Belief Reconstruction in Cooperative Dialogues

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Abstract. We investigate belief change in the context of man-machine dialogue. We start from Perrault's approach to speech act theory [12], which proposes on default mechanisms for the reconstruction of the agents' beliefs. Then we review Sadek's [16] critiques and introduce his approach. We point out some shortcomings and present a new framework for the reconstruction of beliefs which contrarily to Perrault's and Sadek's is monotonic. We focus on a particular application, viz. cooperative man-machine dialogues. Our basic notion is that of a topic: we suppose that we can associate a set of topics to every agent, speech act and formula. This allows to speak about the competence of an agent, and the preservation of beliefs. We give an axiomatics and a possible worlds semantics, and we show how the belief state of an agent can be reconstructed after a speech act.

1 Introduction

Cohen et Levesque [5] have proposed an analysis of speech acts in terms of a multi-modal logic, which in particular contains operators of belief, mutual belief, intention and action. Perrault [12] points out that Cohen and Levesque's speech act theory does not take into account that an agent's beliefs after an utterance depends on his beliefs prior to it. He argues that old beliefs should be preserved, and that beliefs of other agents should be adopted when communicated, provided that they do not conflict with the agent's old beliefs. Perrault uses Reiter's default logic [14] to formalize such a principle.

Clearly, systematic preservation of old beliefs might be undesirable: human agents change their mind, make errors (voluntary or not), forget information, etc. Perrault mentions that problem and discusses a solution where systematic preservation is replaced by a default. He notices that in this case, whenever e.g. the hearer believed A before the speaker informs him that $\neg A$, there are at least two mutually inconsistent extensions: one where the hearer preserves his belief that A, and one where the hearer abandons it and adopts the belief that $\neg A$. To overcome this problem, Sadek [16] defines his preservation axiom as: if an agent's old belief does not contradict a new one coming from a speech act, then the old belief is preserved.

Such a policy is problematic if the speaker is incompetent at the propositional content of the speech act: the agent should not adopt the new belief, but preserve those before the act. This motivates our exposition in the sequel.

We consider man-machine dialogues between the system s and a human user u. We restrict ourselves to cooperative dialogues where u typically wants s to give some information. We make the assumption of rationality both for system and user. Consequently the content of a speech act is consistent. Our running example will be a short dialogue where the user wants the system to inform him about the price of a train ticket to Paris. During the dialogue the user first asks for a first class ticket, and subsequently changes his mind and asks for a second class ticket. Moreover, he puts forward a wrong price.

The paper is organized as follows: first we introduce our formal framework in terms of a multi-modal logic (Sect. 2) where topics are associated to every item of the language (Sect. 3). Then we present topic-based axioms for belief reconstruction (Sect. 4), and associate a semantics (Sect. 5). Finally, we present the relationship between competence and preservation, and give the definition of the coherence (Sect. 6).

2 The Multi-Modal Framework

Just as Cohen, Levesque, Perrault, and Sadek we work in the multi-modal framework, with modal operators for belief, mutual belief, intention and action. We start from Sadek's beliefs reconstruction [16], and describe briefly the relevant features of our framework.

Our language is that of first order multi-modal logic without equality and without function symbols [4, 10, 13]. We suppose that $\wedge$, $\neg$, $\top$ and $\forall$ are primitive, and that $\vee$, $\rightarrow$, $\perp$ and $\exists$ are defined as abbreviations in the usual way. There are the belief operators Bel_u, Bel_s and $Bel_{s,u}$ which respectively stand for "the user believes that", "the system believes that" and "the system and user mutually believe that". There are two operators of intention: $Intend_u$ and $Intend_s$ which respectively stand for "the user intends that" and "the system intends that".

Speech acts [1, 17] are represented by tuples of the form either $\langle FORCE_{u,s} A\rangle$ or $\langle FORCE_{s,u}\, A\rangle$, where $FORCE$ stands for the illocutionary force of the act, and A for the propositional content. Example: $\langle INFORM_{u,s}\, A\rangle$ represents a declarative utterance of the user informing the system that A.

