Interpretation of line drawings of complex objects

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Before research on the machine interpretation of line drawings can find practical applications it is essential to study realistic models of real-world objects. This paper extends the work of Malik on curved objects with piecewise C^3 surfaces. In particular, a new catalogue of junction labellings is given when smooth edges (discontinuities of surface curvature) are permitted on object surfaces.

Keywords: line drawing interpretation, labelling, smooth edges

The machine interpretation of line drawings (pioneered by Guzman[1]) has a privileged position in the computer vision literature. It has proved to be the inspiration and testing ground of important techniques of much wider application, such as discrete relaxation[2]. Ironically, the simplified and idealized nature of the line drawing interpretation problems studied, which make them so attractive as toy problems, also renders the solutions almost irrelevant for the interpretation of real images (although Walters and Krishnan[3] describe an application in the automatic colouring of cartoons). Typical simplifications are that the objects viewed are polyhedral[2,4–8], have quadric/planar faces[9–12] or are composed of C^3 surface patches[13]. Almost all authors assume a general viewpoint (Shapira[8] being an exception) and that objects have Lambertian surfaces.

The line drawing to be analysed is usually taken to be a perfect representation of the corresponding three-dimensional scene, in the sense that it is the projection of the set of surface-normal discontinuities and depth discontinuities in the scene. Notable exceptions are the work of Falk[14] and Shapira and Freeman[9,10], which allow lines or parts of lines to be missing. In these cases, further information is used to interpret the line drawing: Falk matches known object models with the line drawing; Shapira and Freeman find correspondences between two imperfect projections of the same object. Missing lines and junctions are not the only possible imperfections in the line drawing. Other possible errors include misclassification of junctions (e.g. confusion of Y and T junctions), extra junctions (e.g. an L junction being mistakenly merged with a nearby line to form a K junction), extra edges, and errors in the positioning of lines or junctions.

This paper only discusses the over-simplification of models of real-world objects, and does not address the problem of the idealization of the derivation of a line drawing from a real image. Possible approaches to this latter problem include:

1 Interpretation algorithms which allow for imperfect input. For example, the algorithms of Sugihara[7] and Kanatani[15] solve ill-posed problems in the interpretation of line drawings of polyhedra (in which, for example, projections of parallel lines are not perfectly parallel in the line drawing).
2 Probabilistic algorithms, such as probabilistic relaxation[16] with a probabilistic line drawing as input: for example, junction J is of type Y with probability 0.7 and of type T with probability 0.3.
3 Heuristic algorithms, applying rules which are valid for perfect line drawings, under the assumption that a small number of errors in the line drawing will give rise to only a small number of errors in the interpretation.

For simplicity of presentation, it is assumed throughout the rest of the paper that the line drawing is a perfect projection of the three-dimensional scene. We consider the study of perfect line drawings to be the essential foundation on which algorithms for the interpretation of imperfect line drawings will be built. The aim of this paper, the generalization of classical work on the interpretation of perfect line drawings to cover more realistic objects, is part of a long-term project to develop algorithms for the interpretation of imperfect line drawings of realistic objects.

Expanding the class of objects that may occur in the line drawing, to allow for more realistic objects, increases the potential area of application, but draw-

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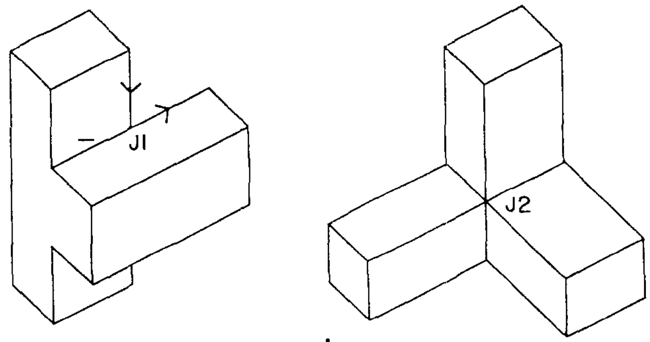
ings which previously had a unique interpretation may become ambiguous. We therefore need to find new sources of information, such as smooth edges, which can be used to reduce ambiguity.

