Extracting decision rules from qualitative data using Sugeno integral: a case study

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Abstract. This paper deals with knowledge extraction from experimental data in multifactorial evaluation using Sugeno integrals. They are qualitative criteria aggregations where it is possible to assign weights to groups of criteria. A method for deriving such weights from data is recalled. We also present results in the logical representation of Sugeno integrals. Then we show how to extract if-then rules expressing the selection of good situations on the basis of local evaluations, and rules to detect bad situations. We illustrate such methods on a case-study in the area of water ecosystem health.

1 Introduction

Sugeno integrals are aggregation functions that make sense on any completely ordered scale, and can then be called qualitative aggregation operations. Like many aggregation operations in multifactorial evaluation, they return a global evaluation lying between the minimum and the maximum of the partial ratings. In a Sugeno integral each group of criteria receives an importance weight, whereby interactions between criteria can be modeled.

Sugeno integrals are used both in multiple criteria decision making and in decision under uncertainty [8, 2, 6]. While many results exist proposing formal characterizations of Sugeno integral [11, 10], fewer papers address the identification of Sugeno integrals from data, and the interpretation of this aggregation method in terms of decision rules. The former problem is addressed by Prade *et al.* [12, 13]: they calculate a family of capacities, if any, that determine Sugeno integrals that account for a set of empirically rated objects both locally with respect to criteria, and globally, each object receiving an overall evaluation. The second issue was first addressed by Greco *et al.* [9]. Representing a Sugeno integral by a set of rules make it more palatable in practical applications. More recently, a possibilistic logic rendering of Sugeno integral has been proposed, in the form of weighted formulas the satisfaction of which is sufficient to ensure a minimal global evaluation [5].

Such a possibilistic logic base can be used to obtain some rules associated to the given data modeled by a Sugeno integral. This paper combines both a technique for identifying a family of capacities at work in a Sugeno integral applied to subjective multifactorial evaluation data, and a technique for extracting decision rules from the obtained family of Sugeno integrals. At the theoretical level it completes the results obtained in [5] by considering the extraction of decision rules that give conditions for an object to have a global evaluation less than a given threshold. Overall, we then get rules that can accept good objects and rules that can discard bad ones. As an illustration the paper presents an application of these results on a case-study on the effects of rainwater pollution on the development of algae. In a nutshell, this application focuses on the following question: what do we learn about the given data on algae when representing the global evaluation by an aggregation of local ones via a discrete Sugeno integral?

The paper is structured as follows: Section 2 begins with a brief reminder about some theoretical results concerning Sugeno integral. Next it presents results on the identification of Sugeno integral and its expression in the form of rules. Section 3 presents the data of the case-study. Section 4 deals with the application of the theoretical results to the given dataset.

2 Interpreting evaluation data using Sugeno integrals

We use the terminology of multiple criteria decision-making where some objects are evaluated according to criteria. We denote by $C = \{1, \cdots, n\}$ the set of criteria, 2^C the power set and L a totally ordered scale with top 1, bottom 0, and the order-reversing operation denoted by ν. An object is represented by a vector $x = (x_1, \cdots x_n)$ where x_i is the evaluation of x according to the criterion i.

In the definition of Sugeno integral the relative weights of the set of criteria are represented by a capacity (or fuzzy measure) which is a set function $\mu : 2^C \to L$ that satisfies $\mu(\emptyset) = 0$, $\mu(C) = 1$ and $A \subseteq B$ implies $\mu(A) \leq \mu(B)$. In order to translate a Sugeno integral into rules we shall also need the notion of conjugate capacity. More precisely, the conjugate capacity of μ is defined by $\mu^c(A) = \nu(\mu(A^c))$ where A^c is the complementary of A. The Sugeno integral of function x with respect to a capacity μ is originally defined by [15, 16]: $S_\mu(x) = \max_{\alpha \in L} \min(\alpha, \mu(x \geq \alpha))$, where $\mu(x \geq \alpha) = \mu(\{i \in C | x_i \geq \alpha\})$. It can be equivalently written under various forms [3, 11, 10], especially:

$$S_\mu(x) = \max_{A \subseteq C} \min(\mu(A), \min_{i \in A} x_i) = \min_{A \subseteq C} \max(\mu(A^c), \max_{i \in A} x_i) \quad (1)$$

2.1 Eliciting Sugeno integrals

In this paper, our first aim is to elicit a family of Sugeno integrals that are compatible with a given dataset. Let us recall how to calculate the bounds of this family.

