A rich discrete labeling scheme for line drawings of curved objects

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Abstract

We present a discrete labeling scheme for line drawings of curved objects which can be seen as an information-rich extension of the classic line-labeling scheme in which lines are classified as convex, concave, occluding or extremal. New labels are introduced to distinguish between curved and planar surface-patches, to identify orthogonal edges and to indicate gradient directions of planar surface-patches.

keywords: scene analysis, shape, line drawing analysis, line labeling, soft constraints.

1 Regularities in man-made objects

Most man-made objects have certain characteristic shape features which distinguish them from natural objects such as rocks, clouds or trees. In this paper we concentrate on planarity and orthogonality, although clearly other regularities (such as symmetry and isometry) can also provide useful visual clues in the interpretation of drawings of man-made objects.

We say that an edge E, whether curved or planar, is *orthogonal* if at each point on E the tangents to the two surfaces which meet at E are orthogonal. Consider, as an illustration, the two drawings in Fig. 1. Of the 20 visible faces of these two objects, 13 are planar. Of the 57 visible edges, 51 are orthogonal, the six non-orthogonal edges being marked by an asterisk. Both of these drawings appear unambiguous to a human viewer. In particular, we immediately identify surface A in Fig. 1(a) as planar and surface B as curved, although most people have great difficulty in explaining why they came to this conclusion. This paper is concerned with setting down some basic local geometrical rules which will allow a computer to automatically identify planar surfaces and orthogonal edges. Possible applications include sketch interpretation [21], automatic indexing of databases of line drawings [19] and 3D object retrieval from the web [1].

We follow in the tradition of Kanade [9] who advocated the use of more information about the physical world, thus avoiding overstrict constraints based on unrealistic assumptions (such as planar surfaces meeting at trihedral vertices [8, 2]) or the optimization of a fairly arbitrary objective function. An important point is that features that are common in man-made objects, such as orthogonal edges or planar faces, are essentially discrete properties. For example, 90 degree angles are common, but 85 degree angles are no more common than 65 degree angles. Thus an objective function F, based on orthogonality (or other essentially discrete properties), which is a true reflection of the likelihood of a complete 3D reconstruction should take on the same constant value over most of the solution space, since the vast majority of solutions involve no orthogonality. Soft Constraint Satisfaction [16] provides a more appropriate method for optimizing F rather than search methods in real-valued parameter space [15, 12, 21].

In a previous paper we have shown how soft constraint satisfaction provides a framework in which we can combine strict geometrical constraints and preference constraints [6]. An example of a strict geometrical constraint is that parallel 3D lines cannot intersect; an example of a preference constraint is that we prefer a pair of lines which are parallel (within a given error tolerance) in the drawing to be projections of 3D parallel edges, since parallel edges are common features of man-made objects. Our approach is similar to that of Ding & Young [7] who used a Truth Maintenance System for the complete 3D reconstruction of polyhedral objects from imperfect line drawings. We introduce new constraints for the analysis of line drawings of *curved* objects, but we do not attempt hidden-part reconstruction.