We study line drawings of regular opaque curved objects composed of C^3 Lambertian surface patches, without surface markings or shadows. We only consider trihedral vertices, and accidental alignments are not permitted. A complete list of our assumptions is given below. We conclude that the increase in ambiguity, caused by allowing a wider, more realistic class of objects, can be compensated by using other sources of information in the intensity image or probabilistic information about the likely shapes of objects.



a b

Figure 2. Two vertices which are not permitted in the Huffman–Clowes labelling scheme

TRIHEDRAL VERTICES

The interpretation of line drawings can be defined as inferring information about the corresponding three-dimensional scene. Most attention has been given to the assignment of semantic labels to each line in the drawing[4,5] or to each point on each line[13]. Examples of semantic labels are: convex (+), concave (−), occluding (→), shadow and crack[2].

Sugihara[6] has given necessary and sufficient conditions for a labelled line drawing of a polyhedral scene to be physically realizable. Unfortunately, his algebraic technique does not easily generalize to curved objects.

The Huffman–Clowes labelling scheme (e.g. Huffman[4], Clowes[5], Winston[17] or Sugihara[7]) is based on a catalogue of the vertices which can be formed by the intersection of three distinct planes at a point P in three-dimensional space. Figure 1 shows an example of a line drawing labelled according to the Huffman–Clowes labelling scheme. In fact, this drawing contains examples of each legal junction labelling. To disallow highly unlikely objects, such as an object consisting of a pair of cubes C_1, C_2 connected only at a single point, a vertex of both C_1 and C_2, the simple circuit condition is imposed. This states that the faces surrounding a vertex V must form a simple circuit, as we 'walk' around V on the object faces that meet at V. Generalized to curved objects, it states that, within a sufficiently small neighbourhood N of V, all the surface patches in N have distinct tangent planes at V.

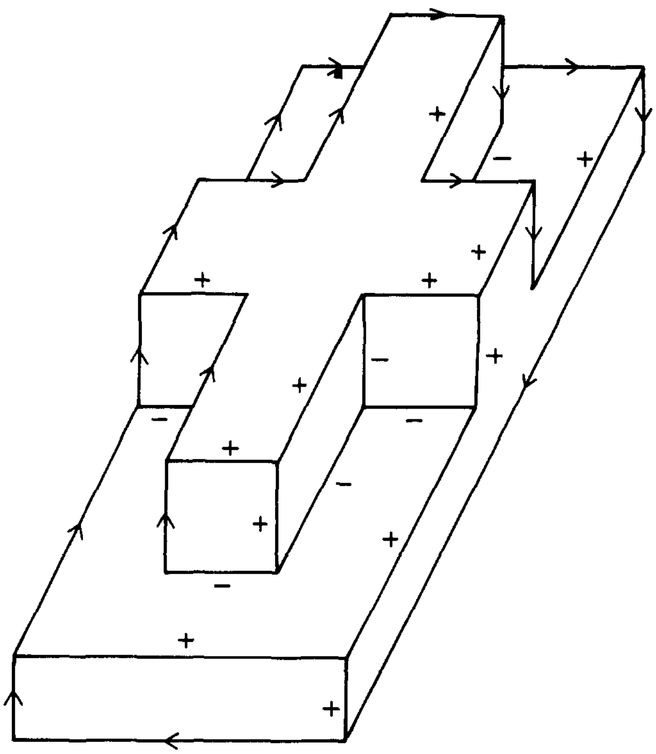


Figure 1. Example of a line drawing labelled according to the Huffman–Clowes labelling scheme

However, the simple circuit condition also eliminates two quite reasonable vertices. These vertices are shown in Figures 2a and b, where they project into junctions J1 and J2. The six faces which meet at J2, for example, lie in only three distinct tangent planes.

Extending the junction catalogues to include all projections of the vertex in Figure 2a has the effect of invalidating the T-junction rule that the cross-bar of a T-junction in the line drawing is the projection of an occluding edge in the scene. This can clearly be seen in Figure 2a, where the labelling of junction J1 is given. The T-junction rule is especially useful, in that it allows the line drawing to be segmented into sets of lines corresponding to distinct objects, which can then be analysed separately and independently. Given the possibility of the vertex shown in Figure 2a this rule is clearly heuristic.

