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Toward a formal representation of space in language: A commonsense reasoning approach

Michel Aurnague*, Laure Vieu**

IRIT 118, route de Narbonne 31062 Toulouse cedex France
tel.: (33) 61 55 67 64 - fax: (33) 61 55 62 58

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Abstract: This paper proposes some formal tools for representing the semantic content of French expressions referring to space. These tools consist of first, a relational --or qualitative-- geometry encompassing topology, distance notions and projective geometry, as well as temporal notions, thus constituting a rather complete theory of naive space-time; and second, several formalisms to deal with functional aspects of entities such as intrinsic orientation, internal structure and categorizing; more generally, it is claimed that a representation in three levels (geometric, functional and pragmatic) is required to account for spatial expressions' semantics. To analyze the semantics of some of these expressions, this work systematically looks for valid reasoning schemata involving them; this approach also enables the testing of the proposed formal system as well as the evaluation of the definitions obtained for the studied lexemes.

0 Introduction

The aim of our work is to elaborate some formal tools for representing the semantic content of French expressions referring to space. In this paper we considered the topological preposition *dans* (in), as well as the projective prepositions *dessus* (above), *dessous* (below), *devant* (in front of), *derrière* (behind), in the schema "N_{target} *est* (ILN de) prep $N_{landmark}$". In the category of spatial referents we also took into account several Internal Localization Nouns (ILNs) such as *haut* (top), *bas* (bottom), *devant* (front extremity) which are all lexical items pointing out the different portions of an object [Borillo A. 92].

A detailed semantic analysis has been carried out so as to distinguish the different spatial configurations each of these lexemes allows us to refer to. What we think is more specific of our methodology is that this linguistic study also identifies the different inferential schemata underlying the combination of spatial expressions in discourse. According to this analysis and to recent works on linguistic and cognitive space [Herskovits 82] [Vandeloise 86] [Lang 90], we claim that semantics of NL spatial expressions cannot be represented with geometric notions alone.

* Equipe de Recherche en Syntaxe et en Sémantique, URA 1033 (CNRS), Université Toulouse-Le Mirail
and Equipe Langue, Raisonnement, Calcul, IRIT. email: aurnague@irit.fr
** email: vieu@irit.fr

The description of the shape of some spatial entities and of the geometric relations holding between them is not sufficient to determine the applicability of spatial expressions. Otherwise, it would not be possible to explain, for example, why we make a difference between some uses of *sur* (on) and the uses of *contre* (against):

*L'affiche est sur / *contre le mur*	*La planche est contre / *sur le mur*
'The poster is on / *against the wall'	'The board is against / *on the wall'

In the same way, one could not account for the impossibility of stating *le livre est dans le gant* 'the book is in the glove' to describe a situation for which we can yet state the two sentences:

Le livre est dans la main	*La main est dans le gant*
'The book is in the hand'	'The hand is in the glove'

In fact functional notions such as support (in the first example, the wall supports the poster totally but doesn't support the board or only partially) and pragmatic knowledge (illustrated in the second example by the "fixation principle" fixing the inside of the glove) have to be taken into account beyond geometric data. These remarks together with several others of the same kind, have led us to adopt the methodology of analyzing and representing the meaning of French spatial expressions on three levels organized into a hierarchy [Aurnague & Vieu, 93].

1 The geometric level

The purpose of this level is to give a formalism in which to represent geometric information found in the spatial expressions and to reason over it. Below we present the kind of formalism we have chosen (its primitive elements and how they are structured), and in the following subsections we give the three parts composing it.

1.1 A relational representation system based on individuals

On the geometric level we deal with the spatio-temporal referents of concrete entities, that is, the space-time portions determined by their matter during their "life" or during the length of time determined by a particular event. The distinction between spatio-temporal referents and entities facilitates the drawing of the line between what is "objective" and purely geometric and what is "subjective" and depends on the nature and function of the entities. In Section 1, spatio-temporal referents are simply called "individuals" and are quantified over, whereas, in the other sections, they are always designated through the pseudo-function stref(*ent*)[1] to point out the entities that determine them. stref is not a function because there is no realm of individuals given beforehand (since space-time is built from the text, cf. below); in fact, it only groups spatio-temporally equivalent entities into equivalence classes (e.g., groups of entities like the pair *the deck of cards* and *the cards*, or, *the snowman* and *the heap of snow* [Link 83]).

The actual use of prepositions like *dans* (in) or *devant* (in front of) which allows us to situate a trajector with respect to a landmark shows the relational nature of the spatial structures handled in language. This property of space in NL has been emphasized in various works related to psychology and linguistics like [Miller & Johnson-Laird 76], [Herskovits 82], [Talmy 83] or [Vandeloise 86]. Some works formalizing commonsense spatial reasoning in AI make a similar assumption: [Hayes 85a] [Hayes 85b] [Davis 88]

[1] Or stref(*ent*, *e*) for the spatio-temporal referent of the entity *ent* temporally bounded by the event e, that is, a "temporal slice" of stref(*ent*).

[Güsgen 89] [Randell & Cohn 89] [Mukerjee & Joe 90] [Egenhofer & Franzosa 91] [Frank 92] [Freksa 92] [Hernández 93]. Then, absolute spaces (i.e., in which entities are localized by means of coordinates) are not the most appropriate representational structure for our purposes although the most common both in mathematics and AI (robotics and vision being the major areas having developed spatial reasoning). Moreover, two properties of these absolute spaces seem to contradict the use of spatial expressions in NL. First, whereas the position of every entity needs to be known exactly to be represented in a coordinate system, the information expressed in a text is partial and imprecise. Second, the variable granularity of the expression of space in NL (for instance, the same entity can be considered at one time as a point and later as a volume) is not compatible with the discreteness of any implementable coordinate system, in which the minimum units are defined *a priori*. So, knowledge representation on the geometric level will be done within a relational structure and not within a coordinate system.

Within the various existing approaches to a relational represention of space, a further distinction between three classes can be made depending on the chosen kind of primitive entities. A first approach takes points as primitive elements. In [Frank 92] and [Freksa 92] all objects are punctual but only orientation and distance relations are considered, for which the notion of point is clearly well suited. In [Davis 88], [Egenhofer & Franzosa 91] and [Buschbeck-Wolf 93] points are just abstract entities assumed to belong to r^3, and concrete objects are represented as sets of points; the authors thus take advantage of classical topology and Euclidean geometry's soundness and of their known theorems, but this kind of points have no ontological support. A second approach takes intervals as basic entities, concrete objects being represented as pairs or triplets of intervals (according to the space dimension required), and thus extends Allen's temporal logic [Allen 84] to several dimensions, as in [Güsgen 89] and [Mukerjee & Joe 90]; the main drawbacks of this approach are the impossibility to represent non-rectangular (or non-parallelepipedic) objects and the impossibility to treat correctly topological relations between objects out of alignment. The last approach takes directly "individuals" or "extended bodies" to represent concrete objects, as in [Randell & Cohn 89], [Randell et al. 92] and [Hernández 93]. In our work the spatio-temporal referents of the entities will be taken as primitive elements, thus taking this third approch. The actual structure of space-time is therefore a result of the combination of spatio-temporal relations, that is, space-time is completely built from the information present in the text in such a way that the existence of no abstract entity nor organization (such as a particular dimension of space) has to be assumed beforehand.

To organize rigorously the various relations linking spatio-temporal referents, we wanted our formalism to be an axiomatic theory of space-time. Unlike period-based theories of time [Allen 84] [van Benthem 83], relatively few formal theories have been proposed for modeling space or space-time with individuals as primitive elements. Ours is an extension of the theory proposed by B. L. Clarke [Clarke 81, 85], on which [Randell & Cohn 89] [Randell et al. 92] are also based. This theory comes in three main parts. The first describes mereological and topological relations between individuals. This part is adapted from [Clarke 81], eliminating the fusion operator that made it second-order; it also includes temporal relations taken from [Kamp 79]. The second part builds the "points" to be considered in a given situation and then describes orientation and distance relations between these points. The third part develops projective geometry based on individuals and a new kind of primitive elements, directions.

1.2 Mereology, Topology and Time

1.2.1 Mereology

This first part of the theory is grounded on a new version of mereology. Mereology is a theory first introduced by Lesniewski early in this century as a component of a system proposed as an alternative to set theory [Lesniewski 27-31]. Since then, mereology has repeatedly been taken up as a theory of part-whole relation in formal ontology [Leonard & Goodman 40] [Simons 87]. In [Clarke 81, 85] and [Tarski 72] it constitutes the basis of axiomatic theories for topology and geometry, taking "individuals" or "bodies" as primitive entities instead of points, with part-whole relation replacing set-theoretical inclusion.

The theory we present is based on the system proposed in [Clarke 81] for modeling topology. This mereological-topological system is built on the sole primitive relation of "connection". Clarke's choice of connection as primitive makes his system quite distinct from classical mereology [Leonard & Goodman 40] [Lesniewski 27-31] which is usually based on one of the following relations: "proper part", "overlap" or "discrete from".

In this theory, two individuals are "connected" when they share some part or when they are in contact (i.e., "joined" in the terminology of [Hayes 85b]). Connection is axiomatized as follows in [Clarke 81]:

Connection is reflexive and symmetric:

A1 $\forall x\ C(x,x)$

A2 $\forall x\ \forall y\ (C(x,y) \Rightarrow C(y,x))$

Two individuals are (spatio-temporally) equal when they connect with the same individuals:

A3 $\forall x\ \forall y\ (\forall z\ (C(z,x) \Leftrightarrow C(z,y)) \Rightarrow x =_{st} y)$

Several mereological relations are then defined:

D1 $P(x,y) \equiv_{def} \forall z\ (C(z,x) \Rightarrow C(z,y))$ "x is part of y"

D2 $PP(x,y) \equiv_{def} P(x,y) \wedge \neg P(y,x)$ "x is a proper part of y"

D3 $O(x,y) \equiv_{def} \exists z\ (P(z,x) \wedge P(z,y))$ "x overlaps y"

Contact, also called "external connection", tangential part and non tangential part are not relations belonging to classical mereology. Their definition was made possible by Clarke's choice of the connection relation as the primitive:

D4 $EC(x,y) \equiv_{def} C(x,y) \wedge \neg O(x,y)$ "x is externally connected to y"

D5 $TP(x,y) \equiv_{def} P(x,y) \wedge \exists z\ (EC(z,x) \wedge EC(z,y))$ "x is a tangential part of y"

D6 $NTP(x,y) \equiv_{def} P(x,y) \wedge \neg\exists z\ (EC(z,x) \wedge EC(z,y))$ "x is a non tangential part of y"

These relations can be represented pictorially[2] as in Figure 1:

[2] Although we believe in their usefulness, figures like this may be misleading. Individuals are not subsets of the Euclidean plane; moreover, they are not necessarily connected (i.e., one-piece).

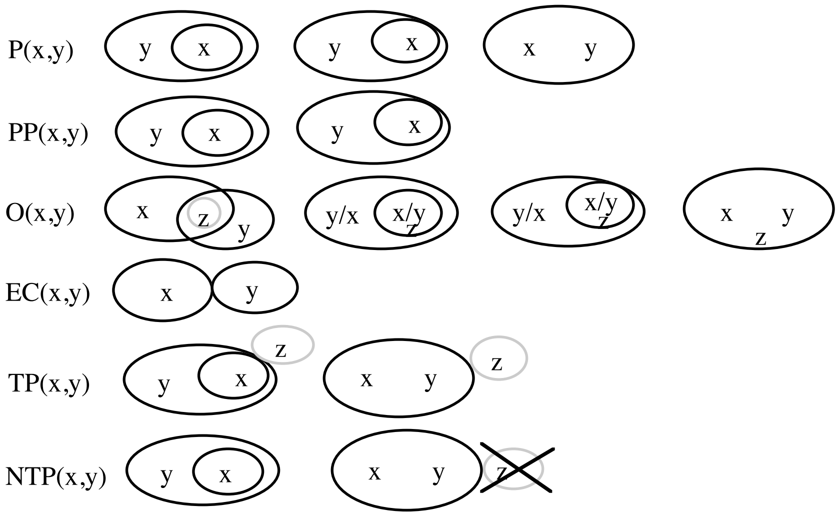


Fig. 1. Mereological relations.

Those definitions together with the three axioms confer the desired inferential properties to the relations: part, proper part and non tangential part are transitive, overlap and external connection are symmetric... Many properties are also obtained combining different relations; for instance we have:

$$\forall x\ \forall y\ \forall z\ ((P(x,y) \wedge O(x,z)) \Rightarrow O(y,z))$$

which states that an individual overlapping a part of another individual also overlaps the latter.

Classical mereology as well as Clarke's contains a fusion operator for summing up any collection of individuals into a new individual. It is principally used for defining Boolean operators such as intersection or complement. We felt that this general operator was here unnecessary. Indeed, it is possible to introduce the sum and the intersection of two individuals, the complement of an individual, and the universal individual without going second-order in this first part of the theory. We achieved this with the following axioms:

A4 $\forall x\ \forall y\ \exists z\ \forall u\ (C(u,z) \Leftrightarrow (C(u,x) \vee C(u,y)))$
this[3] individual z is noted x+y and represents the sum of x and y.

A5 $\forall x\ (\exists y\ \neg C(y,x) \Rightarrow \exists z\ \forall u\ (C(u,z) \Leftrightarrow \exists v\ (\neg C(v,x) \wedge C(v,u))))$
this individual z is noted -x and represents the complement of x.

A6 $\exists x\ \forall u\ C(u,x)$
this individual x is noted a* and represents the universal individual.

A7 $\forall x\ \forall y\ (O(x,y) \Rightarrow \exists z\ \forall u\ (C(u,z) \Leftrightarrow \exists v\ (P(v,x) \wedge P(v,y) \wedge C(v,u))))$
this individual z is noted $x \cap y$ and represents the intersection of x and y.

We get then a pseudo-Boolean algebra over individuals.[4]

[3] Note that axiom A3 guarantees the unicity of the individuals whose existence is asserted, and thus justifies the notations we introduce.

[4] It isn't a Boolean algebra because there is no null individual, otherwise any two individuals would overlap. Note that -x exists iff $x \neq a^*$ and that $x \cap y$ exists iff O(x,y).

1.2.2 Topology

The notion of non tangential part, stemming from the notion of external connection, enables Clarke to introduce the topological notions of interior, closure, and open or closed individual. Here again, Clarke's definitions based on the fusion operator are replaced by first-order axioms:

A8 $\forall x\ \exists y\ \forall u\ (C(u,y) \Leftrightarrow \exists v\ (NTP(v,x) \wedge C(v,u)))$
this individual y is noted ix and represents the interior of x.