The set of data is a collection of $(x^k, \alpha_k)_k$ where x^k are tuples of local evaluations of objects $k = 1, \ldots, N$ and α_k is the global evaluation of object k. This data set is supposed to be provided by some expert. We want to know if there exists a capacity μ such that $S_\mu(x^k) = \alpha_k$ for all k, and if so, we want to calculate at least one solution. In [14], the following result is proved:

Proposition 1 *For a given data item (x, α), $\{\mu | S_\mu(x) = \alpha\} = \{\mu | \check{\mu}_{x,\alpha} \leq \mu \leq \hat{\mu}_{x,\alpha}\}$ where $\check{\mu}_{x,\alpha}$ and $\hat{\mu}_{x,\alpha}$ are capacities defined by*

$$\check{\mu}_{x,\alpha}(A) = \begin{cases} \alpha \text{ if } \{i | x_i \geq \alpha\} \subseteq A \\ 0 \text{ otherwise} \end{cases} \text{ and } \hat{\mu}_{x,\alpha}(A) = \begin{cases} \alpha \text{ if } A \subseteq \{i | x_i > \alpha\} \\ 1 \text{ otherwise.} \end{cases}$$

Remark: It is easy to see that $\check{\mu}_{x,\alpha}$ is a necessity measure with respect to the possibility distribution $\check{\pi}_{x,\alpha}(i) = \begin{cases} 1 \text{ if } x_i \geq \alpha \\ \nu(\alpha) \text{ otherwise} \end{cases}$, and $\hat{\mu}_{x,\alpha}(A)$ is a possibility measure with respect to the possibility distribution $\hat{\pi}_{x,\alpha}(i) = \begin{cases} 1 \text{ if } x_i \leq \alpha \\ \alpha \text{ otherwise} \end{cases}$.

Hence we can calculate the bounds of the compatible Sugeno integrals:

Proposition 2 *The set of compatible capacities with the given data $(x^k, \alpha_k)_k$ is $\{\mu | \max_k \check{\mu}_{x^k,\alpha_k} \leq \mu \leq \min_k \hat{\mu}_{x^k,\alpha_k}\}$.*

As a consequence, $\max_k \check{\mu}_{x^k,\alpha_k}$ and $\min_k \hat{\mu}_{x^k,\alpha_k}$ can be any kind of capacity, since any capacity is the eventwise minimum of necessity measures and the eventwise maximum of possibility measures [4]. Moreover, it is not always the case that $\{\mu | \max_k \check{\mu}_{x^k,\alpha_k} \leq \mu \leq \min_k \hat{\mu}_{x^k,\alpha_k}\}$, that is, the set of solutions can be empty. [4]

2.2 Extracting if-then rules using possibilistic logic

Based on the above results and procedures described in [12], suppose we have a family of Sugeno integrals compatible with the evaluation data. Now, we try to express if-then rules associated to these integrals, thus facilitating the interpretation of the data.

Selection rules First let us recall how Sugeno integral can be encoded by means of a possibilistic logic base with positive clauses (see [5] for more details).

We need to use the inner qualitative Moebius transform of a capacity μ which is a mapping $\mu_\# : 2^C \to L$ defined by

$$\mu_\#(E) = \mu(E) \text{ if } \mu(E) > \max_{B \subset E} \mu(B) \text{ and 0 otherwise.}$$

A set E such that $\mu_\#(E) > 0$ is called a focal set. The set of the focal sets of μ is denoted by $\mathcal{F}(\mu)$. Moreover we denote by $\mathcal{F}(\mu)^\alpha$ the set of the focal sets E such that $\mu(E) = \alpha$. Note that Sugeno integral can be expressed in terms of $\mu_\#$ using Equation (1) as follows: $S_\mu(x) = \max_{E \in \mathcal{F}(\mu)} \min(\mu_\#(E), \min_{i \in E} x_i)$.