C-JUNCTIONS

Malik[13] has published a mathematically rigorous paper on the semantic labelling of line drawings of curved objects. This was later extended to incorporate shape-from-shading information in the vicinity of junctions and lines[18]. However, a small correction needs to be made to the labelling scheme given by Malik[13]. He believed that at the projection of an invisible vertex (a C-junction, in the terminology of Shapira and Freeman[9,10] and Chakravarty[11]), the line is always concave. In other words, the invisible vertex was assumed to be always of the form illustrated in Figure 3a (junction C). In fact, a convex C-junction is also possible: the junction J in Figure 3b is an example. The broken line is a hidden line which is shown for illustrative purposes only, and is not actually present in the line drawing. This example involves the *saddle* surface:

$$z = (x+1)^2\sqrt{y+1} - 2\sqrt{2}x$$

An example of a *convex* surface, which also projects

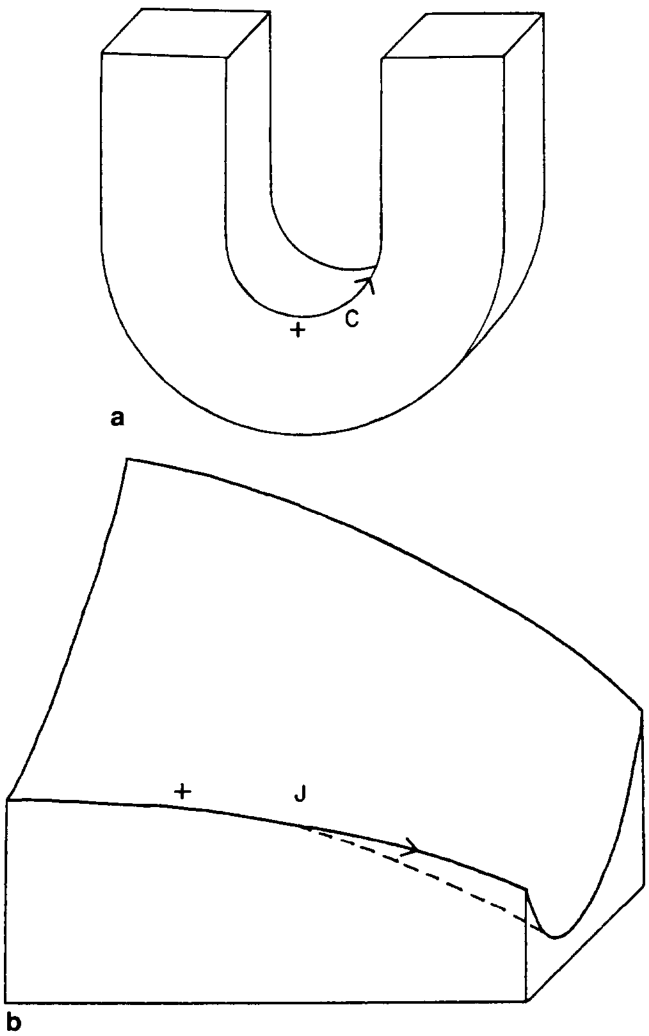


Figure 3. (a) Concave C-junction C; *(b) convex C-junction* J

into a convex C-junction, from the appropriate viewing angle, is:

$$z = \sqrt{y+1}\,\sqrt{x+1} - z$$

Given that the line labellings of the mirror-images of the line drawings in Figures 3a and b are also possible, it can be seen that each of the four combinations:

$$+\leftarrow \quad +\rightarrow \quad \leftarrow+ \quad \rightarrow+$$

of the labels 'convex' and 'occluding' is a physically possible labelling of a C-junction. Clearly, the labelling in Figure 3b is much less likely than the labelling in Figure 3a.

Malik and Maydan[18] have shown that shape-from-shading analysis[19] of the intensity image, when combined with line labelling, can often indicate the presence of C-junctions. Let P be a point on a 3D edge E. If shape-from-shading analysis indicates that P is an isolated point at which the normal to one of the surfaces which meet at E is orthogonal to the viewing direction, then (under a general viewpoint assumption) the projection of P in the line drawing is a C-junction.

Figure 4 illustrates a transition from a convex edge to a concave edge. Although this transition-point Q