A9 $\forall x\ (\exists y\ \neg C(y,x) \Rightarrow \exists z\ \forall u\ (C(u,z) \Leftrightarrow \exists v\ (\neg C(v,i(-x)) \wedge C(v,u))))$
this individual z is noted cx and represents the closure of x.[5]

D7 $OP(x) \equiv_{def} x =_{st} ix$ "x is open"

D8 $CL(x) \equiv_{def} x =_{st} cx$ "x is closed"

One additional axiom is necessary to ensure that the intersection of two open individuals is itself open:

A10 $\forall x\ \forall y\ (\forall z\ [(C(z,x) \Rightarrow O(z,x)) \wedge (C(z,y) \Rightarrow O(z,y))] \Rightarrow \forall z\ (C(z,x \cap y) \Rightarrow O(z,x \cap y)))$

These topological notions may not be directly useful by themselves; indeed, the difference between a closed individual and its interior (the former includes its "skin" or boundaries[6] while the latter doesn't) is not always intuitive. Still, it is important to prove that they can be introduced in this theory, for they guarantee that a number of important spatial concepts (e.g., connectedness, continuity) can in turn be defined. We recall below Clarke's definitions of separation and connectedness:

D9 $Sp(x,y) \equiv_{def} \neg C(cx,y) \wedge \neg C(x,cy)$ "x and y are separated".

D10 $Con(x) \equiv_{def} \neg \exists y\ \exists z\ ((y+z) =_{st} x \wedge Sp(z,y))$ "x is a connected individual"

Clarke's calculus introduced a novelty in mereology, namely the possibility to represent contact, with the relation EC. However, this relation is stronger than needed for modeling contact as it is used in NL. Indeed, an individual and its complement are not connected, which doesn't seem to match common sense. Thus, we introduce the relation of "weak contact" to describe the situation in which two objects touch (to match the terminology of [Hayes 85b]) without sharing any boundary, as opposed to the external connection that can be seen as a relation of "strong contact":

D11 $WCont(x,y) \equiv_{def} (\neg C(x,y) \wedge \forall z\ ((z \neq_{st} a^* \wedge P(x,z) \wedge OP(z)) \Rightarrow C(cz,y)))$ "x touches y"

An individual touches its complement (but does not connect with it). Two closed individuals[7] may touch each other, each being part of the other's complement. Weak contact has proved to be useful in modeling the semantics of *sur* (on) [Aurnague 91] and should be more appropriate than external connection in modeling worlds evolving through time since the occurrence or disappearance of EC(x,y) alters x and y spatio-temporal identity (because of the theorem $\forall x\ \forall y\ (EC(x,y) \Rightarrow C(x,y))$ and axiom A3) whereas the occurrence or disappearance of WCont(x,y) does not necessarily entail such an alteration.

[5] Note that cx exists iff $x \neq a^*$.

[6] Note, though, that these "boundaries" cannot be handled directly as individuals in this theory, otherwise no difference between external connection and overlap could be made. After the introduction of points in the following section, boundaries may be defined as sets of boundary points, see [Aurnague & Vieu, in press] and [Vieu 91].

[7] We assume that the spatio-temporal referents of concrete objects are closed, whereas the spatio-temporal referents of space portions such as interiors are open or semi-open.

1.2.3 Time

Time is then introduced by two primitive relations taken from [Kamp 79]: complete precedence ($<$) and temporal overlap ($\mathcal{O}$); axioms are recalled below:

A11 $\forall x\ \forall y\ (x<y \Rightarrow \neg y<x)$
A12 $\forall x\ x\mathcal{O}x$
A13 $\forall x\ \forall y\ (x\mathcal{O}y \Rightarrow y\mathcal{O}x)$
A14 $\forall x\ \forall y\ (x<y \Rightarrow \neg x\mathcal{O}y)$
A15 $\forall x\ \forall y\ \forall z\ \forall t\ (x<y \wedge y\mathcal{O}z \wedge z<t \Rightarrow x<t)$

Linearity ($\forall x\ \forall y\ \forall z\ (x<y \vee x\mathcal{O}y \vee y<x)$), the last (optional) axiom given by Kamp, cannot be retained here because, due to the sum axiom A4, there are temporally non-convex individuals.

Temporal inclusion and temporal equivalence can be defined:

D19 $x\subset_t y \equiv_{def} \forall z\ (z\mathcal{O}x \Rightarrow z\mathcal{O}y)$
D20 $x\equiv_t y \equiv_{def} x\subset_t y \wedge y\subset_t x$

Adding the following axiom of monotonicity [van Benthem 83]:

A16 $\forall x\ \forall y\ (x\subset_t y\Rightarrow (\forall z\ (z<y \Rightarrow z<x) \wedge \forall z\ (y<z \Rightarrow x<z))$

It is now important to introduce axioms to link temporal relations to spatio-temporal relations. First, if two individuals connect spatio-temporally, they must overlap temporally:

A17 $\forall x\ \forall y\ (C(x,y) \Rightarrow x\mathcal{O}y)$

This axiom, together with A14, yields $\forall x\ \forall y\ (x<y \Rightarrow \neg C(x,y))$ as a theorem.
The following axioms are needed to achieve a complete integration of temporal notions in the spatio-temporal structure:

A18 $\forall x\ \forall y\ (P(x,y) \Rightarrow x\subset_t y)$
A19 $\forall x\ \forall y\ \forall z\ ((x<y \wedge z<y) \Leftrightarrow (x+z)<y)$
A20 $\forall x\ \forall y\ \forall z\ (((x+y)\mathcal{O}z \Leftrightarrow (x\mathcal{O}z \vee y\mathcal{O}z))$

Note that Kamp's application of his theory of time to discourse interpretation is extended to any kind of individuals, so that not only spatio-temporal referents of eventualities can be compared temporally but also spatio-temporal referents of objects, locations, substances... (see §2.2.3 below).

1.2.4 Limits

One of the main spatial concepts handled in language, limit or boundary, couldn't be handled in Clarke's calculus in a straightforward fashion (cf. footnote 6). In order to make up for this lack, we introduce the notion of empty individual and regard a limit as a tangential part having an empty interior, empty in the sense that it has no closed individual as part. The principal reason why limits can be considered as individuals comes from the fact that we assume we work in a finite domain, which is realistic in our context of text understanding. If we had added a non-atomicity condition (as Clarke does), it would not have been possible to define limits in this way since no individual would verify these definitions. Moreover, we can characterize the limits in terms of empty individuals because the spatio-temporal referents of objects are closed and not open. An individual is empty if all its parts are open:

D12 $\mathrm{Empty}(x) \equiv_{def} \forall y\ (P(y,x) \Rightarrow OP(y))$ "x is empty"

Using this property, we build three types of limits (Lim1, Lim2, Lim3) through which surfaces, lines and points can be differentiated. To restrict these definitions to spatial limits only[8], we force the individuals to be temporally equivalent.

A part x of y is a limit of type 1 if it is a tangential part of y, has an empty interior, and any tangential part of x is also a tangential part of y (which means that everything inside the limit is flush with it):

D13 $\quad Lim1(x,y) \equiv_{def} x{\equiv_t}y \wedge Empty(i(x)) \wedge TP(x,y) \wedge \forall z\, ((x{\equiv_t}z \wedge TP(z,x)) \Rightarrow TP(z,y))$ "x is a limit of y"

The envelope of an individual is its maximal limit of type 1:

D14 $\quad Env(x,y) \equiv_{def} Lim1(x,y) \wedge \forall z\, (Lim1(z,y) \Rightarrow P(z,x))$ "x is the envelope of y"

The notion of limit of type 1 being thus defined, we can now state that a limit of type 2 is a "boundary" between two individuals that are themselves limits of type 1:

D15 $\quad Lim2(x,y) \equiv_{def} \exists z\, [Lim1(y,z) \wedge \neg Env(y,z) \wedge x{\equiv_t}y \wedge Empty(i(x)) \wedge TP(x,y) \wedge \forall t\, [x{\equiv_t}t \wedge TP(t,x) \Rightarrow \exists w\, (Lim1(w,z) \wedge EC(w,t) \wedge EC(w,y))]]$ "x is a type 2 limit of y"

Similarly we regard a limit of type 3 as a "boundary" between two individuals of the limit of type 2. In short, surfaces, lines and points satisfy the conditions of a limit of type 1, lines and points those of a limit of type 2 whereas individuals being limits of type 3 are exclusively points.

On the basis of this information it is possible to determine the nature of the spatial referents handled in the system:

D16 $\quad Surface(x,y) \equiv_{def} Con(x) \wedge Lim1(x,y) \wedge \neg\exists z\, Lim2(x,z)$ "x is (a portion of) surface of y"

D17 $\quad Line(x,y) \equiv_{def} Con(x) \wedge Lim2(x,y) \wedge \neg\exists z\, Lim3(x,z)$ "x is (a portion of) line around y"

D18 $\quad Point(x,y) \equiv_{def} Con(x) \wedge Lim3(x,y)$ "x is an end point of y"

These limit concepts play a great part in the formalization of ILNs like *dessus* (top extremity), *bord* (edge), *angle* (corner) [Aurnague 91] [Aurnague & Vieu, 93].

1.3 Points, Instants and Geometry

1.3.1 Points

Up to now, this theory of space-time doesn't say anything about spatio-temporal points. Set-theoretical and topological concepts such as inclusion and closure have been introduced in a non standard way (mereological tradition aside). As explained in §1.1, this presents significant advantages concerning representational issues for we need not having a function assigning particular sets of points to every entity. Yet, some spatial notions are more easily grasped when referring to points. This is the case for alignment and distance and, more generally, for all geometric notions resting on the concepts of straight line, circle, etc. Introducing points in the theory while avoiding the arbitrariness described above can be done by *defining* each point as a set of individuals, the exact opposite to the classical point of view. In this approach, the only points present in the representation are those we can "talk about" given the specific situation described; they recover an ontological and cognitive justification. In addition, the exact granularity needed is obtained automatically: a point may be subdivided into several points when further information is added at the

[8] The purpose is not to exclude temporal limits but anomalous individuals such as partly temporal and partly spatial limits.

mereological-topological level. There are two ways of conceiving point construction in this approach. One is to build maximal sets of nested individuals (for any two individuals of this set, one is part of the other) as put to use in [Tarski 72]; the other is to apply the ultra-filter technique (for any two individuals, their intersection belongs to the set; any individual having as part a member of the set belongs to the set; the set is maximal). In [Clarke 85], Clarke gives a definition of points that matches the ultra-filter construction approximately. However, Clarke's definition is flawed: only very strange structures[9] are models for the resulting theory. We then split his unique definition in two and obtain the following definitions for interior points (IPt) and boundary points (BPt):[10]

D21 $\quad \mathrm{IPt}(\alpha) \equiv_{\mathrm{def}} \neg\, \alpha{=}\Lambda \wedge$ (a)

$\forall x\, \forall y\, ((x{\in}\alpha \wedge y{\in}\alpha) \Rightarrow (O(x,y) \wedge x{\cap}y{\in}\alpha)) \wedge$ (b)

$\forall x\, \forall y\, ((x{\in}\alpha \wedge P(x,y)) \Rightarrow y{\in}\alpha) \wedge$ (c)

α maximal (i.e., $\forall\beta$ ((β verifies (a), (b) and (c)) $\wedge\ \alpha{\subseteq}\beta) \Rightarrow \alpha{=}\beta$))

D22 $\quad \mathrm{BPt}(\alpha) \equiv_{\mathrm{def}} \exists x\, \exists y\, (x{\in}\alpha \wedge y{\in}\alpha \wedge EC(x,y)) \wedge$

$\forall x\, \forall y\, ((x{\in}\alpha \wedge y{\in}\alpha) \Rightarrow ((O(x,y) \wedge x{\cap}y{\in}\alpha) \vee$

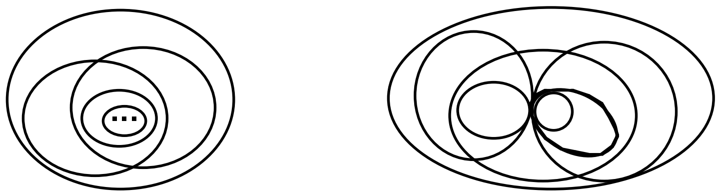
$\exists z\, \exists t\, (z{\in}\alpha \wedge t{\in}\alpha \wedge P(z,x) \wedge P(t,y) \wedge EC(z,t)))) \wedge$

$\forall x\, \forall y\, ((x{\in}\alpha \wedge P(x,y)) \Rightarrow y{\in}\alpha) \wedge$

α maximal

D23 $\quad \mathrm{Pt}(\alpha) \equiv_{\mathrm{def}} \mathrm{IPt}(\alpha) \vee \mathrm{BPt}(\alpha)$ "α is a point"

Figure 2 illustrates these two kinds of points:



interior point boundary point

Fig. 2. The two kinds of point.

For the sake of generality, we add an axiom stating the existence of points although it is needed only when there is an infinite number of individuals:

A21 $\quad \forall x\, \forall y\, (C(x,y) \Rightarrow \exists\alpha\, (\mathrm{Pt}(\alpha) \wedge x{\in}\alpha \wedge y{\in}\alpha))$

1.3.2 Instants

Instants, i.e., temporal points, are built in a similar fashion; they are maximal sets of temporally overlapping individuals:

D24 $\quad \mathrm{Inst}(\iota) \equiv_{\mathrm{def}} \forall x\, \forall y\, ((x{\in}\iota \wedge y{\in}\iota) \Rightarrow x\,\mathcal{O}\,y) \wedge \forall x\, (\neg x{\in}\iota \Rightarrow \exists y\, (y{\in}\iota \wedge \neg x\,\mathcal{O}\,y))$ "ι is an instant"

[9] As soon as there are two externally connected individuals, the closure of any other individual (universe excepted) must connect with those two individuals [Vieu 91].

[10] The Greek alphabet is used to quantify over sets of individuals; Λ is the null set.