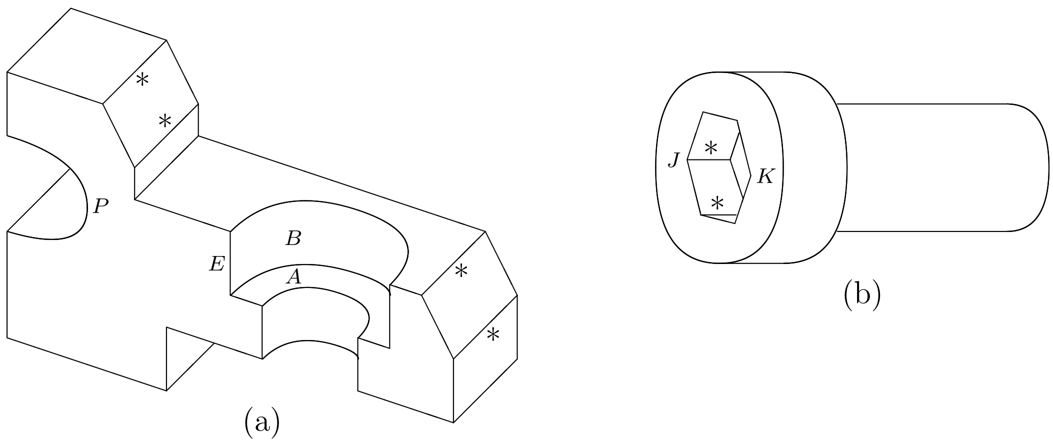


Figure 1: Examples of drawings of curved man-made objects.

2 Line labels for curved objects

Line-drawing labeling, in which each line is assigned a semantic label ('+' for convex, '−' for concave or '→' for occluding), was pioneered by Huffman [8] and Clowes [2]. At a convex (concave) edge E, the external angle between the two visible surfaces which intersect at E is greater (less) than π; at an occluding edge, only one surface is visible. They established the catalogue of legal labelings of junctions created by the projection of trihedral vertices. These constraints, together with the Outer Boundary Constraint (which simply says that the outer boundary of the drawing is necessarily an occluding line), were often found to be sufficient to uniquely determine the correct semantic labeling of the drawing of a polyhedral object. Catalogues of labeled junctions have also been established for non-trihedral vertices [20], but in this case extra visual cues, such as parallel lines, are required to avoid an exponential number of legal global labelings [6].

Malik [14] extended semantic labeling to line drawings of curved objects composed of smooth surface patches separated by surface-normal discontinuity edges. His catalogue was refined and extended to include objects with tangential edges and surfaces [3, 4]. When surfaces are curved, the semantic label of a line may change at any point due to the presence of phantom junctions. For example, transitions from occluding to convex labels, such as the point P in Fig. 1(a), are common in line drawings of curved objects. Other transitions may occur. For example, as pointed out in [3], a convex-concave transition can occur if distinct

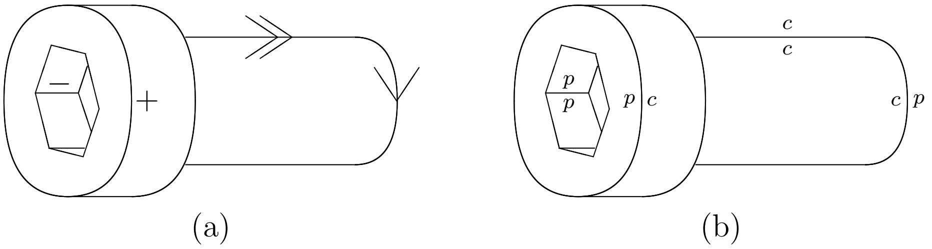


Figure 2: Illustration of (a) semantic line labels, and (b) planarity labels.

surfaces may be tangential to each other. Under the Straight Edge Formation Assumption, which says that a straight edge is formed by the intersection of two locally-planar surfaces [5], the semantic label of a line L is invariant along any straight segment of L. In particular, a straight line has a unique semantic label. Each global semantic labeling can be individually checked for physical realisability via Sugihara's linear programming approach [18] generalized to curved objects with some linear features [5]. Unfortunately, although mathematically elegant, Sugihara's approach does not help us find the most likely interpretation of a drawing.

3 Planarity constraints

Consider a 3D edge E formed by the intersection of two surfaces S, T. We say that S is *locally planar* along E if the tangent to S at each point of E is identical. Planar surfaces are necessarily locally planar, but a curved surface may also be locally planar. As an example, consider surface B in Fig. 1, which is locally planar along the edge E, since it has an invariant tangent plane along the whole length of E.