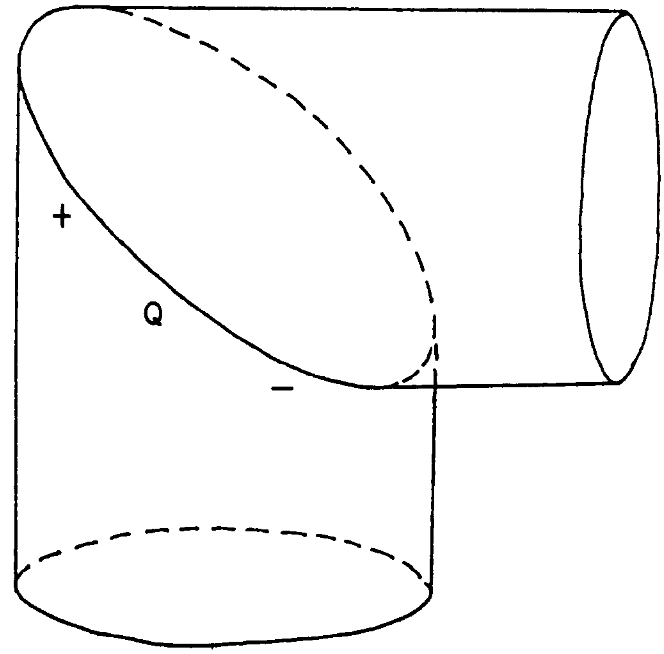


Figure 4. Transition Q *from a convex edge to a concave edge*

appears to project into a C-junction, labelled + − in the line drawing, the edge actually fades out as we approach Q from either direction. There is no difference in the surface normal on either side of the point Q, and hence there will be no discontinuity in the intensity images at Q. In fact, assuming general light-source positions and a general viewpoint, the intensity discontinuity will pass from positive to negative (or *vice versa*) as we follow the edge through Q. Such transitions Q are therefore detectable in the intensity image.

TIGHTER CONSTRAINTS FOR CURVED OBJECTS

Another refinement can be made to the list of legal junction labellings given by Malik[13], this time in a more positive direction, by tightening the constraint on the labelling of a curvature-L junction. A curvature-L junction is simply a discontinuity of curvature of a curved line.

The only two 3D configurations that can cause a curvature-L junction in the drawing are illustrated in Figures 5a and b. The curvature-L junctions are marked 'cL'. This is under Malik's assumptions, which include the restriction that objects are composed of C^3 surface patches. The intersection of two C^3 surfaces, in the neighbourhood of a point at which they are not tangential, is also C^3 (see Malik[13]). Under a general viewpoint assumption disallowing accidental alignment, the projection of this curve of intersection is also C^3. Therefore we assume that the line L_1–L_3 is C^3 in the neighbourhood of the junction. Similarly, assuming a general viewpoint, L_2 is also C^3.

As with the C-junction, two discontinuity edges and one extremal edge meet in 3D space, and one edge is not visible due to occlusion, but here it is a discontinuity edge which is occluded. (A discontinuity edge is a surface-normal discontinuity; an extremal edge, or limb, is the locus of points of intersection of the object

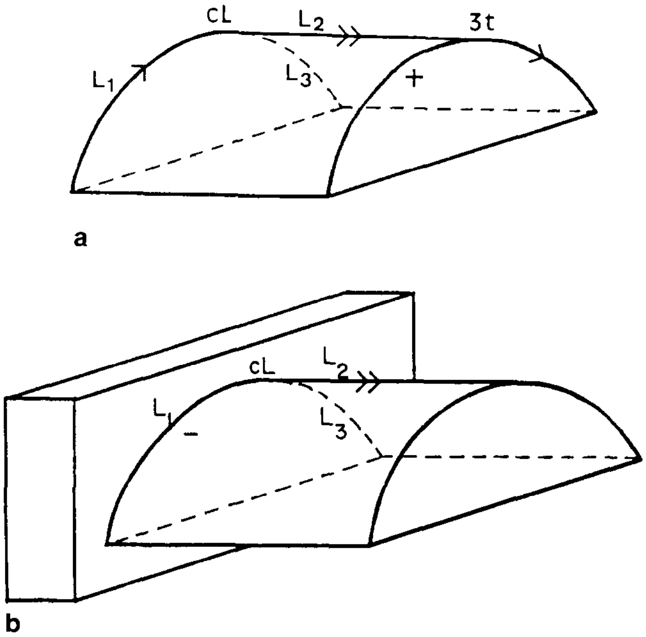


Figure 5. Two types of curvature-L junction (cL)

