i = 1, \ldots, n+1$.

This case does not hold KKT_1, since each ω_i is not the same.

Case 2: $\lambda_1 = 0$, and $\lambda_i \neq 0 \ \ \forall i = 2, \ldots, n+1$.

This case does not hold, similarly to Case 2 in Theorem 3.2.1.

Case 3: $\lambda_1 = 0$ and there exists at least one but not for all $\tilde{i} \in \{2, \ldots, n+1\}$ such that $\lambda_{\tilde{i}} \neq 0$, otherwise $\lambda_i = 0$.

According to KKT_3, we have $a_i^* \leq m_i^*$ for any i. We also obtain $a_{\tilde{i}}^* = m_{\tilde{i}}^*$. By putting this result into KKT_1, we obtain $\sum_{i \in N} a_i^* \leq \frac{\alpha - \omega_{\tilde{i}}}{2\beta}$ where $\hat{i}$ is the largest $\tilde{i}$ such that $\sum_{j=1}^{\hat{i}} m_i^* \leq \frac{\alpha - \omega_{\tilde{i}}}{2\beta}$. Furthermore, according to KKT_1, we obtain that $a_{i+1}^* =$

$\max\left\{\frac{\alpha-\omega_{\tilde{i}}}{2\beta} - \sum_{j=1}^{\hat{i}} m_j^*, 0\right\}$. For the rest of $i \neq \tilde{i}$, $a_i^* = 0$. According to KKT$_2$, we have

$K \geq \sum_{i=1}^{\hat{i}} m_i^*$. Therefore, we have a solution a^* such that

$$a_i^* = \begin{cases} m_i^* & \text{if } i \leq \hat{i}, \\[2ex] \max\left\{\frac{\alpha-\omega_{\tilde{i}}}{2\beta} - \sum_{j=1}^{\hat{i}} m_j^*, 0\right\} & \text{if } i = \hat{i}+1 \\[2ex] 0 & \text{otherwise,} \end{cases}$$

where $\hat{i}$ is the largest $\tilde{i}$ which is chosen from the lowest cost retailers if $K \geq \sum_{i=1}^{\hat{i}} m_i^*$.

Case 4: $\lambda_i \neq 0 \quad \forall i = 1, \ldots, n+1$.

This case does not hold, similarly to Case 4 in Theorem 3.2.1.

Case 5: $\lambda_1 \neq 0$ and $\lambda_i = 0 \quad \forall i = 2, \ldots, n+1$.

According to KKT$_2$, we have $\sum_{i \in N} a_i^* = K$. This result contradicts KKT$_1$, since we have different ω_i.

Case 6: $\lambda_1 \neq 0$, there exist at least one but not for all $\tilde{i} \in \{2, \ldots, n+1\}$, such that $\lambda_{\tilde{i}} \neq 0$ otherwise $\lambda_i = 0$.

According to KKT$_3$, we have $a_i^* \leq m_i^*$ for any i. We also obtain $a_{\tilde{i}}^* = m_{\tilde{i}}^*$. According to KKT$_1$, we obtain $\alpha - 2\beta \sum_{i \in N} a_i^* - \omega_i - \lambda_1 \leq 0$ which induces $\tilde{i}$ must be allocated from the lowest cost retailers, since $\alpha - 2\beta \sum_{i \in N} a_i^* - \omega_i - \lambda_1 \leq 0$ for all $i \neq \tilde{i}$. Let $\hat{i}$ be the largest $\tilde{i}$ such that $\sum_{j=1}^{\hat{i}} m_i^* \leq \frac{\alpha-\omega_{\tilde{i}}}{2\beta}$. Furthermore, according to KKT$_1$, we obtain that $a_{\hat{i}+1}^* = \max\left\{\frac{\alpha-\omega_{\tilde{i}}}{2\beta} - \sum_{j=1}^{\hat{i}} m_j^*, 0\right\}$. For the rest of $i \neq \tilde{i}$, $a_i^* = 0$. According to

KKT$_2$, we obtain $K \leq \frac{\alpha - \omega_i}{2\beta}$. Therefore, we have a solution a^* such that

$$
a_i^* = \begin{cases} m_i^* & \text{if } i \leq \hat{i}, \\[2ex] \max\left\{ \frac{\alpha - \omega_i}{2\beta} - \sum_{j=1}^{\hat{i}} m_j^*, 0 \right\} & \text{if } i = \hat{i} + 1 \\[2ex] 0 & \text{otherwise,} \end{cases}
$$

where $\hat{i}$ is the largest $\tilde{i}$ such that $\sum_{j=1}^{\hat{i}} m_j^* \leq \frac{\alpha - \omega_i}{2\beta}$ and $\tilde{i}$ is chosen from the lowest cost retailers if $K \leq \frac{\alpha - \omega_i}{2\beta}$.

According to the above six cases, we obtain the conditions of allocations to maximise the total retailers profit if cost is strongly asymmetric. $\qquad\square$

This theorem presents the conditions to maximise the total profit of the retailers under cost asymmetric case. Allocations must fully fulfil the orders from the low cost retailers. Furthermore, the sum of allocations should not exceed either the capacity size of the supplier or the joint monopoly quantity at the $\hat{i}$-th lowest cost retailer such that $\sum_{i=1}^{\hat{i}} \leq \frac{\alpha - \omega_i}{2\beta}$. That is the supplier does not allocate capacities even when they have enough capacities, similarly to the case of symmetric costs. The main difference is that allocation is prioritised by the cost of the retailers. In order to maximise the total retailer profit, the allocation should be prioritised by the efficient retailers for the distributions.

Now, we investigate specific Pareto optimal allocations in the cost asymmetric case. First, we check Pareto optimality of proportional allocation which also partially holds in the symmetric cost case as shown in Theorem 3.3.2. The following theorem

shows that Pareto optimality does not hold in the asymmetric case.

Theorem 3.5.2. *Assume that the retailers submit the truthful orders* $m^* = q^{c\omega}$. *Proportional allocation is not Pareto optimal in the cost asymmetric case.*

Proof. According to the definition of Proportional allocation in Equation (2.3.1) in Section 2.3, if the retailers submit truthful orders m^*, allocation for retailer i under proportional allocation g is

$$g_i(m^*) = \begin{cases} \frac{m_i^*}{\sum\limits_{i \in N} m_i^*} & \text{if } K \leq \sum\limits_{i \in N} m_i^*, \\ m_i^* & \text{otherwise} \end{cases}$$

which does not satisfy the conditions to maximise the total profits of the retailers presented in Theorem 3.5.1. Therefore, proportional allocation is not Pareto optimal in the case of symmetric costs. $\qquad \square$

In contrast to the case of the symmetric costs or the monopolistic downstream markets like Cachon and Lariviere in [11], Proportional allocation is no more Pareto optimal in the case of asymmetric costs. This result is due to the fact that the lowest cost retailers are not fully prioritised under proportional allocation. In the asymmetric cost case, the lowest cost retailer is the best performer. Therefore, the way to maximise the total retailer profits is to give priorities to such retailers.

According to Theorem 3.5.1, giving priorities to the low cost retailers has a positive effect on the total retailer profits. In the following theorem, we prove the conditions that max-max allocation is Pareto optimal,

Theorem 3.5.3. *Assume that the retailers submit the truthful orders $m^* = q^{c\omega}$. Let $\hat{i}$ be the largest retailer that satisfies $\sum_{j=1}^{\hat{i}} m_j^* \leq \frac{\alpha - \omega_{\hat{i}}}{2\beta}$. Max-max allocation is Pareto optimal allocation mechanism in the cost asymmetric case if $K \leq \frac{\alpha - \omega_{\hat{i}}}{2\beta}$.*

Proof. According to the definition of max-max allocation in page 30, max-max allocation is exactly same as the optimality conditions in Theorem 3.5.1 if $K \leq \frac{\alpha - \omega_{\hat{i}}}{2\beta}$. Meanwhile, max-max allocation is an efficient allocation if $K \leq \sum_{i=1}^{n} m_i^*$. Therefore, max-max allocation is Pareto optimal if $K \leq \frac{\alpha - \omega_{\hat{i}}}{2\beta}$. □

According to Theorem 3.5.3, max-max allocation is Pareto optimal if the capacity is strictly limited. In order to enforce max-max allocation mechanism to be Pareto optimal, it is necessary that the retailers submit truthful order quantities. However, the problem is individually responsive mechanisms are not incentive compatible allocation in this model. As a result, the retailers inflate orders from the truthful orders and the total retailer profits are not maximised.

The overall retailers' profits are not significant issue for the allocation in the symmetric cost case, since the marginal revenues and the marginal costs are symmetric. However, according to Theorem 3.3.2 and 3.5.2, cost asymmetry has a significant impact on the total retailer profits under proportional allocation. The reason for the difference is that retailers with the lowest cost sell items in a more efficient way. Therefore, it is necessary to prioritise the lowest costs retailers for the efficiency of the total retailer profits.

3.5.2 Truth-Inducing Mechanisms With the Cost Asymmetric Retailers

Now let us investigate truth-inducing mechanism in the case of cost asymmetric retailers. In this section, we evaluate the efficiency of the truth-inducing mechanisms by the total retailer profits. First, we check the truth-inducing properties of uniform allocation and capped allocation presented in Section 3.4 that are the truth-inducing mechanisms for the cost symmetric retailers. Then, we investigate the efficiency of the total retailer profits under those allocation mechanisms.

First, we investigate if uniform allocation mechanism is truth-inducing.

Theorem 3.5.4. *Uniform allocation is truth-inducing in the case of cost asymmetric retailers.*

Proof. The truth-inducing property is obtained similarly to Theorem 3.4.2. □

According to Theorem 3.5.4, uniform allocation is truth-inducing mechanism in a cost asymmetric case. Since the allocation gives priority to the smallest orders, inflating the orders is not beneficial for the retailers. As mentioned in Theorem 3.5.3, this is an opposite characteristic from the max-max allocation which is a partially Pareto optimal mechanism. Therefore, even though uniform allocation is truth-inducing mechanism, the total retailer profits is not efficient under uniform allocation.

Similarly, we prove that capped allocation is truth-inducing in the following theorem.

Theorem 3.5.5. *Capped allocation is truth-inducing in the case of cost asymmetric*

retailers if $\gamma_i \leq m_i^*$ *for all* i.

Proof. Either increasing or decreasing orders from the truthful order quantities do not increase the allocations. Therefore, the proof is obtained similarly to Theorem 3.4.2.

$\square$

In this section, we have seen the properties of allocations in the context of quantity competition. We have seen that cost structures of retailers have a great impact of retailers profit and thus on the efficiency of mechanisms. We have presented an allocation mechanism that achieves stability and efficiency at the same time. However, this mechanism requires to set proper upper limits reflecting the cost information of the retailers.

3.6   Discussion and Related Work

This chapter has dealt with capacity allocation problems in a supply chain where the retailers compete in quantity competition. We have extended the analysis of the properties of allocation mechanisms from supply chains with monopolistic retailers [11, 12] or a single market [114] to supply chains with competitive retailers. Due to strategic interactions of the retailers in our model, the retailers' behaviour is more complex and significantly different from the ones in the existing models. As a result, we observe differences of properties of allocation mechanisms. We analyse specific allocation mechanisms according to evaluation criteria of mechanisms that covers efficiency aspect and stability aspect.

In contrast to the case of the monopolistic downstream markets, the proportional allocation mechanism is no longer necessarily Pareto optimal under quantity competition. We have given a necessary and sufficient condition for the proportional allocation to be Pareto optimal for the symmetric cost retailers. Furthermore, we prove that proportional allocation is not Pareto optimal for the asymmetric cost retailers, which is a significant difference from the Cachon and Lariviere's model. A main reason is an extension of the strategic interaction in the downstream market.

We have introduced max-max allocation, allocation from the largest orders, which satisfies Pareto optimality condition in the cost asymmetric retailers if all retailers submit truthful orders. A characteristic of max-max allocation is a strong prioritisation of the most efficient retailers, i.e. the lowest cost retailer. If we relax this assumption and allow the retailers to submit any orders, the retailers inflate the orders, which is a typical behaviour under individually responsive allocations. To maximise the total retailer's profit, the exclusive distribution to the lowest cost retailer is a solution. We further consider the joint profit between the supplier and the retailer in Chapter 5.

In order to improve the stability of supply chains, we have proposed a truth-inducing allocation mechanism, capped allocation mechanism. Capped allocation is more intuitive than the other truth-inducing mechanisms such as uniform allocation [114] and lexicographic allocation [11]. Under capped allocation mechanism, the supplier allocates capacities up to predetermined upper limit of supplies. In the asymmetric cost case, we reveal a problem of the efficiency with respect to the total retailer profits. We present a condition of to achieve truth-inducing and efficiency for

the total retailer profits.

Even though proportional allocation does not have desirable properties compared to other mechanisms, proportional allocation prevails in industry. We address the following three reasons. First, it is intuitive. Second, it is an equitable distribution and any retailers will not be allocated zero in any time. Third, proportional allocation induces a greater profit for the supplier than the other allocations.

We have assumed that the market price is determined by the total supply quantity in the market. This market mechanism is getting more remarkable in the electronic marketplaces, since the total supply quantity is easily obtained in the electronic market places. Another typical competition rule is price competition where the sellers determine their selling prices. In the following chapter, we compare how two types of market competition rules influence properties of allocation mechanisms.

Chapter 4

Allocation Mechanisms for an Oligopoly under Price Competition

In the previous chapter, we have introduced quantity competition in capacity allocation problems in supply chains. An interesting observation is that proportional allocation, the most popular allocation mechanism, is no longer Pareto optimal for the competitive downstream market due to the strategic interactions of the competing retailers, in contrast to the monopolistic downstream market model [11]. This chapter extends the capacity allocation game to price competition in the downstream market. It is known that price competition induces the agents to make more competitive strategic decisions compared to quantity competition (Chapter 12 in [76] or [26, 27, 3]). Our interest is how market rules, price competition and quantity competition, in the downstream market influence the properties of allocation mechanisms.

As we have presented in Section 2.2, we consider the model of the capacity allocation game: a single supplier allocates its capacity to orders from retailers that

compete in the same market based on price competition. We assume that the upstream market applies a certain allocation mechanism and the downstream market is based on price competition. In such settings, it is interesting how the allocation mechanism applied in the upstream market influences the competition among the retailers in the downstream market and how a variation of prices in the downstream market affects the properties of allocation mechanisms in the upstream market. The order quantity plays an important role for the capacity allocation problem. However, there is another key strategic interaction which is the determination of selling price. This additional interaction makes analysis of properties of allocation mechanisms more complicated compared to the case under quantity competition.

The rest of this chapter is organised as follows. We first consider the case where there is no restrictions on allocations in Section 4.1. In such a case, we obtain desired allocation and pricing strategy in the downstream market. Then we relax this assumption in Section 4.2. We investigate how the retailers behave under price competition with different types of allocations. In other words, we construct a link between the capacity size of the supplier and the pricing strategy of the retailers under efficient allocations. In Section 4.3, we investigate properties of allocation mechanisms with respect to evaluation criteria proposed in Chapter 2. We focus on two representative allocation mechanisms, namely proportional allocation and uniform allocation, and we check their properties. In Section 4.4, we examine how heterogeneous allocations have an impact on the selling price in the downstream market. In Section 4.5, we describe the relations between our work and other work.

4.1 Desirable Market Behaviour under Price Competition

In Section 2.2, we have presented our capacity allocation model where the retailers determine their order quantities and their selling prices corresponding to the allocations executed by the supplier. Similarly to the case of quantity competition in the previous chapter, a key issue is a strategic decision with respect to an order quantity. This is because an allocation is based on the orders from the retailers and the retailers determine their selling prices corresponding to allocations. In other words, a determination of an order is a triggering action of the capacity allocation game. We are interested in the optimal order. However it is not unique, since it depends on the allocation mechanism. In this section, we focus on investigating a desired allocation for each retailer without considering a capacity constraint of the supplier. That is a special case where the supplier is always able to fulfil any orders from the retailers. This problem is closely related to the work given by Kreps and Scheinkamn [64], Boccard and Wauthy [6, 7], and Francesco [40].

Kreps and Scheinkman consider a two-stage duopoly game where two sellers determine their supply limits and thereafter determine their selling prices. Boccard and Wauthy, and also Francesco extend from the Kreps and Scheinkman's model to the oligopoly. A main difference from our model is that the supply limit is determined by the allocation rather than the agents' decisions. Therefore, we can see our model is an extension of their models where a retailer is able to determine its desired allocation.

If the retailer is able to determine its allocation corresponding to the allocation

of its competitors, we can represent this problem with a Cournot model as presented in the previous chapter. The following lemma shows a key characteristic of Cournot best response function represented by Equation (3.2.1):

Lemma 4.1.1. *[6, 7] Suppose $a_i > a_j$. If $a_i \leq r(a_{-i})$ then $a_j < r(a_{-j})$.*

This lemma hinges on the assumption that $a_i P(a_i + a_{-i})$ is concave in a_i for all a_{-i}. Therefore, the profit maximising quantity $r(a_{-i})$ is unique and decreasing in a_{-i}. The above lemma presents the relationship between the allocation and its best response. If we know that an allocation is less than or equal to the best response quantity, the smaller allocations are also less than or equal to the associated best response quantity. Therefore, it implies, if the largest allocation a_l satisfies $a_l \leq r(a_{-l})$, then all other allocations and their corresponding best response have the same relationships.

As we have mentioned above, Francesco investigates the existence of Nash equilibrium in the model of Kreps and Scheinkman in an oligopoly case based on a mixed-strategy analysis where agents make strategic decisions randomly. Based on his work, we describe how a retailer determines its selling price in the downstream market. Let us denote, $\bar{p}$ and $\underline{p}$ as an upper and a lower bound of the price equilibrium in the mixed strategy. The following lemma characterizes the price equilibrium.

Lemma 4.1.2. *[40] Let l be the largest allocated retailer, i.e. $l = \arg\max_{i \in N} a_i$. Given a, there are price equilibria as follows:*

1. *if for all i, $a_i \leq r(a_{-i})$, $P(a_i + a_{-i})$ is a unique equilibrium.*

2. *if $a_l > r(a_{-l})$ and $D(0) > a_{-l}$, then*

$$\bar{p}_l = P(r(a_{-l}) + a_{-l}) \ , \ and$$

$$\underline{p}_i = \frac{P(r(a_{-l}) + a_{-l})r(a_{-l})}{a_l} \ for \ all \ i,$$

3. *if $D(0) \leq a_{-l}$, $p^* = 0$ is the unique price equilibrium.*

According to Lemma 4.1.2, there are three types of price equilibria according to the patterns of allocations. First, if the allocation for all retailer is less than or equal to its best response quantity, the retailer sets the price at $P(a_i + a_{-i})$. Second, if the largest capacity size is greater than its best response quantity and the sum of the allocation for the rest of the retailers does not exceed the maximum demand, the retailers set the price between $\underline{p}$ and $\bar{p}$ in mixed strategy. At any price, the revenue of the largest allocated retailer is $r(a_{-l})P(r(a_{-l}) + a_{-l})$. Third, if the sum of the allocations for all retailers except for the largest one is greater or equal to the maximum demand, the all these retailers set their prices at zero.

Based on the pricing strategy presented in Lemma 4.1.2, Kreps and Scheinkman shows how agents determine their supply limits in a duopoly; Francesco extends it to an oligopoly. We use their results to describe the desired allocations. In the following lemma, we show that there is a unique equilibrium in desired allocations.

Lemma 4.1.3. *[40] Given the pricing rule in Lemma 4.1.2, there is a unique equilibrium in desired allocations that induces the Cournot outcome.*

According to Lemma 4.1.3, the desired allocation is the Cournot quantity, which is an optimal allocation without considering the capacity constraint of the supplier. In other words, we obtain the truthful order quantity of the retailers. An interesting characteristic is that if all the truthful orders are fully allocated, each retailer sets its

selling price equal to the price in the quantity competition. In the following sections, we deal with the capacity constraint of the supplier to analyse market behaviours.