Using the definition of the Sugeno integral it is easy to get the following result [5]:

Proposition 3 *The inequality $S_\mu(x) \geq \gamma$ is equivalent to $\exists T \in \mathcal{F}(\mu)^\gamma$ such that $\forall i \in T$, $x_i \geq \gamma$.*

[4] In order to compare $\max_k \check{\mu}_{x^k,\alpha_k}$ and $\min_k \hat{\mu}_{x^k,\alpha_k}$ it is not necessary to calculate their values and to compare them on each subset of criteria. It is proved in [14] that the set of compatible capacities is not empty if and only if for all $\alpha_k < \alpha_l$ we have $\{i | x_i^l \geq \alpha_l\} \not\subseteq \{i | x_i^k > \alpha_k\}$.

So each focal T of μ with level $\mu_{\#}(T)$ corresponds to the selection rule:

R^s_T: If $x_i \geq \mu_{\#}(T)$ for all $i \in T$ then $S_\mu(x) \geq \mu_{\#}(T)$.

This set of rules can be encoded in possibilistic logic as a set of weighted cubes. Define for each criterion i a family of Boolean predicates $x_i(\alpha), \alpha > 0 \in L$ such that $x_i(\alpha) = 1$ if $x_i \geq \alpha$ and 0 otherwise. Then we consider weighted Boolean formulas of the form $[\phi, \alpha]$ which are interpreted as lower possibility distributions on the set of objects: $\pi^-_{[\phi,\alpha]}(x) = \begin{cases} \alpha \text{ if } x \models \phi; \\ 0 \text{ otherwise} \end{cases}$. Then the lower possibility distribution associated to a weighted cube is $[\wedge_{j\in T} x_j(\alpha), \alpha]$ interpreted as $\pi_{[T,\alpha]}(x) = \begin{cases} \alpha \text{ if } x_i \geq \alpha, \forall i \in T; \\ 0 \text{ otherwise} \end{cases}$. Each weighted cube $[\wedge_{j\in T} x_j(\mu(T)), \mu_{\#}(T)]$ for a focal set T corresponds to a rule R^s_T as stated above.

The lower possibility distributions associated to a set of such weighted formulas is interpreted as the maximum of the lower possibility distributions associated to each weighted formula. Now consider the possibilistic base

$$B^-_\mu = \{[\wedge_{j\in T} x_j(\alpha), \alpha] : \mu(T) \geq \alpha > 0, T \in \mathcal{F}(\mu)\}$$

with lower possibility distribution $\pi^-_\mu(x) = \max_{\mu(T)\geq\alpha>0, T\in\mathcal{F}(\mu)} \pi^-_{[\phi,\alpha]}(x)$.

Proposition 4 *(Proposition 4 in [5])* $S_\mu(x) = \pi^-_\mu(x)$.

The proof takes advantage of the max-min form of Sugeno integral in equation (1).

Elimination rules The above rules and their logical encoding are tailored for the selection of good objects. Symmetrically, we can obtain rules for the rejection of bad objects associated to the Sugeno integral. In the following we prove results similar to those in [5] for the inequality $S_\mu(x) \leq \gamma$.

The idea is to use the min-max form of Sugeno integral in equation (1), which is the form of possibility distributions in standard possibilistic logic [1]. The focal sets of the conjugate of μ are sufficient to calculate the Sugeno integral:

Proposition 5 $S_\mu(x) = \min_{T\in\mathcal{F}(\mu^c)} \max(\nu(\mu^c_{\#}(T)), \max_{i\in T} x_i)$.

Proof. Note that we can write $S_\mu(x) = \min_{T\subseteq\mathcal{C}} \max(\nu(\mu^c(T)), \max_{i\in T} x_i)$. Hence, $S_\mu(x)$ is the minimum between $\min_{T\notin\mathcal{F}(\mu^c)} \max(\nu(\mu^c(T)), \max_{i\in T} x_i)$ and $\min_{T\in\mathcal{F}(\mu^c)} \max(\nu(\mu^c_{\#}(T)), \max_{i\in T} x_i))$.