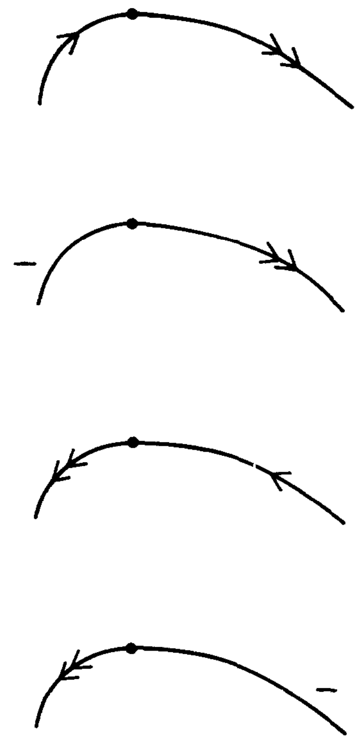


Figure 6. Four labellings of a curvature-L junction

surface with rays emanating from the viewpoint and tangential to the object surface). The corresponding hidden line, shown as the broken line L_3 in Figures 5a and b, is not actually present in the line drawing. It must therefore be occluded by the curved surface whose extremal edge projects into L_2. Malik[13] has already shown that there is a discontinuity of curvature between L_1 and L_2 at the junction. Since L_1, L_2 and L_3 are tangential and are C^3, this means that the curvature of L_3, which is equal to the curvature of L_1, must be greater than the curvature of L_2. Curvature here means signed curvature at the junction, where a line, such as L_1 in Figure 5a, has positive curvature if it is a convex contour of the image of the object in the drawing. This eliminates exactly half of the combinatorially possible labellings of a curvature-L junction.

The remaining legal labellings are given in Figure 6. There are four legal labellings for each curvature-L junction. However, as was the case for the C-junction, some of these legal labellings are quite unlikely: the latter two labellings in Figure 6, which represent concave junctions, are less likely than the first two labellings, which represent convex junctions. For example, the third labelling in Figure 6 can only be caused by a configuration such as that illustrated in Figure 7. The curvature-L junction is marked 'cL' in Figure 7. The object is bounded by the five planes:

$$x = 0, \quad x = 1, \quad y = 0, \quad y = 4, \quad z = 12$$

and bounded below by the surface:

$$z = \sqrt{y}(x+1)^2 + \sqrt{4-y}(2-x)^2 + 2$$

As usual, the broken lines are shown for illustrative purposes only, and are not actually present in the line drawing. Given the unlikelihood of such objects, compared to those in Figure 5, this means that, given a curvature-L junction, it is possible to assign unique labels (apart from the ambiguity between $-$ and $\rightarrow$) with a fairly high degree of certainty.

Another junction which can occur in a line drawing of curved objects is the three-tangent junction, in which three lines meet tangentially. In fact, two tangential 'input' lines merge to become a single 'output' line, and exactly one of the two 'input' lines has continuous curvature with the 'output' line. The only possible three-dimensional configuration giving rise to a three-tangent junction (again, under Malik's assumptions, including piecewise C^3 surface) is illustrated in Figure 5a, where the junction is marked '3t'. The labelling shown is the only possible labelling of a three-tangent junction. The lines labelled $+$ and $\rightarrow$ are the lines with the same curvature.

Figure 7 also contains a three-tangent junction, marked '3t'. This shows that it is possible, although

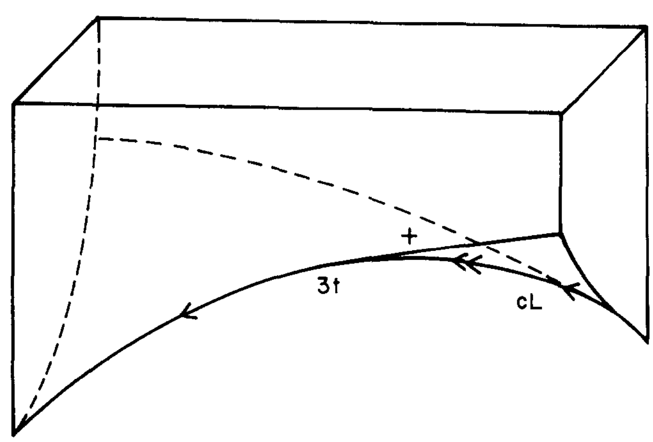


Figure 7. Unlikely, but physically possible, labellings of 3-tangent and curvature-L junctions