4.2 Effects of Capacity Allocation to the Downstream Market

Our interest is how capacity allocation in the upstream market influences market behaviour in the downstream market and how market rules in the downstream market influence properties of allocation mechanisms in the upstream market. In this section, we investigate price equilibria according to the capacity size. That is, we show how a retailer determines its price with respect to different capacity settings. For this, we assume that the order quantity is given and the total order quantity exceeds the capacity, i.e. g is efficient. Since the order quantity is given, we treat the purchase cost as sunk cost.

In the followings, we use some typical economical indicators that are shown in Chapter 2. Let Q^M be the monopoly quantity of the downstream market such that

$$Q^M = \arg\max_Q QP(Q). \tag{4.2.1}$$

Let Q^c be the total Cournot quantity in the oligopoly market, let $\hat{Q}$ be the maximum demand of the downstream market, and let $P(K)$ be the market price corresponding to the quantity fully utilising capacity K.

We consider four capacity ranges for the analysis.

- Strictly scarce capacity ($K \leq Q^M$): the supplier's capacity K is less than the

monopoly quantity Q^M of the downstream market.

- Relatively scarce capacity ($Q^M < K \leq Q^c$): the capacity is greater than the monopoly quantity and less than or equal to the total Cournot quantity Q^c.

- Enough capacity ($Q^c < K < \hat{Q}$): the capacity is greater than the total Cournot quantity and less than the maximum demand of the downstream market.

- Excessive capacity ($\hat{Q} \leq K$): the capacity is greater than the maximum demand.

Recall that the supplier constructs its capacity based on its own forecast. It means that all these four cases may happen. If the capacity is less than the total Cournot quantity, i.e., the first two cases above, it is significant for the supply chain members to evaluate how the limited capacity is allocated.

The following theorem deals with the first situation where the capacity of the supplier is strictly limited (i.e. its capacity size is less than the monopoly quantity).

Theorem 4.2.1. *Given an allocation mechanism g. Suppose that $K \leq Q^M$ and g is efficient. If g is feasible, then $P(K)$ is an equilibrium, i.e., $p^* = P(K)$*

Proof. According to Lemma 4.1.2, it is sufficient to show that for any i, $g_i(m) \leq r(g_{-i}(m))$. Since g is efficient and $K \leq Q^M$, we have $g_i(m) + g_{-i}(m) = K \leq Q^M = \arg\max_q qP(q)$, with respect to Equation (4.2.1).

Case 1: If $r(g_{-i}(m)) + g_{-i}(m) \geq Q^M$, we have

$$g_i(m) + g_{-i}(m) \leq r(g_{-i}(m)) + g_{-i}(m).$$

It follows that $g_i(m) \leq r(g_{-i}(m))$ as desired.

Case 2: Assume that $r(g_{-i}(m)) + g_{-i}(m) < Q^M$. According to the definition of Cournot best response function, we have $r(g_{-i}(m)) = \arg\max_{q_i} q_i P(q_i + g_{-i}(m)) = \arg\max_{q_i}((q_i + g_{-i}(m))P(q_i + g_{-i}(m)) - g_{-i}(m)P(q_i + g_{-i}(m)))$. Let $y = q_i + g_{-i}(m)$ and $y^* = \arg\max_y(yP(y) - g_{-i}(m)P(y))$; we have:

$$r(g_{-i}(m)) = \arg\max_y(yP(y) - g_{-i}(m)P(y)) - g_{-i}(m)$$

$$= y^* - g_{-i}(m) \tag{4.2.2}$$

According to the assumption of Case 2, Equation (4.2.2) implies $y^* < Q^M$. It follows that $P(y^*) > P(q^M)$ because P is strictly decreasing in quantity. It turns out that

$$-g_{-i}(m)P(y^*) < -g_{-i}(m)P(q^M)$$

Notice that the case $g_{-i}(m) \leq 0$ is ruled out since $y^* < q^M$. On the other hand, $y^*P(y^*) \leq q^M P(q^M)$ because $Q^M = \arg\max_q qP(q)$. Therefore, we have,

$$y^*P(y^*) - g_{-i}(m)P(y^*) < Q^M P(Q^M) - g_{-i}(m)P(Q^M)$$

This contradicts the definition of y^*. That is $r(g_{-i}(m)) + g_{-i}(m) \geq Q^M$ and we have $g_i(m) \leq r(g_{-i}(m))$. $\qquad\square$

This theorem shows that the equilibrium price is $P(K)$, when the capacity size of the supplier is less than the market monopoly quantity Q^M and allocation mechanism is efficient. In this case, no retailers can make greater profit by charging higher price than $P(K)$, which is the monopoly price when $K \leq Q^M$. Therefore, the allocation mechanism does not affect the market price within this capacity range. In other

words, in this context, the supplier has no interests to choose a specific allocation mechanism as long as the mechanism is efficient.

Next, we consider the case where the capacity is relatively scarce that is $Q^M < K \leq Q^c$. We have the following result:

Theorem 4.2.2. *Suppose $Q^M < K \leq Q^c$. Under any efficient allocation mechanism g, price equilibria are,*

1. *$p^* = P(K)$, if for all i $g_i(m) \leq r(g_{-i}(m))$;*

2. *$p^* > P(K)$, if there exists i such that $g_i(m) > r(g_{-i}(m))$.*

Proof. Let l be the most allocated retailer. Since g is efficient, we have $\frac{K}{n} \leq g_l(m) \leq K$. It implies $\frac{n-1}{n}K \geq g_{-l}(m) \geq 0$. Since $K < Q^c$, we have $\frac{n-1}{n}K \leq q^c_{-i}$. According to the assumption that $r(a_{-i})$ is decreasing in a_{-i}, we obtain $r(q^c_{-i}) \leq r(\frac{n-1}{n}K) \leq r(g_{-l}(m)) \leq r(0) = Q^M$. Therefore, we have either $g_l(m) \leq r(g_{-l}(m))$ or $g_l(m) > r(g_{-l}(m))$. In the former case, according to Lemma 4.1.1 and Lemma 4.1.2 case 1, we have $p^* = P(K)$. In the later case, according to Lemma 4.1.2 case 2, we have $p^* > P(K)$. $\qquad\square$

According to Theorem 4.2.2, there are two types of pricing strategies for the retailers, when the capacity is relatively scarce. If allocation quantities for each retailer do not exceed the best response quantities, the market price is stable and the equilibrium price reaches the price at the capacity size, which is the statement 1. Otherwise, the market price becomes unstable and higher than $P(K)$, which is the statement 2.

This is a very interesting phenomenon. If there exists a retailer enjoying privilege due to unbalanced allocations, the equilibrium price in the downstream market is higher than the balanced case. The following two extreme cases help to understand the difference. If the supplier allocates the capacity exclusively to a retailer which is one case of the statement 2, the retailer should set the resale price as $P(Q^M) > P(K)$ to maximize its profit even if the retailer is not able to sell all the products. On the other hand, if the supplier allocates the capacity to all retailers equally, which is the case of the statement 1, the mechanism of the price competition works properly and the retailers are not able to increase their profits by charging higher prices than $P(K)$.

Let us now investigate the case where $Q^c < K < \hat{Q}$:

Theorem 4.2.3. *Suppose $Q^c < K < \hat{Q}$. For any efficient allocation mechanism g,* $p^* > P(K)$.

Proof. Let l be the largest allocated retailer. Since $K < \hat{Q}$, for any g, we have $g_l(m) \leq K < \hat{Q}$. Suppose, for all i, we have $g_i(m) \leq r(g_{-i}(m))$. According to the definition of Cournot best response function, we obtain $g_i(m) \leq q_i^c$. It implies $\sum_{i=1}^{n} g_i(m) \leq Q^c$ which is a contradiction of the hypothesis of an efficient allocation $\sum_{i=1}^{n} g_i(m) = K$. Hence, for retailer l, we have $g_l(m) > r(g_{-l}(m))$. According to Lemma 4.1.1 and Lemma 4.1.2 case 2, we have $p^* > P(K)$. $\square$

According to Theorem 4.2.3, the retailers set the resale price greater than $P(K)$ if the capacity is in $Q^c < K < \hat{Q}$. In this case, even if the allocation is the same for all retailers, the allocation quantity is greater than the best response quantity. Hence, the

retailer sets the price higher than $P(K)$ similarly to the Statement 2 in Theorem 4.2.2.

Finally, we consider the case $K \geq \hat{Q}$.

Theorem 4.2.4. *Suppose $K \geq \hat{Q}$. For any efficient allocation mechanism,*

1. $p^* = 0$, if $g_{-l}(m) \geq \hat{Q}$.

2. $p^* > 0$, otherwise.

Proof. Since $K \geq \hat{Q}$ and g is efficient, g satisfies either $g_{-l}(m) \geq \hat{Q}$ or $0 \leq g_{-l}(m) < \hat{Q}$. The first case is the condition of Lemma 4.1.2 case 3. Hence, we have $p^* = 0$. In the later case, since $\frac{K}{n} > q^c$, we obtain $p^* > P(K)$ as same as Theorem 4.2.3. □

According to Theorem 4.2.4, if $\hat{Q} \leq K$ and if the sum of all retailers allocations excepts the largest one exceeds the maximum demand, the equilibrium price is zero. This is similar to the result of Bertrand price competition. On the other hand, if the sum of the all retailers allocations except the largest one does not exceed the maximum demand, these retailers are able to earn some profit by setting a price higher than zero, but the profits are very limited. This is because the total supply exceeds the market size and the market price is closed to zero.

Now, let us summarize the results of this section. We have classified how capacity allocation in the upstream market affects the pricing strategy and the price equilibria in the downstream market as shown in Figure 4.1. The effective range of the capacity for the pricing strategy selection is $Q^M < K \leq Q^c$ and the special case: $\hat{Q} < K$ and
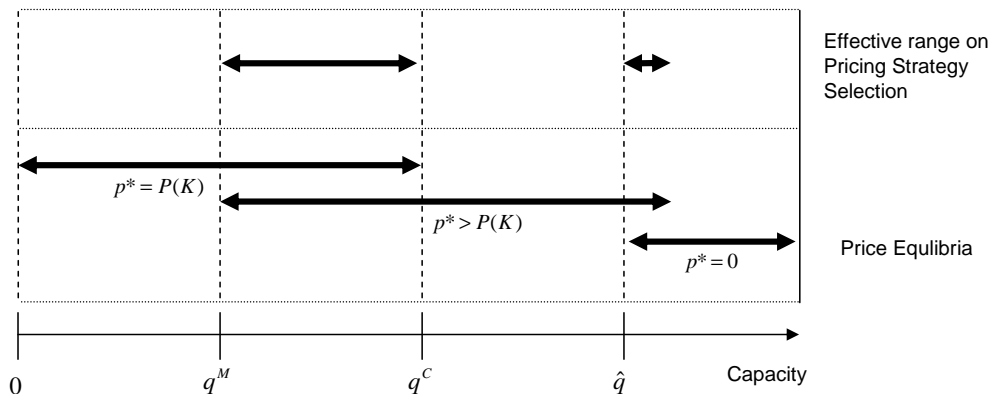
Figure 4.1: Capacity and Corresponding Pricing

$g_{-l}(m) < \hat{Q}$. A crucial range of capacity that affects the pricing strategy selection is $Q^M < K \leq Q^c$. Within this capacity range, the allocation mechanism selection is remarkably sensitive to the downstream market. If $K \leq Q^M$, the equilibrium price is the monopoly price. If $Q^M < K \leq Q^c$, allocations gives a significant impact on the price equilibria. If $Q^c < K < \hat{Q}$, the price equilibria are sensitive to allocations. In the next section, we relax the assumptions about order quantities and condition requiring that total order quantity should exceed the capacity.

4.3   Effects of Allocation Mechanism in Supply Chain

In this section, we consider how the retailers set order quantities based on allocation mechanisms and how the retailers determine selling prices according to allocated quantities in the supply chain. Particularly, we focus on two popular allocation mechanisms in industry, uniform allocation and proportional allocation. Remind that the truthful order quantity is q_i^{cw} as shown in Section 4.1.

One of the main goals of this model is to obtain a truth-inducing mechanism. A representative truth-inducing mechanism is uniform allocation proposed by Sprumont [114]. Cachon and Lariviere [12] extends the mechanism to the supply chain model with the monopolistic downstream markets and we extends to the supply chain model with the oligopolistic downstream market under quantity competition in the previous chapter. In the following theorem, we prove that uniform allocation is a truth-inducing mechanism in the supply chain model with the oligopolistic downstream market under price competition:

Theorem 4.3.1. *Under uniform allocation, there is a unique equilibrium order quantity $m_i^* = q^{cw}$ in the upstream market, which induces price equilibrium $p_i^* = \max\{P(K), P(Q^{cw})\}$ in the downstream market.*

Proof. If $m_i^* = q_i^{cw}$ for all i, according to the definition of uniform allocation, we have $g_i(m^*) = \min\{q_i^{cw}, K/n\} \le q^{cw}$. Since $g_i(m^*) \le r_w(g_{-i}(m^*)) < r(g_{-i}(m^*))$, the profit of retailer i is

$$\pi_i = P(g_i(m^*) + g_{-i}(m^*))g_i(m^*) - wg_i(m^*). \tag{4.3.1}$$

If $m_i' > m_i^*$ and $m_{j \ne i}^* = q^{cw}$, we have $g_i(m') = g_{j \ne i}(m') = K/n$, which is the same allocation quantity to the case at m^*. Hence, by increasing order m_i', retailer i cannot increase its profit. If $m_i' < m_i^*$, we have $g_i(m') \le \frac{K}{n} \le g_{j \ne i}(m') \le q^{cw}$. We check whether this case fits to the condition of case 2 of Lemma 4.1.2. According to the assumption that $r(a_{-i})$ is decreasing in a_{-i}, we have $r(g_{-i}(m')) \le r(g_{-j}(m'))$. Since $g_{j \ne i}(m') \le q_j^{cw} = r_w(q_{-j}^{cw}) < r(q_{-j}^{cw}) \le r(g_{-i}(m')) \le r(g_{-j}(m'))$, this case does not satisfies the condition of case 2 of Lemma 4.1.2. Hence, we only consider the case of the

pure strategy. The profit of retailer i is same as Equation (4.3.1). Recall that Equation (4.3.1) is concave in $g_i(m)$ and maximized at $r_w(g_{-i}(m))$. Since $g_i(m') \leq r_w(q_{-i}^{cw})$, by decreasing m_i', π_i is not increased. Therefore, we have an equilibrium order quantity $m_i^* = q_i^{cw}$, an allocation quantity at $g_i(m^*) = \min\{K/n, q_i^{cw}\}$, and an equilibrium price $p_i^* = \max\{P(K), P(Q^{cw})\}$. $\qquad\square$

According to Theorem 4.3.1, all retailers place truthful order quantities q^{cw} under uniform allocation, since no retailer is able to increase its profit by considering a quantity different with the truthful order quantity. An interesting property of uniform allocation is its robustness of allocation at the truthful order quantity. Even if competitors increase their order quantities, the allocation quantity for the retailer that submits the truthful order is not decreased. At the equilibrium order quantity m^*, we have allocation quantities $g_i(m^*) = \min\{K/n, q_i^{cw}\}$. Since the allocation quantity for each retailer does not exceed the best response quantities, we have the price equilibria $p_i^* = \max\{P(K), P(Q^{cw})\}$ under uniform allocation.

In industry, the most commonly used allocation is proportional allocation, which is a representative IR allocation mechanism. We have seen that proportional allocation is Pareto optimal in the monopolistic downstream market shown by Cachon and Lariviere [11]. We have also seen that it is no longer true in our supply chain model with quantity competition in the previous chapter. A key issue of proportional allocation is that it leads the retailers to inflate orders. In the following theorem, we show that proportional allocation leads the retailers to inflate orders and there is no

order equilibrium.

Theorem 4.3.2. *There does not exist an order equilibrium m^* under proportional allocation, if $K \leq Q^{cw}$.*

Proof. Suppose there exists symmetric m^*. Let us denote $\pi_i(g(m)) = \pi_i(g_i(m), g_{j \neq i}(m))$. Since $K \leq Q^{cw}$, we have $g_i(m^*) \leq q^{cw}$ and $g_{-i}(m^*) \leq q_{-i}^{cw}$. It turns out $r(g_{-i}(m^*)) \geq r(q_{-i}^{cw})$. Since $g_i(m^*) \leq q_i^{cw} = r_w(q_{-i}^{cw}) < r(q_{-i}^{cw}) \leq r(g_{-i}(m^*))$, the profit of retailer i is $\pi_i(g(m^*)) = P(g_i(m^*) + g_{-i}(m^*))g_i(m^*) - wg_i(m^*)$, according to Lemma 4.1.2 case 1. Let $m' = (m_i', m_{j \neq i}^*)$. Equation (2.3.1) implies $g_i(m') > g_i(m^*)$ where $m_i' > m_i^*$.

If $\sum\limits_{i \in N} m_i^* < K$, we have $g_l(m')$ such that $g_l(m') < q_l^{cw}$. Equation (2.3.1) implies $g_{-l}(m') \leq g_{-l}(m^*)$ and $r(g_{-l}(m')) \geq r(g_{-l}(m^*))$. It turns out that $g_l(m') < q_l^{cw} = r_w(q_{-l}^{cw}) < r(g_{-l}(m^*)) \leq r(g_{-l}(m'))$. According to Lemma 4.1.2 case 1, we have $\pi_l(g(m')) = P(g_l(m') + g_{-l}(m'))g_l(m') - wg_l(m')$. According to the concavity of the profit function and $g_i(m^*) < g_l(m') \leq r(g_{-l}(m'))$, we have $\pi_l(g(m')) > \pi_i(g(m^*))$. Hence, symmetric m^* does not exist in the case of $\sum\limits_{i \in N} m_i^* < K$.

If $\sum\limits_{i \in N} m_i^* \geq K$, Equation (2.3.1) implies $g_l(m') - g_l(m^*) = -(g_{-l}(m') - g_{-l}(m^*))$ where $m_l' > m_l^*$. It follows $g_l(m') < r(g_{-l}(m'))$ and $\pi_l(g(m')) = P(g_l(m') + g_{-l}(m'))g_l(m') - wg_l(m')$. The difference between $\pi_l(g(m'))$ and $\pi_i(g(m^*))$ is $(g_l(m') - g_i(m^*))(P(K) - w)$. According to the assumption of w, we have $\pi_l(g(m')) > \pi_i(g(m^*))$. Hence, symmetric m^* does not exist in the case of $\sum\limits_{i \in N} m_i^* \geq K$.

Suppose there exists asymmetric m^* such that $g_l(m^*) < r(g_{-l}(m^*))$. The supposition is contradicted similarly to the case of symmetric m^* and $\sum\limits_{i \in N} m_i^* < K$. Suppose

there exists asymmetric m^* such that $g_l(m^*) > r(g_{-l}(m^*))$. The profit of retailer l is

$$\pi_l(g(m^*)) = P(r(g_{-l}(m^*)) + g_{-l}(m^*))r(g_{-l}(m^*)) - wg_l(m^*),$$

according to Lemma 4.1.2 case 2. Similarly, the profit of retailer i where $m_i^* < m_l^*$ is

$$\pi_i(g(m^*)) = \frac{P(r(g_{-l}(m^*)) + g_{-l}(m^*))r(g_{-l}(m^*))}{g_l(m^*)}g_i(m^*)$$

$$- wg_i(m^*).$$

The difference of profits between retailer l and i is

$$\pi_l(g(m^*)) - \pi_i(g(m^*)) = \frac{g_l(m^*) - g_i(m^*)}{g_l(m^*)}$$

$$(P(r(g_{-l}(m^*)) + g_{-l}(m^*))r(g_{-l}(m^*)) - wg_l(m^*)).$$

At the equilibrium, $\pi_l(g(m^*))$ must be positive. Notice that $\pi_l(g(m^*)) = P(r(g_{-l}(m^*)) + g_{-l}(m^*))r(g_{-l}(m^*)) - wg_l(m^*)$. Hence, we have $\pi_l(g(m^*)) > \pi_i(g(m^*))$. It follows that asymmetric m^* such that $g_l(m^*) > r(g_{-l}(m^*))$ does not exist. $\qquad\square$

According to Theorem 4.3.2, there is no equilibrium since the retailers tends to increase their order quantities more than they need. Under proportional allocation, each retailer is able to decrease the competitors' allocations by increasing its order that makes greater profits for each retailer. Thus, proportional allocation is not robust and the supplier receives more order quantities than the truthful demand of the retailers.