Here, the existence of instants can be proved, i.e., $\forall x\ \forall y\ (x \mathcal{O} y \Rightarrow \exists \iota\ (\mathrm{Inst}(\iota) \wedge x \in \iota \wedge y \in \iota))$ is a theorem. Another theorem states that for each spatio-temporal point, there is a corresponding instant: $\forall \alpha\ (\mathrm{Pt}(\alpha) \Rightarrow \exists \iota\ (\mathrm{Inst}(\iota) \wedge \alpha \subseteq \iota))$. The unicity of this instant is guaranteed by the next axiom:

A22 $\forall \alpha\ (\mathrm{Pt}(\alpha) \Rightarrow \forall x\ \forall y\ (\forall z\ (z \in \alpha \Rightarrow (x \mathcal{O} z \wedge y \mathcal{O} z)) \Rightarrow x \mathcal{O} y))$

A partial order between instants is induced by the relation $<$:

D25 $\iota < \varphi \equiv_{def} \mathrm{Inst}(\iota) \wedge \mathrm{Inst}(\varphi) \wedge \exists x\ \exists y\ (x \in \iota \wedge y \in \varphi \wedge x < y)$ "ι precedes φ"

1.3.3 Distance

Spatial relations can now be introduced in order to deal with relative distances and orientation. Note that a spatial relation between two spatio-temporal points makes sense only if the two points correspond to the same instant. First of all, we define the notion of "null distance" between individuals, which in turn enables the definition of "null distance" between points. This last concept is very different from the classical distance notion, for two points may be at a null distance and still be different:

D26 $\mathrm{IND}(x,y) \equiv_{def} C(x,y) \vee \mathrm{WCont}(x,y)$ "x is at a null distance from y"
D27 $\mathrm{ND}(\alpha,\beta) \equiv_{def} \mathrm{Pt}(\alpha) \wedge \mathrm{Pt}(\beta) \wedge \exists \iota\ (\mathrm{Inst}(\iota) \wedge \alpha \subseteq \iota \wedge \beta \subseteq \iota))) \wedge \forall x\ \forall y\ ((x \in \alpha \wedge y \in \beta) \Rightarrow \mathrm{IND}(x,y))$ "α is at a null distance from β"

Then two primitive relations are needed: "α is closer to β than to γ" ($K(\alpha,\beta,\gamma)$) and "α is between β and γ" ($T(\alpha,\beta,\gamma)$), both adapted from [van Benthem 83]. K is axiomatized as follows in [van Benthem 83] (only A27 had to be modified to account for the notion of null distance):

A23 $\forall \alpha\ \forall \beta\ \forall \gamma\ (K(\alpha,\beta,\gamma) \Rightarrow (\mathrm{Pt}(\alpha) \wedge \mathrm{Pt}(\beta) \wedge \mathrm{Pt}(\gamma) \wedge \exists \iota\ (\mathrm{Inst}(\iota) \wedge \alpha \subseteq \iota \wedge \beta \subseteq \iota \wedge \gamma \subseteq \iota)))$

K is transitive and irreflexive:
A24 $\forall \alpha\ \forall \beta\ \forall \gamma\ \forall \delta\ ((K(\alpha,\beta,\gamma) \wedge K(\alpha,\gamma,\delta)) \Rightarrow K(\alpha,\beta,\delta))$
A25 $\forall \alpha\ \forall \beta\ \forall \gamma\ \forall \delta\ ((K(\alpha,\beta,\gamma) \wedge K(\gamma,\alpha,\beta)) \Rightarrow K(\beta,\alpha,\gamma))$
A26 $\forall \alpha\ \forall \beta\ \neg K(\alpha,\beta,\beta)$

Any point situated at a null distance from a given point is closer to it than any other point:
A27 $\forall \alpha\ \forall \beta\ ((\mathrm{ND}(\alpha,\beta) \wedge (\alpha=\beta\neq\gamma \vee \neg \mathrm{ND}(\alpha,\gamma))) \Rightarrow K(\alpha,\beta,\gamma))$

Space is "connected" for distance:
A28 $\forall \alpha\ \forall \beta\ \forall \gamma\ \forall \delta\ (K(\alpha,\beta,\gamma) \Rightarrow (K(\alpha,\beta,\delta) \vee K(\alpha,\delta,\gamma)))$

From the relation K, it is possible to define a 3-place relation of equidistance (E):

D28 $E(\alpha,\beta,\gamma) \equiv_{def} \neg K(\alpha,\beta,\gamma) \wedge \neg K(\alpha,\gamma,\beta)$ "α is at an equal distance from β and γ", adding an axiom for transitivity of equality:
A29 $\forall \alpha\ \forall \beta\ \forall \gamma\ \forall \delta\ ((E(\alpha,\beta,\gamma) \wedge E(\gamma,\alpha,\beta)) \Rightarrow E(\beta,\alpha,\gamma))$.

Other distance-related definitions such as equidistance and order between pairs of points can also be introduced [Vieu 91]. We then obtain a strict total order between equivalence classes of pairs of points.

Since points are defined as sets of individuals and conversely, the set of points "incident" in an individual can be reconstructed, distance notions between individuals are induced by the axioms on K. For instance, IK(x,y,z), "x is closer to y than to z", is defined as follows:

D29 $\mathrm{IK}(x,y,z) \equiv_{def} x \mathcal{O} y \wedge x \mathcal{O} z \wedge y \mathcal{O} z \wedge \exists \alpha\ \exists \beta\ (\mathrm{Pt}(\alpha) \wedge \mathrm{Pt}(\beta) \wedge x \in \alpha \wedge y \in \beta \wedge \forall \gamma\ \forall \delta\ ((\mathrm{Pt}(\gamma) \wedge \mathrm{Pt}(\delta) \wedge x \in \gamma \wedge z \in \delta) \Rightarrow \mathrm{Inf}(\alpha,\beta,\gamma,\delta)))$ "x is closer to y than to z" where $Inf(\alpha,\beta,\gamma,\delta)$ stands for "the distance between α and β is lesser than the distance between γ and δ", i.e., it's the order relation between pairs of points mentioned above.

1.3.3 Orientation

The relation T was first introduced in [van Benthem 83] through the following definition: $T(\alpha,\beta,\gamma) \equiv_{def} \forall\delta\ (\alpha=\delta \vee K(\beta,\alpha,\delta) \vee K(\gamma,\alpha,\delta))$. However, van Benthem rightly notes that the axioms for K don't suffice to establish some desired properties on T (in particular, the property corresponding to axiom A34 below). We'll then take T as another primitive relation, with the following axioms:

A30 $\forall\alpha\ \forall\beta\ \forall\gamma\ (T(\alpha,\beta,\gamma) \Rightarrow (Pt(\alpha) \wedge Pt(\beta) \wedge Pt(\gamma) \wedge \exists\iota\ (Inst(\iota) \wedge \alpha\subseteq\iota \wedge \beta\subseteq\iota \wedge \gamma\subseteq\iota)))$
A31 $\forall\alpha\ \forall\beta\ \forall\gamma\ (T(\alpha,\beta,\gamma) \Leftrightarrow T(\alpha,\gamma,\beta))$
A32 $\forall\alpha\ \forall\beta\ \forall\gamma\ (T(\alpha,\beta,\gamma) \Rightarrow \neg T(\beta,\alpha,\gamma))$
A33 $\forall\alpha\ \forall\beta\ \forall\gamma\ (T(\alpha,\beta,\gamma) \Leftrightarrow \forall\delta\ (\alpha=\delta \vee K(\beta,\alpha,\delta) \vee K(\gamma,\alpha,\delta)))$
A34 $\forall\alpha\ \forall\beta\ \forall\gamma\ \forall\delta\ (T(\alpha,\beta,\gamma) \Rightarrow (T(\gamma,\alpha,\delta) \Leftrightarrow T(\gamma,\beta,\delta)))$
A35 $\forall\alpha\ \forall\beta\ \forall\gamma\ \forall\delta\ ((T(\alpha,\beta,\gamma) \wedge T(\delta,\beta,\gamma)) \Rightarrow (\alpha=\delta \vee T(\alpha,\beta,\delta) \vee T(\delta,\beta,\alpha)))$
A36 $\forall\alpha\ \forall\beta\ \forall\gamma\ \forall\delta\ ((T(\alpha,\beta,\gamma) \wedge T(\alpha,\beta,\delta)) \Rightarrow (\gamma=\delta \vee T(\gamma,\alpha,\delta) \vee T(\delta,\alpha,\gamma)))$
A37 $\forall\alpha\ \forall\beta\ \forall\gamma\ \forall\delta\ \forall\varepsilon\ ((E(\gamma,\alpha,\beta) \wedge E(\delta,\alpha,\beta) \wedge (T(\varepsilon,\gamma,\delta) \vee T(\gamma,\varepsilon,\delta) \vee T(\delta,\varepsilon,\gamma))) \Rightarrow E(\varepsilon,\alpha,\beta))$

From both relations K and T, we introduce alignment between three points and straight lines as maximal sets of points that are three by three aligned (given that these three are not at a null distance one from the other):

D30 $Al(\alpha,\beta,\gamma) \equiv_{def} T(\alpha,\beta,\gamma) \vee T(\beta,\alpha,\gamma) \vee T(\gamma,\alpha,\beta)$ "α, β and γ are on a line"

D31 $SLine(\Delta) \equiv_{def} \exists\alpha\ \exists\beta\ (\alpha\in\Delta \wedge \beta\in\Delta \wedge \neg ND(\alpha,\beta)) \wedge \forall\alpha\ \forall\beta\ \forall\gamma\ ((\alpha\in\Delta \wedge \beta\in\Delta \wedge \gamma\in\Delta \wedge \neg ND(\alpha,\beta) \wedge \neg ND(\alpha,\gamma) \wedge \neg ND(\gamma,\beta)) \Rightarrow (Al(\alpha,\beta,\gamma) \wedge \forall\delta\ (Al(\alpha,\beta,\delta) \Rightarrow \delta\in\Gamma)))$ "Δ[11] is a straight line"

Several relations between straight lines can then be introduced. The following definition shows how we can deal with the notion of perpendicularity. Two straight lines are perpendicular iff there is a pair of points in each line such that each of the two points of the second line is situated at an equal distance from the two points of the first:

D32 $Per(\Delta1,\Delta2) \equiv_{def} SLine(\Delta1) \wedge SLine(\Delta2) \wedge \exists\alpha\ \exists\beta\ \exists\gamma\ \exists\delta\ (\alpha\in\Delta1 \wedge \beta\in\Delta1 \wedge \neg ND(\alpha,\beta) \wedge \gamma\in\Delta2 \wedge \delta\in\Delta2 \wedge \neg ND(\gamma,\delta) \wedge E(\gamma,\alpha,\beta) \wedge E(\delta,\alpha,\beta))$ "$\Delta1$ and $\Delta2$ are perpendicular"

1.3.4 Toward a geometry based on individuals

This theory can be developed further; for instance, we can define squares, circles, etc., in terms of the notions already introduced and thus start a description of simple shapes. Section 1.3 has shown that many (naive) geometric concepts can be dealt with in a relational theory of space. However, not everything is perfect here.

Points are here built on the basis of mereological and topological data only. This means we assume that the structure of space is completely determined by this kind of data, and that other data (such as distance and orientation) only describe this structure further, without altering it. This assumption has not been secured, though. We are now defining another version of this theory in which distance and orientation concepts are introduced by relations over individuals. This new version should make clear whether our assumption was true or not. Another advantage of this new version will be that the construction of points will then be

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[11] Uppercase Greek letters take for values sets of sets of individuals.

postponed until all the data has been represented, and merely play the part of an interface between the theory and its models —as done in [Kamp 79] for time. The points themselves will no longer be dealt with within the theory, so the theory will be first-order throughout and its computational tractability will benefit of the completeness of predicate logic. Moreover, the relations really useful for representing Natural Language expressions involving distance or alignment are relations between individuals, that is, relations such as IK and not the primitive relations K and T chosen here.

To start building a geometry based on individuals throughout, we first tackled projective geometry, as shown in the following section. For the moment, this is done introducing directions as primitive elements of a new kind, although we conjecture these elements could actually be defined in terms of pairs of individuals.

1.4 Projective geometry

As a first step towards the definition of a geometry based on individuals we introduce here several projective relations. In particular we try to characterize the relative position of two individuals with respect to some direction of the considered space and we propose a formal definition of the extremity of an individual in a direction. We have to make it clear that in this third part of the theory, restrictions on the domain of individuals apply, essentially concerning their shape. Up to now, this part of the theory has been applied to the modeling of the semantics of ILNs and projective prepositions. For the analysis of ILNs we had to restrict the research field of spatial entities to solid, undeformable and connected objects that also have a "normal usefulness". This is why, here, we deal with a class of individuals whose shape is roughly parallelepipedic, cylindrical or spherical.

We complete our ontology by introducing directions as new primitive elements. Directions have been already used in various works in the field of Qualitative Physics [Davis 89] or in semantic studies intended for example to handle the spatial information contained in car accident reports [Jayez 92].

A direction is viewed here as a primitive element which can be linked to ordered pairs of points by the following axiom, $d(\alpha,\beta)$ being a new primitive function giving the direction determined by two points α and β:

A38 $\quad \forall\alpha,\beta\ (PT(\alpha) \wedge PT(\beta) \wedge \neg ND(\alpha,\beta)) \Rightarrow \exists D\ d(\alpha,\beta)=D$

Henceforth, directions will be denoted by uppercase characters so as to differentiate them from individuals and entities which are noted with lowercase ones.

Another axiom indicates that symmetric ordered pairs of points determine opposed directions:

A39 $\quad \forall\alpha,\beta\ (Pt(\alpha) \wedge Pt(\beta)) \Rightarrow (d(\alpha,\beta)=D \Leftrightarrow d(\beta,\alpha)=-D)$

Then we specify the orthogonality between two directions using the definitions of straight lines and perpendicular straight lines in terms of points that we gave above (§ 1.3)[12]:

D33 $\quad Ortho(D,D') \equiv_{def} \exists D1,D2,a1,b1,a2,b2\ SLine(D1) \wedge SLine(D2) \wedge Per(D1,D2) \wedge a1\in D1 \wedge b1\in D1 \wedge a2\in D2 \wedge b2\in D2 \wedge ((d(a1,b1)=D \vee d(a1,b1)=-D) \wedge (d(a2,b2)=D' \vee d(a2,b2)=-D'))$

In consequence, two directions are called orthogonal iff two perpendicular straight lines supporting them can be found.

[12] In fact, it is also possible to define a straight line from a point and a direction. Then, a special axiom links the concept of direction to the relation "between" so as to make calculations involving both kinds of constructs possible.

We are studying the possibility of introducing a primitive relation between directions Kd(D1,D2,D3) which may express that "D2 is closer to D1 than D3 is" (in terms of angular values). Such a relation, similar to the already shown primitive K between points, would be irreflexive (cf. A26) and transitive (cf. A24). It should allow us to define the notions of opposite and orthogonal directions without referring to points and straight lines. We could as well define the median of two directions which can be considered as a kind of sum or composition between these two directions[13]. The behavior of this new primitive having not been completely analyzed and axiomatized we preferred not to integrate it in this paper.

To formalize correctly the orientational process, we also have to introduce at the geometric level a set of thirteen predicates constituting an extension of Allen's relations (R) [Allen 84]. Each formula R(x,y,D) indicates the configuration holding between the maximum intervals filled by the individuals x and y in the direction D[14]. Besides the classical axioms related to Allen's relations we introduce a postulate stating that for every pair of connected individuals x and y and every direction D, one of the relations m, o, s, d, f or = stands between them :

A40 $\forall x,y,D\ (C(x,y) \Rightarrow mosdf{=}m_i o_i s_i d_i f_i(x,y,D))$[15]

The analysis of orientation shows that giving an intrinsic orientation to an entity in a determined direction amounts to say that for “functional reasons” a particular portion of this entity constitutes an extremity in this direction (e.g., usually the neck of a bottle is up). Consequently, the extremity of an individual in a direction seems to be an important notion to grasp in order to formalize intrinsic orientation.

We state that y is an extremity of x in a direction D if y is a limit of x (as underlined above the concept of limit has been already formalized at the geometric level of our representation system) and furthermore if every individual included in x (and not included in y) precedes or meets y in this direction D:

D34 $Ext(y,x,D) \equiv_{def} Lim1(y,x) \wedge \forall v\ ((P(v,x) \wedge \neg P(v,y)) \Rightarrow {<}m(v,y,D))$

It can be observed that, in some cases, for two given individuals x and y (for instance when we are faced with the vertex y of a triangle x) several directions may verify this relation. Generally, this occurs when a tangent to the surface cannot be associated with some particular point.

If we wanted a unique direction to be selected, we would have to introduce more constraints or conditions. That is exactly what we do by introducing a relation "Exts", which indicates that y is an extremity of x in the direction D and z an extremity (of a part u of x) in the opposed direction:

D35 $Exts(y,z,x,D) \equiv_{def} Ext(y,x,D) \wedge \exists u\ (P(u,x) \wedge P(y,u) \wedge Ext(z,u,-D) \wedge Salient(z,x) \wedge (\neg\exists v\ Point(z,v) \vee \neg\exists v\ Point(y,v)))$

[13] The opposite of a direction is the direction which is the farthest. A direction orthogonal to a given one is situated at an equal distance from this direction and its opposite. The median of two directions is equidistant between these two directions and it is nearer to them than other directions verifying the same equidistance property (except for opposed directions which have several medians: two in 2D space and a whole plane in 3D space).