If we consider $T \notin \mathcal{F}(\mu^c)$ then there exists $F \in \mathcal{F}(\mu^c)$ such that $F \subseteq T$ and $\mu^c(F) = \mu^c(T) = \mu^c_{\#}(F^c)$. Moreover $\max_{i\in F} x_i \leq \max_{i\in T} x_i$ which implies that $\max(\mu^c_{\#}(F), \max_{i\in F} x_i) \leq \max(\mu^c_{\#}(T), \max_{i\in T} x_i)$. □

Note that $S_\mu(x)$ takes the form "$\min \rightarrow$" using Kleene implication, like weighted minimum.

Proposition 6 $S_\mu(x) \leq \alpha$ *if and only if* $\exists F \in \mathcal{F}(\mu^c)$ *with* $\mu^c(F) \geq \nu(\alpha)$ *s.t.* $\forall x_i \in F$ $x_i \leq \alpha$.

Proof. $S_\mu(x) \leq \alpha$ implies $\exists F \in \mathcal{F}(\mu^c)$ such that $\mu^{\#}(F^c) \leq \alpha$ and $\max_{i \in F} x_i \leq \alpha$. So we have $\nu(\mu^c_{\#}(F)) \leq \alpha$, i.e., $\mu^c_{\#}(F) \geq \nu(\alpha)$ and $\forall x_i \in F\ x_i \geq \alpha$. □

This proposition shows that for each focal set of the conjugate μ^c we have the following elimination rule:

$$R^e_F\text{: If } x_i \leq \nu(\mu^c_{\#}(F)) \text{ for all } i \in F \text{ then } S_\mu(x) \leq \nu(\mu^c_{\#}(F)).$$

Let us give a possibilistic logic view of elimination rules associated to Sugeno integral, now as set of weighted clauses. Define for each criterion i a family of Boolean predicates $x_i(\alpha), \alpha > 0 \in L$ such that $x_i(\alpha) = 1$ if $x_i > \alpha$ and 0 otherwise. It is slightly different from the previous case. It is easy to check that $x_i = \min_{\alpha<1} \max(x_i(\alpha), \alpha)$.

Here we consider weighted Boolean formulas of the form (ϕ, β) which are interpreted as upper possibility distributions on the set of objects:

$$\pi^+_{(\phi,\beta)}(x) = \begin{cases} 1 \text{ if } x \models \phi; \\ \nu(\beta) \text{ otherwise} \end{cases}.$$

The upper possibility distributions associated to a set of such weighted formulas is interpreted as the minimum of the upper possibility distributions associated to each weighted formula. Then the set of weighted clauses $\{(\bigvee_{j \in F} x_j(\alpha), \nu(\alpha)) : \alpha < 1\}$ induces an upper possibility distribution:

$$\pi_F(x) = \min_{\alpha<1} \max(\alpha, \max_{j \in F} x_j(\alpha)) = \max_{j \in F} x_j.$$

Each weighted clause $(\bigvee_{j \in F} x_j(\mu^c(F)), \nu(\mu^c(F)))$ for a focal set F of μ^c corresponds to the elimination rule R^e_T stated above.

A logical rendering of the Sugeno integral in the min-max form is obtained as follows. First consider the following base of clauses $B^F_\mu = \{(\bigvee_{j \in F} x_j(\alpha), \nu(\alpha)) : \nu(\mu^c_{\#}(F)) \leq \alpha < 1\}$. We claim it encodes the term $\max(\nu(\mu^c_{\#}(F)), \max_{i \in F} x_i)$

Proposition 7 $\pi^+_{B^F_\mu}(x) = \max(\nu(\mu^c_{\#}(F)), \max_{i \in F} x_i)$.

Proof. $\pi^+_{B^F_\mu}(x) = \min_{1>\alpha\geq\nu(\mu^c_{\#}(F))} \max(\alpha, \max_{j \in F} x_j(\alpha)) =$
$\min_{1>\alpha} \max(\nu(\mu^c_{\#}(F)), \max(\alpha, \max_{j \in F} x_j(\alpha))) = \max(\nu(\mu^c_{\#}(F)), \max_{i \in F} x_i)$. □

Now consider the possibilistic base

$$B^+_\mu = \{(\bigvee_{j \in F} x_j(\alpha), \nu(\alpha)) : \nu(\mu^c(F)) \leq \alpha < 1, F \in \mathcal{F}(\mu^c)\}$$

with upper possibility distribution $\pi^+_\mu(x) = \min_{F \in \mathcal{F}(\mu^c)} \pi^+_{B^F_\mu}(x)$.