One way to obtain an order equilibrium under proportional allocation is to assume a maximum order quantity denoted by $\tilde{m}$. In reality, it is common to consider that there exists a maximum order quantity, which is set by either the supplier side or the

retailer side. In that case, according to Theorem 4.3.2, we obtain that the equilibrium order quantity $m_i^* = \tilde{m}_i$. The interesting case of the capacity range is $K \leq \sum_{i=1}^{n} \tilde{m}_i$. If the maximum order quantity is symmetric, we have allocation $g_i(m^*) = \frac{K}{n}$ and the equilibrium price $p^* = P(K)$, which is the same price as the case of quantity competition as shown in the previous chapter. If it is asymmetric, we obtain allocation $g_i(m^*) = Km_i^* / \sum_{i=1}^{n} m_i^*$. The equilibrium price is either $p^* = P(K)$ or $p^* > P(K)$, which depends on a relationship between the largest allocation $g_l(m^*)$ and its best response $r(g_{-l}(m^*))$. If the maximum order quantities are heterogeneous and the allocation for the largest allocated retailer exceeds its best response quantity, the retailers set their resale price greater than $P(K)$ which is a unique phenomenon in our model compared to the quantity competition in [42].

4.4 Heterogeneous Allocations

In industry, the allocated quantities are not always equal. For example, under proportional allocation with maximum order quantity shown in the previous section, it may be observed that some prioritized customers are assigned the greater maximum order quantities than the others. As we have seen in Theorem 4.2.2, the difference of allocation quantities to the retailers affects the pricing strategies and the largest allocated retailer increases the selling price to enjoy the benefit of the privileged allocation. Let us further investigate how heterogeneous allocations affect the total retailer profits.

As shown in Section 4.2, we focus on the interesting capacity range $Q^M < K \leq Q^c$

where allocations influence the pricing strategies of retailers. In the above capacity range, we have shown in Theorem 4.2.2 that there are two types of price equilibria according to an allocation and its corresponding best response quantity. We classify allocation mechanisms into two types of mechanisms based on the condition of Theorem 4.2.2:

- strong heterogeneous allocation $g^{\mathcal{M}}$: if there exists i such that $g_i(m) > r(g_{-i}(m))$,

- non-strong heterogeneous allocation $g^{\mathcal{P}}$: otherwise.

Strong heterogeneous allocation reflects the case where at least one retailer has been privileged with respect to allocation. Notice that the symmetric allocation is included in $g^{\mathcal{P}}$, since $K \leq Q^c$. It is certain that the largest allocated retailer earns greater profit under strong heterogeneous allocation than the profit under non strong heterogeneous allocation. However, we have one question how heterogeneous allocations influence the total retailer profit. The following theorem corresponds to this question.

Theorem 4.4.1. *Suppose that $Q^M < K \leq Q^c$. For any efficient $g^{\mathcal{P}}(m)$ and $g^{\mathcal{M}}(m)$,*

$$\sum_{i \in N} \pi_i(g^{\mathcal{M}}(m)) > \sum_{i \in N} \pi_i(g^{\mathcal{P}}(m)).$$

Proof. Let $\pi_i(g(m)) = \pi_i(g_i(m), g_{j \neq i}(m))$. First we show the total retailer profits under $g^{\mathcal{P}}$. According to the case 1 of Lemma 4.1.2, the total retailers' profit is

$$\sum_{i \in N} \pi_i(g^{\mathcal{P}}(m)) = \sum_{i \in N} \left(P(K)g_i^{\mathcal{P}}(m) - wg_i^{\mathcal{P}}(m) \right)$$

$$= P(K)K - wK. \tag{4.4.1}$$

Now we show the case of $g^{\mathcal{M}}$. If $g_l^{\mathcal{M}}(m) \neq K$, according to the case 2 of Lemma 4.1.2, the total retailers profits are

$$\sum_{i \in n} \pi_i(g^{\mathcal{M}}(m)) = \sum_{i \in n} \left(\underline{\mathrm{p}}_i g_i^{\mathcal{M}}(m) - w g_i^{\mathcal{M}}(m) \right).$$

We have $p^* > P(K)$ under $g^{\mathcal{M}}$ according to Theorem 4.2.2. Hence, we have

$$\sum_{i \in n} \pi_i(g^{\mathcal{M}}(m)) = \underline{\mathrm{p}}K - wK > P(K)K - wK. \tag{4.4.2}$$

According to Equation (4.4.1) and (4.4.2), we obtain

$$\sum_{i \in n} \pi_i(g^{\mathcal{M}}(m)) > \sum_{i \in N} \pi_i(g^{\mathcal{P}}(m)).$$

If $g_l^{\mathcal{M}}(m) = K$, retailer l is a monopolist. Hence, we have $\pi_l > \sum_{i \in n} \pi_i(g^{\mathcal{P}}(m))$ according to Equation (4.4.1) and $K > Q^M$. $\qquad \square$

According to Theorem 4.4.1, under strong heterogeneous allocation $g^{\mathcal{M}}$, the total retailer profits are greater than the one under non-strong heterogeneous allocation $g^{\mathcal{P}}$, when the capacity of the supplier is relatively scarce. Since there is no difference with respect to the total cost of all retailers between $g^{\mathcal{M}}$ and $g^{\mathcal{P}}$, we have focused on the revenue. The total revenue is $P(K)K$ under $g^{\mathcal{P}}$. Meanwhile, the total revenue is $\underline{\mathrm{p}}K$ under $g^{\mathcal{M}}$. At first glance, it seems inconsistent, because the market demand cannot be K if the retail price is $\underline{\mathrm{p}} > P(K)$ under $g^{\mathcal{M}}$. However, $\underline{\mathrm{p}}K$ consists of the revenues of all retailers and they do not set price $\underline{\mathrm{p}}$ at once. In fact, the total selling quantity under $g^{\mathcal{M}}$ is less than K, since the selling quantity of retailer i at price $\bar{p}$ is less than $g_i^{\mathcal{M}}$. Notice that the profit of the mixed strategy is equal at any price between $\underline{\mathrm{p}}$ and $\bar{p}$. Therefore, it implies that even if the less prioritized retailers under

$g^{\mathcal{M}}$ decrease their profits compared to the ones under $g^{\mathcal{P}}$, the increasing amount of the profits of the prioritized retailers exceeds the decreasing amount of the less prioritized retailers.

The analysis in this section reveal that strong heterogeneous allocations may affects the total retailer profits even in a case of cost symmetric case. The similar phenomenon is only observed in a case of asymmetric cost in the quantity competition model in the previous chapter.

4.5 Discussion and Related Work

This chapter has dealt with capacity allocation problems in a supply chain where a supplier allocates capacity to a set of retailers and the retailers compete in price. We have shown how the capacity allocation in the upstream market affects the pricing strategy and the price equilibrium in the downstream market. According to the classification of the capacity size, we are able to characterise the capacity ranges that influence the pricing strategies.

We have investigated the effects of allocation mechanisms in the supply chain, especially for two popular allocation mechanisms: uniform allocation and proportional allocation. We have found that the equilibrium order quantity is not always the Cournot quantity in our model. This is a significant difference compared to the results in Kreps and Scheinkman's model and their variants. This occurs if allocation mechanisms allow retailers that may decrease competitors' allocations according to increasing their orders, such as proportional allocation.At the opposite, we have shown

that uniform allocation induces truthful order quantity from the retailers; hence we have the same equilibrium price as the model presented in Chapter 3.

We observe a unique phenomenon in our model, which is heterogeneousness of allocations leads to higher market price if capacity is relatively scarce. Furthermore, the total retailer profit is increased, even though some retailers decrease their profits. In quantity competition, the total market supply determines the market price, thus the price is not affected by heterogeneousness of allocations. On the other hand, each retailer determines the selling price in price competition with allocation constraints. Therefore, the privileged retailers are able to take an opportunity to set higher price for the residual demand, because less prioritized retailers are not able to fulfil all the demand. In our model, the total retailer profits are maximized, where the supplier exclusively allocates to a single retailer. It means that one important criteria, Pareto optimality, in Cachon and Lariviere's model [11] is not a significant criteria in our model. Hence, truth-inducing property enhances the significance of mechanism design criteria in the competitive market.

In this model, choosing an allocation is a trade-off between the efficiency goal and the stability goal. IR allocation leads to order inflation which contributes for a higher utilization of the capacity and a greater profit for the supplier. However, the supplier lacks of the accurate demand information. This is a serious problem, since the accurate demand is a fundamental input for all business planning for the supplier. Furthermore, the retailers encounter uncertain allocations and unstable prices under IR allocations. In contrast, the truth-inducing allocations let the supplier obtain the truthful demand and the stable outcomes for the retailers. However it may not be

the profit-maximizing.

In our model, we assume that the end-users follow surplus maximising rule where the end-users choose from the lowest price retailers similarly to [69, 64, 6, 7, 40]. This market rule is significantly relevant to electronic marketplaces, since the end-users have an access to the price gathering site where selling prices are listed.

There are extensive researches regarding to pricing strategies with supply limits subsequent to Kreps and Scheinkman. A first stream of research extends the results of the Kreps and Scheinkman. Vives [121] shows price equilibria in a symmetric oligopoly case with common capacity constraints among sellers. Francesco [40] extends the Kreps and Scheinkman model from a duopoly to an oligopoly. Madden [71] shows the conditions to obtain the same results with the KS model with respect to an elastic demand. A second stream of research shows the limits of the Kreps and Scheinkman model by assuming asymmetric cases, including asymmetric cost in duopoly [34] and imperfect capacity pre-commitment [6, 7], uncertain demand [106], and dynamic capacity accumulation [5]. The difference of the two streams is caused by the symmetric behaviour in the models. Our model is relevant to both streams of these literatures, since feasible allocations cover both symmetric and asymmetric cases. However, all the above models do not consider allocation mechanisms in the linked market.

Overall, choosing an allocation mechanism is not just choosing one policy for allocation, but also it influences the market behaviours through a supply chain.

Chapter 5

Coordinating Contract in Online Markets

In the previous chapters, we have investigated whether allocation mechanisms entail that the supply chain members adopt a selfish behaviour or a behaviour taking into account of the supply chain profit. We have focused on the case where the supplier and the retailers agree on wholesale contract. One of the key conclusions is that the competition among the retailers makes the supply chain members do not behave for the total supply chain profit. In this chapter, we relax the restriction of wholesale contract and consider joint work in online markets under numbers of different contracts. We are particularly interested in how contracts make the agents to behave for the global profit.

5.1   Introduction

Undertaking joint work frequently fails to achieve the overall optimal performance. The main reason for poor performance is that members of the supply chain mainly concern by their own objectives. However, it is known that optimal performance can be achieved if the members coordinate by contracting on a set of transfer payments such that each member's goal becomes aligned with the global objective [9].

This chapter deals with the problems of the goal alignment of the members of online markets, i.e. the online market owner and the sellers. We point out four reasons in favor of coordination contracts. First, at the opposite of the traditional markets, the partnership between traders and the market owner in an online market can be loosely tied and dynamic in most situations. A trader can enter the market any time and could leave the market any time, even during a transaction. The market owner has poor information about the traders. Therefore, it is necessary to design contracts that attract the members to build solid relationships between the members.

Second, an online market can normally accommodate thousands of traders to trade in the market. Monitoring the behaviour of each trader is costly. Therefore the mechanism that coordinates the market owner and traders is critical for efficiency, effectiveness and stability of the market.

Third, unlike traditional supply chains, the partners involved in online trades do not always necessary to incur marginal costs, since the products do not actually go through online markets. As a result, the selling price does not need to take care of marginal cost of all the members. This issue is known as a double marginalisation

problem, and which is a typical reason for the failure of a coordination [82]. In that sense, a question is whether coordinating contracts in traditional supply chains hold the coordination property.

Fourth, one may enjoy the free rider effect in online market which prevents achieving coordination. One's sales effort, for instance an advertisement action perfromed by the online market owner or an investment on inventories performed by the sellers, may increase the partner's profit, which is an advantage of joint work. However, it may prevent putting the maximum effort for the overall profit. Hence, the contract that restricts free riding behaviours is critical for the coordination.

We consider the typical situation where a seller registers to an online market to sell a product. The seller and the owner of the market form an alliance of business to generate revenue through online sales. However, the efficiency and stability of the alliance relies on the contract (mechanism) that specifies the way to split the revenue and costs over the members of alliance. Thus we are interested in contracts that give each party positive expected profit such that there is no other contract which gives a better profit to one party without sacrificing the other party's profit. In other words, there is no better contract such that both parties of the alliance are happy to move to. Therefore the concept of coordination specifies the efficiency and stability of an alliance.

In this chapter, we consider some typical contracts and examine whether they lead coordinating behaviours in the online market. One of the simplest and widely used contract in e-business is fixed-fee charging contract. Under the contract, the market owner always gets a fixed amount from the revenue regardless the amount of overall

revenue and costs of the alliance. We prove that fixed-fee contracts coordinates the online market. However, this result is limited to a case where the sales effort of the market owner is ignorable. A typical example of the sales effort is an advertisement. A characteristic of this model is that the online market owners pay the cost of advertisement. Therefore, we consider strategic decisions of the seller and the online market owner.

Next, we consider a commonly used contract – revenue sharing, under which each party receives a certain percentage of revenue from the overall revenue. Unfortunately, the contract is unable to coordinate an alliance even if a variable cost of the market owner are ignorable.

Finally we represent market behaviour under *profit sharing contract* where the revenue and the costs are shared between the online market owner and the seller with a predetermined portion. Each member pays their own cost at its portion and their partner's cost at the inverse portion. We show that this contract coordinates an online market owner and sellers.

As far as we know, few works have tackled this coordination problem in online market. In [88], Netessine et al. consider the problem of coordination for online retailers that behave similar to the traditional supply chains [9, 13]. That is, online retailers buy products and sell them online. In our model, we do not assume this behaviour (the online market owner do not buy these products). As a result, our proposal differs from coordination in the traditional supply chains.

In Section 5.2.1, we present the concepts of contract and alliance coordination based on a generic market model. In Section 5.2.3, we detail the characteristics

of online market. In the next two sections, we formally characterize the properties
of fixed-fee contract (Section 5.3) and revenue-sharing contract (Section 5.4) in the
context of online market. In Section 5.5, we propose a new contract that achieves
alliance coordination. Finally, in Section 5.6, we discuss some related works and
conclude the paper.

5.2 Online Market Coordination

5.2.1 Alliance Coordinating Contracts

Performing actions involve incurring costs. If some revenue is generated by the result
of a joint-action of multiple parties, the way to split the revenue is a fundamental issue
for the concerned parties. Typically, this way is specified in contracts. To establish
an alliance consists of a contract. The aim of a contract is to reduce uncertainties for
the decision makings and aim at ensuring smooth confirmation of profits among the
members of the alliance. Knowing the contract, members of the alliance determine
their actions. In the following, we formally describe all these basic concepts.

5.2.2 Contract in Alliance

Consider an alliance $\mathcal{A}$ in which there are n agents. Each agent i ($1 \leq i \leq n$) has
a strategy space S_i. Let $\mathcal{S} = \prod_{i=1}^{n} S_i$. Each agent i chooses a strategy $\sigma_i \in S_i$. Let
$\sigma = (\sigma_1, \cdots, \sigma_n) \in \mathcal{S}$ be a strategy profile and σ_{-i} be the strategy profile for all
agents except i. We assume that alliance $\mathcal{A}$ earns an alliance revenue as the result

of the strategic choices of all agents in $\mathcal{A}$. Let $R : \mathcal{S} \to \Re$ be a function representing alliance revenue. Usually, the strategy is interpreted as investments, efforts, or actions incurring costs to generate revenue. Since the alliance revenue is generated as a result of joint-work among the members, the way to share the revenue is significant for the alliance members. In general, the way of sharing is specified in a contract defined as follows.

Definition 5.2.1. A contract of an alliance $\mathcal{A}$ is a function $\tau : \mathcal{S} \times \Re \to \Re^n$ which satisfies $\sum_{i \in \mathcal{A}} \tau_i(\sigma, r) = r$ for any $\sigma \in \mathcal{S}$ and $r = R(\sigma)$.

That is, w.r.t. an alliance revenue, a contract defines individual revenues for each agent based on its strategic choices. We assume the alliance revenue is non-negative. Individual revenue for each agent may include fees and incentives, hence it may be negative.

Example 5.2.1. *Consider a joint investment alliance on natural resource development consisting of three investors, $\mathcal{A} = \{a, b, c\}$. Let σ be a set of monetary investments of all alliance members. According to the investments, natural resource $r = R(\sigma)$ is mined. Suppose all members agree on the following contract before the investments: Each share of r is proportional to the individual amount of investment; that is $\tau_i(\sigma, r) = r \left(\sigma_i / \sum_{i \in \mathcal{A}} \sigma_i \right)$ for all i.*

The above example is an ideal case, since the profits are shared among the alliance members. However, the profit-sharing is not always effective or implementable in industry. For instance, an individual cost cannot always be associated to specific

alliance revenue or cost information is usually private information.

The definition of contract in Definition 5.2.1 shows that individual revenues are based on strategies and contracts. It means that agents may choose their strategies with respect to the contract. In order to choose the strategies agents need criteria to evaluate their profits in terms of strategies and contracts. Let $\pi_i^\tau(\sigma)$ be the profit of agent i at strategy profile σ and contract τ. Profit is based on revenue and cost. Let $c_i(\sigma_i)$ be the cost function of agent i. Then the profit function is $\pi_i^\tau(\sigma) = \tau_i(\sigma, r) - c_i(\sigma_i)$. We assume that agents choose their own strategies to maximize their own profits. Since the profit of agent i depends on the other agents' strategies, the evaluation of profits is explained by Nash equilibrium. Formally:

Definition 5.2.2. Given alliance $\mathcal{A}$, contract τ, the set of all strategy spaces $\mathcal{S}$, and profits $\{\pi_i^\tau(\sigma)\}_{i \in \mathcal{A}}$, the profile $\hat{\sigma} \equiv (\hat{\sigma}_1, \cdots, \hat{\sigma}_n)$ is called a Nash equilibrium if for all $i \in \mathcal{A}$, and for all $\sigma_i \in S_i$, $\pi_i^\tau(\hat{\sigma}) \geq \pi_i^\tau(\sigma_i, \hat{\sigma}_{-i})$.

It means that at Nash equilibria, no agent can increase its profit by changing strategies w.r.t. contract τ. However, even if we have a Nash equilibrium, the overall profit may not be maximized. Alliance optimal profit is gained by Pareto optimal contract as follows.

Definition 5.2.3. A contract τ is *Pareto optimal* if there is no other contract τ' such that for any strategy profile $\sigma \in \mathcal{S}$ such that $\pi_i^{\tau'}(\sigma) \geq \pi_i^\tau(\sigma)$ for all i with at least one strict inequality.

It means that if a contract is Pareto optimal, then there is no other contract where

all the agents' profits can increase. Notice that our definition of Pareto optimal contract avoids the case where increasing one agent's profit is possible without decreasing others agents' profits. To investigate Pareto optimal contracts, we have to compare profits for each strategy profile, each contract and each agent. In order to simplify this task, we introduce the notion of alliance optimal profit. Let the alliance profit be $\pi(\sigma) = R(\sigma) - C(\sigma)$, where $C(\sigma)$ is a linear combination of each cost $c_i(\sigma_i)$. Let the alliance optimal profile be $\sigma^* \equiv (\sigma_1^*, \cdots, \sigma_n^*)$ such that $\sigma^* = \arg \max_{\sigma^* \in \mathcal{S}} \pi(\sigma)$. The following proposition shows how to check Pareto optimality of contracts.