A surface-normal discontinuity edge (a convex, concave or occluding edge) is formed by the intersection of two surfaces S, T. A line L which is the projection of a surface-normal discontinuity edge has a semantic label $+, -, \leftarrow$ or $\rightarrow$, as illustrated in Fig. 2(a). We also label each side of L by a planarity label p or c to indicate whether S, T are locally planar or not (p for 'planar' and c for 'curved'). At an occluding edge, one of the surfaces, say T, is invisible. The planarity label of S is written on the side of the line into which the surface S projects and the planarity label of T on the other side. This is illustrated in Fig. 2(b): the rightmost label p indicates that the hidden base of the cylindrical part of the object is

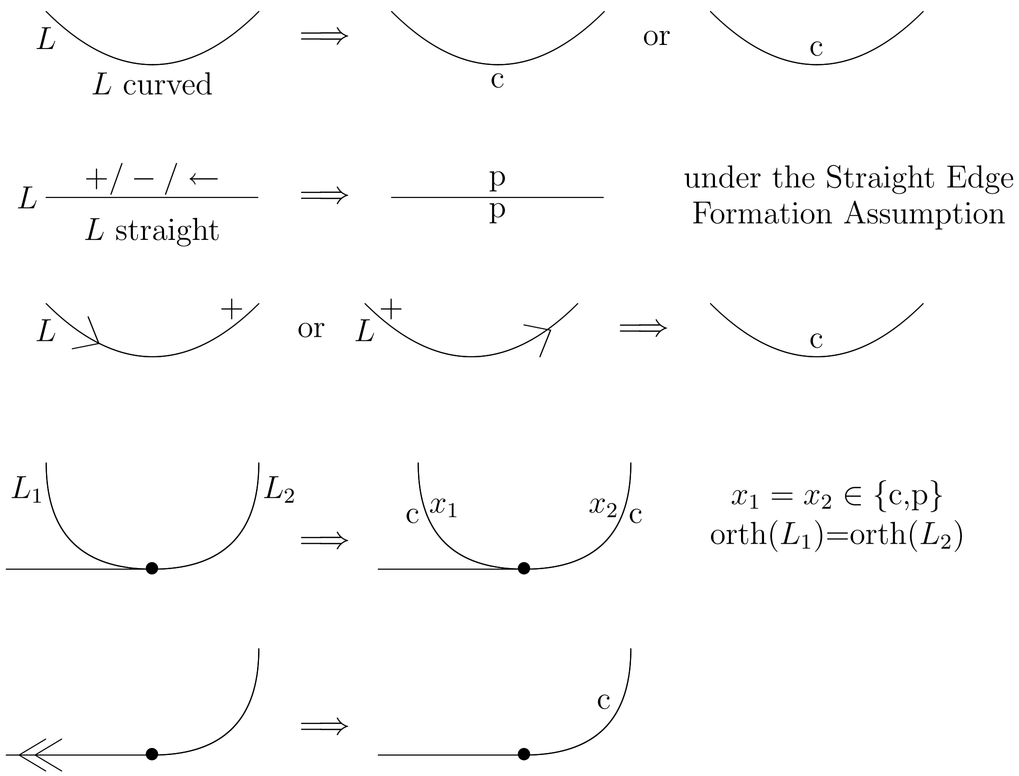


Figure 3: Planarity constraints.

locally planar. An extremal edge is the locus of points of intersection of the line of sight with a curved visible object surface. The projection of an extremal edge, known as an extremal line, is labeled by a double-headed arrow as shown in Fig. 2(a). By convention, both sides of an extremal line are always labeled c, as shown in Fig. 2(b).

Having introduced the new planarity labels, we can now give the planarity constraints which relate planarity and semantic line labels. These are given in Fig. 3. We say that the drawing satisfies the General Viewpoint Assumption (GVA) if no small perturbation in the position of the viewpoint changes the configuration of the drawing (junction-types, presence of straight lines and parallel lines). Under the GVA, the projection of a 3D edge E is a straight line if and only if E is a straight edge. The first constraint in Fig. 3 simply says that a curved line cannot be the projection of the intersection of two locally planar surfaces. The second constraint is a translation of the Straight Edge Formation Assumption.