Proposition 8 $S_\mu(x) = \pi^+_\mu(x)$.

3 Data for the case study

Samples were collected on retention basins in the Eastern suburbs of Lyon before groundwater seepage (see [7] for more details). Some samples are obtained by rainy weather and others are obtained by dry weather. We then speak about "rain waters" and "dry waters" respectively. The waters contain many pollutants (like heavy metals, pesticides, hydrocarbons, PCB, ...) and our aim is to assess their impact on the water ecosystem health. This is why the unicellular algal compartment is considered hereafter. Algae are chosen for their high ecological representativeness at the first level of the food chain.

First, algal growth (C) was measured as a global indicator of algal health with standardized biossay (NF EN ISO 8692), then bioassays more specific of different metabolic pathways were carried out: chlorophyll fluorescence (F) as phosynthesis indicator and two enzymatic activities, Alkaline phosphatase Activity (APA) and Esterase Activity (EA) as nutrients metabolism indicators. Assays were performed after 24 hours exposure to samples collected during 7 different rainfall events and for different periods of the year for dry weather. Results, presented in Table 1 in which each row represents a sample, are expressed as percent of activity of control (control being algae before exposure to rain waters). The effects are considered significant when the values are far

data under rainy weather

AE	APA	F	C
83	36, 46	185, 45	45, 39
131, 64	25, 88	10, 69	0
35, 6		167, 06	0
16, 36	81, 25	194, 97	7, 17
107, 82	72, 64	167, 04	0
58, 18		116, 57	63, 39
698, 37	42, 15	90, 18	92, 70

AE	APA	F	C
24, 65	104, 93	153, 51	67, 58
17, 6	466, 4	123, 62	15, 76
33, 22	47, 6	163, 58	55, 17
96, 78	35, 17	21, 51	9, 71
74, 06	92, 3	123, 43	26, 59
64, 55	73, 32	163, 08	0
5, 12	206, 87	111, 56	92, 17

data under dry weather

AE	APA	F	C
509	55, 02	111, 64	110, 69
209, 28	109, 1	73, 18	102, 30
1964, 58	95, 93	6, 96	0
122, 62	98, 61	137, 09	69, 30
5, 6		143, 12	38, 81
45, 35	78, 27	129, 45	56, 82
64, 88	331, 37		0
143, 63			65, 52
	31, 92	44, 23	75, 78

Table 1. Original data

from 100. The values obtained are less than 100 in the case of inhibition and greater than 100 in the case of activation. The expert translates the results to the totally ordered scale $L = \{15, 25, 50, 85, 100\}$. Level 100, interpreted as a complete lack of effect of the rainwater, is the best evaluation; and the farther an evaluation is from 100, the worse it is. More precisely we have the following interpretation:

15	25	50	85	100
very strong effect	strong effect	effect	weak effect	no effect

With these rescaled data the expert can give a global evaluation (global eval.) in the scale L. The results are presented in Table 2.

The evaluation scale is equipped with the reversing order map ν defined by: $\nu(15) = 100$, $\nu(25) = 85$, $\nu(50) = 50$.

These experiment results can be modeled by an aggregation operation: the four criteria will be APA, AE, F and C, and we try to elicit Sugeno integrals which represent

data under rainy weather

AE	APA	F	C	global eval.
85	25	50	50	50
85	25	15	15	25
25		50	15	25
25	85	25	15	25
100	85	50	15	50
50		85	50	50
15	50	100	85	50

AE	APA	F	C	global eval.
15	100	50	50	50
15	15	85	25	25
25	50	50	50	50
100	25	25	15	25
85	100	85	25	85
50	85	50	15	50
15	15	100	100	50

data under dry weather

AE	APA	F	C	global eval.
15	50	100	100	85
15	100	85	100	85
15	100	15	15	25
100	100	85	50	85
15		85	25	50
50	85	85	50	50
50	15		15	25
85			50	50
	25	50	85	50

Table 2. Rescaled data

the given global evaluation. Next the obtained Sugeno integrals are translated into rules whose conditions use the criteria.