Proposition 5.2.2. *Let σ^* be an alliance optimal strategy profile and let $\hat{\sigma}^\tau$ be the unique Nash equilibrium under a contract τ. If $\hat{\sigma}^\tau = \sigma^*$, then contract τ is a Pareto optimal.*

Proof. The assumptions of the proposition entail that we have $\sum_{i \in \mathcal{A}} \pi_i^\tau(\hat{\sigma}^\tau) = R(\hat{\sigma}^\tau) - \sum_{i \in \mathcal{A}} c_i(\hat{\sigma}^\tau) = R(\sigma^*) - C(\sigma^*) = \pi(\sigma^*)$. Since a strategy profile σ^* maximizes the alliance profit, there is no other way to increase the alliance profit. Hence, contract τ is Pareto optimal. $\square$

Choosing contract and setting its parameters is a way to give incentive to agents for their participation in an alliance. For instance, contract may guarantee some revenue. This is especially significant if demand has to be considered as uncertain. Taking into account uncertainty of demand is mandatory, since agents have to decide their behavior before they realize the actual demand. Decision criteria of agents are dependent on risk tendencies. Since we assume all the agents are risk neutral, these decisions are only dependent on the expected profits. We denote E as expectation for

stochastic variables. Based on the above settings, we define an alliance coordinating contract as follows.

Definition 5.2.4. Given alliance $\mathcal{A}$, contract τ coordinates alliance $\mathcal{A}$, if it satisfies the following conditions,

1. contract τ is Pareto optimal,

2. there exists a strategy profile σ such that $E\left[\pi_i^\tau(\sigma)\right] > 0$ for all $i \in \mathcal{A}$.

The definition shows that relevant contracts must be acceptable by every agent. That is, in addition to the Pareto optimality, the definition of alliance coordination requires that all expected profits should be positive; this constraint is called participation constraint.

5.2.3 Online Market Model

In the previous section, we have shown the definition of contract and the alliance coordinating contract. Let us instantiate this framework in the context of online market where sellers and buyers trade items. In this online market, an alliance consists of an online market owner and a seller.

For strategic choices of the two alliance members, we suppose that the seller chooses the listing quantity of a single type of items on the online market and the online market owner chooses advertisement amount. As mentioned, we suppose that the sales depend of a given stochastic demand at given price. The revenue is shared between the two alliance members according to the contract. The seller directly ships the item to the buyers and thus buyers are not considered as members of the alliance.

Formally, let $\mathcal{A} = \{o, s\}$ be the alliance such that o is the online market owner and s is the seller. The strategy of o is to choose advertisement amount a and the strategy of s is to choose listing quantity q. For a given price p, we assume a random demand $X(a)$ which is affected by advertisement amount a. Let F be the cumulative distribution function of the demand and f be its probability distribution function. If the listing quantity is q and the advertisement amount is a, we obtain the expected revenue $R(q, a) = pQ(q, a)$ where $Q(q, a)$ is the expected sales quantity such that $Q(q, a) = E[\min\{X(a), q\}]$. It means that if listing quantity q is greater than demand, then sales quantity is a demand, otherwise the sale quantity is inventory quantity q. Stochastic demand entails that $E[\min\{X(a), q\}]$ and thus $Q(q, a)$ are equal to:

$$Q(q, a) = \int_0^q x f(x|a) dx + \int_q^\infty q f(x|a) dx. \tag{5.2.1}$$

By differentiating $Q(q, a)$ w.r.t. q, we get that the increase of inventory quantity for one unit results in the increase of the expected sales quantity less than one unit (since $\frac{\partial Q(q,a)}{\partial q} = 1 - F(q|a) < 1$). For the advertisement effect, we assume positive effect on the expected sales (that is $\frac{\partial Q(q,a)}{\partial a} \geq 0$). Furthermore, we assume that the expected sales is diminishing concave (i.e. $\frac{\partial^2 Q(q,a)}{\partial a^2} \leq 0$ and $\frac{\partial^3 Q(q,a)}{\partial a^3} \geq 0$).

For online trades, we consider the following costs for online market owner o and seller s. For agent o the cost is equal to the fixed cost $c_{\bar{o}}$ plus advertisement cost. Let $g(a)$ be a cost function for advertisement a where $g(0) = 0$, $\frac{dg(a)}{da} > 0$ and $\frac{d^2 g(a)}{da^2} \geq 0$. In other words, the cost of advertisement and the marginal advertisement cost both increase w.r.t. advertisement amount. For seller s, at the time of listing quantity

on the online market, we assume that items are already prepared as inventories with unit cost c_p. We also consider fixed cost for seller s as $c_{\bar{s}}$ and shipment cost c_s per unit.

At the opposite of listing quantity, advertisement can be null to generate revenue. In such a case, the alliance strategy (q, a) is equal to $(q, 0)$ and whenever it is clear q stands for the strategy. Let us detail the case where there is no advertisement. By choosing listing quantity q, the alliance expects to earn revenue $r = R(q) = pQ(q)$ at given price p. Suppose the alliance agrees on a contract τ, the expected profit of online market owner o is

$$E\left[\pi_o^\tau\right] = \tau_o(q, r) - c_{\bar{o}} \tag{5.2.2}$$

and the expected profit of seller s is

$$E\left[\pi_s^\tau\right] = \tau_s(q, r) - c_s Q(q) - c_p q - c_{\bar{s}} \tag{5.2.3}$$

These two functions show that the profits of alliance members depend on the choice of listing quantity and the contract. Meanwhile, the choice of the listing quantity incurs variable cost for the seller, however it does not incur any variable cost for agent o. This setting is very unique for online market compared to the traditional supply chains [9]. While physical distributions are executed among alliance members that incur variable cost in the traditional supply chains, online market does not incur any variable costs due to direct shipment from the seller to the buyer.

Based on the above settings, we first show that there exists an alliance optimal profit for this model.

Lemma 5.2.3. *Let q^* be an alliance optimal listing quantity in the online market model without advertisement effect. There exists a unique optimal listing quantity $q^* = F^{-1}\left(\frac{p-c_s-c_p}{p-c_s}\right)$ in the online market model.*

Proof. According to the definition of the profit, we obtain the alliance expected profit as follows, $E\left[\pi(q)\right] = (p - c_s)Q(q) - c_p q - (c_{\bar{o}} + c_{\bar{s}})$. The first-order derivative of the expected profit is

$$\frac{dE\left[\pi(q)\right]}{dq} = (p - c_s)(1 - F(q)) - c_p \tag{5.2.4}$$

The second-order derivative of Equation (5.2.4) is $\frac{d^2 E[\pi(q)]}{dq^2} = -(p - c_s)f(q)$. Since $f(q)$ is positive, we obtain that the alliance profit function is concave in quantity q. Therefore, the alliance optimal quantity q^* must be the solution of Equation (5.2.4) such that $q^* = F^{-1}\left(\frac{p-c_s-c_p}{p-c_s}\right)$. □

Lemma 5.2.3 shows that the unique alliance optimal quantity exists in this model. Therefore, an equilibrium listing quantity under a certain contract must be equal to q^*.

Now let us relax the assumption of no advertisement. Taking into account advertisement entails to redefine the Pareto optimality checking, since the alliance profit function is different. The alliance expects to earn revenue $r = R(q, a) = pQ(q, a)$ at given price p. Suppose the alliance agrees on a contract τ, the expected profit of agent o is

$$E\left[\pi_o^\tau(q, a)\right] = \tau_o(q, a, r) - c_{\bar{o}} - g(a), \tag{5.2.5}$$

and the expected profit of agent s is

$$E\left[\pi_s^\tau(q, a)\right] = \tau_o(q, a, r) - (1 - \alpha)c_s Q(q, a) - c_p q - c_{\bar{s}} \tag{5.2.6}$$

In order to check the alliance coordination, we define the expected alliance profit as follows,

$$E\left[\pi(q,a)\right] = (p - c_s)Q(q,a) - c_p q - (c_{\bar{o}} + c_{\bar{s}}) - g(a). \qquad (5.2.7)$$

According to the definitions of Q, Equation (5.2.7) is concave in listing quantity q and a. Therefore, there exists an alliance optimal pair $\{q^*, a^*\}$. Notice that this pair is not necessary unique. We suppose that for any given fixed listing quantity q, there exists optimal advertisement amount $a^*(q)$. Formally, this is represented by the following first-order condition, similarly to [98, 13]:

$$\frac{\partial \pi(q, a^*(q))}{\partial a} = (p - c_s)\frac{\partial Q(q, a^*(q))}{\partial a} - \frac{dg(a^*(q))}{da} = 0 \qquad (5.2.8)$$

For this online market model, we now investigate the properties of alliance coordinating contracts. We focus on two typical contracts: *fixed-fee contract* and *revenue-sharing contract*. For these two contracts, we study the advertisement effect.

5.3   Fixed-fee Contract

Fixed-fee contract is employed in many online markets. Fixed-fee contract is a contract where one agent always gets the same individual revenue regardless the alliance revenue, formally:

Definition 5.3.1. A contract τ of an alliance $\mathcal{A}$ is called to be fixed-fee contract by agent i_0 if it satisfies the following conditions: for any $\sigma \in \mathcal{S}$ and $r \in \Re$,

1. $\tau_{i_0}(\sigma, r) = \alpha$

2. $\sum_{i \neq i_0} \tau_i(\sigma, r) = r - \alpha$

where $\alpha \in \Re$ is constant and interpreted as the charging fee.

Under fixed-fee contract, agent i_0 charges fixed-fee α to the other agents and the returns of the alliance is taken by agents except for agent i_0. In the context of online market, we have $\tau_o(\sigma, r) = \alpha$ and $\tau_s(\sigma, r) = r - \alpha$ s.t. $\sigma = (q, a)$. Notice that charging a membership fee is a similar contract.

Example 5.3.1. *Consider an online market where sellers sell second-hand items to buyers. In this online market, the owner o charges $2 fixed-fee to seller s for each listing. The contract can be represented as follows:*

$$\tau_o(\sigma, r) = 2; \qquad \tau_s(\sigma, r) = r - 2.$$

It means for any seller's strategy the market owner's share of revenue is constant. This contract is used in eBay BuyItNow option.

5.3.1 No Advertisement

The following proposition shows that the online market model without advertisement effect achieves alliance coordination.

Proposition 5.3.2. *Let α be a fixed-listing fee of the online market. Fixed-fee contract τ achieves alliance coordination in online market model without advertisement effect, if $c_{\bar{o}} < \alpha < (p - c_s)Q(q^*) - c_p q^* - c_{\bar{s}}$.*

Proof. If $q > 0$, under fixed-fee contract, the expected profit of online market owner o and seller s are respectively,

$$E\left[\pi_o^\tau(q)\right] = \alpha - c_{\bar{o}} \tag{5.3.1}$$

$$E\left[\pi_s^\tau(q)\right] = r - \alpha - c_s Q(q) - c_p q - c_{\bar{s}}$$

$$= (p - c_s)Q(q) - c_p q - \alpha - c_{\bar{s}}, \tag{5.3.2}$$

otherwise, we have $\pi_o^\tau(q) = \pi_s^\tau(q) = 0$. Since the expected profit of online market owner o is always α if $q > 0$, a concern of the online market owner is if the seller lists items at quantity $q > 0$. Hence, we check the optimal listing quantity for seller s denoted as $\hat{q}_s$. By differentiating the profit function of seller s w.r.t. q, we obtain $\frac{dE[\pi_s^\tau(q)]}{dq} = (p - c_s)(1 - F(q)) - c_p$. According to Equation (5.2.4), we get $\frac{dE[\pi_s^\tau(q)]}{dq} = \frac{dE[\pi(q)]}{dq}$ and according to Lemma 5.2.3, there is a unique alliance optimal quantity q^* in this model. Thus we obtain $\hat{q}_s = q^*$. In order to satisfy participation constraints, Equation (5.3.1) and (5.3.2) must be positive at $q = q^*$. Therefore, fixed-fee contract achieves alliance coordination, if fixed-fee α satisfies $c_{\bar{o}} < \alpha < (p - c_s)Q(q^*) - c_p q^* - c_{\bar{s}}$. □

Proposition 5.3.2 shows that the alliance coordination is due to the cost structure of the online market owner which does not incur variable cost for the listing quantity. As long as the fee is greater than the owner's cost and lower than the seller's expected profit, the alliance coordination holds.

5.3.2 Advertisement Effect

According to the advertisement effect, strategies are now pairs (q, a) and the contracts are $\tau_o(q, a, r) = \alpha$ and $\tau_s(q, a, r) = r - \alpha$. The following proposition shows that fixed-fee contract does not achieve alliance coordination.

Proposition 5.3.3. *Fixed-fee contract τ does not achieve alliance coordination in the online market model with advertisement effect.*

Proof. Under fixed-fee contract τ, if $q > 0$, according to Equation (5.2.5) and the definition of the contract, the profit function of online market owner o is: $E\left[\pi_o^\tau(q, a)\right] = \alpha - c_{\bar{o}} - g(a)$. The first-order derivative of profit function of online market owner o w.r.t. a is $\frac{d\pi_o(a)}{da} = -\frac{dg(a)}{da} < 0$. Hence, under fixed-fee contract, online market owner o does not have incentive to place any positive advertisement amount which is the assumption of the online market model without advertisement effect shown in the previous section. Therefore, fixed-fee contract does not achieve alliance coordination in this model. $\square$

According to Proposition 5.3.3, seller s enjoys a benefit of advertisement effect as a free rider under fixed-fee contract. Furthermore, the online market owner does not have any incentive to place advertisement in a context of alliance coordination.

We have shown that fixed-fee contract achieves alliance coordination in the limited case where advertisement is not considered. The next question we address is whether another popular contract, revenue-sharing contract, achieves alliance coordination.

5.4 Revenue-Sharing Contract

The following contract, *revenue-sharing contract*, is also frequently used in the online markets. Individual revenue is a proportion of the alliance revenue [13].

Definition 5.4.1. A contract τ of an alliance $\mathcal{A}$ is called to be revenue-sharing contract if there exists $\alpha_1, \cdots, \alpha_n$ s.t. $\sum_{i \in \mathcal{I}} \alpha_i = 1$ and for any $(\sigma_1, \cdots, \sigma_n) \in \mathcal{S}$ and $r = R(\sigma)$,

$$\tau_i(\sigma_1, \cdots, \sigma_n, r) = \alpha_i r \text{ for all } i$$

Example 5.4.1. *Consider an online music market for selling songs. The alliance consists of online music store o and music label s. The contract specifies the following royalties on revenue r: 20% of r for agent o and 80% for agent s. Hence, the contracts are:*

$$\tau_o(\sigma, r) = 0.20r; \qquad \tau_s(\sigma, r) = 0.80r$$

5.4.1 No Advertisement Effect

Under revenue-sharing contract, the online market owner charges a certain portion of the sales amount of the seller. Portion α ranges in $0 < \alpha < 1$. Hence $\tau_o(q, r) = \alpha r$ and $\tau_s(q, r) = (1 - \alpha)r$. The following proposition shows that this contract does achieve alliance coordination.

Proposition 5.4.2. *Let α be a portion that online market owner o earns from the revenue r. Revenue-Sharing contract τ does not achieve alliance coordination in the online market model without advertisement effect.*

Proof. Under revenue-sharing contract, the expected profit of online market owner o is

$$E\left[\pi_o^\tau(q)\right] = \tau_o(q, r) - c_{\bar{o}}$$

$$= \alpha p Q(q) - c_{\bar{o}}$$

and the expected profit of seller s is

$$E\left[\pi_s^\tau(q)\right] = \tau_o(q, r) - c_s Q(q) - c_p q - c_{\bar{s}}$$

$$= (1 - \alpha)p Q(q) - c_s Q(q) - c_p q - c_{\bar{s}}.$$

The first-order condition for the profit maximizing quantity of seller s is $\frac{dE[\pi_s^\tau(q)]}{dq} = ((1-\alpha)p - c_s)(1 - F(q)) - c_p = 0$. Let $\hat{q}_s$ be the profit maximizing quantity of the seller under revenue-sharing contract. We obtain $\hat{q}_s = F^{-1}\left(\frac{p(1-\alpha)-c_s-c_p}{p(1-\alpha)-c_s}\right) > q^*$. Hence, revenue-sharing contract does not achieve alliance coordination. $\square$

As mentioned, the online market owner does not incur any variable cost or any procurement cost. Therefore, revenue-sharing contract does not achieve alliance coordination. Even though the proposition shows that revenue-sharing contract is not an alliance coordinating contract, it is a popular contract in online market. Parameter α is usually set at a small value in the online markets. Therefore, it entails that seller s may list slightly greater quantities than the alliance optimal quantity. This also means that the listed quantity entailed by a revenue-sharing contract may be greater than the one entailed by fixed-fee contract. Therefore, the online market owner may sell greater quantities under revenue-sharing contract compared to fixed-fee contract.

5.4.2 Advertisement Effect

Let α be the online market owner o's portion of revenue. The contracts are $\tau_o(q, a, r) = \alpha r$ and $\tau_s(q, a, r) = (1 - \alpha)r$. Again we show that revenue-sharing contract does not achieve alliance coordination.

Proposition 5.4.3. *Revenue-sharing contract τ does not achieve alliance coordination in the online market model with advertisement effect.*

Proof. Under revenue-sharing contract τ, for a given listing quantity q, let $\hat{a}_o(q)$ be the optimal advertisement amount for online market owner o corresponding to listing quantity q. It entails that the first-order condition represented in Equation (5.2.8) holds for $(q, \hat{a}_o(q))$. Hence, for the optimal profit function π_o^τ, we have $\frac{\partial E[\pi_o^\tau(q,\hat{a}_o(q))]}{\partial a} = \alpha(p - c_s)\frac{\partial Q(q,\hat{a}_o)}{\partial a} - \frac{dg(\hat{a}_o)(q)}{da} = 0$. According to Equation (5.2.8) , if $\hat{a}_o = a^*$ we have $\frac{\partial E[\pi_o^\tau(q,\hat{a}_o)]}{\partial a} < \frac{\partial E[\pi(q,a^*(q))]}{\partial a}$. Thus we have $\hat{a}_o \neq a^*$. Hence, revenue-sharing contract does not achieve alliance coordination in the case of individual advertisement.

$\square$

According to Propositions 5.4.2 and 5.4.3, revenue-sharing contract does not achieve the alliance coordination regardless of advertisement effect. This is mainly due to the lack of relation between marginal cost and marginal profit. In the next section, we propose a contract that takes care of this relation.

5.5 Profit Sharing Contract

The aim of this contract is to balance revenue and variable costs between alliance members. That is each member does not only consider its cost to define its profit, but also the other members' costs. The revenue is $r = R(\sigma_i, \sigma_j)$. Let $\chi_i > 0$ be a parameter for setting at first the portion of revenue for agent i and, second the portion of cost that agent j charges to agent i. We assume that $\sum_{i \in \mathcal{A}} \chi_i = 1$. The following contract is in the scheme of profit sharing contract in [56],

Definition 5.5.1. Let $\chi_i > 0$ be a portion parameter of profit sharing contract τ. A contract τ of an alliance $\mathcal{A}$ is a profit sharing contract if $\tau_i(\sigma_i, \sigma_j, r) = \chi_i r - \chi_i c_j(\sigma_j) + \chi_j c_i(\sigma_i)$ for all $i \in \mathcal{A}$.

In the context of online market, profit sharing contract is interpreted as follows:

$$\tau_o(q, a, r) = \chi r - \chi c_s S(q, a) - \chi c_p q + (1 - \chi) g(a)$$

$$\tau_s(q, a, r) = (1 - \chi) r + \chi c_s S(q, a) + \chi c_p q - (1 - \chi) g(a)$$

This profit sharing contract is a combination of revenue-sharing, sales discount, listing incentive and advertisement cost sharing.

The following theorem shows that profit sharing contract achieves alliance coordination.