Let ACT be the set of all speech acts. To every speech act $\alpha \in ACT$ there is associated a modal operator $Done_\alpha$. $Done_\alpha A$ is read "the speech act α has been performed, before which A was true". [1] In particular, $Done_\alpha \top$ is read "α has been performed". Using this operator, the beliefs of the system at state S_k can be kept in *memory* at state S_{k+1}.

Formally, acts and formulas are defined by mutual recursion. (This allows to have propositional contents of speech acts that are non-classical formulas.) For example, $Bel_s Done_{\langle INFORM_{u,s}\ Bel_u Bel_s p\rangle} Bel_s Bel_u Bel_s \neg p$ is a formula.

[1] $Done_\alpha A$ is just as $\langle \alpha^{-1} \rangle A$ of dynamic logic [9].

Just as in [5] and [12], to each operator of belief there is associated the modal logic KD45. Hence e.g. $Bel_u A \rightarrow Bel_u Bel_u A$ is a theorem. We suppose that common belief is related to belief by the axiom

$$Bel_{i,j} A \rightarrow Bel_i A$$

To keep things simple we suppose that the logic of each operator of intention is K.[2]

Each $Done_\alpha$ operator obeys the principles of modal logic K. As $Done_\alpha$ is a modal operator of type possible, the rule of necessitation takes the form

$$\frac{A}{\neg Done_\alpha \neg A} .$$

With Sadek [15] we suppose that speech acts cannot be performed simultaneously. Hence $Done_\alpha A$ and $Done_\beta B$ are inconsistent whenever α is a speech act different from β, i.e. we have the axiom:

$$\neg(Done_\alpha \top \wedge Done_\beta \top) \text{ if } \alpha \text{ and } \beta \text{ are different.}$$

Then from the system's point of view, the dialogue is a sequence of speech acts $(\alpha_1, \ldots, \alpha_n)$. Each α_{k+1} maps a mental state S_k to a new mental state S_{k+1}: $S_0 \xrightarrow{\alpha_1} S_1 \xrightarrow{\alpha_2} \ldots \xrightarrow{\alpha_n} S_n$. S_0 is the system's initial belief state (before the dialogue starts). Given S_k and α_{k+1}, our task is to construct the new mental state S_{k+1}.

3 Topics

The notion of a topic is central in our approach. We start from the idea that to every agent, speech act and formula, there is associated some set of topics among a fixed set of topics $\mathcal{T}$. This will allow in particular to formulate a topic-based axiom of preservation, saying that a formula A is always true after a speech act α if A was true before α and the topics of A are not among those associated to α.

As the interpretation of formulas is in general context-dependent, the topic associated to formulas, speech acts, and agents will be so as well: if the speaker informs the hearer that Venus is beautiful, he might speak about the morning star or about Greek statues. Here we shall suppose – as often done in dialogue systems – that the topics can be determined independently of the context. (We suppose that our agents work in a specialized domain, where the words of their technical language have one and only one interpretation.)

[2] Our notions of intention and common belief are oversimplified: first, we offer no particular principle for intentions. We did this because the existing analyses of intention vary a lot, and the systems that have been put forward in the literature are rather complex. Second, our condition linking belief and common belief is weaker than the usual induction axiom. We argue that such an inductive principle is not necessary at least in a first approach: as Cohen and Levesque, we suppose that common belief directly comes as the indirect effect of a speech act. (This is different from Perrault's view, where mutual belief is constructed inductively via default rules.)

3.1 The Subject of a Formula

The subject of a formula is what the formula is about. For example, the formula *Class*(*first*) is about train classes, but not about destinations or prices. $\mathcal{S}ubject(A)$ denotes the set of topics A is about. We give the following axioms:

Axiom 1. *$\mathcal{S}ubject(p) \neq \emptyset$ if p is atomic.*

Axiom 2. *$\mathcal{S}ubject(\top) = \emptyset$.*

Axiom 3. *$\mathcal{S}ubject(\neg A) = \mathcal{S}ubject(A)$.*

Axiom 4. *$\mathcal{S}ubject(A \wedge B) = \mathcal{S}ubject(A) \cup \mathcal{S}ubject(B)$.*

Axiom 5. *$\mathcal{S}ubject(\mu A) = \mathcal{S}ubject(A)$ where μ is any modal operator.*