The last three planarity constraints in Fig. 3 (showing a phantom, a 3-tangent and a curvature-L junction respectively) allow us to deduce that a surface is curved. Consider a

3D point P at which a surface-normal discontinuity edge E intersects an extremal edge E_{ext}, and let S be the surface in which the extremal edge lies and T_P the tangent-plane to S at the point P. By the definition of an extremal edge, the viewpoint lies in the plane T_P. If S were locally planar along E, then T_P would be the tangent plane to this locally planar segment of E, which would contradict the General Viewpoint Assumption. This reasoning allows us to deduce the c labels shown in the last three planarity constraints in Fig. 3. There are three distinct constraints depending on whether the extremal edge E_{ext} or part of the surface-discontinuity edge E is occluded. The dot represents a discontinuity of curvature between the projections of E and E_{ext}. These planarity constraints are valid even in the case of objects with tangential edges and surfaces [4].

Since planar surfaces are common in man-made objects, it is natural to try to maximize the number of planar surfaces in our interpretation of the drawing. However, simply maximizing the number N_p of p labels is not always sufficient to determine the correct planarity labeling: for example, applying this criterion to the drawing in Fig. 1(a) does not allow us to distinguish between the two labelings $\frac{c}{p}$ and $\frac{p}{c}$ for the edge separating faces A and B. Considering the drawing as a planar graph G, the faces of G are projections of visible (partial) surface-patches of the 3D object. A face F of G can only be the projection of (part of) a planar surface if all its planarity labels are p. We say that F is *locally planar* if all its planarity labels are p. Maximizing the criterion $N_p + N_{lpf}$, where N_{lpf} is the number of faces of G which are locally planar, allows us to refine the partial order defined by N_p and hence to find the correct planarity labeling of the drawing in Fig. 1(a).

4 Constraints from orthogonal edges

It is well known that identifying a labeled junction as the projection of a cubic corner allows us to calculate the 3D orientation of the three faces that meet at the corresponding vertex [17, 10]. This section shows that we can extend this to curved objects, by determining certain information about the surfaces which meet at viewpoint-dependent vertices, based only on the assumption that the 3D edge E is orthogonal.

Consider the first constraint shown in Fig. 4, reading the implication from left to right.

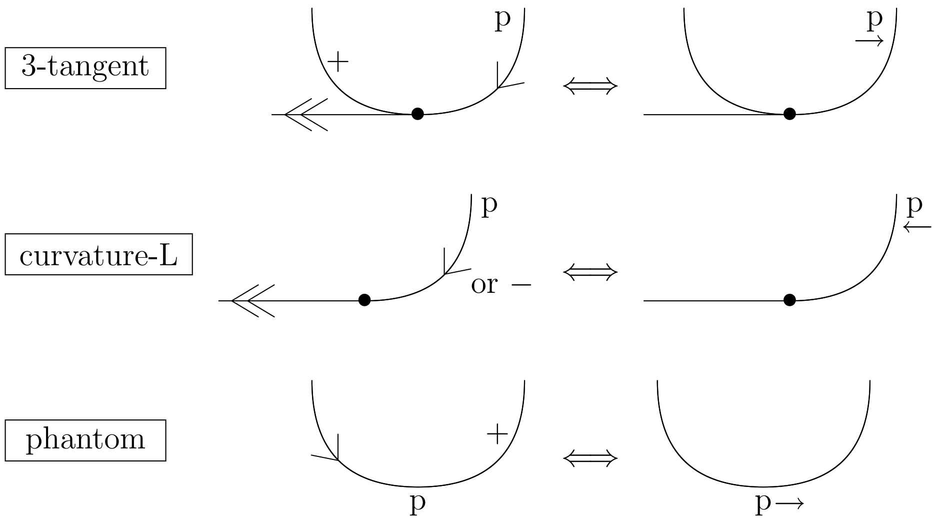


Figure 4: Gradient-direction constraints assuming that the surface-normal discontinuity edges are orthogonal. The $\Leftarrow$ implications are consequences of the GVA.