Theorem 5.5.1. *Profit sharing contract τ achieves alliance coordination in the online market model.*

Proof. W.r.t. τ, the profit function for online market owner o is

$$E\left[\pi_o^\tau(q, a)\right] = \chi((p - c_s)S(q, a) - c_p q - g(a)) - c_{\bar{o}}, \qquad (5.5.1)$$

and the profit function of seller s is

$$E\left[\pi_s^\tau(q,a)\right] = (1-\chi)((p-c_s)S(q,a) - c_p q - g(a)) - c_{\bar{s}}. \tag{5.5.2}$$

By differentiating Equation (5.5.1) w.r.t. a, we obtain a marginal profit of online market owner w.r.t. advertisement $\frac{\partial E[\pi_o^\tau(q,a)]}{\partial a} = \chi \left((p-c_s)\frac{\partial S(q,a)}{\partial a} - \frac{dg(a)}{da}\right)$. Since $(p - c_s)\frac{\partial S(q,a)}{\partial a} - \frac{dg(a)}{da} = \frac{\partial E[\pi(q,a)]}{\partial a}$, we have $\frac{\partial E[\pi_o^\tau(q,a)]}{\partial a} = \chi \frac{\partial E[\pi(q,a)]}{\partial a}$. Therefore, it satisfies the first-order condition shown in Equation (5.2.8). A condition to satisfy participation constraint is Equation (5.5.1) and (5.5.2) must be positive at a pair $\{q^*, a^*\}$ as follows:

$$E\left[\pi_o^\tau(q^*,a^*)\right] = \chi\left((p-c_s)S(q^*,a^*) - c_p q^* - g(a^*)\right) - c_{\bar{o}} > 0$$

$$E\left[\pi_s^\tau(q^*,a^*)\right] = (1-\chi)\left((p-c_s)S(q^*,a^*) - c_p q^* - g(a^*)\right) - c_{\bar{s}} > 0$$

Therefore, we obtain that profit sharing contract achieves alliance coordination, if

$$\frac{c_{\bar{o}}}{(p-c_s)S(q^*,a^*) - c_p q^* - g(a^*)} < \chi < \frac{(p-c_s)S(q^*,a^*) - c_p q^* - g(a^*) - c_{\bar{s}}}{(p-c_s)S(q^*,a^*) - c_p q^* - g(a^*)}$$

$$\square$$

Profit sharing contract charges their costs to the alliance partner with prefixed portion each other. At the same time, based on the opposite portion, the revenue is shared. According to Theorem 5.5.1, profit sharing contract balances out the costs of the alliance members by sharing them. This contract can be used as a benchmark contract to implements contracts in online markets. In practice, it is not always the case that the alliance members are able to exchange cost information. In such a case, we are able to use profit sharing contract, however it is possible to obtain an approximate contract, which is a combination of revenue-sharing contract and fixed-fee contract. Typical implementations are the cost sharing part in profit sharing

contract is approximated as fixed fee.

5.6 Discussion and Related work

In this chapter, we have focused on the coordination problem between the online market owner and the seller. We have treated contracts as market mechanisms to attract the sellers to participate in listing items for online sales. We have designed contracts in online markets to induce both the online market owner and the seller make strategic decisions to maximise the overall profit rather than the individual profit.

At first, we have presented the notion of contract and alliance coordination. Next we have shown how this framework can be used to describe a specific kind of market namely online market. Notice that our definitions of contract and alliance coordination are not restricted in online market and we further discuss this point later. Then, we have studied behaviour of this market wit respect to two popular contracts: fixed-fee contract and revenue-sharing contract. We have shown that it is difficult to obtain coordination for these two contracts: only fixed-fee contract with no advertisement achieves alliance coordination. Revenue-sharing contract leads sellers to list greater quantities compared to the case of fixed-fee contract; this property may be a desirable one for gaining market shares. We finally exhibit a profit sharing contract that enables to achieve coordination. In order to implement this contract, it is necessary to share cost information between the alliance members. If exchanging cost information is not possible, profit-sharing contract can be used as a target performance of the

approximated contracts. For instance, profit sharing can be approximated to revenue-sharing plus fixed-fee if we set the fixed fee at the cost of the optimal quantities. In eBay, revenue-sharing plus fixed-fee contract is implemented, while the parameter of the contracts are categorised by the product group.

Our definition of a coordinating contract differs from the one given by Gan, et al. in [43]. They define a contract as a proportion of the alliance revenue in the traditional supply chains. Their definition mainly focuses on revenue-sharing contract. Our framework is more flexible to describe different types of contract such as fixed-fee contract. With their definition, even such a simple contract is not straightforward to describe .

We have assumed that all agents are risk neutral similar to others settings such as a supplier and a retailer model [9, 13] and a supplier and an online retailer model [88]. Therefore, we consider that the decision making criteria of the agents are their expected profits. If we consider risk averse agents, our definition of alliance coordination may be extended for taking it account utilities of agents as proposed in the cases of risk averse agent modeled by expected exponential utility objectives and the mean-variance objective in [43] or conditional value-at-risk (CVaR) describing the worst possible outcome [110].

This chapter focuses on the coordination problems in online markets. There are extensive studies on coordination problems in traditional supply chains. Cachon [9] compares several different contracts that achieve supply chain coordination instead

of wholesale price contract [8, 67]. Typical coordinating contracts are buy-back contract [96, 75], revenue-sharing contract [30, 45, 13, 44] and quantity-flexibility contract [79, 2]. These contracts are solutions for the the traditional supply chains with double marginalisation problem. However, in our online market model, double marginalisation problem is not a key problem, since the products are not actually distributed through the online market. Therefore, a simple contract, fixed-fee contract, achieves an alliance coordination in our model. Most of them consider a single supplier and a single retailer relationship under price dependent stochastic demand similarly to ours.

The coordination problem in online markets is particularly relevant to B2B spot markets. Netessine and Rudi [88] propose coordinating contracts for online retailers that do not hold inventory and the sales orders are fulfilled by direct shipping from the manufacturers. Indeed, the online retailers reduce risks by not holding inventories. However they purchase the products from the manufacturers. Hence, the problems in the model are still very similar to the ones in the traditional supply chain models discussed in the online markets above. Furthermore, the environment is exactly similar to traditional catalogue retailers [37]. We consider electronic markets that do not purchase items for online sales. In that context, we focus on how electronic market should charge from the sellers for the coordination.

The emergence of e-business leads to problems of the choice of channels between e-commerce channel and traditional supply chain channel. Considering coordination, there are very few research about the condition for the choice either between the online retailers and the traditional retailers in [88, 89] or between online direct sales

and the traditional retailers in [118]. However, these studies are not for the general electronic marketplace. The former is for the online retailers and the latter is for the manufacturer's online sales. Wang and Benanoch [125] also consider the choice between the electronic markets and the traditional retailers. The coordinating contract proposed is limited for the transaction-based commission fee which is generally a percentage of the transaction value. In our study, we do not restrict the fee structure of the electronic markets such as the transaction-based commission fee.

Recently, affiliate program is paid attention as a new marketing tool especially in B2C and B2B spot markets. In the affiliate programs, affiliate websites such as price comparison websites, product review sites, personal websites, personal e-mail, and video sharing sites, receive rewards for leading the customers to the electronic markets. Typical reward is based on revenue-sharing contract. Libai et al. [70] propose the coordinating contract between the electronic market owner and the affiliate sites. We set the coordination with the affiliate programs as future work.

Coalition formation is a similar area to the coordination problem in this chapter. A central issue in coalition formation is how the gains from the cooperation are to be distributed among the agents [25]. However, our research deals with a problem not just whether to participate into an alliance, but we consider how agents choose strategies according to the chosen contracts. For instance, the market owner has number of choices for advertisement amount in our model.

Chapter 6

Market Mechanism Design for

Autonomous Trading Agents

In the previous chapters, we have focused on theoretical analysis on market mechanism design for supply chain management. This chapter and the following chapter deal with market mechanism design based on experimental approach. This chapter formally represents our market model and specifies a way to evaluate our market mechanisms.

6.1 Background

In order to design market mechanisms, there are two types of approaches: theoretical approach and experimental approach. From Chapter 2 to Chapter 5, we design market mechanisms theoretically. An advantage of the theoretical approach is clearness, predictability and stability of the solutions. However, there is a criticism on some too

strict assumptions on the strategic decisions and knowledge of agents in theoretical approach. For instance, Nash equilibrium is a central piece in game theory, which assumes complete information and rational choices. With respect to these assumptions, Goeree and Holt [48] mention that:

> if we admit any noises on strategic decisions, it is easily inaccessible to a mainstream of theoretical works such as Nash equilibrium for static games with complete information, Bayesian Nash equilibria for static games with incomplete information, subgame perfectness for dynamic games with complete information, and some refinement of the sequential Nash equilibrium for dynamic games with incomplete information.

An extremely purified environment may cause severe problems for an implementation of mechanisms in practice. In order to check the theoretical results, economists have conducted experiments with human subjects [111, 29, 112, 113, 101].

Recently, *agent-based computational economics (ACE)* has been intensively studied for the similar purpose. ACE is the computational study of economic processes modelled as dynamic systems of interacting agents who do not necessarily possess perfect rationality and information [117]. ACE supplements theoretical approach with respect to economic processes, local interactions among traders and out-of-equilibrium dynamics that may or may not lead to equilibria in long run [117].

Traders in ACE are autonomous agents that may not always have enough information to take a theoretical solution. Hence, a main concern in ACE is how to converge

to a theoretical solution by learning. Typical ACE study has focused on design of autonomous trading agents under "fixed" market mechanisms [47, 22, 20, 107, 46, 100]. Normally, well-known market mechanisms for human traders are used in experiments, such as English auction, second price sealed bid auction, continuous double auction and so on. In the advance of design of such agents, one question may arise, whether well-known market mechanisms for human traders suit for these autonomous agents. In this thesis, we investigate market mechanisms that suit for autonomous trading agents. In particular, we focus on double auction under which traders (sellers and buyers) are extensively interact each other, as illustrated in Figure 1.4 in page 14. We design market mechanisms as an automated match maker.

Typical approach to design market mechanisms focus on a specific function in market mechanisms to improve market performance [128, 23, 99]. In contrast, we are interested in an effect of a combination of small components in market mechanisms. To be more specific, we address the following five issues,

1. *How to estimate the desirable outcome according to actions of the agents that make decisions with limited information?* The market maker does not have enough information about the desired outcome. It estimates the desired outcome according to the actions of the agents that may not always be able to take the ex-post optimal choice.

2. *How to navigate those agents in proper manners by market mechanisms?* Even though the market maker does not know the desirable outcome, the market maker needs to navigate agents to induce proper manners.

3. *How market mechanisms influence the market behaviour of autonomous trading agents and the market performance?* Normally, a market mechanism consists of several different key components namely *market policies*. A combination of market policies works as a market mechanism. Therefore, it is necessary to comprehend how a combination of market policies influences market behaviour. In our environment, there is not only a single way to find the optimal choice. Therefore, there are many different types of agents that implement own strategies in a market.

4. *Which agents enjoy the benefits from specific mechanisms?* A difficulty is that the effects of market mechanisms are not uniformly work for those agents. Hence, it is significant to identify how specific policies influence specific types of agents.

5. *How should we evaluate market mechanisms?* Since interactions of the market maker and the traders are complex, it is not simple to identify how to measure the effects in market performance indexes.

For these issues, we design several market mechanisms and conduct experiments with autonomous trading agents. Our approach consists of the following five tasks:

1. **Specification of market mechanisms**: we decompose market mechanisms into market policies. For each policy, we formally represent to specify the design space.

2. **Specification of evaluation criteria of market mechanisms**: we specify

market performance indexes to evaluate market mechanisms.

3. **Design of experiments**: we design our experiments to control experimental environment.

4. **Conduct of experiments**: we implement our market mechanisms and conduct our experiments with these market mechanisms. We use CAT platform to implement our market mechanisms and to conduct our experiments.

5. **Interpretation of experimental results**: we analyse and interpret the experimental results.

We present these the first two tasks in this chapter and the rest of the tasks are presented in the following chapter.

The organisation of this chapter as follows. In order to introduce characteristics of double auction, we present several classic auction types and classify them in Section 6.2. Then, in Section 6.3, we detail our market model and decompose a market mechanism into market policies with formal representations. In Section 6.4, we present a typical analytical tools to analyse market performances. In Section 6.5, we propose our evaluation criteria of market performance. In Section 6.6, we give a summary and discussions of this chapter.

6.2 Classic Auction Types

In this thesis, we focus on market mechanism design with *double auction* where sellers and buyers intensively interacts for trades. In order to introduce characteristics of

double auction, we present several classic auction types. Based on surveys by Wurman [127] and Friedman [41], we classify classic auction types in Figure 6.1 according to the following three market rules,

- bidder,

- transparency,

- pricing.

The first distinction is whether bidders are single-sided or double-sided. In single-sided auction, either sellers or buyers submit their preferable prices. In double-sided auction or double auction, sellers and buyers both submit their preferable prices. The second distinction is whether bidding price is open to others (outcry) or not (sealed-bid). In single-sided and sealed-bid category, there are first-price sealed-bid (FPSB) auction and second-price sealed-bid (SPSB) auction. The characteristic of the SPSB auction is an incentive compatibility where the bidders tend to submit truthful their valuations [120, 18, 49]. Contrary, the transaction prices under FPSB tend to be higher. In single-sided and outcry category, English auction and Dutch auction have long history in practice. English auction is used for rare items such as fine arts, while Dutch auction is used for perishable items, such as flowers and fish.

In double auction, it is common to distinguish continuous double auction (CDA) and clearinghouse (CH), or call market. Typical CDAs are outcry and continuous clearing, while typical CHs are sealed-bid and periodical clearing. Many market institutions combine both types of auction rules.
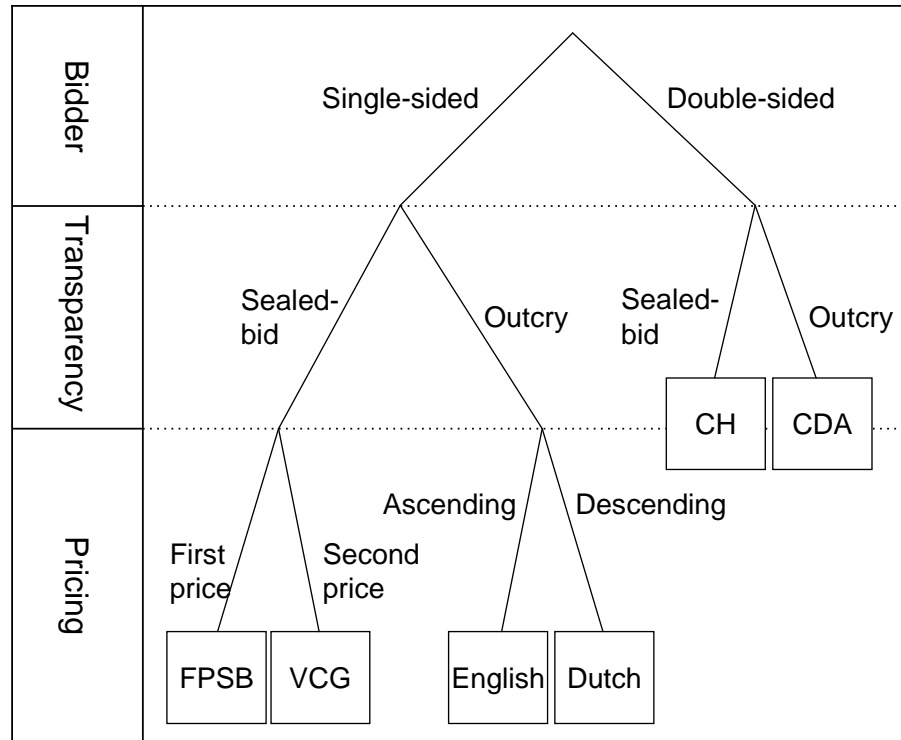
Figure 6.1: A Classification of Classic Auction Types.

In online markets, some new types of auction mechanisms have emerged that combine some classic auction types of single-sided auctions above. For instance, eBay implements a new auction type *proxy bidding* that combines auction rules of English auction and second-price auction [108]. In a similar sense, we do not restrict ourselves to design market mechanism for double auction. We combine several different market rules to develop a new market mechanism.

6.3 Experimental Context

Hereafter, we focus on market mechanism design which implements double auctions. Let us detail the model:

6.3.1 Overview of the Model

Consider a market as illustrated in Figure 6.2. Each seller and buyer is an autonomous trading agent. For the market maker, each seller and buyer submit an ask order and a bid order, respectively. Each order has a limit price which is the least preferable price for the trader. This type of orders is called as limit order. The market maker matches incoming orders and determines market prices for the matched orders. These orders are then executed and notified to the traders. The market maker charges some fees to traders. Hereafter, we focus on the actions of the market maker. The framework is mainly based on CAT.

We detail market mechanisms and traders. We also describe the sequence of the actions in detail.

Market Mechanism

A market mechanism consists of several market policies (components). We specifies these policies as follows:

- Accepting policy: the conditions to accept incoming orders,

- Matching policy: the way to match between accepted ask orders and bid orders,
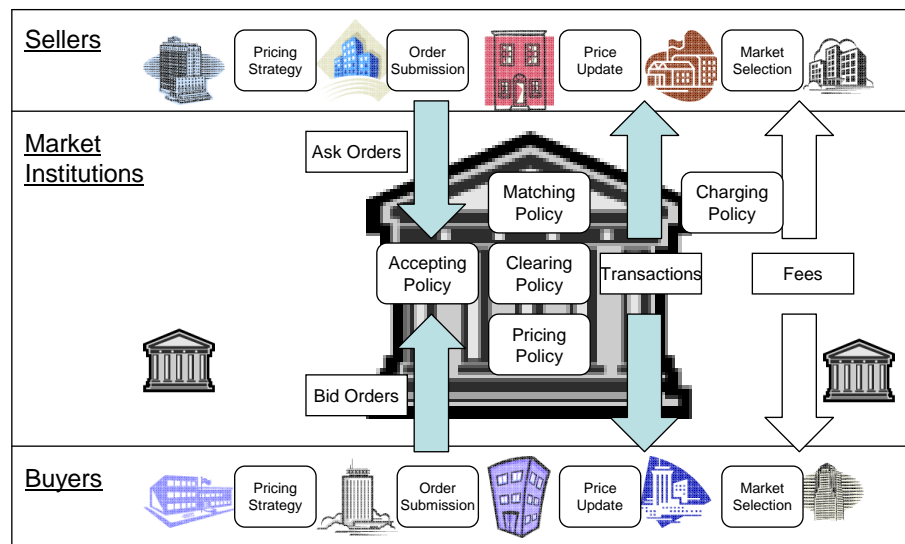
Figure 6.2: A General Double Auction Market.

- Clearing policy: the timing to clear the market,

- Pricing policy: the way to determine a clearing price or a transaction price,

- Charging policy: the way to charge fees to the traders.

These policies are classified into two types of policies, (i) internal policies such as accepting policy, matching policy, clearing policy, and pricing policy, and (ii) a external policy such as charging policy. In this thesis, we focus on internal policies. We formally represent these policies in Section 6.3.2 and implement specific policies in Section 7.2. These policies are controlled in our experiments and we analyse how

they work as a market mechanism.

Traders

Actions of each trader are mainly order submission and update of prices. For these actions, each trader implements the following strategies:

- Pricing strategy: determination of a limit price (the least preferable price),

- Order submission: submission and control of feasible orders to market institutions,

- Price update: updating price during a negotiation between market makers,

- Market maker selection: selection of market institutions for trades.

The order submission is controlled in experiments. The price update strategy is fundamentally same as pricing strategy. In our experiment, we consider a single market maker. Hence, we do not focus on market maker selection.

Interaction Between a Market Maker and Traders

In our experiment, we consider a synchronised time frame. In such context, a auction type is called *synchronised double auction* SDA [41]. In the SDA, a market is divided into periods referred to as days. Each day consists of one or several rounds to accept orders from the traders. As Rust et al. mention in [109], the reason for structuring of multiple rounds is to control traders' abilities to learn from the market information.

At each time point (round), a trader has the opportunity to submit an order. Then the market maker determines whether it accepts the incoming order. Once the market maker has accepted orders, it determine the way to match them (or it matches the ask and bid orders). Finally, orders are cleared.