4 Experimental results

In this section, we try to interpret the above data in terms of selection and elimination rules built via a Sugeno integral.

Data under rainy weather We consider the data under rainy weather and we compute the bounds of the set of compatible capacities $\check{\mu}$, $\hat{\mu}$ and their conjugate capacities.

$criteria$	$\check{\mu}$	$\hat{\mu}$	$\check{\mu}^c$	$\hat{\mu}^c$	$criteria$	$\check{\mu}$	$\hat{\mu}$	$\check{\mu}^c$	$\hat{\mu}^c$	$criteria$	$\check{\mu}$	$\hat{\mu}$	$\check{\mu}^c$	$\hat{\mu}^c$
$\{AE\}$	15	25	50	15	$\{APA\}$	15	25	50	15	$\{F\}$	15	25	85	15
$\{C\}$	15	50	25	15	$\{AE,APA\}$	25	50	50	50	$\{AE,F\}$	15	100	100	15
$\{AE,C\}$	15	100	100	15	$\{APA,F\}$	15	100	100	15	$\{APA,C\}$	15	100	100	15
$\{F,C\}$	50	50	85	50	$\{AE,APA,F\}$	85	100	100	50	$\{AE,APA,C\}$	25	100	100	85
$\{AE,F,C\}$	50	100	100	85	$\{APA,F,C\}$	50	100	100	85	$\{AE,APA,F,C\}$	100	100	100	100

Table 3. Weights for criteria groups for rainy weather

We have $\check{\mu} \leq \hat{\mu}$ so it is possible to represent the data with a Sugeno integral. Let us denote by μ a capacity with $\check{\mu} \leq \mu \leq \hat{\mu}$.

Remark 1. As $\check{\mu} \leq \mu \leq \hat{\mu}$, $\mu(\{F, C\}) = 50$. Since $\mu(F) \leq 25$, either C is a focal element with level 50 or $\{F, C\}$ is a focal element with level 50.

Remark 2. Since $S_{\check{\mu}} \leq S_{\mu} \leq S_{\hat{\mu}}$, then we are going to consider $\check{\mu}$ (resp. $\hat{\mu}^c$) to obtain selection (resp. elimination) rules. Indeed, testing $\check{\mu}$ is larger than a threshold and $\hat{\mu}^c$ less than this threshold give sure decisions despite the limited knowledge about μ.

– Let us consider $\check{\mu}$. The focal sets are $\mathcal{F}(\check{\mu})^{100} = \{\{AE, APA, F, C\}\}$, $\mathcal{F}(\check{\mu})^{85} = \{\{AE, APA, F\}\}$, $\mathcal{F}(\check{\mu})^{50} = \{\{F, C\}\}$, $\mathcal{F}(\check{\mu})^{25} = \{\{AE, APA\}\}$, and we obtain the following selection rules
  - If $x_{AE} \geq 85$, $x_{APA} \geq 85$ and $x_F \geq 85$ then $S_{\check{\mu}}(x) \geq 85$.

  - If $x_F \geq 50$ and $x_C \geq 50$ then $S_{\check{\mu}}(x) \geq 50$.
  - If $x_{AE} \geq 25$ and $x_{APA} \geq 25$ then $S_{\check{\mu}}(x) \geq 25$.
- Let us consider $\hat{\mu}^c$. We have $\mathcal{F}(\hat{\mu}^c)^{50} = \{\{APA, AE\}, \{F, C\}\}$, $\mathcal{F}(\hat{\mu}^c)^{85} = \{\{AE, APA, C\}, \{AE, F, C\}, \{APA, F, C\}\}, \mathcal{F}(\hat{\mu}^c)^{100} = \{\{AE, APA, F, C\}\}$ which produces the following elimination rules:
  - if $x_{APA} \leq 50$ and $x_{AE} \leq 50$ then $S_{\hat{\mu}}(x) \leq 50$.
  - if $x_F \leq 50$ and $x_C \leq 50$ then $S_{\hat{\mu}}(x) \leq 50$.
  - if $x_{APA} \leq 25$ and $x_{AE} \leq 25$ and $x_C \leq 25$ then $S_{\hat{\mu}}(x) \leq 25$.
  - if $x_{AE} \leq 25$ and $x_F \leq 25$ and $x_C \leq 25$ then $S_{\hat{\mu}}(x) \leq 25$.
  - if $x_{APA} \leq 25$ and $x_F \leq 25$ and $x_C \leq 25$ then $S_{\hat{\mu}}(x) \leq 25$.