It concerns a labeled 3-tangent junction formed by the projection of a curved orthogonal edge E which is the intersection of a curved surface S_c with a planar surface S_p. Let P represent the 3D point on E at which the tangent-plane to S_c passes through the viewpoint, and let T_P represent this tangent-plane. If $\mathbf{n}$ is the normal to the planar surface S_p, then by orthogonality of E, $\mathbf{n}$ is parallel to T_P. It follows that the projection of $\mathbf{n}$ in the drawing is parallel to the extremal edge (which is the projection of T_P). We symbolize the direction of the projection of $\mathbf{n}$ by a short arrow next to the p on the right hand side of Fig. 4.

Similar constraints exist, and are given in Fig. 4, for curvature-L junctions and phantom junctions. The arrowhead indicates the direction of $\mathbf{n}$. By convention, the 3D orientation of $\mathbf{n}$, the normal to the planar surface S_p, is always towards the viewpoint. This convention means that in the curvature-L constraint of Fig. 4, the gradient direction is the same whether E is a concave or an occluding edge. For the phantom junction shown in Fig. 4, the gradient direction can only be determined accurately if the position of the phantom junction is precisely known. However, this is an important constraint when read from right to left, since it allows us to locate phantom junctions at the points where the gradient-direction is tangential to the line. Under orthographic projection, the gradient-direction is invariant on a planar surface, and hence propagates through junctions and along locally-planar surfaces. Under perspective projection, the gradient directions of a surface meet at a vanishing point;

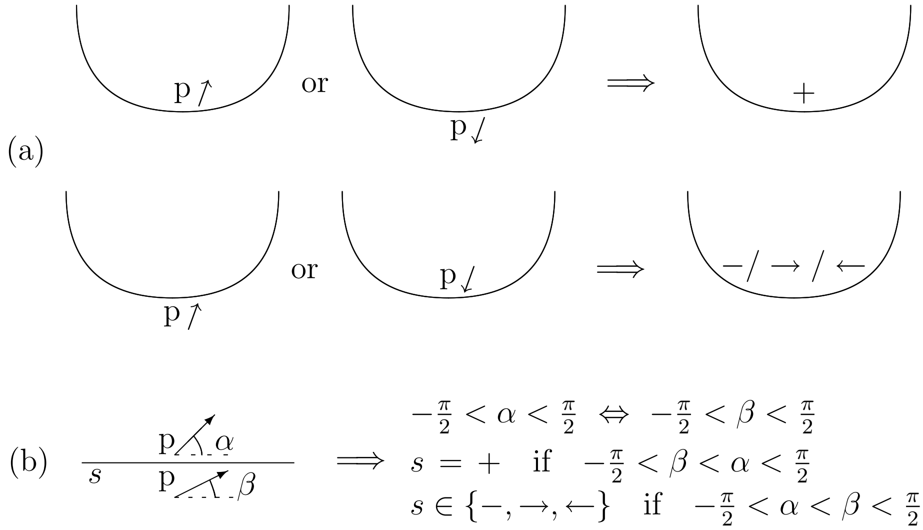


Figure 5: The gradient-direction/semantic-label constraints: (a) assuming that the edge is orthogonal; (b) for any edge.

at least two gradient directions are required to locate the vanishing point of the normals to a planar surface.

For each line segment L in the drawing we create a boolean variable orth(L) which takes on the value `true` if and only if L is the projection of an orthogonal edge segment. We then impose the hard constraints

$$\text{orth}(L) \;\Rightarrow\; \begin{array}{l}\text{the gradient-direction constraints apply at each}\\ \text{3-tangent, curvature-L and phantom junction on } L.\end{array}$$

together with the soft constraint which imposes a penalty of w_o if orth$(L) =$ `false`. Following the discussion in Section 3, there is also a penalty of w_{ph} for each phantom junction, a penalty of w_p for each c planarity label, a penalty of w_{lpf} for each face which is not locally planar, and a penalty w_t for each planarity label transition (p to c) between the two ends of a line segment. The objective function to be minimized is the sum of these penalties. Hard constraints can be viewed as penalty functions taking values in $\{0, \infty\}$ [16].