6.3.2 Formal Representation

In this section, we formally represent our model. Let $I = S \cup B$ be a set of traders where S is a set of sellers and B is a set of buyers s.t. $S \cap B = \emptyset$, respectively. Trader $i \in I$ has a fixed valuation for the commodity denoted as v_i. Let X be a set of orders from traders. An order $x \in X$ consists of two components, an order owner $\mathfrak{I}(x) \in I$ and price $p(x)$. We consider limit order deals where the traders set the limit prices; then, the market maker determines market price that satisfies the limit prices. We assume that the limit price satisfies $p(x_i) \geq v_i$ for seller $i \in S$ and $p(x_j) \leq v_j$ for buyer $j \in B$. If the order is rejected by the market institution, the trader has an opportunity to resubmit the order in the following round if there exists.

Four internal policies are represented as follows.

Accepting Policies

The decision rules whether the market maker accepts or rejects is called as *accepting policy* defined as follows.

Definition 6.3.1. Let X be a set of incoming orders. An accepting policy $\mathfrak{A}$ is a function that assigns true or false to each incoming order i.e., $\mathfrak{A} : X \to \{true, false\}$.

According to the accepting policy, the market makers filter the incoming orders. The goal of an accepting policy is to reduce price fluctuation and thus to urge the traders to improve their limit prices. Given an accepting policy $\mathfrak{A}$ and the set of orders X, let $A = \{x \in X : \mathfrak{A}(x) = \text{true}\}$. Let us denote the set of accepted ask orders as $A^{ask} = \{x \in A : \mathfrak{I}(x) \in S\}$ and the set of accepted bid orders as $A^{bid} = \{x \in A : \mathfrak{I}(x) \in B\}$, respectively.

Matching Policies

The market makers matches from the accepted orders. A matching policy is defined as follows

Definition 6.3.2. Given $X = X^{ask} \cup X^{bid}$, a matching policy $\mathfrak{M}$ is a function $\mathfrak{M} : 2^X \to 2^{X^{ask} \times X^{bid}}$ such that:

1. for any $H \subseteq X$, if $(x, y) \in \mathfrak{M}(X)$ then $x \in H$, $y \in H$ and $p(x) \leq p(y)$,

2. for any $H \subseteq X$, if $(x_1, y), \ (x_2, y) \in \mathfrak{M}(X)$, then $x_1 = x_2$, and

3. for any $H \subseteq X$, if $(x, y_1), \ (x, y_2) \in \mathfrak{M}(X)$, then $y_1 = y_2$

Matching policies make feasible matches between ask orders and bid orders. Each matched pair satisfies two conditions: a sell price is less or equal to a buy price, and each order is not matched with multiple orders at once. Let M be a set of matched pairs obtained from accepting policy $\mathfrak{M}$ that is $M = \mathfrak{M}(A)$. We denote $|M|$ as a number of matched pairs in M.

Clearing Policies and Pricing Policies

Clearing policies rule both trigger timing and conditions to clear the market. At the market clearance, the matched orders are cleared from the market and the unmatched orders are kept in the market. Meanwhile, the market maker determines a clearing price based on clearing policies defined as follows,

Definition 6.3.3. Given X, a pricing policy $\mathfrak{P}$ is a function that assigns a positive real number to a pair of an ask order and a sell order, $P : X^{ask} \times X^{bid} \to \mathfrak{R}$ satisfies $p(x) \leq \mathfrak{P}(x, y) \leq p(y)$ for any $x \in X^{ask}$, $y \in X^{ask}$ such that $p(x) \leq p(y)$.

A pricing policy specifies a clearing price which is between a sell price and a buy price of each pair.

According to the combinations of the four policies above, we evaluate the performance of the market institution. In Section 6.5, we propose our evaluation criteria to design the policies of the double auction market.

6.4 Analytical Tools for Double Auctions

In this section, we present the tools used to analyse the performance of our experiments. We show an example where the market maker receives orders consisting of 12 sellers and 10 buyers as shown in Figure 6.3. In the figure, the supply and demand schedule is depicted. The S-S line indicates the supply schedule and the D-D line indicates the demand schedule. The schedule lines indicate the accumulated number

of traders that agree on trades w.r.t. a price. For example, there is one seller that agrees on the trade if the transaction price exceeds \$18 and two sellers above \$25. The two lines cross at \$60. Hence, we obtain 6 pairs at \$60.
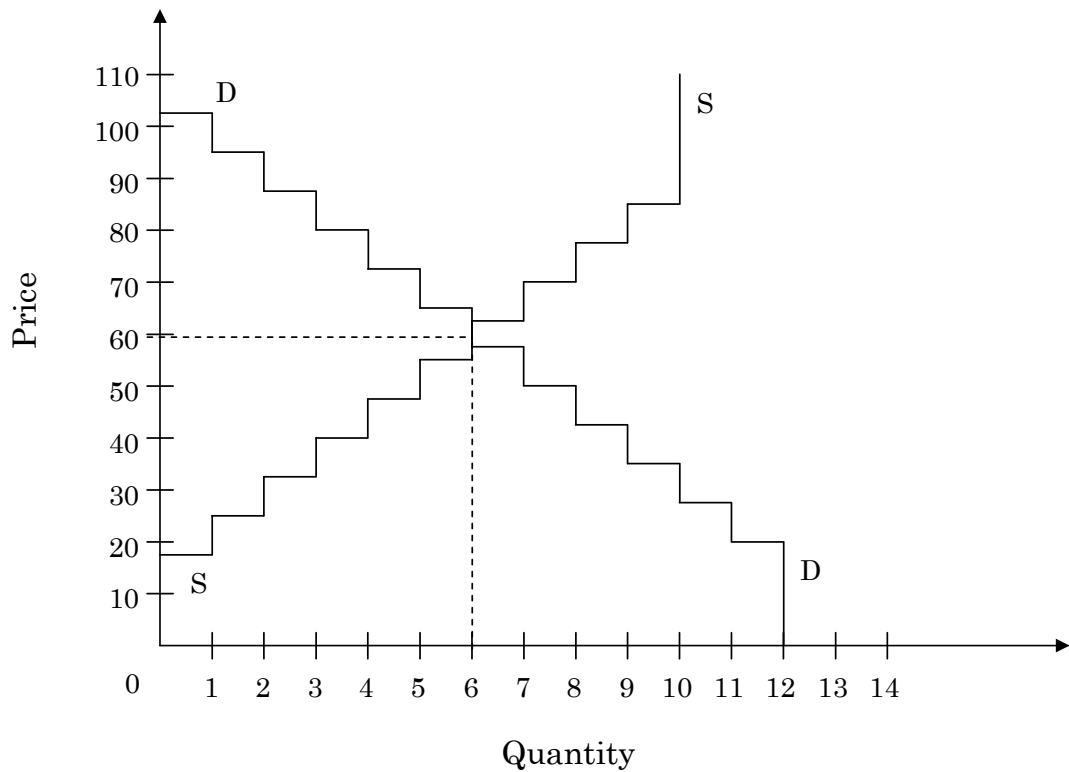
Figure 6.3: An Example of Theoretical Supply and Demand Schedule

We consider the special case where the market is perfectly competitive like Bertrand competition presented in Section 2.2. In such a case, the traders set their prices at their valuations.

Let v_i be a valuation of the item by trader i. That is $p(x_i) = v_i$. Let us denote $X^{ask} = \{x_1, \ldots, x_n\}$ such that $p(x_1) \leq \ldots \leq p(x_n)$ and $X^{bid} = \{y_1, \ldots, y'_n\}$ such that $p(y_1) \geq \ldots \geq p(y'_n)$. In order to avoid a trivial situation, we assume $p(x_1) \leq p(y_1)$.

That is there is at least one pair of matchable orders. If we make matches from (x_1, y_1), the last number of the match is equal to the crossing point in Figure 6.3. In this case, the price at crossing point is the equilibrium price. Let us denote $\hat{q}$ be the number of matches at the equilibrium. Let $\hat{p}$ be the equilibrium or the competitive price such that $\hat{p} = \frac{p(x_{\hat{q}}) + p(y_{\hat{q}})}{2}$. Any trader $i \leq \hat{q}$ is an *intra-marginal trader*. The rest of the traders are *extra-marginal traders*.

The overall outcome of the trades in the market institution is measured by the sum of distance of the valuations between the matched pairs. That is, given M, we obtain the overall system profit $V_M = \sum_{(x,y) \in M} (v_{\Im(y)} - v_{\Im(x)})$. If a match $\hat{M}$ consists of all the intra-marginal traders, the overall system profit $V_{\hat{M}}$ is maximised.

We explain these characteristics in the following example. Suppose 3 sellers that submit truthful values $\{p(x_1), p(x_2), p(x_3)\} = \{1, 2, 3\}$ and 3 buyers submit orders at their truthful valuations $\{p(y_1), p(y_2), p(y_3)\} = \{2, 3, 4\}$. There are several ways to match pairs in the example. The most economical way of matching is to match from the best prices each other. In such a case, we obtain a following two matches $\{(1, 4), (2, 3)\}$. The sum of price spreads is $4 = (4 - 1) + (3 - 2)$. The number of matches is able to increase if we match from the highest prices of both sides such as $\{(3, 4), (2, 3), (1, 2)\}$. Even though the number of matches increases, the sum of price spreads is decreased from 4 to $3 = (4 - 3) + (3 - 2)$. This trade-off between the number of matches and the system efficiency is described in the following section.

6.5 Market Performance Indexes for Evaluation

As we have mentioned in Section 6.3, tthe market maker operates the market based w.r.t the rules that specify the policies. The challenge is to design market mechanisms by considering the balance of all policies. In our analysis, we adopt the following four performance indexes to evaluate market mechanisms:

- Transaction volume (TV),

- Allocation efficiency (AE),

- Convergence coefficient (CC),

- Transaction profit (PR).

Our aim is to show how these market performance indexes are affected by a combination of market policies. The above indexes cover the following three perspective, liquidity of the market (TV), the overall trader profits (AE), the stability of prices (CC), and the profit of the market maker (PR). In the followings, we formally represent each market performance index.

6.5.1 Transaction Volume

We observe liquidity of a market according to transaction volume (TV). The transaction volume is obtained at the time of clearing. In such timing, the transaction volume is number of match M,

$$TV = |M|.$$

The number of transaction indicates how actively traded in the market. As shown later, the transaction volume is limited to evaluate the performance of the market maker, since it is not correlated to the aspect of the profit and social welfare. However, this criterion is a powerful measure to check the low performance of the market. As long as the transaction volume is significantly small, the other indexes also have poor results. An advantage to use transaction volume is the easiness to check the performance.

6.5.2 Allocation Efficiency

The second index measures the efficiency of the system output. Matching pairs from the given set of orders is frequently referred to as allocation. The most efficient case of matching with respect to social welfare is $V_{\hat{M}}$ shown in the previous section, where $\hat{M}$ is a matching that include all the intra-marginal traders at competitive equilibrium. The allocation efficiency (AE) is measured by a ratio of actual allocation per the most efficient one as follows,

$$AE = \frac{\sum\limits_{(x,y)\in M} \left(v_{\Im(y)} - v_{\Im(x)}\right)}{\sum\limits_{(x,y)\in \hat{M}} \left(v_{\Im(y)} - v_{\Im(x)}\right)} 100.$$

According to allocation efficiency, we are able to evaluate overall efficiency. This index provides significant information. Notice that, in real markets, it is not possible to evaluate this ratio since the valuations of the traders are unavailable. Therefore, this index is an advantage of the experimental approach.

6.5.3 Convergence Coefficient

The third index is convergence coefficient (CC) proposed by Smith [111]. The convergence coefficient indicates how clearing prices are spread out from the competitive equilibrium price $\hat{p}$ as follows,

$$CC = \frac{100}{\hat{p}} \sqrt{\frac{\left(\sum\limits_{(x,y) \in M} \left(\mathfrak{p}(x,y) - \hat{p} \right) \right)^2}{n}}.$$

The convergence coefficient is calculated from the ratio of the standard deviation of the transaction price to the competitive equilibrium price. According to this index, the volatility of the price in the market is observed. In other words, this index indicates the traders' uncertainties about their profits in the future. If CC is closed to 0, most transactions are cleared at a value closed to the competitive equilibrium price. Hence, the pricing policy strongly influences this index.

If CC is closed to 0, the most transactions are executed closed to the competitive equilibrium price. Since clearing price $\mathfrak{p}(x,y)$ is a key variable in CC, the pricing policy strongly influences this index. Therefore, convergence coefficient is an advantage of the experimental approach similarly to allocation efficiency.

6.5.4 Transaction Profits

The fourth index we use is the sum of transaction profits (PR) as defined follow,

$$PR = \sum_{(x,y) \in M} \left(p(y) - p(x) \right).$$

The index PR is the sum of the bid-ask spread in a match M. This amount represents the chargeable amount for the transaction fee. At the same time, it also shows

the maximum transaction profits for the traders. Meanwhile, this is the maximum transaction profits for the traders. No matter how the market institution charges fee, the total profit is obtained from this criterion.

This section introduces our market performance indexes to evaluate market mechanisms. These indexes cover both the market makers perspective and the traders' perspectives.

6.6   Summary and Discussion

In this chapter, we have presented our model to design market mechanisms for autonomous trading agents. We have classified traditional auction mechanisms based on [127, 41].

In order to investigate how market mechanisms influence market behaviours, we decompose a market mechanism into several fundamental market policies. We formally represent each policy that makes possible to analyse the effect of policies. This approach is similar to [94], however they only present specific policies.

With respect to evaluations of market mechanisms, we do not focus on a single type of performances as a design goal like allocation efficiency [116] or profit of the traders [100]. As we have mentioned in Section 6.5, our evaluation criteria covers the views of market makers and traders. Niu et al. [94] take similar approach to ours, however our approach is different. The main difference is our market performance indexes cover the profit of the market makers and the transaction volumes in addition

to allocation efficiency and convergence coefficient. In the CAT competition, market share, profit share ,and transaction success rate are used to evaluate how specialists (market makers) perform well to attract traders with earning profits and an accurate estimation to accept incoming orders [93, 91, 123]. In our experiment, we focus on a single market maker, therefore we do not consider market share and profit share.

In this chapter, we have briefly observed how evaluation criteria correspond to market policies. However, it is not sufficient, since the traders behaviours are significantly related to the market performances. In the experiments, the traders bidding strategies are taken into account in the experimental design.

Chapter 7

Experimental Analysis of Market Mechanisms

Based on the market model in the previous chapter, we design and conduct an experiment to show how market mechanisms influence market behaviours of autonomous trading agents in double auction in this chapter.

7.1 Background

In our experiment, we control both market policies and strategies of autonomous trading agents. We use several market policies based on a framework presented in Section 6.3. In the following, we consider three types of biddig strategies; that is random traders, market adaptive traders, and human-like experimental traders. We detail these strategies in this chapter.

In our experiment, we are particularly interested in the following questions:

- How the number of rounds gives opportunities to the autonomous trading agents to learn the market situations?

- How specific market policies influence market performances?

- How combinations of specific market policies influence market performances?

- Which types of bidding strategies are influences by specific market policies?

To investigate the above questions, we develop our market mechanisms on a CAT platform which is used in Trading Agent Competition in Market Design competition (TAC-MD or CAT) in [93]. We use three types of autonomous trading agents embedded in the CAT platform where the parameters are based on the settings of the CAT-2008.

This chapter is organised as follows. In Section 7.2, we present specific market policies. In Section 7.3, we present three types of bidding strategies of autonomous trading agents. In Section 7.4, we present our experimental design. We design five types of experiments to investigate how market mechanisms influences market performances. Then, in Section 7.5, we summarise the experimental results. Finally, in Section 7.6, we describe the discussions of the results and related work.

7.2 Implementation of Market Mechanisms

This section presents specific market policies based on the classification of market policies with formal representations in Section 6.3.2.

7.2.1 Matching Policy

Matching policies determine feasible matched pairs between sell orders and buy orders as we have shown in the definition of matching policy in Definition 6.3.2 in page 122. Since matching policies are relevant to all the evaluation criteria presented as we have mentioned in Section 6.5, matching policy is a key function of the market makers. A well-known algorithm is *4-heap algorithm*, proposed by Wurman et al. in [128], which generates efficient and stable matches. The key idea of the 4-heap algorithm is to make matches between the best prices. As soon as a new order comes, the 4-heap algorithm updates the current matched set and the current unmatched set. The 4-heap algorithm contains four different types of heaps,

- X_{in}^{ask}: Contains all of the ask orders in the current matched set prioritised by maximal price.

- X_{out}^{ask}: Contains all of the ask orders in the current unmatched set prioritised by minimal price.

- X_{in}^{bid}: Contains all of the bid orders in the current matched set prioritised by minimal price.

- X_{in}^{bid}: Contains all of the bid orders in the current unmatched set prioritised by maximal price.

With these heaps, the 4-heap algorithm operates matching as follows. Let x_{in}^{ask}, x_{out}^{ask}, x_{in}^{bid}, and x_{out}^{bid} be the top nodes of X_{in}^{ask}, X_{out}^{ask}, X_{in}^{bid}, and X_{out}^{bid}, respectively. When a new ask order x_{new}^{ask} comes, the algorithm operates one of the following actions. Either

the new order is in a matched set and the best order in X_{out}^{bid} is in a matched set, or it displaces an ask order in X_{in}^{ask}, or it is placed into X_{out}^{ask}. This operation is shown in the following Pseudocode,

if $p(x_{new}^{ask}) \leq p(x_{out}^{bid})$ **and** $p(x_{in}^{ask}) \leq p(x_{out}^{bid})$ **then**
    put x_{new}^{ask} in *heap* X_{in}^{ask}
    $x_{tmp}^{bid} \leftarrow$ **get** 'top heap' from *heap* X_{out}^{bid}
    put x_{tmp}^{bid} in *heap* X_{in}^{bid}
else if $p(x_{new}^{ask}) \leq p(x_{in}^{ask})$ **then**
    $x_{tmp}^{ask} \leftarrow$ **get** 'top heap' from *heap* X_{in}^{ask}
    put x_{tmp}^{ask} in *heap* X_{out}^{ask}
    put x_{new}^{ask} in *heap* X_{in}^{ask}
else
    put x_{new}^{ask} in *heap* X_{out}^{ask}

Similar operation is executed for the incoming bid orders. The algorithm always tries to make pairs if there are feasible pairs. The order at the lowest ask price is matched with the order at the highest bid price. In other words, the 4-heap algorithm makes matches from the largest bid-ask spreads. Therefore, the 4-heap algorithm maximises the criterion PR.

7.2.2   Pricing Policy

Pricing policies determines a clearing price for any feasible matched pair as we have shown in the definition of pricing policy in Definition 6.3.3 in page 123.

A representative unified pricing is the following *mid-point pricing policy*:

Definition 7.2.1. Let M be a match generated by 4-heap algorithm. Let $x_{in,l}^{ask}$ and

$x_{in,l}^{bid}$ be the bottom heaps of the matched sell orders and the matched buy orders. For any $(x, y) \in M$, the mid-point pricing policy sets a clearing price as follows, $\mathfrak{P}(x, y) = \left(p(x_{in,l}^{ask}) + p(x_{in,l}^{bid}) \right) / 2$.

The mid-point pricing policy entails that the clearing price is unique for all pair in the match. As clearing prices, the median of the prices of the matched pairs is taken. Therefore, the mid-point pricing policy is robust to the extreme orders.

In contrast to CHs, CDAs clear their markets continuously. Therefore, the individual pricing is significant in CDAs. We present two typical pricing policies of the individual pricing. The simplest policy in the individual pricing is *k-pricing policy* defined as follows,

Definition 7.2.2. Given any match $(x, y) \in M$ and parameter $k \in [0, 1]$, k-pricing policy sets the following clearing price $\mathfrak{P}(x, y) = kp(x) + (1 - k)p(y)$.

Under k-pricing policy, the clearing price is a weighted average of the bid price and the ask price of the pair. The parameter k controls the weight of the bid order and the ask order for the clearing price. An interesting question is which value is proper for the parameter k. In practice, $k = 0.5$ is selected in most cases for the stability purpose. The experimental results by Phelps [99] support that $k = 0.5$ is the most efficient parameter in CDAs in a steady market situation.