Axiom 6. *$\mathcal{S}ubject(\forall x A) = \mathcal{S}ubject(A)$.*

Axiom 7. *$\mathcal{S}ubject(A[t/x]) \subseteq \mathcal{S}ubject(A)$, where $[t/x]$ is a substitution.*

Hence we consider that atomic formulas are always about something. It is common to consider that the topics of a formula are the same as those of its negation. Demolombe and Jones show in [6] that the axiom for conjunction is too restrictive in some cases. Concerning the axioms for the modal operators, one might argue that intuitively, $Bel_s Price(95\$)$ could be about the system's beliefs relative to ticket prices, while $Bel_s Bel_u Price(95\$)$ is about the system's beliefs about the user's belief relative to ticket price. We will not consider this difference here, and suppose that modalities have no influence on the subject of a formula. The last two axioms concern first-order, and allow to derive $\mathcal{S}ubject(\forall x A) = \mathcal{S}ubject(\forall y (A[y/x]))$.

Together, the above axioms entail the following property.

Property 1. $\mathcal{S}ubject(A) = \bigcup_p \mathcal{S}ubject(p)$ where p is atomic and occurs in A.

This allows to represent the $\mathcal{S}ubject$ function in an economic way, by just giving it for atomic formulas. An intuition which is sometimes useful is to consider that the subject of an atomic formula is its predicate name.

Note that our $\mathcal{S}ubject$ function is not extensional: logically equivalent formulas may have different topics. In particular we may have $\mathcal{S}ubject(p \vee \neg p) \neq \emptyset$, althought $\mathcal{S}ubject(\top) = \emptyset$. We nevertheless have the following corrolary of Prop. 1:

Property 2. If $A \leftrightarrow B$ and A and B are constructed from the same atoms, then $\mathcal{S}ubject(A) = \mathcal{S}ubject(B)$.

The spirit of our $\mathcal{S}ubject$ function is that of Epstein. He defines the *relatedness relation* $\mathcal{R}$ as a primitive relation between propositions because "the subject matter of a proposition isn't so much a property of it as a relationship it has to other propositions" [7, page 62]. Thus, topics are not explicitly represented in the language. Then he defines the *subject matter of a proposition A* as $s(A) = \{\{A, B\} : \mathcal{R}(A, B)\}$. More precisely, s is called *the subject matter of* *set-assignment associated with* $\mathcal{R}$. Epstein shows that we can also define s as primitive, and that we can then define two propositions as being related if they have some subject matter in common. Our $\mathcal{S}ubject$ function can be seen as an extension of this function to a multi-modal language.

Other studies of the notion of topic exist in the literature, in particular those of Lewis [11] and Goodman [8]. Both are quite different from Epstein's. Goodman's notion of "absolute aboutness" is defined purely extensionally. Hence for him logically equivalent formulas are about the same topics, while this is not the case for us. Moreover, as he focusses on the "informative aspect" of propositions, the subject of a tautology is the empty set.

3.2 The Scope of a Speech Act

The scope of a speech act α are those topics of $\mathcal{T}$ on which α gives new information (possibly indirectly). For example, the topics of $\langle INFORM_{u,s}\ Class(first)\rangle$ are not only the class, but also ticket prices. Indeed, information about the class allows the system to deduce the price (via its static laws, cf. Sect. 4.1 and 6). $\mathcal{S}cope(\alpha)$ denotes the set of topics on which speech act α gives new information.

The above example illustrate that it is not always possible to equate the scope of an act with the subject of its propositional content, because the $\mathcal{S}cope$ function must take into account laws. But we consider that we should have at least inclusion. Thus, we have the axiom:

Axiom 8. *If* $\alpha = \langle F_{i,j}\ A\rangle$ *then* $\mathcal{S}ubject(A) \subseteq \mathcal{S}cope(\alpha)$.

In general the scope of an act depends on its illocutionary force and the subject of its propositional content. We shall consider here that the scope of a speech act does not depend on its illocutionary force. Thus we have:

Axiom 9. *If* F *and* G *are two illocutionary forces then* $\mathcal{S}cope(\langle F_{i,j}\ A\rangle) = \mathcal{S}cope(\langle G_{i,j}\ A\rangle)$.