There are also gradient-direction constraints at projections of trihedral vertices V at which surfaces and edges meet non-tangentially. Let L_1, L_2, L_3 be the projections of the edges E_1, E_2, E_3 meeting at V and let F_{12} be the face bounded by E_1 and E_2. The normal $\mathbf{n}$ to F_{12} is parallel to E_3 in 3D iff both E_1 and E_2 are orthogonal at V. Thus, invoking the

GVA which says that parallel 2D lines are projections of parallel 3D lines, we can deduce that the gradient-direction of F is parallel to L_3 iff both E_1 and E_2 are orthogonal at V.

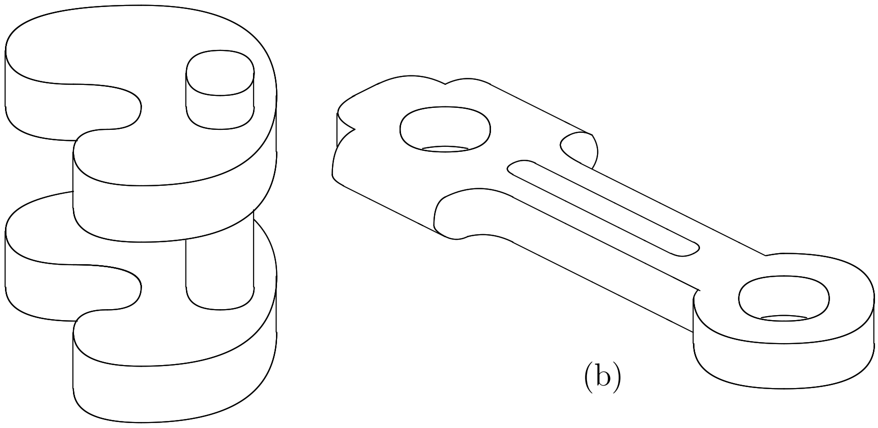
The gradient-direction/semantic-label constraints in Fig. 5(a) show the tight relationship between the gradient direction and the semantic label of a line L, whenever L is the projection of an orthogonal edge. The arrows represent any gradient direction which points away from L in the first constraint and towards L in the second constraint. The line L is shown curved, but could be straight or curved in the opposite direction. At a point P on an orthogonal edge E at which surfaces S_1, S_2 intersect, the normal $\mathbf{n}_1$ ($\mathbf{n}_2$) to surface S_1 (S_2) is parallel to the tangent-plane to the surface S_2 (S_1) at P. If E projects into a convex (concave) line L, it follows that the projections of $\mathbf{n}_1$,$\mathbf{n}_2$ point away from (towards) L. Since, by convention, occluding lines have the same gradient-direction labels as concave lines, this proves the correctness of the gradient-direction/semantic-label constraints given in Fig. 5(a).

One consequence of these constraints is that a closed curve L which is the projection of a locally-planar orthogonal edge cannot have the same semantic label around the whole of L. If L is not a closed curve but instead terminates at a 3-tangent or curvature-L junction, then the gradient-direction constraints allow us to determine the gradient-direction. This, in turn, provides us with the semantic label at each point of L via the gradient-direction/semantic-label constraints.

In Fig. 5(b), the surfaces S_1,S_2 are both locally-planar and hence L is necessarily straight. The right-hand end of L is further away from the viewer than the left-hand end if and only if $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$ and if and only if $-\frac{\pi}{2} < \beta < \frac{\pi}{2}$. Furthermore, for $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$, the dihedral angle between the surfaces S_1 and S_2 is greater than π if $\alpha > \beta$, equal to π if $\alpha = \beta$ and less than π if $\alpha < \beta$. The constraints of Fig. 5(b) follow immediately.