Another well-known pricing policy of the individual pricing is volume weighted average pricing (VWAP) policy which is implemented in Euronext [59]. Contrary to k-pricing policy, VWAP policy is a history-based pricing policy defined as follows.

Definition 7.2.3. Let M' be the set of matched pair in a given fixed time interval.

The clearing price under *volume weighted average pricing (VWAP)* is determined as follows, $\mathfrak{P}(x, y) = \sum\limits_{(x',y')\in M'} \frac{(p(x')+p(y'))/2}{q(x',y')}$, where q is an order quantity for a match.

The clearing price is determined by the weighted moving average in the fixed-time interval. Since VWAP used historical data, randomness of the incoming orders has less impact on the volatility of the clearing price compared to k-pricing policy. Niu, et al. [94] conduct experiments to compare the performance of k-pricing policy and VWAP. Their results support that VWAP is able to clear with less volatility than k-pricing policy.

7.2.3   Clearing Policy

Keeping orders increase chances to match them, and thus it is a way to generate more outcome. However, if the clearing policy has a too long clearing cycle, then the market maker will loss some opportunities. Therefore, it is important to characterize how the performance is influenced by a clearing policy.

Typical clearing policies are classified into two types,

- continuous clearing,

- periodic clearing.

The continuous clearing policy is used in continuous double auction. As soon as the market institution finds matched orders, the market is cleared. Under continuous clearing policy, order timing has a greater significance than prices of orders.

The market is cleared periodically under periodic clearing policy. For instance,

Arizona stock exchange clears the market daily and Euronext uses twice a day clearing policy (round clearing). The orders in the time same bucket are prioritised by the prices rather than order timing.

Since we assume that the traders submit orders round-based, we use round-based clearing policy to represent periodic clearing policies.

7.2.4 Accepting Policy

Accepting policies rule how a market institution accepts incoming orders. As a prerequisite, the market institution expects that the traders of the rejected orders resubmit the orders at an improved price. Therefore, the accepting policy corresponds to the future price of the traders. The accepting policies are relevant to all the evaluation criteria presented in Section 6.5.

One of the most well-known accepting policies is *quote-beating accepting policy* used at NYSE.

Definition 7.2.4. Let X_{out}^{ask} and X_{out}^{bid} be the heaps of the current unmatched ask orders and bid orders, respectively. Let x_{out}^{ask} and x_{out}^{bid} be the top heaps. For any incoming order x_{new}, *quote-beating accepting* policy satisfies the following accepting rule:

$$
\mathfrak{A}(x_{new}) = \begin{cases} \text{true} & \begin{cases} \text{if } \left(\mathfrak{I}(x_{new}) \in S \text{ and } p(x_{new}) < p(x_{out}^{ask}) \text{ or } \nexists x_{out}^{ask}\right) \text{ or} \\ \\ \left(\mathfrak{I}(x_{new}) \in B \text{ and } p(x_{new}) > p(x_{out}^{bid}) \text{ or } \nexists x_{out}^{bid}\right) \end{cases} \\ \\ \text{false} & \text{otherwise.} \end{cases}
$$

Under quote-beating accepting policy, the market institution accepts a new order if it exceeds the current best price among the unmatched orders. Quote-beating accepting policy is mainly used in CDAs to reflect the current market situation.

Niu et al. [94] point out that quote-beating accepting policy frequently fail to reduce the fluctuation of clearing price, since current unmatched orders heavily depends on randomness of individual orders. In order to reduce the fluctuation, they propose the history-based approach as follows.

Definition 7.2.5. Let $\tilde{p}$ be the expected equilibrium price. For given adjustment parameter δ, the set of accepting conditions as follows is called *equilibrium-beating accepting* policy,

$$
\mathfrak{A}(x_{new}) = \begin{cases} \text{true} & \begin{cases} \text{if } (\mathfrak{I}(x_{new}) \in S \text{ and } p(x_{new}) \leq \tilde{p} + \delta) \text{ or} \\[1em] (\mathfrak{I}(x_{new}) \in B \text{ and } p(x_{new}) \geq \tilde{p} - \delta) \end{cases} \\[2em] \text{false} & \text{otherwise.} \end{cases}
$$

According to the experiment in [94], this policy successfully reduces the volatility of clearing price for the randomly price shouting traders under CDA. They show how parameter δ works for the convergence of fluctuations of the clearing price. The problem of this policy is how to estimate the expected equilibrium price $\tilde{p}$ and to set the adjustment parameter δ.

We propose a new accepting policy *learning-based accepting policy* based on the

learning approach similarly to the *linear reward-inaction* algorithm developed by Hilgard and Bower [53]. The key concept of this policy is that the decision of acceptance is based on a probability which is function of the success of the past transactions. If an accepted order at price p has been successfully transacted, then next incoming order at price p will be evaluated with a higher probability. At the opposite if an accepted order at price p has not been matched, then next order at price p will be evaluated with a lower probability. Hence, the key point is that we assume that past experiences can entailed the future decisions (about the acceptance). Let $L(p)$ be a learning function that assigns the expected matching success rate to each price p. Let $L^{ask}(p)$ and $L^{bid}(p)$ be the learning function for ask orders and bid orders. With these functions, we define the learning based accepting policy as follows.

Definition 7.2.6. Let U $= [0, 1]$ be a uniform distribution function. Let Prob(U) be a randomly chosen probability from distribution U. Let x_{new} be an incoming order. The following set of accepting conditions is called *learning-based accepting policy*,

$$\mathfrak{A}(x_{new}) \begin{cases} \text{true} & \begin{cases} \text{if } \mathfrak{I}(x_{new}) \in S \text{ and Prob(U)} \leq L^{ask}(p(x_{new})) \text{ or} \\[2ex] \mathfrak{I}(x_{new}) \in B \text{ and Prob(U)} \leq L^{bid}(p(x_{new})) \end{cases} \\[4ex] \text{false} & \text{otherwise,} \end{cases}$$

where $L^{ask}(p)$ and $L^{bid}(p)$ are updated according to the following three types of algorithms with initial parameter α, price ceiling p^{max}, step parameter ν, update parameter λ, and the lower bound δ:

At the beginning, the learning functions are initialised for all price $p \in [0, p^{max}]$ at α

with step ν as follows,

Initialisation

$\quad p := 0$

$\quad$**while** $p \leq p^{max}$

$\qquad L^{ask}(p) := \alpha$

$\qquad L^{bid}(p) := \alpha$

$\qquad p := p + \nu$

As clearing occurs, for all $(x, y) \in M$, $L^{ask}(p)$ and $L^{bid}(p)$ are are increased w.r.t. λ

where for all p such that $p \geq p(x)$ and $p \leq p(y)$, respectively, as follows,

Clear Market

$\quad$**for** each $(x, y) \in M$

$\qquad p := 0$

$\qquad$**while** $p \leq p(x)$

$\qquad\quad L^{ask}(p) := \min\left\{L^{ask}(p) + \lambda, 1\right\}$

$\qquad\quad p := p + \nu$

$\qquad p := p(y)$

$\qquad$**while** $p \leq p^{max}$

$\qquad\quad L^{bid}(p) := \min\left\{L^{bid}(p) + \lambda, 1\right\}$

$\qquad\quad p := p + \nu$

As order expiration occurs, for all unmatched ask order x, $L^{ask}(p)$ is decreased at λ

such that $p \geq p(x)$; for all unmatched bid order y, $L^{bid}(p)$ are decreased w.r.t. λ such

that $p \leq p(y)$ as follows,

Order Expiration

    for each $x \in X_{out}^{ask}$

      $p := p(x)$

      while $p \leq p^{max}$

          $L^{ask}(p) := \max \left\{ L^{ask}(p) - \lambda, \delta \right\}$

          $p := p + \nu$

      $p := 0$

      while $p \leq p(y)$

          $L^{bid}(p) := \max \left\{ L^{bid}(p) - \lambda, \delta \right\}$

          $p := p + \nu$

To avoid a wrong convergence, we set the lower bound δ to control λ. Once the value of learning function $L(p)$ reaches delta, we shift lambda factor to a smaller valeue be small. A similar proposal has been made by Narendra and Thathachar [87]. Unlike the linear reward-inaction algorithm, our setting keeps monotonicity for the learning functions with respect to price. An advantage of this policy is adaptiveness of the market situations. However, this policy requires certain time periods to learn the history.

7.3   Bidding Strategies of Traders

As we have mentioned in Section 6.3, we assume that valuation of each trader is assigned; it is fixed. Each trader sets a limit price based on its pricing strategy. This section introduces three automated bidding strategies, *zero-intelligence with constraint* (ZI-C), *zero intelligence plus* (ZIP), and *Roth and Erev* (RE) that are used in our experiments. These strategies have been implemented on the CAT platform. These

automated bidding strategies have the following characteristics,

- ZI-C: random strategy with considering its valuation,

- ZIP: learning-based strategy with reactive to market information,

- RE: reinforcement learning strategy mimicking human-like behavior.

These three bidding strategies have a similar structure to determine its limit price according to the valuations and mark-ups (margins) calculated by the bidding strategies. Trader i sets the limit price for order x_i according to its valuations and mark-up ξ_i,

$$p(x_i) = \begin{cases} v_i + \xi_i & \text{if } i \in S \\ v_i - \xi_i & \text{otherwise} \end{cases}$$

In the following, we detail how mark-ups are obtained according to the automated bidding strategies.

7.3.1 ZI-C (Zero Intelligence with Constraint) Strategy

ZI-C strategy is proposed by Gode and Sunder [47] which chooses a mark-up randomly from uniform distribution. Given a maximum mark-up ξ_i^{max}, under ZI-C strategy, trader i sets its limit price $p(x_i)$ as follows

$$p(x_i) = \begin{cases} v_i - \xi_i & \text{if } i \in B \\ v_i + \xi_i & \text{otherwise} \end{cases}$$

where ξ_i is randomly drawn from a uniform distribution ranging in $[0, \xi_i^{max}]$. This strategy is independent from the market situations.

7.3.2 Zero-Intelligence Plus (ZIP) Strategy

While ZI-C strategy ignores the state of the market and past experience, Zero-Intelligence Plus (ZIP) strategy is a representative learning-based bidding strategy designed by Cliff and Bruten [20, 22]. ZIP strategy is reactive to market information and intends to be competitive in the market. Under ZIP strategy, mark-up is adaptive to the market situation which is followed by the adaptive rules for sellers,

if last shout x_j is accepted and transacted with y
    if any seller $i \neq j$ such that $p(x_i) \leq \mathfrak{P}(x_j, y)$
        should raise its mark-up towards $\mathfrak{P}(x_j, y)$
    else any active seller
        should decrease its mark-up towards $\mathfrak{P}(x_j, y)$
else any active seller $i \neq j$ should decrease its mark-up towards $p(x_j)$

The adaptive rules for buyers are similar ways. Under the above adaptive rules, the mark-up is determined as follows,

$$\xi_{i,t} = p(x_{i,t}) + \Gamma_{i,t} - v_i,$$

where $\Gamma_{i,t}$ is a momentum-based update function such that

$$\Gamma_{i,t} = \begin{cases} 0 & t = 0 \\ \gamma_i \Gamma_{i,t-1} + (1 - \gamma_i)\delta_{i,t} & \text{otherwise}, \end{cases}$$

and γ_i is a momentum coefficient and $\delta_{i,t}$ is the Widrow-Hoff delta value [126] which converges to the target price $\tau_{i,t}$ with learning rate η_i determining the rate of convergence,

$$\delta_{i,t} = \eta_i(\tau_{i,t} - p(x_{i,t-1})) \tag{7.3.1}$$

and the target price $\tau_{i,t}$ is determined by the following stochastic function of price $p(z)$,

$$\tau_{i,t} = R_{i,t}p(z) + A_{i,t}$$

where $p(z)$ is either transaction price $\mathfrak{P}(x_j, y)$ or limit price $p(x_j)$, $R_{i,t}$ sets a relative perturbation, and $A_{i,t}$ sets an absolute perturbation. The relative perturbation ensures to be competitive for the large $p(z)$, while the absolute perturbation ensure to be profitable for the small $p(z)$.

ZIP strategy is adaptive to market situations and requires public market information for the updates. According to an experiment with human subjects and autonomous trading agents conducted by Das et al. [31], ZIP traders behave more efficiently than the human traders in CDA.

7.3.3 Roth-Erev (RE) Strategy

Roth-Erev (RE) strategy is based on RE learning algorithm [107], a representative reinforcement-learning algorithm, which relies only on direct feedbacks from the mechanism. Notice that ZI-C strategy does not utilise any market information and ZIP strategy rely on the release of market information including other traders.

Under RE strategy, a mark-up of trader i at time t is randomly drawn from discrete choices ranging $[0, K)$ where K is the number of choices. The way to choose a mark-up is updated according to a direct experience which consists of choice $\kappa \in [0, K)$, experiment parameter θ, payoff of trader i at time t with choice κ, and recency

parameter ω. At time t, the probability of mark-up $\rho(\xi_{i,t})$ is updated as follows:

$$\rho(\xi_{i,t}) = \frac{\chi_{i,t,\kappa}}{\sum\limits_{\kappa} \chi_{i,t,\kappa}}$$

where $\chi_{i,t,\kappa}$ is a propensity function given as follows,

$$\chi_{i,t,\kappa} = \begin{cases} \frac{\nu_i}{K} & t = 0 \\ (1-\omega)\chi_{i,t-1,\kappa} + \Gamma(\theta, r(i, t-1, \kappa)) & \text{otherwise.} \end{cases}$$

At the initial time, propensities are even with scale parameter ν_i. Therefore, the choice κ is uniformly drawn for the mark-up. At time $t > 0$, the propensities are updated by the following experience function Γ, while the previous propensities are succeeded at the weight $1 - \omega$,

$$\Gamma(\theta, r(i, t-1, \kappa)) = \begin{cases} r(i, t-1, \kappa)(1-\theta) & \text{if } \kappa \text{ is a choice at t-1} \\ r(i, t-1, \kappa)\frac{\theta}{K-1} & \text{otherwise.} \end{cases}$$

The experience function consists of two factors, reflecting the direct feedback from the mechanism and including opportunities of other choices. RE strategy chooses mark-ups based on its past experiences and small experiments.

7.4 Experimental Design

To conduct our experiment, we present how we control market policies, strategies of traders and several different parameters.

7.4.1 Basic Settings

We implement our market mechanisms on the CAT platform and evaluate the performance based on the proposed evaluation criteria in Section 6.5. Each game in our

experiments consists of 20 trading days and each day consists of 1, 3 or 5 rounds according to the types of experiments. We set the entitlement for the trade as one. Therefore, each trader submit one unit of order in each day. To reduce the effects of randomness, we use the daily-based average of 20 trading days for 10 games to measure market performance indexes. In each game, there are 20 sellers and buyers, respectively. The valuations of the sellers and the buyers are prefixed as follows: $\{50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100, 105, 110, 115, 120, 125, 130, 135, 140, 145\}$. In this setting, we have the competitive equilibrium price \$97.5 and 10 pairs are corresponding number of matched pairs among the intra-marginal traders according to the analytic tools presented in Section 6.4. Each trader has a single bidding strategy either ZI-C, ZIP or RE.

7.4.2 Strategic Parameters of Trading Agents

In all experiment types, we set the strategic parameters for trading agents that follow the same setting as CAT-2008:

- ZI-C strategy

    - the uniform distribution range: $[0, 50]$

- ZIP strategy

    - the momentum coefficient: 0.5

    - the learning rate: 0.85

    - the uniform distribution for the relative perturbation: $[0, 0.05]$

– the uniform distribution for the absolute perturbation: $[0, 0.05]$

- RE strategy

    – the number of choices: 100

    – the experimentation parameter: 0.2

    – the recency parameter: 0.1

    – the scaling parameter: 1.0

7.4.3 Controlled Policies and Parameters

As a benchmark purpose, we present the following desirable policies,

- pricing policy: competitive equilibrium pricing policy,

- accepting policy: always accepting policy and equilibrium-beating accepting policy with the competitive equilibrium.

In our experiments, we have a unique competitive equilibrium in price. Under the competitive equilibrium pricing policy, the market is cleared at the closest price to the competitive equilibrium price among the feasible clearing prices. The *always accepting policy* never rejects incoming orders anytime. The equilibrium-beating accepting policy rejects incoming orders if ask orders exceed the competitive equilibrium price or bid orders are less than the competitive equilibrium price. Hence, the equilibrium-beating accepting policy is a filter to obtain intra-marginal traders.

Our experiment consists of the following five types of experiments.

- Experiment 1: Number of Round Effect
- Experiment 2: Pricing Policy Effect
- Experiment 3: Clearing Timing Effect
- Experiment 4: Accepting Policy Effect
- Experiment 5: Robustness of Accepting Policies

In Figure 7.4.3, we present the controlled elements such as market policies and bidding strategies in our experiments. In the following, we detail each experiments.

Experiments		Controlled Elements		Fixed Elements	
1	**Round Effect**	**Round**: 1, 3, 5	**Bidding Strategy**: - ZI-C - ZIP - RE	**Pricing**: Comp. Equilibrium **Accepting**: Always Accept. **Clearing**: Continuous	**Matching**: 4-Heap
2	**Pricing Policy Effect**	**Pricing**: - k-Pricing - VWAP - Mid-Point		**Accepting**: Always Accept. **Clearing**: Continuous **Round**: 5	
3	**Clearing Timing Effect**	**Clearing**: - Continuous - Round		**Pricing**: Comp. Equilibrium **Accepting**: Always Accept. **Round**: 5	
4	**Accepting Policy Effect**	**Accepting**: - Learning-Based - Quote-Beating - Always - Equilibrium-Beating		**Pricing**: Comp. Equilibrium **Clearing**: Continuous **Round**: 3	
5	**Robustness of Accepting Policies**	**Accepting**: - Learning-Based - Quote-Beating - Always - Equilibrium-Beating - Equilibrium-Beating (5)			

Figure 7.1: Controlled Variables in the Experiments

In **Experiment 1**, we intend to extract how number of rounds in each day has a positive effect. We consider three cases: 1 round/day, 3 rounds/day and 5 rounds/day. All policies are one of the most simplest: the competitive equilibrium pricing policy, the always accepting policy, and the continuous clearing policy. Hence, our intension is to test even a simple market mechanism improves market performances.

In **Experiment 2**, our interest is to find which performance indexes have a direct impact from specific pricing policies. In this experiment, we use three popularly used pricing policies: the k-pricing policy, the VWAP and the mid-point pricing policy. Since we consider the mid-point pricing policy, we use the round clearing condition in this experiment. For k-pricing policy, we use 0.5 as the weight parameter k, which takes the middle point as the clearing price in each pair. For VWAP, we use 20 as the number of past matches to generate the average. We use the always accepting policy. We set 5 rounds/day.

In **Experiment 3**, we investigate the effect of clearing timing. We compare the effect between continuous clearing and round clearing condition. We set 5 rounds/day. The rest of policies are the same as Experiment 1.

In **Experiment 4**, we investigate how accepting policy affects the market performances. We use four accepting policies: learning-based accepting policy, quote-beating accepting policy, always accepting policy and equilibrium-beating accepting policy. For the learning-based accepting policy, we use probability 1.0 as the initial values for any price. With respect to the update steps, we use two levels, a coarse level: 0.2 and a fine level: 0.01. We use the expected probabilities at 0.005 as a threshold for the change of two levels. If they are above the threshold the update

steps are based on the coarse level, otherwise the update steps are based on the fine level.

In **Experiment 5**, we extend the settings of Experiment 4 and we investigate robustness of accepting policies. In this experiment, we focus on how small fluctuations of clearing prices affect the market performances under different accepting policies. We use VWAP for the pricing policy which is a stable pricing however it is not as stable as the competitive equilibrium pricing policy. In this experiment, we include a special case of the equilibrium-beating accepting policy where the acceptance range is relaxed from the competitive equilibrium for $5.

7.5 Experimental Results

This section provides experimental results.

7.5.1 Experiment 1: Number of Round Effect

In Table 7.1, we present how the number of rounds in each day improves the market performances. In the case of 1 round per day, for all three types of traders, the simplest market mechanism does not provide appropriate learning period for ZIP and RE. For all performance indexes, ZI-C performs much better than ZIP and RE. In the case of 5 rounds per day, this simple mechanism boosts its all performances for all three traders. That is extended rounds provide extra opportunities for handling the unmatched orders. Moreover, it provides time to learn the market situation.