This might be criticized. Consider the acts $\alpha = \langle INFORM_{u,s}\ A\rangle$ and $\beta = \langle DENY_{u,s}\ A\rangle$ performed in a state where $Bel_s A$ holds. α gives no new information on the subject of A, while β does. Having in mind that all those beliefs α gives no information about will be preserved, we cautiously consider here that both α and β give new information on the subject of A. (This is related to the negation-axiom 3 for the $\mathcal{S}ubject$ function.)

3.3 The Competence of an Agent

The competence of an agent are those topics the agent is competent at. For example, the topics of the user are the destination and the class of the ticket he wants to buy. The price is not among the competences of the user: he is supposed to ignore the laws relating destination, classes, and prices (wich are in the system's domain of competence, cf. Sect. 4.1). $\mathcal{C}ompetence(u)$ denotes the set of topics the user is competent at.

We formulate no axioms for the $\mathcal{C}ompetence$ function. In Sect. 6 we study its interaction with the $\mathcal{S}cope$ function.

3.4 A Topic Structure

A topics structure consists of a set of topics together with the $\mathcal{S}ubject$, $\mathcal{S}cope$, and $\mathcal{C}ompetence$ functions. A given topic structure will allow us to reconstruct beliefs by means of two principles: competence and preservation. In the next section we shall present these principles, after introducing some laws.

4 Axioms for Belief Reconstruction

We introduce here general laws about the agents' beliefs, and formulate principles governing presupposition and indirect effects of speech acts, as well as the axiom schemas of competence and preservation.

4.1 Static Laws

Some beliefs cannot be revised. This kind of beliefs must always be preserved in the belief reconstruction process. We call such formulas laws. (They correspond to integrity constraints in databases.) For example, the system never drops its belief that $Dest(Paris) \wedge Class(first) \to Price(95\$)$, and the mutual belief that first and second class are incompatible is always preserved.

We distinguish two types of laws: those known by the system and those common to system and user. Common beliefs of user and system are represented by static laws of the form $Bel_{s,u}A$. An example is $Bel_{s,u}\neg(Class(first) \wedge Class(second))$. Laws that are known by the system but ignored by the user are represented by formulas of the form Bel_sA. For example, $Bel_s(Dest(Paris) \wedge Class(first) \to Price(95\$))$.

4.2 Laws for Presuppositions and Indirect Effects

To every act there is associated a set of preconditions and a set of effects. For example, a precondition of the act $\langle INFORM_{u,s}\, Dest(Paris)\rangle$ is $Bel_u Dest(Paris)$.

From the hearer's point of view, the preconditions become presuppositions.[3] In cooperative dialogues a speech act with informative force presupposes that the speaker believes its propositional contents (condition of sincerity). This can be formulated as the *schema of presupposition*:

$$\neg Done_{\langle INFORM_{i,j}\ A\rangle}\neg Bel_i A.$$

It follows by standard principles of modal logic that the performance of an act allows to deduce the preconditions, i.e. $Done_{\langle INFORM_{i,j}\ A\rangle}\top \to Done_{\langle INFORM_{i,j}\ A\rangle} Bel_i A$. For example, for the act $\langle INFORM_{u,s}\ Class(first)\rangle$ we have:

[3] What we call here *presuppositions* of an act α are the preconditions of α from the point of view of an observer. For example, if $Bel_u A$ is a precondition of an act performed by the user u, then from the fact that u has performed this act, s *presupposes* that u believed A before (i.e. that u satisfied the preconditions of the act). Note that s could nevertheless have believed $\neg A$ before α: the presupposition $Bel_s Done_\alpha Bel_u A$ is consistent with $Bel_s Done_\alpha Bel_s \neg A$ (which comes from the memory).

$$Done_{\langle INFORM_{u,s}\ Class(first)\rangle}\top \rightarrow Done_{\langle INFORM_{u,s}\ Class(first)\rangle} Bel_u\, Class(first).$$

Other schemas of this kind can be formulated saying e.g. that before the act $\langle INFORM_{u,s}\, A\rangle$ the speaker believed that the hearer ignores A. More generally, let A' be the precondition of an act α. Then we have $\neg Done_\alpha \neg A'$.