5 Experimental trials

To test our labeling scheme, we built up a test set of 28 drawings, all produced by orthographic projection: the four drawings in Fig. 1 and Fig. 6, the 15 drawings of curved objects from [3], the four drawings of curved objects from the test-set of [13] and the five drawings given in Fig. 2 of [1]. Any lines which were not projections of visible edges were eliminated



(b)

(a)

Figure 6: Drawings of curved objects with orthogonal edges.

in order to produce perfect projections of opaque objects. In the correct interpretations, 76.7% of the 395 edges are orthogonal, 72.9% of the 229 faces are planar and 67.6% of the 1044 planarity labels are p. The GVA was satisfied in each of the drawings.

We applied the Outer Boundary Constraint together with the appropriate semantic labeling scheme [3, 6]. For simplicity and reproducibility, we set all of w_o, w_{ph}, w_p, w_{lpf} and w_t to 1. We also applied the following simple geometrical constraints: (1) two surfaces which are tangential cannot be orthogonal; (2) a viewpoint-dependent junction (3-tangent, curvature-L or phantom) cannot be concave [3] if it is the projection of the intersection of a planar surface with a curved surface at an orthogonal edge. For each drawing, we found, by exhaustive search, the set of all optimal labelings. A line segment was considered to be *correctly/ambiguously/incorrectly* labeled if it was assigned the correct label in all/some/none of the optimal labelings. Out of 395 orthogonality labels, 94.4% were correct, 2.3% ambiguous and 3.3% incorrect. Out of 1044 planarity labels, 95.2% were correct, 3.4% ambiguous and 1.4% incorrect. Out of 229 visible faces, 96.1% were correctly classified as locally planar/curved, 3.5% were ambiguous and 0.4% were incorrect (ambiguity typically occurring when two regions are separated by a curved line but there is insufficient evidence to determine which of the two faces is planar). If we set $w_p = w_{lpf} = 0$ (i.e. we do not try to maximize planarity), the planarity constraints alone (Fig. 3) correctly identify only 77.6% of the planarity labels and correctly classify only 63.1% of faces as locally-planar/curved.

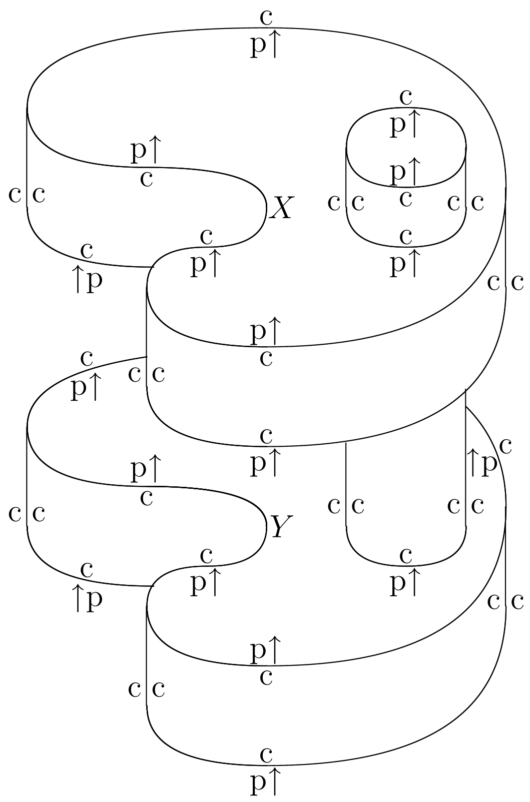


Figure 7: The drawing of Fig. 6(a) with planarity labels and gradient directions.

To illustrate the strength of our constraints, consider the drawing in Fig. 6(a). Using the semantic labeling scheme [14], the planarity constraints and the gradient direction constraints, we obtain the labeling given in Fig. 7 (with semantic labels not shown to avoid cluttering up the figure). This interpretation simultaneously maximizes the number of orthogonal edges, the number of p labels and the number of locally-planar surface patches. All surfaces are correctly labeled as planar or curved and the directions of the projections of the normals to the planar surfaces have been correctly determined. This provides a much richer interpretation than the semantic labeling alone. The phantom-junction gradient-direction constraint (Fig. 4) allows us to precisely locate the two phantom junctions X, Y shown in Fig. 7 at which there is a transition from an occluding label to a convex label.