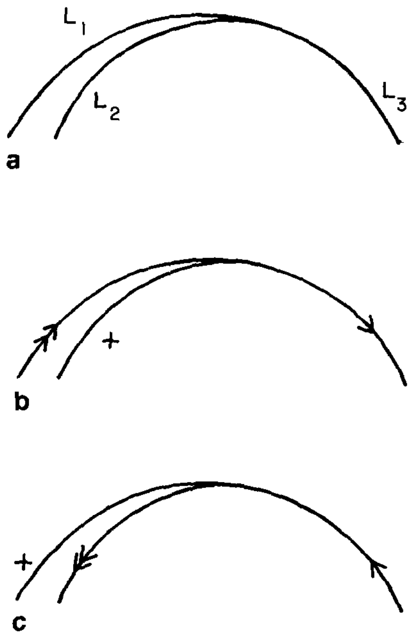


Figure 8. (a) 3-tangent junction; (b) its labelling if it is L$_2$ *and* L$_3$ *which have the same curvature; (c) its labelling if it is* L$_1$ *and* L$_3$ *which have the same curvature*

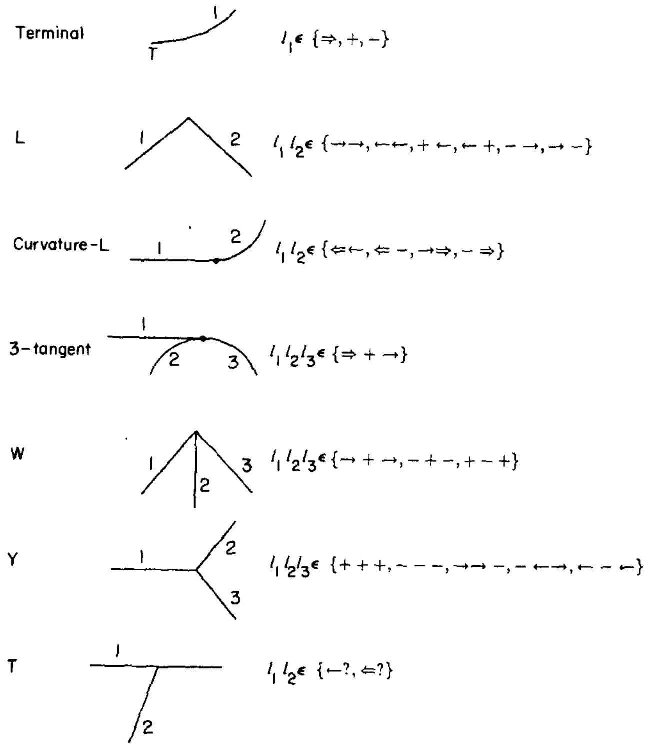


Figure 9. Catalogue of junction labellings for objects with piecewise C^3 *smooth surfaces*

quite unlikely, that the projection of the extremal edge at a three-tangent junction has a greater absolute value of curvature than the projections of the discontinuity edges.

Determining the first derivative of a curve in a line drawing is less error-prone than determining the second derivative. Therefore, it may happen that a three-tangent junction is detected in the drawing, but that which pair of lines have the same curvature cannot be reliably determined. Such a junction is illustrated in Figure 8a. Its two legal labellings are shown in Figure 8b (for the case where it is L_2 and L_3 which have the same curvature) and in Figure 8c (for the case where it is L_1 and L_3 which have the same curvature). As with the C-junction and the curvature-L junction, the labelling in which the projection of the extremal edge has the greater absolute value of curvature (i.e. Figure 8c) is the less likely of the two. Therefore, some (probabilistic) information about the labelling of a three-tangent junction can be obtained without the need to determine the curvature of the lines.

We complete this section by giving, in Figure 9, the complete catalogue of junction labellings, assuming, among other things, that objects have piece-wise C^3 smooth surfaces. A complete list of assumptions is given below. This is the catalogue of Malik[13] corrected according to the discussion above on the labelling of C-junctions, curvature-L junctions and three-tangent junctions. For each junction type, l_i $(i = 1, 2, 3)$ denotes the label for the line i. In Figure 9, we use the symbol $\Rightarrow$ instead of a double-headed arrow to denote the projection of an extremal edge. In the set of labellings for a T-junction, the symbol '?' represents any label. This catalogue does not include the projections of the vertex illustrated in Figure 2a in the set of labelling for a T-junction. Terminal junctions are discussed in detail by Koenderink and van Doorn[20]. A terminal junction is simply a line which terminates at a point T.

DISCONTINUITIES OF SURFACE CURVATURE

In previous sections we have implicitly made several assumptions about the class of line drawings to be analysed. For example, we have inherited the assumption from Malik's work that object surfaces are piecewise C^3. Since edges are discontinuities of the first derivative of surfaces, this leaves discontinuities of second and third derivatives which are neither accepted as edges nor allowed to occur within object surfaces. In this section we will show that a labelling scheme exists for lines which are projections of discontinuities of surface curvature (smooth edges).