After an act, the hearer believes its presuppositions. Let A' be the precondition of an act α. Then we have the *schema of consumption of indirect effects*:

$$Done_\alpha \top \rightarrow A'.$$

For example, for the act $\langle INFORM_{u,s}\ Class(first)\rangle$ we have:

$$Done_{\langle INFORM_{u,s}\ Class(first)\rangle}\top \rightarrow Bel_u\, Class(first).$$

Similar schemas can be given for other kinds of speech acts.[4]

Finally, it is reasonable to suppose that preconditions and propositional content of an act are related in Epstein's sense. Hence we have the following axiome.

Axiom 10. *If A is the propositional content of α and A' a precondition of α, then $\mathcal{S}ubject(A) \cap \mathcal{S}ubject(A') \neq \emptyset$.*

Let $LAWS$ be the set of laws governing speech acts together with the set of static laws. Then it is a basic assumption of our framework that the new mental state S_{k+1} can be constructed solely from memory (i.e. the set of formulas $\{Done_{\alpha_{k+1}} A : A \in S_k\}$) together with the set of laws. This requires appropriate principles of competence and preservation that we shall introduce in the rest of the section.

4.3 Axiom of Competence

In our running example, the user informs the system about the train class he wants. The user is considered by the system to be competent at train classes, in the sense that whenever the system learns something about the user's beliefs on classes, it is prepared to abandon its previous beliefs about classes. More generally, whenever i believes A and is competent at all the subjects of A, then A is true. Therefore we have the topic-based axiom schema

$$Bel_i A \rightarrow A \text{ if } \begin{cases} \mathcal{S}ubject(A) \subseteq \mathcal{C}ompetence(i) \text{ and} \\ A \text{ contains no modal operator.} \end{cases}$$

If A was not modality free, this axiom would allow to deduce formulas that are not intuitive. For example, suppose the user is competent at the class, and due to

[4] In principle, the above schemas are axiom schemas, that can be instantiated by any propositional content. Nevertheless, in order to avoid complex interactions on the semantical level between the accessibility relations respectively associated to acts and beliefs, we have rather chosen to represent these schemas in a particular theory. Such a theory will contain formulas such as $\forall x(\neg Done_{\langle INFORM_{u,s}\ Class(x)\rangle}\neg Bel_u\, Class(x))$. The integration of such schemas into the logic seems to require an analysis similar to that of conditionals. This will be subject of future work.

a misunderstanding we have $Bel_s Class(second) \wedge Bel_s Bel_u Bel_s \neg Class(second)$. Then the system should not adopt $Bel_s \neg Class(second)$.

In our application, as we take the system's viewpoint, we want to apply the *belief adoption axiom* $Bel_s(Bel_u A \rightarrow A)$. Indeed, the belief adoption axiom can be derived from the principle of competence by the necessitation rule of modal logic K.

The other way round, the user's beliefs are not adopted if the user is incompetent at A. This is the case e.g. for ticket prices: the user is supposed not to know the system's law relating destinations, classes, and prices. Therefore, if the user informs the system that the ticket price is 115\$, then the system does not give up it's previous belief. Nevertheless, it keeps track of the user's belief, i.e. the next state contains the formula $Bel_s Bel_u Price(115\$))$. This will be useful when it comes to planning the next speech act.

4.4 Axiom of Preservation

Suppose the system believes A. Whenever the user speech act has "nothing to do" with A, the system should keep on believing that A after the act. This is reminiscent of the frame problem in reasoning about actions. As we have done for competence, we formulate the principle of preservation as an axiom schema in terms of topics, in a spirit close to the solution in [3]:

$$Done_\alpha A \rightarrow A \text{ if } \begin{cases} \mathcal{S}cope(\alpha) \cap \mathcal{S}ubject(A) = \emptyset \text{ and} \\ A \text{ contains no } Done_\beta \text{ operator.} \end{cases}$$

A cannot contain modal operators for speech acts, because we suppose that two speech acts cannot be performed simultaneously. In particular, in the course of the dialogue $Bel_i Done_\beta \; Bel_i Done_\alpha \top$ does not imply $Bel_i Done_\alpha \top$.[5]

The absence of systematic preservation allows us to keep the system's beliefs consistent after the performance of a user speech act. Our approach is motivated by two cognitive considerations: the first corresponds to the necessity of abandoning beliefs. (After α has been performed, we can no longer believe A.) The second corresponds to the fact that we do not want to modify the past (what is done, is done). The aim is to always generate consistent states.