[14] The constraint of spatial connectedness on the studied entities ensures that the extension along a given direction is an interval. Moreover, one may feel necessary to fully express these relations in terms of projections of the individuals onto a straight line and of calculations on the resulting intervals (as usually with Allen's relations). Previously we had in our system a predicate of projection alone with several axioms specifying its behavior. However, this predicate has not been used here because it would have implied manipulating "abstract" straight lines, points and intervals not having the same status than the ones defined in §1.2. We think that such a specification requires a preliminary study of the cognitive processes underlying these operations of projection.

[15] On the basis of this postulate and using the definition of inclusion (relation P of Clarke) as well as several theorems related to Allen's relations it can be proved for instance that :
$\forall x,y,D\ P(x,y) \Rightarrow sf{=}sifi(x,y,D)$

In this definition the predicate "Salient" accounts for the visual and cognitive processes that lead us to select a geometrically salient individual z in the individual x. A further specification of this phenomenon makes a precise study of the underlying processes necessary. The remainder of the definition ensures that this individual z constitutes an extremity in the direction -D and that one of the two extremities is not punctual.

Going back to the case of the triangle, such an additional condition allows us (by taking into account the direction orthogonal to the base of the triangle) to select a unique direction among the first set of directions.

2 The functional level

On this level we deal with the orientation process, plural structures as well as some concepts of naive physics such as stabilization or containment. As all these properties directly concern the entities themselves, we handle variables that represent entities and not simple portions of space-time (as is the case on the geometric level). These are designated through the "function" stref. It is also on this level that we introduce the semantic definitions of the spatial expressions we study. Finally, we show the validity of these definitions by verifying the adequacy of their inferential behavior with the results of human reasoning.

2.1 Intrinsic orientation

Using the different tools we have built up to now at the geometric level, and taking into account the properties of the entities themselves, we can tackle the formalization of the orientational process. In this paper, we consider first the intrinsic case, examining only vertical and frontal orientation, that is, leaving aside the lateral case; deictic case, as well as all contextual cases, is eventually grounded on the intrinsic orientation of some entity, consequently, these are studied in the definition section (§ 2.3). In the same way as we restricted the type of entity processed by the system (cf. the third part of the space-time theory, § 1.4) to connected and roughly parallelepipedic, cylindrical or spherical objects, we introduce some constraints on orientations. First, the texts studied are "instantaneous" in the sense that the entities described as well as the speaker do not change in position with respect to one another. Moreover, we make the hypothesis that an entity is oriented by a unique speaker.

Basing our analysis on the remark we made about the importance of the extremity notion for intrinsic orientation, we introduce a new partial function mapping an extremity y of an entity x (and an extremity z of a portion of x) on the corresponding direction D:

A41 $\forall$x,y,z,D (dir-ext(y,z,x)=D $\Leftrightarrow$ (Part(y,x) $\wedge$ Part(z,x) $\wedge$ Exts(stref(y),stref(z),stref(x),D)))

Henceforth we will say that such a direction is generated by the extremities y and z of x.

The above axiom, which handles directly entities and not simple portions of space,[16] relies on the geometric relation "Exts" (indicating that the individual stref(y) constitutes an extremity of stref(x) in a direction) as well as on the part-whole relations between entities. These relations, also called meronomies, call for many functional notions and are not reducible to simple spatial inclusion. They are fully described in §2.2.2.

[16] The function stref gives us the portion of space-time filled by an entity; as a result of the instantaneity constraint previously mentioned, this portion corresponds here to a specific temporal slice temporally bounded by the event (or state) introduced by the NL spatial expression analyzed.

Starting on with the vertical intrinsic orientation, a particular direction of an entity can be considered as its upper intrinsic direction if, in a canonical position, this direction coincides with the gravitational upper direction. We express these conditions by means of the following definition[17]:

D36 $\quad \text{Orient-haut}(D,x) \equiv_{def} \exists y,z\ (\text{dir-ext}(y,z,x)=D \wedge \text{Can-Us}(x) \wedge (\text{In-Use}(x) > \text{dir-ext}(y,z,x)=\text{haut-grav}))$

In this definition, the predicate "Can-Us" ensures that the entity x has a canonical use. The predicate "In-Use" together with the non-monotonic implication (> denoting an implicature) allows us restricting the coincidence of the directions to "normal" (canonical) uses of x. We think that the non-monotonic logic proposed in [Asher & Morreau 91] could be a good framework for handling such information.

A similar formula specifies what is a lower intrinsic orientation, and a biconditional links it to the previous upper orientation:

D37 $\quad \text{Orient-bas}(D,x) \equiv_{def} \exists y,z\ (\text{dir-ext}(y,z,x)=D \wedge \text{Can-Us}(x) \wedge (\text{In-Use}(x) > \text{dir-ext}(y,z,x)=\text{bas-grav}))$

A42 $\quad \text{haut-grav} = -\ (\text{bas-grav})$

The processing of frontal orientation calls for more complex mechanisms that mirror more complex phenomena. We distinguish three cases, which, as we will see, are not mutually exclusive.

The first case occurs when the frontal orientation of an entity x is induced by what Vandeloise calls the "general orientation" of x [Vandeloise 86], which depends on various factors such as the direction of motion, the arrangement of perception apparatus, etc. So, we first state that a given direction of an entity x can be considered as a front direction of type 1 if that direction of x coincides with its general orientation:

D38 $\quad \text{Orient-avant1}(D,x) \equiv_{def} \exists y,z\ \text{dir-ext}(y,z,x)=D \wedge \text{Orient-gen}(x,D)$

We find in this category human beings, animals, arrows but also cars and vehicles in general[18].

The second kind of frontal orientation covers all the entities that, in a canonical use, have their frontal direction coinciding with the frontal direction of the user. So, by means of this second rule, we state that a specific direction of an entity x constitutes a front direction of type 2 if the front direction of every entity using x in a canonical way coincides with this direction of x:

D39 $\quad \text{Orient-avant2}(D,x) \equiv_{def} \exists y,z\ (\text{dir-ext}(y,z,x)=D \wedge \text{Can-Us}(x) \wedge \forall u,v,D'\ ((\text{Utilize}(x,v) \wedge \text{Avant-i}(u,v,D')) > D'=\text{dir-ext}(y,z,x)))$

This second case of frontal orientation that we call tandem orientation happens with chairs, cars, clothes, etc.

The third (and last) rule corresponds to the entities that, in a canonical use, have their frontal direction opposed to the user's frontal direction (cupboards, computers, TVs, etc.):

D40 $\quad \text{Orient-avant3}(D,x) \equiv_{def} \exists y,z\ (\text{dir-ext}(y,z,x)=D \wedge \text{Can-Us}(x) \wedge \forall u,v,D'\ ((\text{Utilize}(x,v) \wedge \text{Avant-i}(u,v,D')) > D'=-\text{dir-ext}(y,z,x)))$

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[17] We indicated earlier that, in the framework of this work, we consider only "instantaneous" utterances. However, the properties of intrinsic orientation we define here concern the whole life of the entity (or at least a significant part of it) and therefore they must have a spatio-temporal reading. We are at the moment working on a temporal translation of such definitions in which directions should be also considered as extended over time (like the other spatio-temporal individuals). e denoting an event and y/e representing a slice of y whose time matches the time of e, a spatio-temporal version of "orient-haut" should be:
$\text{Orient-haut}(D,x) \equiv_{def} \exists y,z\ \text{dir-ext}(y,z,x)=D \wedge \text{Can-Us}(x) \wedge \forall e\ ((\text{Event}(e) \wedge e \subset_t \text{stref}(x)) \Rightarrow (\text{In-Use}(x,e) > \text{dir-ext}(y,z,x)/e=\text{haut-grav}/e))$

[18] But not a mere bullet which can only take a contextual orientation.

Finally, we express with the following rules that every entity having an intrinsic frontal orientation falls into one of these three cases and that front and back (intrinsic) directions stand in a relation of opposition:

D41 Orient-avant(D,x) $\equiv_{def}$ Orient-avant1(D,x) $\vee$ Orient-avant2(D,x) $\vee$ Orient-avant3(D,x)

A43 $\forall$x,D Orient-avant(D,x) $\Leftrightarrow$ Orient-arriere(-D,x)

As the formalization of the lateral cases is not completely worked out for the moment, we leave it aside in this paper. However, it can be underlined that this lateral modality calls for already more complex representations than frontal orientation does (which, as we just saw is itself more complex than the vertical one). This property of our formal tools seems to match perfectly the observations made by psycholinguists about acquisition and manipulation of orientation notions.

2.2 The structure of the entities

On the geometric level, there is only one relation to link a part to a whole, namely the spatial inclusion (noted P). As a result, it is impossible to describe a spatial referent as having a specific internal structure. However, we often give a particular structure to the entities, sometimes just by choosing the noun phrase used to refer to an object. For instance, *the rice* is conceived as a continuous entity whereas *the rice grains* is definitely a discrete one (i.e., a collection). At the same time, there exist several part-whole relations (also called meronomies) between the entities: "member-collection" (e.g., *Texas is a member of the USA*), "component-assembly" (e.g., *this wheel is a part of this car*), etc. This diversity alone precludes their direct identification with spatial inclusion. Besides, meronomies cannot be represented on the geometric level, because they rely on functional aspects of the entities. In particular, the relation "member-collection" cannot be reduced to a mere inclusion as, among other things, it is not transitive (The USA is a member of UNO whereas Texas is not).

We felt that dealing with the internal structures of the entities was part of our work for several reasons. First, ILNs designate parts of a whole, i.e., they describe a kind of meronomy; in addition, we will see in § 2.3 that the preposition *dans* (in) can be used to describe meronomies. Second, structural relations help explain what the links are between the different entities used to refer to the same physical object (i.e., having the same spatio-temporal referent). At last, we can mention that on the geometric level, any finite sum of individuals is a new individual whereas few of these sums correspond to actual entities. This can be explained by the fact that we conceptualize as entity only what has some type of internal structure; the knowledge of these structures then takes on another sort of importance.

2.2.1 The plural structure

The notion of collection is important in NL: most plural noun phrases (e.g., *Jean et Marie* 'John and Mary', *les arbres* 'the trees'...) refer to a collection, and many singular noun phrases too (e.g., *le couple Dupont* 'the couple Dupont', *la forêt* 'the forest'...). Two types of relations are associated with this notion: the relation "member-collection" (e.g., between one tree and the forest) and the relation "subcollection-collection" (e.g., between a smaller group of trees and the forest). To represent them, we have taken up the lattice structure introduced by G. Link in his "Logic of Plurals and Mass Terms" [Link 83].

In this structure, also called the "plural" lattice, the entities denoted by singular noun phrases in the discourse analyzed are represented as atoms, whereas the entities denoted by plural noun phrases are non-atomic elements. The latter are put in relation to their members and to their sub-collections by the ordering relation (noted $\leq_i$) of the lattice.[19] It must be noted that a collection referred to by a singular noun phrase appears as an atom in this structure, therefore their members are not directly related to them. Still, this atomic entity and the corresponding plural entity (e.g., *the forest* and *the trees*) are linked through the existence of another entity: the portion of matter making both of them up. Portions of matter are atoms in the plural lattice, but form themselves a new semilattice[20] structure —a "matter" semilattice— where the new ordering relation (noted $\leq$) is interpreted as "is a portion of". Obviously, the two ordering relations are not unrelated: if we have $x \leq_i y$ then we have $h(x) \leq h(y)$, where h is the homomorphism between semilattices that gives each entity its corresponding portion of matter. This function h, restricted to the set of entities that are portions of matter (a subset of the set of the plural lattice's atoms), is the function identity. The portions of matter are real entities people can refer to by what is called mass terms (e.g., *the water in my glass*, *the gold of my ring*), and the matter semilattice, together with some other tools, enables Link to deal with properties specific to mass terms such as cumulative reference[21].

However, E. Bach has emphasized the difficulty in [Bach 86] for any entity to pick the right entity corresponding to the portion of matter that makes it up. He gives as an example the entity *the snowman* which should be linked by the function h to *the snow making up the snowman*, but this portion of matter can also be described as being constituted by *the water making up the snow which makes up the snowman*, etc. It must be noted that these portions of matter are different entities since snow and water have different properties. So, even though we accept the presence of portions of matter as atoms of the plural lattice and the fact that any entity (including other portions of matter) may be linked to a portion of matter through a "constitution" relation, we think that the role of the matter semilattice has to be played by the spatial structure introduced on the geometric level.

In our proposition, to be consistent with the pseudo-Boolean structure of the geometric level, we have removed the null element from Link's plural lattice and so on the functional level too, we only have a pseudo-lattice. For this reason, we have chosen to represent this plural structure in a mereologic way instead of representing it classically according to set theory. The chosen primitive is $\leq$ (noted $\leq_i$ in Link's logic), the part-of relation of mereology.

The following are the two axioms required:

A44 $\forall x\ \forall y\ \forall z\ ((x \leq y \wedge y \leq z) \Rightarrow x \leq z))$

A45 $\forall x\ \forall y\ ((x \leq y \wedge y \leq x) \Leftrightarrow x=y)$

along with the following definitions (the last two are in fact axioms, cf. §1.2.1):

D42 $At(x) \equiv_{def} \forall y\ (y \leq x \Rightarrow y=x)$ "x is atomic"

A46 $\forall x\ \forall y\ \exists z\ \forall u\ (u \leq z \Leftrightarrow \exists v\ (v \leq u \Rightarrow \exists w\ (w \leq v \wedge (w \leq x \vee w \leq y))))$ "z, noted $x \oplus y$, is the sum of x and y"

A47 $\forall x\ \forall y\ (\exists v\ (v \leq x \wedge v \leq y) \Rightarrow \exists z\ \forall u\ (u \leq z \Leftrightarrow (u \leq x \wedge u \leq y)))$ "z, noted $x \otimes y$, is the intersection of x and y"

An axiom is necessary to link the plural structure of the entities and the spatio-temporal structure:

A48 $\forall x\ \forall y\ (x \leq y \Rightarrow P(stref(x),stref(y)))$

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[19] In this structure, it is then impossible to make the difference between a singleton and its unique member. We could not find any evidence of NL's behaving otherwise.

[20] Only the join operator is useful with respect to mass term properties. This semi-lattice is non-atomic in G. Link's theory: he assumes that there are no minimum portions of a given substance.

[21] To fully understand this logic, the reader is strongly invited to read [Link 1983]. As for mass terms' behavior, the reader may refer to [Pelletier &Schubert 1989], a good synthesis of the abundant literature on this topic.