For ZIP, in the case of 3 rounds/day, CC is extremely worse than other cases. In

experiment 4 and 5, we further investigate the case of round 3.

Table 7.1: Round Effects

| | ZI-C | | | | ZIP | | | | RE | | | |
Num. of Round	PR	AE	CC	TV	PR	AE	CC	TV	PR	AE	CC	TV
1	11.95	78.74	9.04	6.98	2.65	44.58	6.41	4.50	5.41	39.52	9.98	3.37
3	13.78	92.14	8.86	8.77	10.80	73.45	10.27	6.72	10.75	72.99	10.34	6.71
5	14.91	94.82	8.58	9.29	7.03	93.76	6.37	9.62	11.90	85.27	10.42	8.01

A summary of experimental results of round effects (1, 3, and 5) for ZI-C (zero intelligence with constraint), ZIP (zero intelligence plus) and RE (Roth and Erev) traders evaluated by PR (profit), AE (allocation efficiency), CC (convergence coefficient) and TR (transaction volume) based on the average of 10 games consisting of 20 game days.

7.5.2   Experiment 2: Pricing Policy Effect

This section provides how pricing policies affects on the market performance indexes. In Table 7.2, we present the experimental results with the three different pricing policies: the mid-point pricing, the k-pricing policy, and the VWAP. The three pricing policies that we use are fundamentally similar property that searches for the middle point. Therefore, for all three types of traders, the performance indexes are almost the same, except for the convergence coefficient (CC).

As we have analysed in Section 7.2, pricing policies have direct relation on the CC theoretically. We observe a significant impact of pricing policies on CC for all three different bidding policies. Among the three policies, the VWAP converges well to the competitive equilibrium. A main reason is that the VWAP is able to calculate the average price from the greater samples compared to other two pricing policies. As a result, the VWAP provides stable prices that leads better performance on CC.

Table 7.2: Pricing Effects

Pricing Policy	ZI-C				ZIP				RE			
	PR	AE	CC	TV	PR	AE	CC	TV	PR	AE	CC	TV
MP	15.75	66.08	3.52	4.92	4.64	64.33	15.09	6.01	8.06	34.13	7.90	2.60
k	16.50	67.18	5.21	5.06	4.61	64.79	15.52	6.09	8.16	33.69	7.56	2.56
VWAP	16.50	67.35	1.84	5.06	4.57	63.70	12.72	6.03	7.25	32.38	4.69	2.45

A summary of experimental results of pricing effects (MP (mid-pricing policy), k (k-pricing policy), and VWAP (volume weighted average pricing)) for ZI-C (zero intelligence with constraint), ZIP (zero intelligence plus) and RE (Roth and Erev) traders evaluated by PR (profit), AE (allocation efficiency), CC (convergence coefficient) and TR (transaction volume) based on the average of 10 games consisting of 20 game days.

7.5.3 Experiment 3: Clearing Timing Effect

In this experiment, we investigate how clearing timing has impacts on the performance indexes. We consider two policies: continuous clearing and round clearing. In Table 7.3, we present the experimental results of these two.

We observe that there is a trade-off between two policies. For all three traders, the round clearing has greater profits for the market maker, while the continuous clearing makes higher allocation efficiency and transaction volumes. Therefore, the round clearing is beneficial to the market makers; the continuous clearing is beneficial to the overall traders.

Table 7.3: Clearing Timing Effects

Clearing Timing	ZI-C				ZIP				RE			
	PR	AE	CC	TV	PR	AE	CC	TV	PR	AE	CC	TV
Continuous	14.91	94.82	8.58	9.29	7.03	93.76	6.37	9.62	11.90	85.27	10.42	8.01
Round	22.51	94.01	2.09	8.28	13.03	82.84	10.50	8.22	18.03	80.18	4.63	6.86

A summary of experimental results of clearing timing effects (continuous clearing and round-based clearing)) for ZI-C (zero intelligence with constraint), ZIP (zero intelligence plus) and RE (Roth and Erev) traders evaluated by PR (profit), AE (allocation efficiency), CC (convergence coefficient) and TR (transaction volume) based on the average of 10 games consisting of 20 game days.

7.5.4 Experiment 4: Accepting Policy Effect

In Experiment 4, we show how accepting policies improve market performances. We use four policies: the learning-based accepting policy, the always accepting policy, the quote-beating accepting policy and the equilibrium-beating accepting policy. The result is presented in Table 7.4.

Compared to always accepting policy, learning-based accepting policy improves the profit of the market maker (PR) and allocation efficiency (AE) for learning-based bidding strategy (ZIP and RE), however convergence coefficient (CC) is increased. Hence, learning-based accepting policy navigate properly to the learning-based bidding traders that improve their bidding prices. For ZIP traders, learning-based accepting policy improves PR and AE compared to quote-beating accepting policy. However, learning-based accepting policy has similar effect on PR, AE and TV for ZI-C traders compared to quote-beating accepting policy. A disadvantage of learning-based accepting policy is fluctuations of the clearing prices. Since learning process may accept some orders from the extra-marginal traders, the cost of learning is affected on the high convergence coefficient.

Equilibrium-beating accepting policy is a benchmark accepting policy that extracts orders from the intra-marginal traders. For all bidding strategies, it successfully gains high profits for the market institution with small fluctuations on the clearing price. Hence, the equilibrium-beating accepting policy is a stable and efficient policy for the market institution. However, it is not always preferable for the traders, since it reduces the traders overall profit (AE) and transaction volume (TV).

Table 7.4: Accepting Policy Effects

Accepting Policy	ZI-C				ZIP				RE			
	PR	AE	CC	TV	PR	AE	CC	TV	PR	AE	CC	TV
L-Based	14.80	91.99	13.23	8.75	11.39	75.00	14.09	6.75	11.06	74.36	14.46	6.74
Always	13.78	92.14	8.86	8.77	10.80	73.45	10.27	6.72	10.75	72.99	10.34	6.71
QB	14.76	92.07	8.79	8.78	10.41	72.43	9.77	6.57	10.98	74.59	10.12	6.69
EB	21.92	84.95	0.44	6.84	15.81	60.15	0.49	4.70	16.10	61.67	0.28	4.80

A summary of experimental results of accepting policy effects (L-based (learning-based accepting policy), always accepting policy, QB (quote-beating accepting policy) and EB (equilibrium-beating accepting policy)) for ZI-C (zero intelligence with constraint), ZIP (zero intelligence plus) and RE (Roth and Erev) traders evaluated by PR (profit), AE (allocation efficiency), CC (convergence coefficient) and TR (transaction volume) based on the average of 10 games consisting of 20 game days.

7.5.5 Experiment 5: Robustness of Accepting Policies

This experiment aims to investigate robustness of accepting policies to the fluctuations of the clearing prices. In contrast to Experiment 4, we use relatively stable pricing policy, the VWAP. The results of the experiment is presented in Table 7.5.

Table 7.5: Robustness of Accepting Policies

Accepting Policy	ZI-C				ZIP				RE			
	PR	AE	CC	TV	PR	AE	CC	TV	PR	AE	CC	TV
L-Based	15.12	92.67	7.89	8.68	2.42	93.82	2.40	9.36	11.27	74.38	9.22	6.64
Always	14.40	92.31	8.94	8.89	5.92	84.84	6.60	8.49	10.59	73.38	10.08	6.64
QB	14.09	91.63	9.08	8.78	2.34	92.27	2.95	9.24	10.49	72.79	10.43	6.67
EB	22.13	85.66	1.33	6.93	1.24	7.84	4.14	0.70	15.80	61.60	1.45	4.77
EB(5)	20.15	90.85	1.99	7.91	2.33	90.05	2.30	8.99	15.28	67.58	2.24	5.45

A summary of experimental results of accepting policy effects with VWAP policy (L-based (learning-based accepting policy), always accepting policy, QB (quote-beating accepting policy), EB (equilibrium-beating accepting policy), and EB(5) (equilibrium-beating accepting policy with $ 5 slack)) for ZI-C (zero intelligence with constraint), ZIP (zero intelligence plus) and RE (Roth and Erev) traders evaluated by PR (profit), AE (allocation efficiency), CC (convergence coefficient) and TR (transaction volume) based on the average of 10 games consisting of 20 game days.

According to this experiment, the learning-based accepting policy improves all the market performances for all three bidding strategies compared to the quote-beating accepting policy and the always accepting policy in contrast to the Experiment 4.

Under the learning-based accepting policy, accepting probabilities of orders are adjusted by the learning rules. Therefore, the learning-based accepting policy is a robust policy for the volatility of the clearing prices from the competitive equilibrium price.

An interesting observation is that the equilibrium-beating accepting policy is no longer high-performing mechanism for ZIP traders if the clearing prices are fluctuated. The failure occurs when the clearing price is unbalanced from the competitive equilibrium. In such a case, the strict rejection and the unbalance of the pricing policy make the trader of one-side stop submitting orders, since there is no way to improve.

We also present a case where the rejection range has $5 slack from the competitive equilibrium price. The slack makes the equilibrium-beating accepting policy reduces the failure of match making. However, to set a proper slack is an issue for the equilibrium-beating accepting policy.

7.6 Discussion and Related Work

This chapter presents the implementation, the experimental design, and the experimental results with respect to market mechanisms for autonomous trading agents on the CAT platform. We present several specific market policies and a combination of these policies work as a market mechanism. In the experiments, we have controlled both the market policies and the bidding strategies of the traders. Our findings from the experiments are:

1. *Number of rounds in synchronised double auction improves the overall market*

performances for all three types of agents. The result of Experiment 1 is similar to the result of Cason and Friedman [15] where multiple rounds of clearinghouse improves the market performance with human traders. In supply chains, several rounds in a trading day require high synchronisations between the market makers and the traders that are costly for human agents. As a result, fully automated trading agents are suitable to implement such market mechanisms.

2. *Pricing policies have direct impacts on fluctuations of clearing prices.* The result in Experiment 2 extends to the case of ZIP traders and RE traders from Niu et al. [94] that consider ZI-C traders and GD traders, a belief based bidding strategy designed by Gjerstad and Dickhaut [46]. Furthermore, there is a mutual effects with accepting policies according to the results of Experiment 4 and 5. Equilibrium-beating accepting policy is sensitive to the fluctuations of the clearing prices. For ZIP traders, this sensitivity is critical for all the market performances.

3. *Clearing timing characterises the properties of market mechanisms.*

4. *Accepting policy improves some market performances relying on the bidding strategies of traders.* While round clearing has a positive effect on the profit of the market makers, continuous clearing has a positive effect on the profit of the overall traders and the transaction volume. Since 4-heap engine tends to make matches among the intra-marginal traders, the clearing before the end of the rounds reduces the profit of the market maker. However, it increases the transaction volume. Similar trade-off is briefly mentioned in [123, 92]. In this

thesis, we explicitly present the trade-offs in the experimental results.

5. *Our proposing accepting policy, learning-based accepting policy, is robust to the fluctuation of the clearing prices.* This characteristic is significant for the implementation. Since market makers are not able to obtain information about the equilibrium price, the clearing prices may have some fluctuations.

Agent-based computational economics (ACE) is an emerging area. We use CAT platform, one of the most representative market mechanism design platforms, to implement market mechanisms and to conduct our experiments. There are a number of research with CAT platform. Niu et al. [92] analyse the market mechanisms of the eight participant of the first CAT competition (CAT-2007). With respect to market performances, they present the typical trade-offs between the characteristics of the market policies, such as profit maximising matching or volume maximising matching. Niu et al. [91] compare the participants of the CAT-2007 in a bilateral way. Vytelingum et al. [123] propose a sophisticated market mechanism in CAT-2007 which consists of equilibrium-beating accepting policy with slacks, round-based clearing, and side-biased pricing (k-pricing with weighted by number of sellers and buyers). With respect to equilibrium-beating accepting policy with slacks, they use moving average and variance of the transaction price. This policy is similar to Niu et al. [94], however their approach is based on a static slack. Adjustment of k-pricing policy is important in a CAT competition, since a number of buyers and sellers that come to each market are dynamic. That is supply and demand schedule can be dynamic in

a local market. Similar treatment is proposed by Petric et al. [97]. In our experiment, the supply and demand schedule had been symmetric. Hence, we have only considered static k-pricing policy and fixed slacks for equilibrium-beating accepting policy in our experiment. Gruman and Narayana [50] proposes a method to estimate the type of automated bidding strategies according to the bidding behaviours. This approach is effective if the mechanism is adaptive to the types of bidding strategies of market participants.

One stream of ACE is design of autonomous trading agent itself. Typical research fixes market mechanisms and evaluates performance of agents according to profit of agent. Gode and Sunder [47] propose ZI strategy presented in Section 7.3.1 which is a random strategy normally used as a benchmark strategy. It is observed that ZI-C traders performed better than human traders in the experiment, even ZI-C traders do not have intension to maximise its profit which is a fundamental assumption for economists in the past. This result motivates researchers to develop more intelligent bidding agents. Cliff et al. [20, 22, 23, 21] design ZIP, presented in Section 7.3.2, and its variants for CDA. Preist and van Tol [102] extend ZIP strategy to clearinghouses. Roth and Erev [107] design RE strategy, presented in Section 7.3.3, and Nicolaisen [90] modifies RE with the experience function. Gjerstad and Dickhaut [46] propose a belief based bidding strategy that an agent builds to indicate whether a particular order price is likely to be accepted in the market and Tesauro and Bredin [115] modifies the strategy of Gjerstad and Dickhaut based on dynamic programming.

With these trading agents, there is some research that compares their performances mainly measured by the payoffs of agents. The difference from our approach is the following researches focuses on improving agent's strategy rather than improving market performances according to market mechanisms. Hence, the evaluations are made by the performance of agents rather than the market performances. Gode and Sunder [47] present that even ZI performs well similarly to human traders under CDA. Cason [14] compares human traders and adaptive ZI, random bidding range is updated by the market information in the previous round, in repetitive clearinghouse where they have similar performances. Das et al. [31] show that ZIP and GD perform better than human traders in CDA with k-pricing policy and quote-beating accepting policy. Tesauro et al. [116] compare the performance of ZI, Kaplan, ZIP and GD in CDA with k-pricing policy and quote-beating accepting policy where ZIP and modified GD have the better performances. Walsh et al. [124] consider to choose an autonomous trading agent among ZIP, GD and other strategies in CDA as a strategic choice. They investigate equilibria of strategic choices that are dependent of the choices of other agents. Phelps et al. [100] analyse strategic equilibria of choices of agents similarly to [124] in CDA.

Chapter 8

Conclusions

This thesis investigates the problem of market mechanism design for supply chain management and e-marketplace development. Instead of viewing a market as an isolated entity, we consider each market to be a part of a supply chain and study the effects of interactions between different markets in the supply chain.

8.1 Summary of Contributions

This thesis has made the following contributions to the research of market mechanism design for supply chain management:

1. *Introduced a methodology of market mechanism design that views the mechanism design problem as a supply chain management problem.* We have focused on three important issues: (i) the capacity allocation problem, (ii) the online coordination problem, and (iii) the market design problem for autonomous trading agents, in relation to market mechanism design.

2. *Extended the capacity allocation game to competitive supply chain models.* We have also extended traditional economic analysis to the capacity allocation problem in supply chains. We have found that the proportional allocation, one of the most commonly used allocation mechanism in practice, is sensitive to supply chain settings such as market rules in the downstream market and cost structures of intermediaries. We have designed a new allocation mechanism for supply chains with quantity competition; we have proved that it is a truth-telling mechanism. We have provided a comprehensive analysis of the relation between the properties of allocation mechanisms and pricing strategies of supply chain members.

3. *Introduced an approach to the modelling of online markets as supply chains.* We have designed coordinating contracts in online markets to induce alliance members make strategic decisions to maximise the overall profit rather than the individual profit. We have proved that the fixed-fee contract achieves coordination while the revenue-sharing contract does not achieve coordination if we do not consider the effect of advertisement. We have shown that fixed-fee contracts no longer achieve coordination if the effect of advertisement is taken into account. We have design a new online market contact based on the idea of profit-sharing and prove that it achieves coordination with advertisement.

4. *Introduced a formal description of market mechanisms with double auction for autonomous trading agents.* We have presented a formal framework of market policies, such as accepting policies, matching policies and pricing policies

based on double auction mechanisms. Based on the framework, we have described a variety of specific market policies such as the k-pricing policy and the quote-beating accepting policy. We have combined a number of different market policies to test how these policy influence market behaviours.

5. *Conducted a set of experiments to test the relationship between market mechanisms and trading strategies.* We have found that market performances, which are the profit of the market maker, the transaction volumes and the allocation efficiency, improved with time. The experiments also have shown that the periodic clearing policy have given market makers greater profits than the continuous clearing for all three types of trading agents we have used. For learning and human-like bidding agents, the continuous clearing policy have been preferable to the periodic clearing policy because it have improved allocation efficiency and transaction volume. With those implemented market mechanisms, we have participated in the CAT competition in 2007 and 2008: we achieved fourth place and third place.

In appendix, we present a list of published papers in support of this thesis.

8.2 Future Work

In the capacity allocation model, we have focused on the case of simultaneous decision makings for the retailers. Recently, there has been an emerging research area in mechanism design, namely automated mechanism design or adaptive online mechanism design under a dynamic environment [24, 95]. However, all these works are

focused on a single market case. It is interesting to apply a dynamic environment in the two-connected markets model. In such a case, Stackelberg competition [122], which is based on a sequential move game, plays an important role to analyse the property of market mechanisms.

With respect to the online market model for the coordination problem, we have not considered some specific aspects of online markets like non-cooperative shipment which is the most significant problem in eBay-like market according to Resnick and Zeckhauser [104]. An interesting approach for this problem is to embed the concept of reputation, which is a set of information shared by the community and dealing with an individual's competences and expected behaviour. In such a case, reputation mechanisms, collecting and using feedbacks (reports representing trust level between two-parties) play key roles, especially how to induce truthful feedbacks from the agents. Dellarocas et al. propose some mechanisms which induce truthful feedbacks [32, 33, 58, 81], however, they do not consider coordination in the community. Muller [85] and Hirschman [54] consider how reputation achieves coordination in simple iterated games.

In our experiments, we have considered some typical automated bidding strategies. Our framework is sufficiently general to those specific strategies. Our approach is applicable to other bidding strategies such as variants of ZIP strategy [21, 102], a variant of RE strategy [90] and Gjerstad and Dickhaut's strategy [46] and its variant [115]. Furthermore, an interesting extension is that market makers are in competitive environments like CAT competition [93, 92].

As an experimental platform, we have used the CAT platform. It is interesting

to extend the experimental analysis for supply chains. Moyaux et al. [84] used some auction mechanisms in the CAT platform to control supply chains.

In our experiment, we have made some limitations on order types. We have assumed that quantity of each order is one unit. If quantity matters, an issue is how to determine the bidding price and the bidding quantity for an autonomous trading agent. In finance, algorithmic trading is an emerging topic that considers how large amount of trades are absorbed by the market without changing the market price [62, 4].

Published Work

Part of the results presented in Chapter 2, 3, 4 and 5 in this thesis have been published in the following papers:

- *M. Furuhata and D. Zhang*, Capacity allocation with competitive retailers, *Proceedings of the 8th International Conference on Electronic Commerce (ICEC-06), pages 31-37, 2006.* This paper presents part of the results in Chapter 3.

- *M. Furuhata and L. Perrussel and D. Zhang*, Mechanism Design for Capacity Allocation with Price Competition, *Proceedings of the 10th International Conference on Electronic Commerce (ICEC-08), article number 43, 2008.* This paper presents part of the results in Chapter 4.

- *M. Furuhata and D. Zhang and L. Perrussel*, Online Market Coordination, *Proceedings of the 11th Pacific Rim International Conference on Multi-Agents (PRIMA-08), Lecture Notes in Computer Science 5357, 185-196, 2008.* This paper This paper presents part of the results in Chapter 5.

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