Now we can formally express how the new state S_{k+1} is constructed:

$$S_{k+1} = \{Done_{\alpha_{k+1}} A : A \in S_k\} \cup LAWS.$$

Hence we have $S_{k+1} \rightarrow C$ iff $LAWS \vdash (Done_{\alpha_{k+1}} Done_{\alpha_k} \ldots Done_{\alpha_1} S_0) \rightarrow C$.

Suppose $\alpha = \langle FORCE_{i,j}\; A \rangle$. Then *presuppositions* are beliefs of the form $Bel_j Done_\alpha A'$, where A' is a precondition of α. According to axiom 10 of Sect. 4.2, the preconditions of a speech act are related with its propositional content in Epstein's sense, i.e. $\mathcal{S}ubject(A) \cap \mathcal{S}ubject(A') \neq \emptyset$. It follows from the axiom $\mathcal{S}ubject(A) \subseteq \mathcal{S}cope(\alpha)$ that:

[5] Else $Bel_i(Done_\beta \top \wedge Done_\alpha \top)$ would follow, contradicting our assumption that two speech acts are never performed simultaneously.

Property 3. If A' is a precondition of the speech act α, then

$$\mathcal{S}cope(\alpha) \cap \mathcal{S}ubject(A') \neq \emptyset.$$

An important consequence of the above property is that preconditions are never preserved. This is in accordance with cognitive intuitions: presuppositions are beliefs about the immediately preceding speech act.

5 Semantics

Suppose given a topic structure, i.e. a set of topics together with $\mathcal{S}ubject$, $\mathcal{S}cope$, and $\mathcal{C}ompetence$ functions. Possible worlds models for that topic structure are of the form $M = \langle W, B_s, B_u, B_{s,u}, I_s, I_u, \{D_\alpha : \alpha \in ACT\}, V\rangle$, where W is a set of worlds, $B_s, B_u, B_{s,u}, I_s, I_u$ and every D_α are accessibility relations, D is a domain, and V is a mapping which interprets variable and constant symbols, and associates to each world $w \in W$ an interpretation V_w of predicate symbols. M must satisfy the following restrictions:

1. $B_s, B_u, B_{s,u}$ are serial, transitive, and euclidean
2. $(B_u \cup B_s)^* \subseteq B_{s,u}$
   (mutual belief)
3. for every $\alpha, \beta \in ACT$ such that α is different from β, $D_\alpha^{-1} \cap D_\beta^{-1} = \emptyset$
   ('single past hypothesis')
4. For every $w \in W$ and every agent $i \in \{u, s\}$ there is some $w' \in W$ such that:
   (a) wB_iw', and
   (b) for every atomic formula $p(t_1, \ldots, t_n)$ such that $\mathcal{S}ubject(p(t_1, \ldots, t_n)) \subseteq \mathcal{C}ompetence(i)$, we have:
   $(V(t_1), \ldots, V(t_n)) \in V_w(p)$ iff $(V(t_1), \ldots, V(t_n)) \in V_{w'}(p)$
   (competence of i).
5. For every $w, w' \in W$ and every act α such that $w' D_\alpha w$ there are mappings $f, g : W \to W$ such that:
   (a) $f(w') = w$, $g(w) = w'$, and
   (b) for every u' such that $w'(\Re_1 \circ \ldots \circ \Re_m)u'$, with $m \geq 0$ and $\Re$ among $B_u, B_s, B_{s,u}$, we have:
      i. if $u'\Re v'$ then $f(u')\Re f(v')$, and if $u\Re v$ then $g(u)\Re g(v)$
      ii. $V_{u'}^\alpha = V_{f(u')}^\alpha$, and $V_u^\alpha = V_{g(u)}^\alpha$
   where V_u^α is the restriction of V_u to those interpretations of atomic formulas whose subject is not in the scope of α [6]
   (preservation of p through α)