We mentioned above that an atomic collection and the corresponding plural collection were linked through the identity of their portions of matter: in our case, the link would be made through the identity of their spatial referents. However, this may raise ambiguities because different plural entities can describe the same concrete object (i.e., can have the same spatio-temporal referent), as with the example of the two entities *the decks of cards* and *the cards* given by Link. Moreover, it is not desirable to link mass terms describing objects that can also be seen as collections[22] (e.g., *the rice in this bowl*) to the corresponding plural entities (e.g. *the rice grains in this bowl*) because, as it is claimed by many philosophers and linguists, mass terms do not behave as collections but as if they were describing fully continuous entities [Pelletier & Shubert 89]. Therefore, we introduce the relation "is the collection" that links a singular (i.e., atomic) entity designating a collection to the right plural entity. This relation is noted Is-coll. It satisfies the following axioms:

A49 $\forall x \forall y\ (\text{Is-coll}(x,y) \Rightarrow (\text{At}(x) \wedge \neg\text{At}(y)))$

A50 $\forall x \forall y\ (\text{Is-coll}(x,y) \Rightarrow \text{stref}(x) =_{\text{st}} \text{stref}(y))$

A51 $\forall x \forall y \forall z\ ((\text{Is-coll}(x,y) \wedge \text{Is-coll}(x,z)) \Rightarrow y=z)$

2.2.2 Meronomies

With the help of the preceding plural structure, we are now able to define the various meronomies that can be expressed in NL. We already mentioned the two meronomies: "member-collection" and "subcollection-collection". Here, we consider two more, namely "component-assembly" (e.g., *my nose is part of my head*) and "piece-whole" (e.g., *this shard was part of my cup*). Actually, we have shown in [Vieu 91] that another pair of meronomies should be added in order to cover the whole range of part-whole relations: "portion-whole" (e.g., *this is a portion of the strawberry pie mum made*) and "substance-whole" (e.g., *there is butter in this cake*), but as the study of mass term properties goes beyond the scope of this paper, we decided to set them aside.

The relation "component-assembly" has to be distinguished from the relation "piece-whole" on the grounds that in the first, the part has a well-defined function with respect to its whole, whereas in the latter, the part is arbitrarily defined in its whole (e.g., *South-West of France*, *the top of the ball*). In the first case, the function of the part usually gives it well-defined boundaries, but in the second case the part must often be designated by describing its boundaries, for instance, using an ILN. Moreover, a component can be unconnected (for instance, a set of spanners in a tool box), whereas a piece is only a connected part.

D43 $\text{Member}(x,y) \equiv_{\text{def}} \text{At}(x) \wedge ((\neg\text{At}(y) \wedge x \leq y) \vee \exists z\ (\text{Is-coll}(y,z) \wedge x \leq z))$ "x is a member of the collection y"

D44 $\text{Subcoll}(x,y) \equiv_{\text{def}} (\neg\text{At}(x) \wedge \neg\text{At}(y) \wedge x \leq y \wedge \neg x=y) \vee \exists z(\neg\text{At}(y) \wedge \text{Is-coll}(x,z) \wedge z \leq y) \vee \exists z(\neg\text{At}(x) \wedge \text{Is-coll}(y,z) \wedge x \leq z) \vee \exists z\ \exists u(\text{Is-coll}(x,z) \wedge \text{Is-coll}(y,u) \wedge z \leq u)$ "x is a subcollection of y"

D45 $\text{Component}(x,y) \equiv_{\text{def}} \text{At}(y) \wedge \neg\exists z\ \text{Is-coll}(y,z) \wedge \text{Funct}(x,y) \wedge \text{PP}(\text{stref}(x),\text{stref}(y))$ "x is a component of the assembly y"

D46 $\text{Piece}(x,y) \equiv_{\text{def}} \text{At}(y) \wedge \neg\exists z\ \text{Is-coll}(y,z) \wedge \neg\text{Funct}(x,y) \wedge \text{PP}(\text{stref}(x),\text{stref}(y)) \wedge \text{Con}(\text{stref}(x))$ "x is a piece of the whole y"

Finally, we group all four cases in one predicate in order to represent the part-whole relations in an unspecified way:

D47 $\text{Part}(x,y) \equiv_{\text{def}} \text{Member}(x,y) \vee \text{Subcoll}(x,y) \vee \text{Component}(x,y) \vee \text{Piece}(x,y)$ "x is a part of y"

[22] This is not the case for any mass term. For example, it is impossible to determine what is an elementary particle of 'dirt', 'mud', 'milk shake'... Nevertheless, the "set approach" has been taken by a number of authors such as Laycock [1972].

We also need two axioms for the relation Funct(x,y) to express that x has some kind of function with respect to y. This relation is not deeply analyzed in this work, but it is enough for our immediate purposes to state the transitivity of "Funct" and the fact that if a part has no function with respect to a whole, it also has no function with respect to an all-inclusive entity:

A52 $\forall x\ \forall y\ \forall z\ ((\mathrm{Funct}(x,y) \wedge \mathrm{Funct}(y,z)) \Rightarrow \mathrm{Funct}(x,z))$

A53 $\forall x\ \forall y\ \forall z\ ((\mathrm{Part}(x,y) \wedge \neg\mathrm{Funct}(x,y) \wedge \mathrm{part}(y,z)) \Rightarrow \neg\mathrm{Funct}(x,z))$

It is worth noting that we do not have as a theorem $\forall x\ \forall y\ (\mathrm{PP}(\mathrm{stref}(x),\mathrm{stref}(y)) \Rightarrow \mathrm{Part}(x,y))$, in particular, since if y is a collection, only its members and its subcollections can be considered as parts.

2.2.3 Classification of entities

We will see, in particular for the preposition *dans* (in), that the semantic study of spatial markers shows the necessity of distinguishing several classes of entities. These are: the objects (e.g., *a tree*, *a forest*, *Mary*, *a glass of water*), the locations (e.g., *a country*, *a crossroads*, *the Atlantic Ocean*, *the Pyrénées*, *my garden*, *a forest*) and the space portions (e.g., *the inside of a box*, *a hole in the sheet*, *a cave in the mountain*, *the middle of (the inside of) the room*)[23].

There are three important facts to note. First, the space portions are always described with the help of another entity that is an object or a location. It has been underlined in §1 that the spatio-temporal referents of the entities are determined by their physical matter. This is not directly the case for space portions, for they are immaterial; still, they are functionally determined from some material entity(ies), so their spatio-temporal referents are geometrically determined from its (their) spatio-temporal referent(s)[24].

Second, a last class of entities is needed, here as in most areas of NL semantics: the eventualities (events and states). However, eventualities (identified by the predicate Event()) are not dealt with in detail in this work, and in particular, the various part-whole relations occurring among them have not been studied. As a consequence, the meronomies above only apply to entities of the three other kinds.

Third, the same noun phrase can designate different entities. For instance, *the forest* can be an object: the collection of trees, and it can be a location: the portion of ground on which the trees grow. The same holds for many noun phrases: a city is either a collection of houses, or a location; a sea or a river is either a water body, or a location; a mountain is either a heap of earth, or a location.

But not every entity can belong to a definite class. There is no rule to prevent someone from forming either the plural noun phrase *John and the USA* in NL or the corresponding plural entity in the plural structure. However, we make the assumption that every atomic entity always belongs to one of the three classes. This assumption is rather strong because it implies that there is no singular noun phrase describing a "mixed" collection such as *John and the USA*. This may be a drawback of our formalism, but we did not find any counter-example to this assumption. As a consequence, we add the axioms:

A54 $\forall x\ ((\mathrm{At}(x) \wedge \neg\mathrm{Event}(x)) \Rightarrow (\mathrm{Obj}(x) \vee \mathrm{Loca}(x) \vee \mathrm{Sp\text{-}port}(x)))$

A55 $\forall x\ (\mathrm{Obj}(x) \Rightarrow (\neg\mathrm{Loca}(x) \wedge \neg\mathrm{Sp\text{-}port}(x)))$

[23] Actually, there are two more classes of entity: portions of matter and substances. They are needed to represent mass terms and the two meronomies we have set aside.

[24] The space portions could be considered as being of the same type as spatial referents, that is, as being no entity, since their spatial referent as well as all of their properties derive from those of objects or locations. This position was not chosen, on the grounds that we can talk about space portions on the same level as for other entities, and we can even state some of their properties, e.g. *the cave is dark and humid*.

A56 $\forall x\ (Loca(x) \Rightarrow (\neg Obj(x) \wedge \neg Sp\text{-}port(x)))$

A57 $\forall x\ (Sp\text{-}port(x) \Rightarrow (\neg Obj(x) \wedge \neg Loca(x)))$

Finally, we can state that in a part-whole relation the part and the whole are of the same type, restricting this axiom to the cases where the whole is not a plural entity (which could be a mixed entity):

A58 $\forall x\ \forall y\ ((Part(x,y) \wedge At(y)) \Rightarrow ((Obj(x) \wedge Obj(y)) \vee (Loca(x) \wedge Loca(y)) \vee (Sp\text{-}port(x) \wedge Sp\text{-}port(y))))$

The role of this axiom is in particular to exclude the interpretation of a mere spatio-temporal inclusion between an object and a space portion as a piece-whole relation. In fact, in *la boîte est dans l'intérieur de l'armoire* 'the box is in the cupboard's inside', the box is certainly not conceived as a part of the inside.

In addition, it is necessary to modify the above definition of the "piece-whole" meronomy (§2.2.2), which was only conceived for material entities (objects and locations). For space portions, we actually need to take into account the material entity that determines it. The inside of a drawer of a cupboard is a part (piece-whole) of the inside of the cupboard: the two space portions are determined by two objects linked by a meronomy (here, "component-assembly"). This is not the case for the inside of a box situated in the cupboard: the two space portions have no conceptual link, neither do the two objects. To allow for this modification, we add the following axioms, where the relation Det(x,y) should be read as "x determines y":

A59 $\forall x\ (Sp\text{-}port(x) \Rightarrow \exists y\ ((Obj(y) \vee Loca(y)) \wedge Det(y,x)))$

A60 $\forall x\ \forall y\ \forall t\ \forall u\ ((Piece(x,y) \wedge Sp\text{-}port(x) \wedge Det(t,x) \wedge Det(u,y)) \Rightarrow (t=u \vee Part(t,u))$

2.3 Definitions

Thanks to all the geometric and functional tools introduced above, we can now express the raw (i.e., not taking into account the context; this is done on the pragmatic level) semantics of various spatial lexemes. We will especially consider the formalisms proposed for dealing with orientational lexemes and the preposition *dans* (in).

2.3.1 Orientational lexemes

Here we would want to illustrate how the intrinsic orientational tools we have proposed previously can be used for the representation of the semantic content of both internal and external localization expressions. As we saw, the axes and directions arising in such intrinsic orientations are linked to the entity itself. In the case of a contextual process the orientation is the result of the interaction between the entity concerned and another entity in the context. This means that the relevant axes and directions in contextual orientation derive from the interaction between these two entities. In the framework of this study, we only consider a particular case of contextual orientation, namely the deictic one, in which the orienting entity is the speaker. However, interpretations relying on vertical contextual orientation (which is given by gravity) will be formalized because, very often, vertical deictic uses are restricted to situations where the speaker is standing up.

2.3.1.1 Internal Localization Nouns (ILNs)

Let us start with Internal Localization Nouns (ILNs) and more precisely with the definition of the *haut* (top) of an entity. Intuitively, the intrinsic top corresponds to the portion of the entity situated in the pole whose direction is the intrinsic upper direction. In consequence, we state by means of the following definition, that an entity y constitutes the intrinsic top of an entity x if y is the maximal element situated in the pole of x whose direction is D, and furthermore, if this direction corresponds to the intrinsic upper direction of x:

D48 $\text{Haut-i}(y,x,D) \equiv_{def} \text{Orient-haut}(D,x) \wedge \text{In-pole}(y,x,D) \wedge \forall w\ (\text{In-pole}(w,x,D) \Rightarrow \text{Part}(w,y))$

The direction D appearing in this predicate "Haut-i" plays a very important part for the distinction between intrinsic and deictic top cases. In the case of an intrinsic top this direction comes from the entity itself whereas in a deictic situation it is given by another element of the context (the speaker) and does not have any special relation with the entity[25]:

D49 $\text{Haut-d}(y,x,D) \equiv_{def} \exists v\ (\text{Orient-haut}(D,v) \wedge v \neq x \wedge \text{Speaker}(v) \wedge \text{In-pole}(y,x,D) \wedge \forall w\ (\text{In-pole}(w,x,D) \Rightarrow \text{Part}(w,y)))$

As we pointed out before, and according to some experiments made by psychologists and psycholinguists [Carlon-Radvansky & Irwin 93], these vertical deictic uses are much more acceptable when they coincide with vertical contextual uses that is to say when the intrinsic upper direction of the speaker coincides with the gravitational up. In consequence, although in this study as a whole, we do not consider contextual cases other than deictic ones, the contextual use of *haut* (top) seems an important configuration to describe:

D50 $\text{Haut-c}(y,x,\text{haut-grav}) \equiv_{def} \text{In-pole}(y,x,\text{haut-grav}) \wedge \forall w\ (\text{In-pole}(w,x,\text{haut-grav}) \Rightarrow \text{Part}(w,y)))$

We give below the definitions corresponding to the concept of pole (and inclusion in a pole). Basically we can say that the pole y of an entity x in a direction D is constituted by the portion of x extending from the middle of x to its extremity in the direction D. These rules essentially rely on Allen's relations between the spatio-temporal referents of the previously mentioned entities (middle, extremity, etc.) in the direction D:

D51 $\text{Pole}(y,x,D) \equiv_{def} \exists e,m\ (\text{Part}(y,x) \wedge \text{Ext'}(e,x,D) \wedge \text{mid}(m,x) \wedge \text{m}(\text{stref}(m),\text{stref}(y),D) \wedge \text{f}(\text{stref}(e),\text{stref}(y),D))$

D52 $\text{In-pole}(y,x,D) \equiv_{def} \exists u\ (\text{Pole}(u,x,D) \wedge \text{Part}(y,u))$

On the basis of our orientational tools, we can introduce similar formal representations for the ILNs *bas* (bottom), *avant* (front), *arrière* (back). It is also possible to specify the semantic content of ILNs such as *dessus* (top extremity), *dessous* (bottom extremity), *devant* (front extremity), *derrière* (back extremity) using the same formalization of orientational phenomena. The only difference between the semantic definition of these lexemes and the representations associated to the ILNs *haut, bas, avant, arrière,* etc., concerns the topological and geometric aspects. For instance, the *dessus* (top extremity) of an entity is the uppermost surface (roughly) perpendicular to the upper direction and in contact with the exterior of the entity. We obviously need here topological and geometric concepts that are much more complex than the sole notion of pole in a direction. In [Aurnague 91], several definitions are introduced in order to characterize what is an external surface perpendicular to a direction D and most advanced in this direction.