Sugeno integral $S_\mu(x)$ complies with all rules, hence the following comments:

- If criteria AE, APA and F are satisfied enough then the global evaluation is good;
- When criterion C has a bad rating, two other criteria also need to have a bad rating in order to obtain a bad global evaluation.
- However if criteria other than C get bad ratings it is not enough to get a bad global evaluation.

Let us consider fictitious examples of data and predict the global evaluation given with $S_{\check{\mu}}$ and $S_{\hat{\mu}}$ obtained above. We get an interval-valued evaluation given by the range of compatible capacities $S_{\check{\mu}} \leq S_\mu \leq S_{\hat{\mu}}$. In the left-hand table we consider that only one criterion is perfect and the others get the worst value. In the right-hand table we consider that only one criterion has the worst value and the other are satisfied.

AE	APA	F	C	$S_{\check{\mu}}$	$S_{\hat{\mu}}$
15	15	15	100	15	50
100	15	15	15	15	25
15	15	100	15	15	25
15	100	15	15	15	25

AE	APA	F	C	$S_{\check{\mu}}$	$S_{\hat{\mu}}$
100	100	100	15	85	100
100	100	15	100	25	100
100	15	100	100	50	100
15	100	100	100	50	100

Some comments concerning the global evaluation: Criterion C is not sufficient to downgrade it under 85, and it is not sufficient to bring it above 50. No other criterion is sufficient to alone bring it above 25. Criterion F is not sufficient to downgrade it under 25, and criteria AE and APA are not sufficient to downgrade alone it under 50. These remarks give a good idea of the relative importance of criteria.

Data under dry weather This section is similar to the previous one. First we compute the bounds of the capacities as per Table 4. Since $\check{\mu} \leq \hat{\mu}$, it is possible to represent the data with a Sugeno integral. We remark that the set of solutions $\check{\mu} \leq \mu \leq \hat{\mu}$ is not compatible with the previous one since they have an empty intersection.

Remark 3. We have $\mu(\{F, C\}) = 85$ and since $\mu(F) \leq 50$, either C is a focal element with level 85 or $\{F, C\}$ is a focal element with level 85.

- Let us consider $\check{\mu}$. The focal sets form $\mathcal{F}(\check{\mu})^{100} = \{\{AE, APA, F, C\}\}, \mathcal{F}(\check{\mu})^{85} = \{\{F, C\}, \{AE, APA, F\}\}$, $\mathcal{F}(\check{\mu})^{25} = \{\{APA\}\}$. It produces the following rules:
  - If $x_{AE} \geq 85$, $x_{APA} \geq 85$ and $x_F \geq 85$ then $S_{\check{\mu}}(x) \geq 85$;
  - If $x_F \geq 85$ and $x_C \geq 85$ then $S_{\check{\mu}}(x) \geq 85$.

$Criteria$	$\check{\mu}$	$\hat{\mu}$	$\check{\mu}^c$	$\hat{\mu}^c$	$Criteria$	$\check{\mu}$	$\hat{\mu}$	$\check{\mu}^c$	$\hat{\mu}^c$	$Criteria$	$\check{\mu}$	$\hat{\mu}$	$\check{\mu}^c$	$\hat{\mu}^c$
$\{AE\}$	15	85	25	15	$\{APA\}$	25	25	25	15	$\{F\}$	15	50	85	15
$\{C\}$	15	85	25	15	$\{AE,APA\}$	25	85	25	25	$\{AE,F\}$	15	100	85	25
$\{AE,C\}$	15	100	85	50	$\{APA,F\}$	25	50	100	15	$\{APA,C\}$	25	85	100	15
$\{F,C\}$	85	85	85	25	$\{AE,APA,F\}$	85	100	100	25	$\{AE,APA,C\}$	25	100	100	50
$\{AE,F,C\}$	85	100	85	85	$\{APA,F,C\}$	85	100	100	25	$\{AE,APA,F,C\}$	100	100	100	100

Table 4. Weights for criteria groups for dry weather

- If $x_{APA} \geq 25$ then $S_{\check{\mu}}(x) \geq 25$.