Condition 4 says that for every world there is a world compatible with i's beliefs such that all atomic formulas i is competent at are interpreted in the same way in both worlds. Condition 5 says that whenever w results from the

[6] Hence $V_{u'}^\alpha = V_{f(u')}^\alpha$ if and only if for all atomic formulas $p(t_1, \ldots, t_n)$ such that $\mathcal{S}ubject(p(t_1, \ldots, t_n)) \cap \mathcal{S}cope(\alpha) = \emptyset$, $(V(t_1), \ldots, V(t_n)) \in V_{u'}^\alpha(p)$ if and only if $(V(t_1), \ldots, V(t_n)) \in V_{f(u')}^\alpha(p)$.

performance of α in w' then w and w' interpret those formulas whose subject is not in the scope of α in the same way [7].

Satisfaction of a formula in a world of a model is defined as usual. B_s, B_u, $B_{s,u}$, I_s, I_u are respectively associated to Bel_s, Bel_u, $Bel_{s,u}$, $Intend_s$, $Intend_u$. For example,

1. $w \models Bel_i A$ iff $w' \models A$ for every w' such that wB_iw'.
2. $w \models Done_\alpha A$ iff there is a $w' \in W$ such that $w'D_\alpha w$ and $w' \models A$.

The other clauses are similar.

A formula A is *true in a model* M if $w \models A$ for every $w \in W$. A is *valid* in a topic structure if A is true in every model for that structure, and A is *satisfiable* in a topic structure if $\neg A$ is not valid. A is a *logical consequence* of a set of formulas Γ in a topic structure (noted $\Gamma \models A$) if for every model M of that structure, if every element of Γ is true in M then A is true in M.

6 Topic Structure and Laws

Competence and preservation interact during belief construction. What is the relationship between them? Suppose the system believes A, and the user performs a speech act whose propositional content is $\neg A$. Whenever the user is competent at the subject of $\neg A$, $\neg A$ is adopted by the system. Hence the system should preserve neither A nor – most importantly – any of the consequences of A that it has deduced via its laws. In the opposite case where the user is incompetent at $\neg A$, the system might keep on believing A and its consequences.

Let us look closer at the subtle balance between laws, competence and preservation. If an act α_{k+1} may lead to an inconsistent state S_{k+1}, then this is due to a belief coming from α_{k+1} and to one that comes from S_k. (We suppose here that the system is consistent in state S_k, and that the propositional content of the act is itself consistent.) If – due to competence of the speaker – the belief coming from α_{k+1} has priority over the system's beliefs, then the belief coming from S_k must not be preserved. On the contrary, if the speaker is incompetent the system refuses the incoming information. In both cases, the system will not become inconsistent.

Let the set of laws be:

$$Bel_s(Dest(Paris) \wedge Class(first) \rightarrow Price(95\$)) \quad (1)$$

$$Bel_s(Dest(Paris) \wedge Class(second) \rightarrow Price(70\$)) \quad (2)$$

$$Bel_{s,u}\neg(Class(first) \wedge Class(second)) \quad (3)$$

$$Bel_{s,u}\neg(Price(70\$) \wedge Price(95\$)) \quad (4)$$

Let S_k be $Bel_s Dest(Paris) \wedge Bel_s Class(second)$. Then $Bel_s Price(70\$)$ can be deduced from (2).

[7] Because the 'belief-subtrees' respectively rooted in w and w', i.e. the worlds accessible via B_i and $B_{i,j}$, interpret the atomic formulas whose subject is not in the scope of α in the same way.