2.3.1.2 External prepositions

Now, we are going to show how our orientational formalism allows us to express the meaning of the external preposition *devant* (in front of). We can say that an entity y is situated (intrinsically) in front of an entity x if y is included in the space portion situated in front of x (that is to say the space portion delimited by means of x and its intrinsic frontal direction). In order to grasp such a configuration, we introduce the predicate In-sp(y,x,D) which specifies that an entity y is included in the space delimited by the entity x and

[25] It can be deduced (from the definition of vertical intrinsic orientation) that when the speaker is in a canonical position, the direction applied to the spatial configuration coincides with the gravitational upper direction.

the direction D. From a more formal point of view this is expressed by stating that a relation m_i or > stands between the spatio-temporal referents of y and x in the direction D[26]:

D53 $\quad$ In-sp(y,x,D) $\equiv_{def}$ m_i >(stref(y),stref(x),D)

Then we can characterize the fact that an entity y is situated intrinsically in front of an entity x setting down that y has to be contained in the space delimited by x and the direction D, which in turn constitutes the intrinsic frontal direction of x:

D54 $\quad$ Etre-devant-i(y,x,D) $\equiv_{def}$ Orient-avant(D,x) ∧ In-sp(y,x,D)

Here again the deictic use of the preposition *devant* (in front of) differs from the intrinsic use in the underlying direction given by a speaker describing the scene situated in front of him:

D55 $\quad$ Etre-devant-d(y,x,D) $\equiv_{def}$ ∃w (Orient-avant(-D,w) ∧ w≠x ∧ w≠y ∧ Speaker(w) ∧ In-sp(y,x,D) ∧ Etre-devant-i(x,w,-D))

The fact that the speaker is facing the landmark to which he gives a frontal orientation means that we consider a mirror configuration (between the orienting speaker and the landmark) and is expressed by the minus sign associated with the underlying direction of the predicate "Orient-avant". In fact, mirror deictic configurations are very frequent in French as opposed to the tandem ones, less often used.

Before finishing the presentation of the lexeme *devant*, it may be mentioned that the notions of distance and relative size between the trajector and the landmark play a great part in the semantics of most spatial prepositions. Actually, the importance of these notions increases when we consider combinations of the same relation (as in the utterances above) because they constitute factors that can block the application of transitivity. However, although distance and relative size rely on geometric tools, their part is heavily affected by contextual factors. Consequently such phenomena have to be described and formalized at the pragmatic level.

2.3.2 The preposition *dans*

The preposition *dans* (in), used in the given syntactic structure and in a concrete spatial sense, can refer to three different configurations between trajector and landmark. In the first configuration, probably the main case, *dans* links a trajector that is completely included in the inside of the landmark, as in the following examples. These cases are called "total dans".

(1) a. *L'eau est dans le verre* 'The water is in the glass'
b. *La ville est dans le brouillard* 'The city is in the fog'
c. *Paul est dans l'île*[27] 'Paul is in the island'
d. *Il y a un trou dans ce morceau de fromage* 'There is a hole in this piece of cheese'
e. *L'île est dans la mer* 'The island is in the sea'

In the second one, the trajector is only partially included in the inside of the landmark. We refer to these cases as "partial dans".

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[26] This specification of "In-sp" is sufficient because we only consider parallelepipedic, spherical or cylindrical entities. If we wanted to take into account more complex shapes (amphitheaters, arches and more generally curved objects) we would have to state a much more complicated formula. We tested the latter for some particular entities and we showed that some interesting inferential properties obtained on the basis of this simple version of "In-sp" were lost.

[27] With nouns referring to types of location like countries or regions, the use of *dans* is impossible and *en* 'in' has to be used (e.g., *Paul est en France* 'Paul is in France'). Because this fact cannot be explained on semantic grounds, we will deal with these cases as well.

(2) a. *La rose est dans le vase* 'The rose is in the vase'
b. *L'arbre est dans une toute petite cour* 'The tree is in a very small yard'

Finally, *dans* can be used to describe cases of meronomy; the trajector is then a part of the landmark. These cases are called "dans part-of".

(3) a. *Mon cerveau est dans ma tête* 'My brain is in my head'
b. *La Bretagne est en France* 'Brittany is in France'
c. *Paul est dans le jury* 'Paul is in the jury'

2.3.2.1 "total dans", "partial dans" and the "inside" function

The first two configurations involve the notion of inside besides geometric relations (inclusion and overlap). The inside of an entity must be distinguished: the entity and its inside must not be considered as a whole, since we want to differentiate between "total dans" and "dans part-of". It is worth noting that the inside, being a space portion, behaves in a very different way from the object: an object included in the inside is not a part of this inside since it can be removed without altering the inside's identity and shape, whereas a (connected) object included in another is a part of it, fully contributing to its nature. However, the inside of an entity is completely determined by this entity, and we always perceive one with the other. This fact has led many authors (including us in an earlier work [Borillo, M. & Vieu 89]) to consider the inside as being a part of the entity and then to formalize *dans* or *in* as the mere inclusion or overlap between the spatial referents of the trajector and the landmark.

The inside of an object can be of two types, depending on its ability to contain another object. If the object is a "container" (solid, offering resistance to gravity and lateral movements)[28], then its inside is defined by functional properties, besides geometric ones. Otherwise, the inside can only be defined geometrically. The sentences (1a) and (2a) are examples of the "container" case. As Herskovits and Vandeloise have shown, the "place" of the inside of the glass (the space taken by the inside) cannot be defined only by removing the "place" of the glass in the convex hull of the "place" of the glass, which is a geometric function. Indeed, in Figure 3 below, the fly is not *dans le verre* (in the glass). Restricting the scope of the convex hull to the containing parts of the object (as Vandeloise does) is not satisfactory either because in Figure 3, the fly would remain *in the glass*. We need to take each of the object's concavities into account and either "close" it or not, depending on its containing capability.

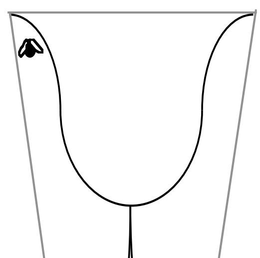

Figure 3

[28] This property is actually complex and goes deeply into several "clusters" of naive physics [Hayes 85]. For example, friction is involved in the situation of a bulb contained by a socket.

In the second case, the object is not a "container"; this occurs with collections: *la foule* (the crowd), with non-solid objects: *l'eau* (water), *le brouillard* (fog) as in (1b), and with solid objects occupying a volume evenly without filling it: *l'arbre* (the tree) (as in *l'oiseau est dans l'arbre* 'the bird is in the tree'). In these cases, the spatial referent of the inside is determined by an "outline" function. It should be noted that here too, this function is not simply the convex hull. To begin with, the outline clings closer to the sides of the object than the convex hull does [Herskovits 82]. What is more, a sizable "hole" may be excluded; for instance, if a crowd forms a circle, what is situated *dans le cercle* (in the circle) is not situated *dans la foule* (in the crowd), i.e., the inside of this circle is not a part of the inside of the crowd.[29] There are also a few cases where the landmark is not a container, yet it is solid and it is not a collection. For instance, in *la boîte est dans le papier-cadeau* 'the box is in the wrapping paper', the paper is not a container but is actually supported by the box. In these cases, the landmark completely surrounds its inside: the spatial referent of the inside is a connected part of the complement of the spatial referent of the landmark.

The inside of an object is used to locate other objects as in sentences (1a), (1b) and (2a). It is also used to locate space portions as in (1d). The interpretation of such a sentence is that the space portion (here the hole) is a part of the inside of the object (here the piece of cheese), the meronomy being a "piece-whole" case. It is not just an inclusion, for the hole is "attached" to the object just as the inside is, i.e., both are determined by the object.

The inside of a location, as expressed in sentences (1c) and (2b), is determined by the location's boundaries and the vertical axis: it is the space portion situated above the ground portion considered. But its exact thickness is not determined. We cannot go too high: a plane flying over a town is not situated *dans la ville* (in the town). We cannot stay too low: a bird can be flying *dans la plaine* (in the plain). When the same noun phrase describes both an object and a location (a town, a forest...) the inside of the location includes the inside of the object.

It should be clear by now that the "inside" function does not embody a unique concept. However, we will not represent it in such detail because our perspective is not to generate discourses from perception and world knowledge, e.g., to decide what is the inside of a given object. Rather, we need to understand how the function inside behaves deductively. For this purpose, examining in detail what is conceived as an inside was helpful. We were led to the conclusion that for a first step, considering the inside as a whole was possible, as its deductive behavior is almost homogeneous. We must admit that we are leaving some cases aside[30], but we hope that in the future we can model the various notions of inside more precisely.

In addition to such cases, there are occurrences of "total dans" or "partial dans" that do not call for the use of an inside. The localization of an object in a space portion (e.g., *la chouette est dans le creux de l'arbre* 'the owl is in the tree's hollow') is just explained by the inclusion ("total dans") or overlap ("partial dans") of the object's spatial referent with the space portion's spatial referent. The localization of a location in another location, without any part-whole relation between the two is slightly more complicated. Sentence (1e) is an example of such localization; *Berlin-ouest était en RDA* 'West-Berlin was in DDR' is another one. In these (few) cases, the landmark totally and tightly surrounds the trajector. Another type of location's inside for situating locations could be defined, but its interest would be limited because not every location situated in such an inside would be *dans* the first location. For instance, a field in the island is not in the sea. A location is "total

[29] Here again, to be able to distinguish between several entities having the same spatial referent can be useful. Note that *the circle* is not linked to the plural entity which *the crowd* is linked to, so it is not a collection.

[30] For instance, from *la ville est dans l'enceinte* 'the city is in the (surrounding) wall' and *l'enceinte est dans les arbres* 'the wall is in the trees', the sentence *la ville est dans les arbres* 'the city is in the trees' does not follow, and yet our model does infer this fact. The characterization of an inside we give actually covers quite well all cases except the case of an "outline" function applied to a landmark having a "hole" (in this example, it is the case for *the trees*)

dans" ("partial dans" is not possible) another location if all the boundaries of the trajector are also boundaries of the landmark.

An important fact to note is that even though it is impossible to deal with the function "inside" on the geometric level (containment and classification of entities are involved in its definition), the preposition *dans* cannot be reduced to the relation of containment. There are many examples without any containment between the landmark and the trajector: in *la mouche est dans la volière* 'the fly is in the aviary', the cage does not prevent the fly from getting out of it, in any direction whatsoever; in (1b), the city is unable to move anyway, but the fog does not even support it.

The definition obtained for "total dans" is:

D56 $TDs(x,y,e) \equiv_{def} ((Obj(x) \wedge (Obj(y) \vee Loca(y)) \wedge P(i(stref(x,e)), stref(int(y),e))) \vee (Sp\text{-}port(x) \wedge Obj(y) \wedge (Piece(x, int(y)) \vee x=int(y))) \vee (Obj(x) \wedge Sp\text{-}port(y) \wedge P(i(stref(x),e), stref(y),e)) \vee (Loca(x) \wedge Loca(y) \wedge \forall z\ ((TP(stref(z,e), stref(x,e)) \wedge EC(stref(z,e), c(stref(ground,e)\text{-}stref(x,e)))) \Rightarrow EC(stref(z,e), stref(y,e)))))$[31]

and the definition for "partial dans" is:

D57 $PDs(x,y,e) \equiv_{def} ((Obj(x) \wedge (Obj(y) \vee Loca(y)) \wedge O(i(stref(x,e)), stref(int(y),e))) \vee (Obj(x) \wedge Sp\text{-}port(y) \wedge O(i(stref(x,e)), stref(y,e)))$

where e is the eventuality (in this case, a state) described by *x est dans y* in the discourse analyzed.

These definitions are accompanied by the following axioms describing the function "inside":

A61 $\forall x\ \forall y\ (y=int(x) \Rightarrow ((Obj(x) \vee Loca(x)) \wedge Sp\text{-}port(y) \wedge Det(x,y) \wedge \forall z\ (z=int(x) \Rightarrow y=z) \wedge \neg C(stref(x), stref(y)) \wedge (Obj(x) \Rightarrow P(i(stref(y)), preint(stref(x))))))$

A62 $\forall x\ \forall y\ \forall t\ \forall u\ \forall r\ ((t=int(x) \wedge u=int(y) \wedge Part(x,y) \wedge Rest(y,x,r)) \Rightarrow P(stref(t), stref(u)+stref(r)))$

A63 $\forall x\ \forall y\ \forall t\ \forall u\ \forall r\ ((t=int(x) \wedge u=int(y) \wedge P(i(stref(x)), stref(u)) \Rightarrow P(stref(t), stref(u)+stref(y)))$

With the geometric function "preint" designating the convex hull of an individual minus this individual, and the relation Rest(y,x,r) expressing that r is the rest of the whole, y, once its part, x, has been taken out.

2.3.2.2 "dans part-of"

The preposition *dans* cannot be used to describe any meronomy. Vandeloise showed that we cannot say **mon nez est dans ma tête* 'my nose is in my head'. We think that this fact is better accounted for by a "contrast principle" than by the explanation he gives (the trajector and the landmark should not share any boundary).

This principle expresses that when someone uses a spatial expression to describe a meronomy, he (as well as the hearer) conceives the part as detached from the whole and situates this part with respect to the rest of its whole. In the case of sentence (3a), we can see that detaching the brain from the head creates an inside where we situate the brain. Detaching the nose from the head does not create such an inside: the nose cannot be in the head. With this principle, we can even explain why in sentences like *l'escargot / la noix est dans sa coquille* 'the snail / the nut is in its shell', the whole is described as being in one of its parts! Note that only "total dans" is then possible between the part and the rest of the whole. The same principle can be found in many sentences corresponding to the syntactic structure *Ntraj est sur ILN de Nland*, such as *la poignée est sur le devant de l'armoire* 'the handle is on the front of the cupboard'. The relation *être sur* really links the handle and the front of the cupboard without its handle.

[31] The difference operator, noted x -y , is the short for the intersection of x with y's complement.

However, this principle is only required for a "dans part-of" describing the meronomies "component-assembly" and "piece-whole", and only between objects. In all other cases every occurrence of a meronomy can be described by the preposition *dans* (as in sentence (3b)).

The definition of "dans part-of", the last one for *dans* (in), is then:

D58 $DPt(x,y,e) \equiv_{def} Part(x,y) \wedge ((Obj(x) \wedge (Component(x,y) \vee Piece(x,y))) \Rightarrow \exists z\ (Rest(y,x,z) \wedge TDs(x,z,e)))$

Before finishing the presentation of formal definitions let us indicate that we also used this theoretical framework for representing the semantic content of the internal preposition *sur* (on) [Aurnague 91]. Its definition is essentially grounded on the geometric notions of contact, relative position on the vertical axis and relative size (or comparable category). It also calls for the Naive Physic concept of "support" whose behavior has been characterized by means of several axioms. We distinguish three configurations of *sur* (on) according to the position on the vertical axis of the target in relation to the landmark. If the target is placed higher than the landmark (e.g., *le livre est sur la table* 'the book is on the table') the situation is referred to as *sur1*. The case in which the target is at the same level than the landmark (e.g., *l'affiche est sur le mur* 'the poster is on the wall') is called *sur2*. *Sur3* applies when the target is placed lower than the landmark (e.g., *la mouche est sur le plafond* 'the fly is on the ceiling'). We have distinguished these three meanings because they do not introduce the same kind of conditions concerning relative size and support and because they have different inferential properties (in particular concerning transitivity). For more details, the reader may refer to [Aurnague & Vieu, 93]

2.4 Inferences

As we said previously in the description of our methodological choices, we wish to obtain a semantic representation of utterances allowing us to draw inferences that have to be in accordance with the deductions made by human beings. We already showed in [Aurnague & Vieu 93] that the inferences we can draw with the formal definitions of *sur* (on) combined with ILNs such as *haut* (top), *devant* (front extremity), *dessous* (bottom extremity) match our commonsense intuitions. For instance from *le vase est sur le dessus de l'armoire* (the vase is on the top extremity of the cupboard) we can deduce that *le vase est sur le haut de l'armoire* (the vase is on the top of the cupboard). We will not give here the different steps of such a reasoning and rather we will focus our attention on the inferences and calculations we are able to make using the semantic definitions proposed in the above section for the external preposition *devant* (in front of) and the internal one *dans* (in).