– Let us consider $\hat{\mu}^c$. We have $\mathcal{F}(\hat{\mu}^c)^{25} = \{\{AE, APA\}, \{AE, F\}, \{F, C\}\}, \mathcal{F}(\hat{\mu}^c)^{50} = \{\{AE, C\}\}, \mathcal{F}(\hat{\mu}^c)^{85} = \{\{AE, F, C\}\}, \mathcal{F}(\hat{\mu}^c)^{100} = \{\{AE, APA, F, C\}\}$, which produces the following rules:
  - if $x_{AE} < 100$ and $x_{APA} < 100$ then $S_{\hat{\mu}}(x) < 100$;
  - if $x_{AE} < 100$ and $x_F < 100$ then $S_{\hat{\mu}}(x) < 100$;
  - if $x_F < 100$ and $x_C < 100$ then $S_{\hat{\mu}}(x) < 100$;
  - if $x_{AE} \leq 50$ and $x_C \leq 50$ then $S_{\hat{\mu}}(x) \leq 50$;
  - if $x_{AE} \leq 25$ and $x_F \leq 25$ and $x_C \leq 25$ then $S_{\hat{\mu}}(x) \leq 25$.

A Sugeno integral $S_\mu(x)$, where $\check{\mu} \leq \mu \leq \hat{\mu}$, complies with all rules, hence, if C, F and AE have a bad evaluation then the global evaluation is bad. As previously, consider we fictitious examples and derive the bounds of the global evaluation given by S_μ.

AE	APA	F	C	$S_{\check{\mu}}$	$S_{\hat{\mu}}$
15	15	15	100	15	85
100	15	15	15	15	85
15	15	100	15	15	50
15	100	15	15	25	25

AE	APA	F	C	$S_{\check{\mu}}$	$S_{\hat{\mu}}$
100	100	100	15	85	100
100	100	15	100	25	100
100	15	100	100	15	100
15	100	100	100	85	100

Some comments concerning the global evaluation: Each of C and AE is not sufficient to alone bring it above 85 or to downgrade it under 85. F is not sufficient to bring alone it above 50 or to downgrade it under 25. APA is not sufficient to bring it above 25 but it can downgrade it to 15. If C and NF have a good evaluation the global evaluation will be good. It is the same if C is replaced by AE and APA.

Discussion The rules presented in this section includes pieces of knowledge familiar to experts in the application area. For example, parameters C and F are used to evaluate the global health of algae, unlike APA and AE which refer to specific pathways metabolism. So, when C and F show no effect or weak effect, the global evaluation is good, while a significant effect on the APA and EA only, is known not to allow degradation of the overall score. Moreover, rules extracted from the obtained Sugeno integrals show stronger effects with rain samples than those obtained after dry weather samples exposure. These results are in perfect agreement with those obtained directly with bioassays.

5 Conclusion

This paper shows the usefulness of qualitative aggregation operations such as Sugeno integrals to extract knowledge from data. The key asset of the approach is the capability of Sugeno integral to lend itself to a complete logical rendering of its informative content, which is typical of qualitative approaches, while a direct handling of the numerical data would make this step more difficult to process. A comparison between the results obtained by this approach and results obtained by standard data-mining methods like e.g., Apriori would be worthwhile in a future work. Of course one objectionis that only special kinds of rules can be expressed by Sugeno integral:a single threshold is used in all conditions of each rule [9]. This limited expressive power may be a cause of failure of the approach if no capacity can be identified from the data. Extracting more expressive rules would need qualitative aggregation operations beyond Sugeno integrals.

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