Now suppose $\alpha_{k+1} = \langle INFORM_{u,s}\ Class(\mathit{first})\rangle$ is performed. As the user is competent about the class, $Bel_s Class(second)$ cannot be preserved. In terms of topics, we have thus that:

$$\mathcal{S}ubject(Bel_s Dest(Paris)) \cap \mathcal{S}cope(\alpha_{k+1}) = \emptyset$$
$$\mathcal{S}ubject(Bel_s Class(second)) \cap \mathcal{S}cope(\alpha_{k+1}) \neq \emptyset$$
$$\mathcal{S}ubject(Class(\mathit{first})) \subseteq \mathcal{C}ompetence(u)$$

Then the state S_{k+1} contains the following formulas:

$Bel_s Done_{\alpha_{k+1}}(Bel_s Dest(Paris) \wedge Class(second) \wedge Bel_s Price(70\$))$	(memory)
$Bel_s Dest(Paris)$	(preservation)
$Bel_s Done_{\alpha_{k+1}} Bel_u Class(\mathit{first})$	(admission)
$Bel_s Bel_u Class(\mathit{first})$	(indirect effects)
$Bel_s Class(\mathit{first})$	(belief adoption)
$Bel_s Price(95\$)$	(from (1))

Thus, if we suppose that $\mathcal{S}ubject(Bel_s Price(70\$)) \cap \mathcal{S}cope(\alpha_{k+1}) = \emptyset$ then $Bel_s Price(70\$)$ is preserved, and then S_{k+1} is inconsistent (from (4)). Thus, we must have $\mathcal{S}ubject(Bel_s Price(70\$)) \cap \mathcal{S}cope(\alpha_{k+1}) \neq \emptyset$.

The example illustrates that the $\mathcal{S}cope$ function must respect laws: we must have topics for act α_{k+1} and formula $Bel_s Price(70\$)$ such that $\mathcal{S}cope(\alpha_{k+1}) \cap \mathcal{S}ubject(Bel_s Price(70\$)) \neq \emptyset$.

To sum it up, given a set of laws we should not accept *any* combination of $\mathcal{S}ubject$, $\mathcal{S}cope$, and $\mathcal{C}ompetence$ functions. The following definition expresses that: a topic structure should be such that it does not forbid the performance of speech acts in states that are compatible with the laws.

Definition 1. *Given a $\mathcal{S}ubject$, $\mathcal{S}cope$, and $\mathcal{C}ompetence$ function and a set of laws, we say that they are coherent iff for every formula A that is satisfiable with the set of laws and every speech act α such that $\mathcal{S}ubject(A) \cap \mathcal{S}cope(\alpha) = \emptyset$, $Done_\alpha A$ is satisfiable with the set of laws.*

For example, the above topic structure is coherent, while we would get incoherent if the user was competent at prices. As well, we would get incoherent if prices were not in the scope of the act α_{k+1} of informing that the user wants a second class ticket.

7 Conclusion

We have presented a fairly standard multi-modal logic for belief reconstruction in cooperative dialogues that is based on the notion of topic. Several simplifying hypotheses have been made. We did this in order to enable future mechanisation of our framework, where we plan to use a tableau method theorem prover in the style of [2].

In particular we have supposed that an agent's competence is fixed a priori. This is satisfactory in short dialogues, but there are examples of longer dialogues which indicate that this constraint is sometimes too strong. (For example, the user might forget things he is supposed to be competent at.) Hence the most general case requires a dynamic competence function, where the system can re-evaluate user competence. Nevertheless, we are convinced that dynamic competence can be integrated later on, and that our choice is a restriction rather than a conceptual limitation of system.

A belief reconstruction module as sketched in this paper should be designed to facilitate integration into a dialogue system. In particular, the system's belief state should make it possible to generate its next speech act. We think that the planning of a new speech act can be based on our framework as well. Indeed, we can identify several what may be called reactive patterns, that are linked to the axiom schemas of preservation and competence. For example, suppose an informative act $\alpha = \langle INFORM_{u,s}\ A \rangle$ took place. First, consider a state containing $Bel_s \neg A \wedge Bel_s Bel_u A$. Here, u is incompetent at the subject of A. This may trigger an informative act of the system, of the sort "You are wrong in believing $\neg A$" (possibly giving an explanation why this is so). Second, consider a state containing $Bel_s Bel_u A \wedge Bel_s Done_\alpha Bel_s Bel_u \neg A$. Here, $Bel_s Bel_u \neg A$ has not been preserved. This may trigger a question of the system why the user changed his mind (but note that this is not the case if the act is a denial).

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