2.4.1 External prepositions

We consider two sentences in which the preposition *devant* (in front of) appears and we examine the results obtained by applying transitivity to their formal representations. We split the verification into three cases according to the deictic or intrinsic nature of the relation involved in each of the two sentences we combine.

2.4.1.1 Intrinsic-intrinsic case

An example of an utterance made up of two intrinsic *devant* (in front of) prepositions is[32]:

La tabouret est devant le fauteuil 'The stool is in front of the armchair'

[32] Obviously these two sentences may be also interpreted in a deictic way. We make here the hypothesis that, when a lexeme pointing out a target with an intrinsic frontal orientation is identified in the text analyzed, the intrinsic interpretation of the preposition *devant* is chosen by default.

Le fauteuil est devant Max 'The armchair is in front of Max'

Using the formal tools introduced for the preposition *devant*, we can give the following representation of these two sentences in which t, f and m respectively denote the stool, the armchair and Max:

Etre-devant-i(t,f,d1)

Etre-devant-i(f,m,d2)

From the predicate "In-sp" appearing in the definition of "Etre-devant" we can deduce the following Allen's relations between the spatio-temporal referents of t, f and m:

mi>(stref(t),stref(f),d1)

mi>(stref(f),stref(m),d2).

A very important parameter in the sense that it affects the overall deduction process concerns the identity of the directions d1 and d2 underlying the two relations "Etre-devant". If we know that these directions coincide (which is formally expressed by d1=d2) we can, on the basis of the axioms associated with the Allen's relations (here we use the theorem ∀ x,y,z mi>(x,y,D) ∧ mi>(y,z,D) => >(x,z,D)), deduce that >(stref(t),stref(m),d2), which, according to the definition of "In-sp", entails In-sp(t,m,d2). Associating this fact with the information about frontal intrinsic orientation of m: Orient-avant(d2,m) contained in the definition of Etre-devant-i(f,m,d2), we have finally:

Etre-devant-i(t,m,d2) ⇔ Orient-avant(d2,m) ∧ In-sp(t,m,d2)

Consequently, we succeed in calculating that *le tabouret est devant Max* (the stool is in front of Max) intrinsically, given the two previous sentences and the additional constraint ensuring the coincidence between the intrinsic frontal direction of the armchair and Max. The importance of this constraint is illustrated by figures 4 and 5. In the first case the two intrinsic directions coincide and *le tabouret est devant Max* can be uttered whereas in the second configuration they are different so that the previous deduction cannot be drawn. Although, for pictorial facilities, we have represented aligned entities with identical or opposed intrinsic directions, it may be noted that, according to the definitions we proposed for *devant* (in front of), directional prepositions do not require alignment along the underlying direction[33]. Moreover the blocking of the inferences based on transitivity occurs every time the directions associated with the relations are different and not only when they are opposed (as illustrated in the figures).

[33] The relative preference for the alignment of the entities involved in an external or directional preposition greatly depends on contextual factors. For this reason this variable (geometrical) feature has not been integrated in the semantic definition of the preposition *devant* (in front of) and is controlled at the pragmatic level.

Figure 4 Figure 5

2.4.1.2 Deictic-deictic case

In an utterance such as the following, the landmarks involved in the two prepositions *devant* take their orientation from the speaker[34]:

Le tabouret est devant la plante 'The stool is in front of the plant'

La plante est devant le lampadaire 'The plant is in front of the light'

The speaker can linguistically express the fact that this description completely depends on its spatial position with respect to the configuration by adding at the beginning of each sentence, an expression such as *vu d'ici* (seen from here).

The following facts (based on the formal tools we described above) with t, p and l denoting respectively the stool, the plant and the light express the semantic content of the previous sentences:

Etre-devant-d(t,p,d)

Etre-devant-d(p,l,d)

The identity of the directions underlying each relation "Etre-devant" comes directly from the hypothesis we made about the uniqueness of the speaker uttering such sentences and about the instantaneous character of such texts (the speaker doesn't change position). The same direction being associated with the two deictic relations "Etre-devant", we can here again calculate that In-sp(t,l,d) and finally conclude that Etre-devant-d(t,l,d), which means that *le tabouret est devant le lampadaire* (the stool is in front of the light, deictically). We do not specify all the calculation steps because they are very similar to what has been shown in the previous example.

Obviously, if the underlying directions had been different, it would not have been possible to draw such an inference. This may occur only when the spatial configuration is described from different positions or point of views in the two sentences (a same speaker occupying distinct positions at different moments or two speakers situated at distinct positions at the same moment, which is excluded in our work). Figures 6 and 7 bring to the fore that transitive deductions correctly work when a unique speaker (Max) applies his orientation to the landmarks (the plant and the light): in this case (figure 6) the sentence *le tabouret est devant le lampadaire* (the stool is in front of the light, deictically) can be inferred. On the contrary, the presence of two speakers (Max and Arthur in figure 7) describing the spatial configuration from distinct positions makes that the sentence *le tabouret est devant le lampadaire* is neither true from Max's point of view nor from Arthur's one.

[34] In this case there is no more ambiguity because the two landmarks do not have any intrinsic frontal orientation so that only a deictic interpretation of the preposition *devant* is possible.

Figure 6

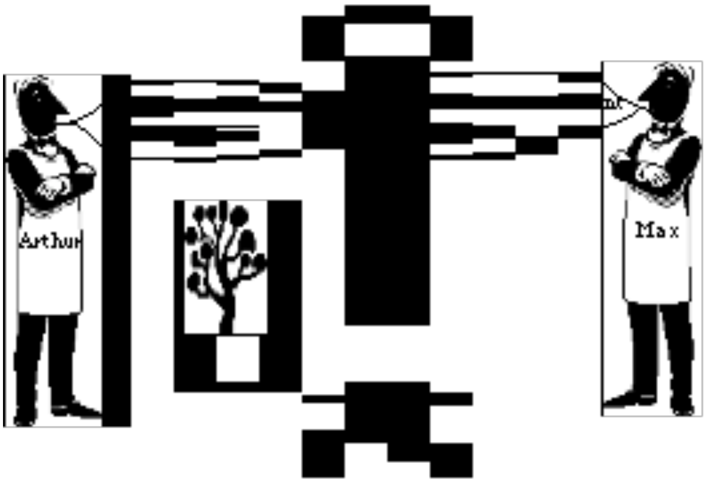

Figure 7

Let us consider now what kind of calculation may occur if the previous spatial configuration was described by means of an utterance composed of a deictic *devant* combined with a deictic *derrière* (rather than two deictic *devant*):

La tabouret est devant la plante 'The stool is in front of the plant'

Le lampadaire est derrière la plante 'The light is behind the plant'

t, p and l denoting here again the stool, the plant and the light the following formulas express the semantic content of the previous sentences:

Etre-devant-d(t,p,d)

Etre-derrière-d(l,p,-d)

From the definitions of "Etre-devant" and "Etre-derrière" and the relation "In-sp" we can state the following facts in terms of extended Allen's relations:

mi>(stref(t),stref(p),d)

mi>(stref(l),stref(p),-d)

The second expression being equivalent to mi>(stref(p),stref(l),d), transitivity axioms associated with Allen's relations allow us deducing that mi>(stref(t),stref(l),d) which means that:

In-sp(t,l,d)

We also know, from the definition of deictic "Etre-devant" that it exists a speaker w different from t and l (the stool and the light) who has an intrinsic front orientation corresponding to the direction -d:

Orient-avant(-d,w) $\wedge$ w$\neq$t $\wedge$ w$\neq$l $\wedge$ Speaker(w)

To prove Etre-devant-d(t,l,d) it remains to state that l is situated (intrinsically) in front of w or in other words that this speaker w faces the light to which she/he is applying her/his frontal orientation.

From the expression Etre-devant-d(t,p,d) we can deduce that the speaker w uttering these sentences is facing p:

Etre-devant-i(p,w,-d).

The predicate "In-sp" appearing in the definition of "Etre-devant-i" tells us that:

mi>(stref(p),stref(w),-d)

This relation combined with mi>(stref(l),stref(p),-d) (previously mentioned) entails by transitivity mi>(stref(l),stref(w),-d) which means that In-sp(l,w,-d).

Consequently we have:

Etre-devant-i(l,w,-d) $\Leftrightarrow$ Orient-avant(-d,w) $\wedge$ In-sp(l,w,-d)

Grouping together all the facts we have stated up to now we can, as in the previous example, infer the following formal expression, which indicates us that *le tabouret est devant le lampadaire* (the stool is in front of the light, deictically):

Etre-devant-d(t,l,d) $\Leftrightarrow$ Orient-avant(-d,w) $\wedge$ w$\neq$t $\wedge$ w$\neq$l $\wedge$ Speaker(w) $\wedge$ In-sp(t,l,d) $\wedge$ Etre-devant-i(l,w,-d))

These calculations show that it is possible to draw identical deductions from utterances describing a spatial configuration by means of different but semantically equivalent external relations.

2.4.1.3 Intrinsic-deictic case

The last case we consider here combines an intrinsic use of the relation "Etre-devant" with a deictic one:

Le tabouret est devant le fauteuil 'The stool is in front of the armchair'

Le fauteuil est devant le lampadaire 'The armchair is in front of the light'

From the formal representation of these sentences (Etre-devant-i(t,f,d1) $\wedge$ Etre-devant-d(f,l,d2)), and with the same kind of calculation as previously applied, we succeed in deducing that *le tabouret est devant le lampadaire* (the stool is in front of the light, deictically: Etre-devant-d(t,l,d2)).

Again, all the deductive process is conditioned by the coincidence between the intrinsic frontal direction of the armchair d1 and the deictic frontal direction d2 given to the light by a speaker w. Figures 8 and 9 illustrate respectively what happens when these directions are identical and when they are not[35].

[35] In figure 8, the deictic orientation of the light given by the speaker Max (facing it) could be also interpreted as a contextual orientation of the light by the armchair. Although it is true that these two orientations coincide in the

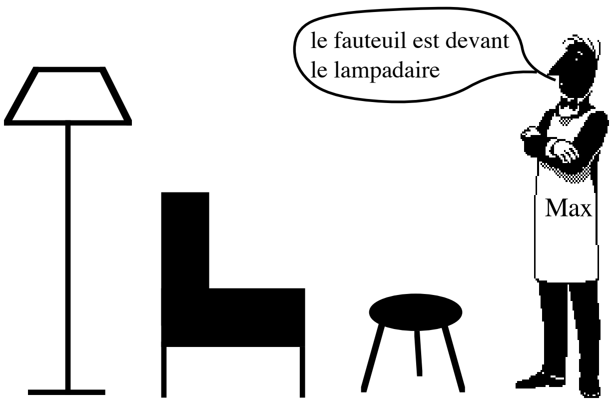

Figure 8

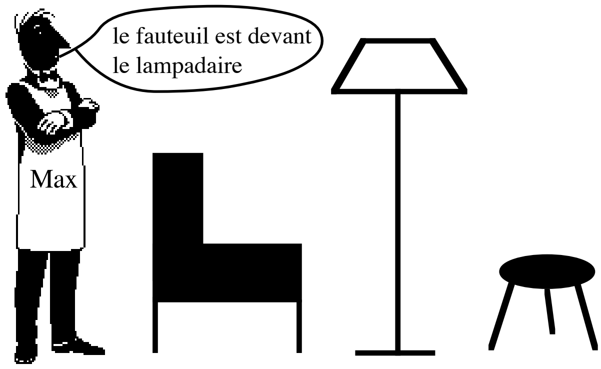

Figure 9

2.4.2 The preposition *dans*: its transitivities

Studying the transitivities of *dans* showed that it was necessary to distinguish the three cases of *dans*: "total dans" is (in some cases) transitive, "partial dans" is not, and the transitivity of "dans part-of" depends on the transitivity of the underlying meronomies.

"Total dans", between two objects and a location, is transitive; this can be seen in the following example:

Paul est dans la maison and *La maison est dans l'île* ⇒
'Paul is in the house' 'The house is in the island'
Paul est dans l'île
'Paul is in the island'

Since Paul and the house are objects and the island a location, the antecedent translates as: TDs(p,m,e1) ∧ TDs(m,i,e2), which in turn gives: P(i(stref(p,e1)), stref(int(m),e1)) ∧ P(i(stref(m,e2)), stref(int(i),e2)), where p, m and i denote Paul, the house and the island, respectively.

First of all, we assume $e1 \subset_t e2$, or at least $e1 \equiv_t e2$ to be able to combine the two sentences. This gives us P(stref(int(m),e1), stref(int(m),e2).

The second clause gives, by axiom A63, P(stref(int(m),e2), stref(int(i),e2)+stref(i,e2)). Then, because we have ¬O(stref(int(m),e2), stref(i,e2)) by a general postulate on the discreteness of the spatio-temporal referents of objects and locations, we obtain P(i(stref(int(m),e2)), stref(int(i),e2)) by the theorem ∀x ∀y ∀z ((P(x,y+z) ∧ ¬O(x,y)) ⇒ P(ix,z)).

Since ∀x ∀y (P(x,iy) ⇒ P(ix,iy)) and ∀x ∀y (P(ix,y) ⇒ P(ix,iy)) are also theorems, by transitivity of P, we finally conclude P(i(stref(p,e1)), stref(int(i),e1)), which is the translation of the consequent.

"Total dans" between three objects x, y and z is also transitive, given the fact ¬O(stref(int(y)), stref(z)), realistic in most contexts.[36]

"Partial dans" is not transitive, however, due to the non transitivity of O. In addition, the combination of a "total dans" and a "partial dans" is not valid as the following example shows:

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configuration depicted by figure 6 we have to recall that, in the framework of this study, we only take into account intrinsic and deictic orientational processes.

[36] Transitivity is not applicable, for instance, to situations in which Part(x,z) is verified.

L'abeille est dans la rose and *La rose est dans le vase* ÷⇒
'The bee is in the rose' 'The rose is in the vase'
**L'abeille est dans le vase*
*'The bee is in the vase'

Again, this is due to the fact that nothing can be inferred from $P(x,y) \wedge O(y,z)$.

As we said, the transitivity of "dans-part-of" depends on the transitivity of the underlying meronomies. For instance, "member-collection" is not transitive whereas "piece-whole" and "component-assembly" are. Moreover, many "mixed" transitivities (e.g., combining Component(x,y) and Member(y,z)) are not valid [Winston et al. 87] [Vieu 91]. The transitivity between two "dans part-of" based on the "component-assembly" or "piece-whole" meronomy involves in addition a transitivity between "total dans" when applied to objects[37].

Apart from the distinction in three cases, we have taken into account the class of each argument in the definitions of *dans*. This was important, as the transitivity of "total dans" depends on its arguments belonging to one or another class.