To illustrate the limitations of our labeling scheme, consider the drawing in Fig. 6(b). Using the semantic labeling scheme, the planarity constraints and the gradient direction constraints, we obtain the labeling illustrated in Fig. 8. To avoid cluttering up the figure, c labels, as well as semantic labels, are omitted. The line L_1, which extends from junction

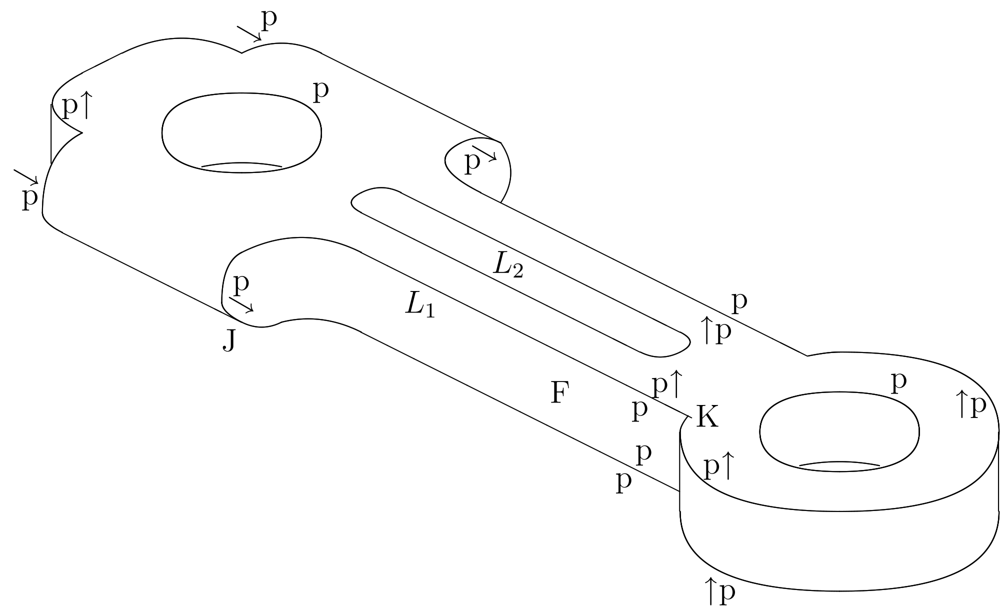


Figure 8: The drawing of Fig. 6(b) with planarity labels and gradient directions.

J to junction K, provides an example of planarity-label transitions. Starting at junction J, the actual planarity label pair changes from $\frac{c}{p}$ to $\frac{c}{c}$ to $\frac{p}{p}$. These changes are not detected by our constraints, which strictly speaking only inform us about planarity labels in the vicinity of junctions or along the length of straight line segments, with the result that face F is incorrectly classified as locally planar. The closed curve L_2 in Fig. 8, which we see as a long hole running down the handle of the object, provides a challenge for our constraints, which produce an incorrect semantic labeling of L_2. It would appear to be the skew symmetry of L_2, whose axis coincides with the axis of symmetry of the whole object, which allows us to identify L_2 as a hole in a planar surface rather than a separate object lying on top of a larger object.

Our complete branch-and-bound search has worst-case exponential time complexity, but this is inevitable given that finding a single semantic labeling is NP-complete [11].

6 Conclusion

We have seen that a rich labeling scheme together with simple local geometrical constraints allow us to obtain a considerable amount of information about the 3D object depicted in a drawing. This information goes much further than traditional semantic line labels (occluding, convex, concave, extremal), but does not in itself provide a complete 3D reconstruction. For this, we can look for various other common features of man-made objects, such as cubic corners, symmetry, parallel curves and isometry.

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