The combination of a "total dans" between an object and a location with a "total dans" between two locations is not valid, as shown by the following example:

Paul est dans l'île and *L'île est dans la mer* ÷⇒
'Paul is in the island' 'The island is in the sea'
**Paul est dans la mer*
*'Paul is in the sea'

The non transitivity is due to the fact that the insides of two locations linked by a "total dans" are not included one in another; they don't even overlap. Remember that "total dans" between two locations is akin to "surrounds", that is, it doesn't situate the first location with respect to the inside of the second location. Note that if we had a "dans part-of" between the last two locations instead (e.g., with Brittany and France), transitivity would hold.

The combination of a "total dans" between a space portion and an object with a "total dans" between two objects is not valid, due to the presence of a meronomy ("piece-whole") in the first relation:

Il y a un trou dans le drap and *Le drap est dans le tiroir* ÷⇒
'There is a hole in the sheet' 'The sheet is in the drawer'

**Il y a un trou dans le tiroir*
*'There is a hole in the drawer'

The two antecedents translate as: Piece(tr, int(d)) $\vee$ tr=int(d)) and P(i(stref(d,e2)), stref(int(ti),e2)), and the consequent as: Piece(tr, int(ti)) $\vee$ tr=int(ti)), where tr, d and ti stand for the hole, the sheet and the drawer, respectively. Even though it is possible to infer P(stref(tr,e), stref(int(ti),e)) (with a similar reasoning process to the one applied above for "total dans" between two objects and a location), this doesn't allow us to conclude Piece(tr, int(ti)), a much stronger formula involving functional aspects.

The complete study of all the possible combinations (about thirty) can be found in [Vieu 91], along with the study of combinations between *dans* and meronomies (not only those expressed by *dans*). As it has just been

[37] This last transitivity, which is valid, depends also on the properties given to the relation Rest, not presented here.

seen, there are many combinations that are not valid, even though many authors have until now taken the transitivity of *dans* (or *in*) for granted, defining it only by the relation of inclusion.

3 The Pragmatic level

We have seen that the adequacy was rather good between what the system is able to infer, and the deductions a human can draw. However, this adequacy is not perfect because, up to now, we have only taken into account the semantic aspects of the spatial expressions and, as in many other fields of language, pragmatic principles play a great part in their interpretation. On top of functional knowledge, they use world knowledge (in particular, knowledge of typical situations) and information about context. The principles we consider here may be seen as instances of more general ones (such as Gricean cooperativity principles [Grice 75]) in the spatial domain.

First, pragmatic principles lead to deduce in some cases (by implicature, which explains why we need a non-monotonic logic on this level e.g., that of [Asher & Morreau 91]) more information than is really present in the text and is represented on the first two levels. For instance, the sentence *Marie est dans la voiture* 'Mary is in the car' is generally understood as *Mary is in the passenger space*, discarding at the same time the alternative *Mary is in the boot*.

Second, they may rule out some expressions (for example, expressions inferred at the previous levels) because, even though their "raw" semantics is verified by the system and in the model, they cannot be uttered, since using the first process mentioned, these expressions would be regarded as conveying information contradictory with what is known. For instance, if we know that *Marie est dans le coffre de la voiture* 'Mary is in the car's boot' is true, then *Marie est dans la voiture* 'Mary is in the car' is not false, and yet, in general we cannot answer to *where is Mary?* with the latter sentence, for in most contexts, it is interpreted as *Mary is in the passenger space*.

A "fixation principle" underlies the examples cited above. This principle, first introduced in [Vandeloise 86] expresses that the typical use of an object "fixes" some of its characteristics. For instance, the front and the back of a car are "fixed" by the usual —not the actual— direction of its motion. Indeed, many intrinsic orientations are determined this way. This principle is also at work when talking about the inside of an object. The inside of a bottle stays the same even when the bottle is handled upside down so that it cannot contain any liquid. Moreover, some containing concavities of an object can be rejected as part of its inside on the grounds that there exists another concavity that corresponds to the "fixed" inside (such a phenomenon explains why, in the previous examples, the boot is not considered as a "normal" inside for a car). Several other principles may be found. We can mention, for instance, the principle of maximum target or the principle of minimum landmark, which are fully described in [Aurnague & Vieu 93].

Third, we must describe the pragmatic phenomenon that enables us to loosen some conditions of the semantic definitions or to add further constraints. An example of loosening occurs with the preposition *sur* (on) for which we may drop the condition of contact between the spatial referents of the target and the landmark. Doing this makes *sur1* transitive; indeed, if we have a pile of books on a table, any of the books may be described as being on the table. If instead of two books, we have a lid on a tea-pot, the tea-pot being on the table does not allow us to say that the lid is on the table [Herskovits 82]. The loosening of some conditions must be controlled by Grice's maxim of relevance: if a relation is more relevant than another, the first cannot be "forgotten" and the second used.

A case of condition addition happens with the preposition *devant* (in front of) whose semantic representation (showed at the functional level) constrains the positions of the trajector and the landmark with respect to the frontal axis. The definition stated that, as soon as the trajector y is further on the frontal direction (associated to x) than the landmark x, y can be described as being *devant* (in front of) x whatever its lateral position with respect to x is (y can be on the left of/in front of/on the right of x). Nevertheless, because of the enunciation context (scene surrounding x and y, intentions of the speaker, etc.), we may want to say that *y est exactement devant x* (y is exactly in front of x) or *y est davantage devant x que ne l'est z* (y is more in front of x than z is), etc. The influence of the context can also be such that only entities situated exactly in front of x could be described as being *devant* x. By reducing the degree of freedom on the lateral axis this pragmatic phenomenon amounts to focus on the frontal direction; we call this "axial priority".

It should be underlined that these pragmatic principles and phenomena are still to be formalized. A non monotonic logical framework in which we could express these principles as well as, more generally, the rules and the knowledge governing discourse, is needed at this third level. In this perspective, we are analyzing the part of spatio-temporal information in the construction of discourse structures and we formalize it within Segmented Discourse Representation Theory [Asher 93]. We hope that this framework and especially the "Discourse Inference in Commonsense Entailment" (DICE, [Lascarides & Asher 91]) component of SDRT will also be appropriate for dealing with pragmatics of spatial expressions.

4 Conclusion

In this paper, we have shown how to build a three-level system (geometric, functional and pragmatic) for representing the meaning of spatial expressions and drawing inferences. This modular construction thus permits us to come closer to the natural reasoning expressed in discourse. We have also shown that it is possible to build a theory of space for representing the geometric aspects of spatial expressions' semantics in a relational framework, and we believe that this is rather new in this domain. In addition, let's note that this theory may be applied to other fields of AI; this has already been done in qualitative physics [Randell & Cohn 89] [Vieu & Martin-Clouaire, 94].

As it has been underlined, we would like to improve this theory so as to define all the geometric concepts we need on the basis of spatio-temporal individuals alone. Then, the introduction of points as sets of individuals should be postponed until we interpret the representations in the framework of model theory.

We expect the geometric level to be fairly comprehensive as far as static spatial expressions are concerned. But the situation is quite different for the functional aspects dealt with in this work. We think this area is boundless, so every new spatial expression will probably require the introduction of new functional concepts. For instance, most of Naive Physics concepts are likely to be needed. In addition, a study of the internal structure of eventualities (meronomies) may be useful.

Concerning the pragmatic level, we plan to carry on with the inventory of the principles that play a part in the domain of spatial expressions. An adequate logical framework should be also elaborated for the manipulation of such information and phenomena.

Beyond simple comparison between deductions made and results of the speaker's reasoning, we now contemplate validating our system (in collaboration with psychologists) by verifying the psycholinguistic anchorage of our representations through various experiments [Borillo, M. 91]. Because our formal definitions isolate and articulate the various components involved in the lexical meaning, testing such parameters should allow psychologists to reach a greater precision degree in the description of mental processes associated with the interpretation of spatial expressions.

References

Allen, J. (1984): Towards a general theory of action and time, *Artificial Intelligence* 23, 2: 123-154

Asher, N. (1993): *Reference to abstract objects in discourse* - Kluwer Academic Publishers, Dordrecht

Asher, N. & M. Morreau (1991): Commonsense Entailment: A Modal Theory of Nonmonotonic Reasoning - In *Proceedings of IJCAI-91*

Aurnague, M (1991): Contribution à l'étude de la sémantique formelle de l'espace et du raisonnement spatial : la localisation interne en français, sémantique et structures inférentielles - PhD dissertation, Université Paul Sabatier, Toulouse

Aurnague, M. & L. Vieu (1993): A three-level approach to the semantics of space - In *The semantics of prepositions: from Mental Processing to Natural Language Processing,* C. Zelinsky-Wibbelt (ed.) - de Gruyter, Berlin

Bach, E. (1986): The algebra of events, *Linguistics and Philosophy* 9: 5-16

van Benthem, J. (1983): *The logic of time* - Reidel, Dordrecht

Borillo, A. (1992): Le lexique de l'espace : prépositions et locutions prépositionnelles de lieu en français - In *Hommage à Nicolas Ruwet*, L. Tasmowski & A. Zrib-Hertz (eds.) - Communication et Cognition, Ghent

Borillo, M. (1991): Sémantique de l'espace et raisonnement spatial - In *Proceedings of the Colloque Sciences de la Cognition* - MRT, Paris

Borillo, M. & L. Vieu (1989): Eléments pour la formalisation du raisonnement spatio-temporel naturel - In *Proceedings of the 7th Congrès RFIA* - AFCET, Paris

Buschbeck-Wolf, B. (1993): A semantic approach to the translation of locative prepositions - In *Working Papers of the 4th Workshop on the Semantics of Time, Space, movement and Spatio-Temporal Reasoning*, M. Aurnague, A. Borillo, M. Borillo & M. Bras (eds.) - Groupe LRC, Toulouse.

Carlson-Radvansky, L.A. & D.E. Irwin (1993): Frames of reference in vision and language: where is above, *Cognition*, 46: 223-244

Clarke, B. (1981): A calculus of individuals based on "connection", *Notre Dame Journal of Formal Logic*, 22, 3: 204-218

Clarke, B. (1985): Individuals and points, *Notre Dame Journal of Formal Logic* 26, 1: 61-75

Davis, E. (1988): A logical framework for commonsense predictions of solid object behaviour, *Artificial intelligence in engineering*, 3, 3: 125-140

Davis, E. (1989): *Representations of commonsense knowledge* - Morgan Kaufmann Publishers, San Mateo (CA)

Egenhofer, M. & R. Franzosa (1991): Point-set topological spatial relations. *Int. J. Geographical Information Systems*, 5, 2: 161-174

Frank, A. (1992): Qualitative Spatial Reasoning about Distances and Directions in Geographic Space. *Journal of Visual Languages and Computing* 3: 343-371

Freksa, C. (1992): Using Orientation Information for Qualitative Spatial Reasoning - In *Theories and Methods of Spatio-Temporal Reasoning in Geographic Space, Proceedings of the International Conference GIS - From Space to Territory,* A. Frank, I. Campari, U. Formentini (eds.) - Springer, Berlin: 162-178

Grice, H.P. (1975): Logic and conversation - In *Syntax and semantics3: Speech Acts.*, P. Cole & J. Morgan (eds.) - Academic Press, New-York: 41-58

Güsgen, H.W. (1989): Spatial reasoning based on Allen's temporal logic, ICSI TR-89-049, International Computer Science Institute, Berkeley.

Hayes, P.J. (1985a): The second naive physics manifesto - In *Formal theories of the commonsense world*, J.R. Hobbs & R.C. Moore (eds.) - Ablex Publishing Corporation, Norwood (NJ): 1-36

Hayes, P.J. (1985b): Naive physics I: Ontology for liquids - In *Formal theories of the commonsense world*, J.R. Hobbs & R.C. Moore (eds.) - Ablex Publishing Corporation, Norwood (NJ): 71-107.

Hernández, D. (1993): Reasoning with qualitative representations: exploiting the structure of space - In *Qualitative Reasoning an Decision Technologies*, N. Piera & M. Singh (eds.) - CIMNE, Barcelona: 493-502

Herskovits, A. (1982): Space and the preposition in English: regularities and irregularities in a complex domain - Ph.D. dissertation, Stanford University, Stanford (CA)

Jayez, J. (1992): Document de travail n°3 - In *Rapport d'activité du groupe inter-PRC "Sémantique des Langues Naturelles"*, June 92.

Kamp, H. (1979): Events, instants and temporal reference - In *Meaning, use and interpretation of language*, R. Bäuerle, U. Egli, A. von Stechow (eds.) - de Gruyter, Berlin: 376-417.

Lang, E. (1990): Primary perceptual space and Inherent proportion schema: two interacting categorization grids underlying the conceptualization of spatial objects, *Journal of Semantics*, 7, 121-141.

Lascarides, A. & N. Asher (1991): Discourse relations and defeasible knowledge - In *Proceedings of the 29th annual meeting of the Association for Computational Linguistics*: 55-63.

Laycock, H. (1972): Some questions of ontology, *Philosophical review* 81: 3-42.

Lesniewski, S. (1927-31): O podstawach matematyki, *Przeglad Filosoficzny* vols. 30-34.

Leonard, H. & N. Goodman (1940): The calculus of individuals and its uses, *The Journal of Symbolic Logic* 5: 45-55

Link, G. (1983): The logical analysis of plurals and mass terms: A lattice theoretical approach - In *Meaning, use and interpretation of language*, R. Bäuerle, C. Schwarze & A. von Stechow (eds.) - de Gruyter, Berlin

Miller, G. & P. Johnson-Laird (1976): *Language and perception* - Belknap Press of Harvard University Press, Cambridge (MA)

Mukerjee, A. & G. Joe (1990): A Qualitative Model for Space - In *Proceedings of the Eighth National Conference on Artificial Intelligence, AAAI 90* - MIT Press, Cambridge (MA): 721-727

Pelletier, F. & L. Schubert (1989): Mass expressions - In *Handbook of philosophical logic*. Vol. 4. D. Gabbay & F. Guenthner (eds.) - Reidel, Dordrecht: 327-407

Randell, D. & A. Cohn (1989): Modelling topological and metrical properties in physical processes - In: *Proceedings of KR'89,* Brachman, Levesque, Reiter (eds.) - Morgan Kaufmann, San Mateo (CA): 357-368.

Randell, D., Z. Cui & A. Cohn (1992): A spatial logic based on regions and connection - In: *Proceedings of KR'92* - Morgan Kaufmann, San Mateo (CA)

Simons, P. (1987): *Parts - A study in ontology* - Clarendon Press, Oxford

Tarski, A. (1972): Les fondements de la géométrie des corps. In *Logique, Sémantique, Métamathématique,* A Tarski -Armand Colin, Paris: 28-34

Talmy, L. (1983): How language structures space - In *Spatial orientation: theory, research and application,* H. Pick & L. Acredolo (eds.) - Plenum, New-York: 225-282

Vandeloise, C. (1986): *L'espace en français : sémantique des prépositions spatiales* - Seuil, Paris.

Vieu, L. (1991): Sémantique des relations spatiales et inférences spatio-temporelles : une contribution à l'étude des structures formelles de l'espace en langage naturel - Ph.D. dissertation, Université Paul Sabatier, Toulouse.

Vieu, L. & R. Martin-Clouaire (to appear in 1994): Spatial and qualitative reasoning in modelling physical processes, *AI Applications*.

Winston, M., R. Chaffin & D. Herrmann (1987): A taxonomy of part-whole relations, *Cognitive Science* 11: 417-444.