A discontinuity of nth derivative of the surface of an object is detectable in the intensity image as a discontinuity of $(n-1)$th derivative. Thus, a discontinuity of surface curvature (a smooth edge) will be detectable in the intensity image as a discontinuity of first derivative, known as a ramp edge (or a roof edge). We use the term *ramp line* to refer to the projection of a smooth edge in the line drawing. Canny[21] has given an optimal linear edge-detector for ramp edges. Of course, the detection of ramp edges will be even less reliable than the detection of step edges (discontinuities of intensity), and this must be taken into account by the labelling algorithm.

The labelling scheme we propose is to label ramp lines by one of two labels: $>$ or $<$. Let E be a locus of discontinuities of curvature of an object surface S (i.e. E is a smooth edge). Let X be a point on E and let N be the plane which passes through X and is normal to E (see Figure 10). Let C_L and C_R be the intersection of N and S to the left and right, respectively, of X. Let κ_L and κ_R be the signed curvatures of C_L and C_R. They are the normal curvatures of the surface S at X in the

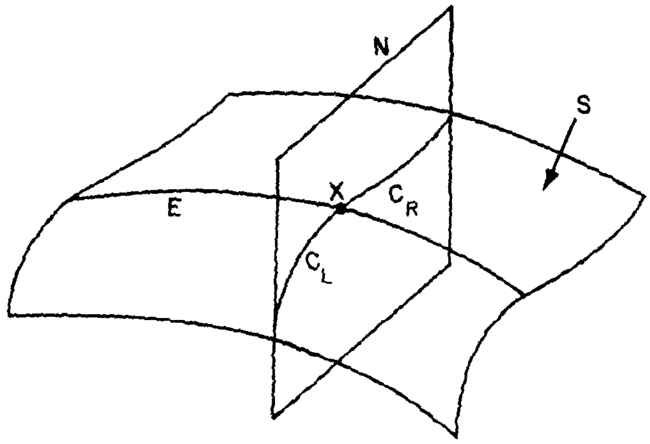


Figure 10. Smooth edge E

direction orthogonal to E. We can now give the meaning of the labels > and <: the projection of E in the line drawing is labelled:

> if $\kappa_L > \kappa_R$

< if $\kappa_L < \kappa_R$

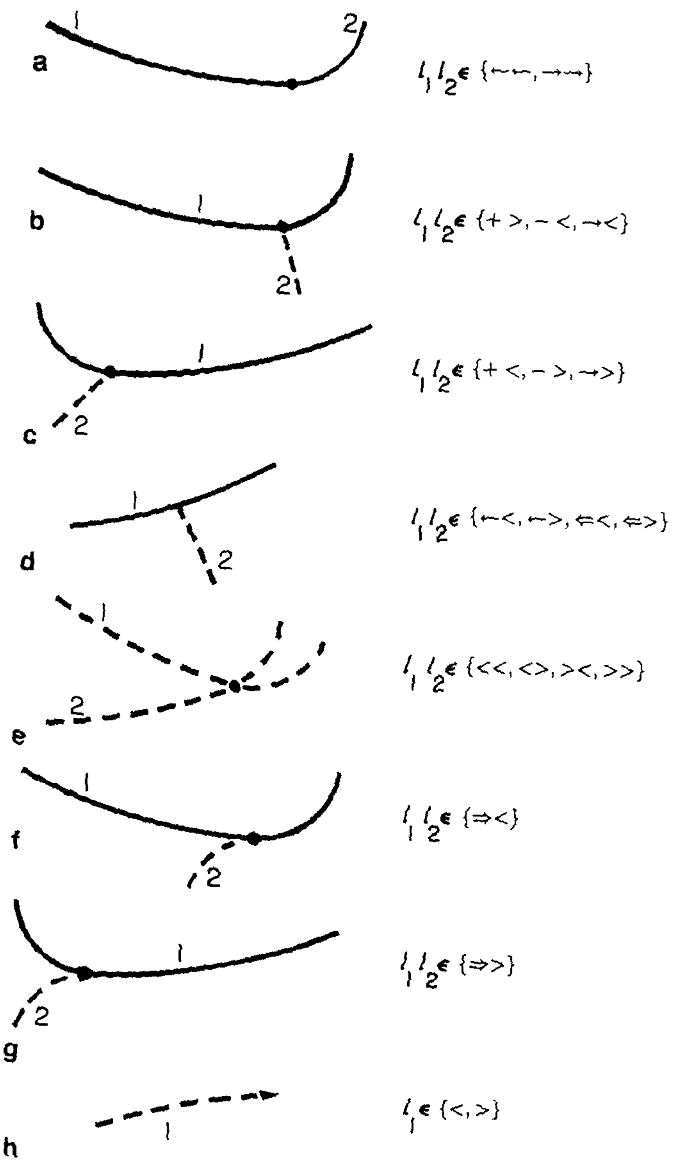


Figure 11. New junction labellings to be added to those in Figure 9

We use the convention that a *concave* surface of an object has *positive* curvature and that a convex surface has *negative* curvature.

The presence of smooth edges leads to the possibility of new types of vertex. To establish a new catalogue of junction labellings (Figure 11), we consider only the two most common:

1 A single smooth edge terminates at a discontinuity edge (i.e. at the boundary of an object surface), e.g. A in Figure 12.
2 Two smooth edges intersect in the interior of an object surface, e.g. B in Figure 12.

Thus, for example, we do not consider the possibility of two smooth edges intersecting on a discontinuity edge. The projections of these two new vertices are shown in the list of labelled junctions in Figure 1a–c, e. Ramp lines are shown as broken lines, unbroken lines, as usual, representing projections of depth discontinuities or of surface-normal discontinuities. New types of junctions are also caused by:

- the intersection of a smooth edge with an extremal edge: e.g. C in Figure 12. All legal labellings of this junction type are given in Figure 11f, g;
- the partial occlusion of a smooth edge by a nearer edge: for example, D in Figure 12. All legal labellings of this junction are given in Figure 11d;

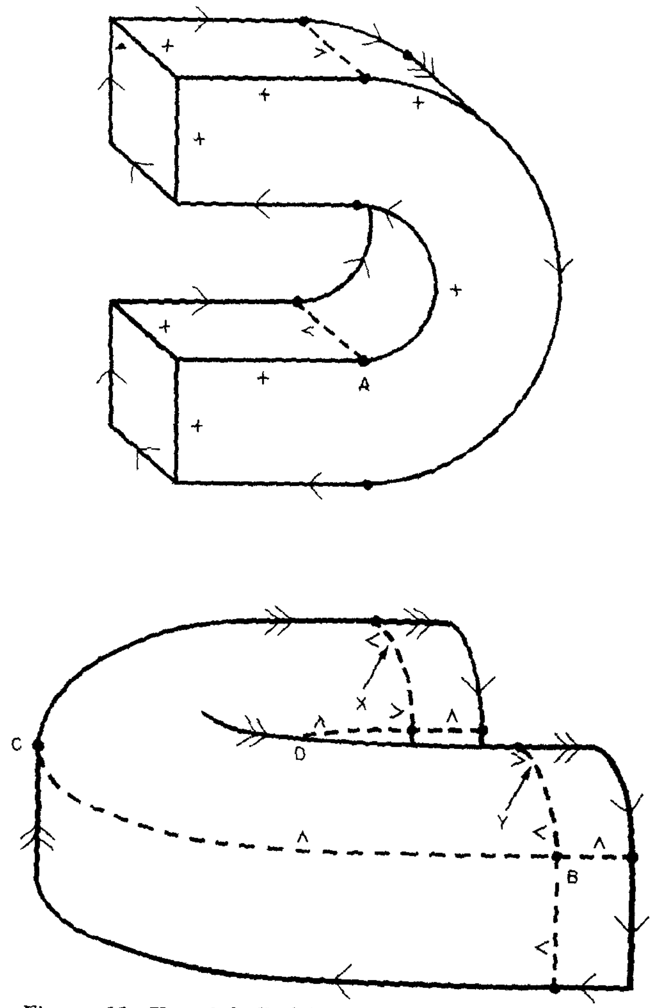


Figure 12. Two labelled line drawings containing ramp lines

- the termination of a smooth edge. The legal labellings of this junction are given in Figure 11h. The arrowhead represents a ramp line that fades out in the drawing (i.e. a ramp edge that fades out in the intensity image).

A dot represents a discontinuity of curvature on the unbroken line in Figure 11a–c, f, g and on both of the broken lines in Figure 11e. The label for the line marked i ($i = 1, 2$) is denoted by l_i. In junctions b–d, f and g the continuous unbroken line has the same label

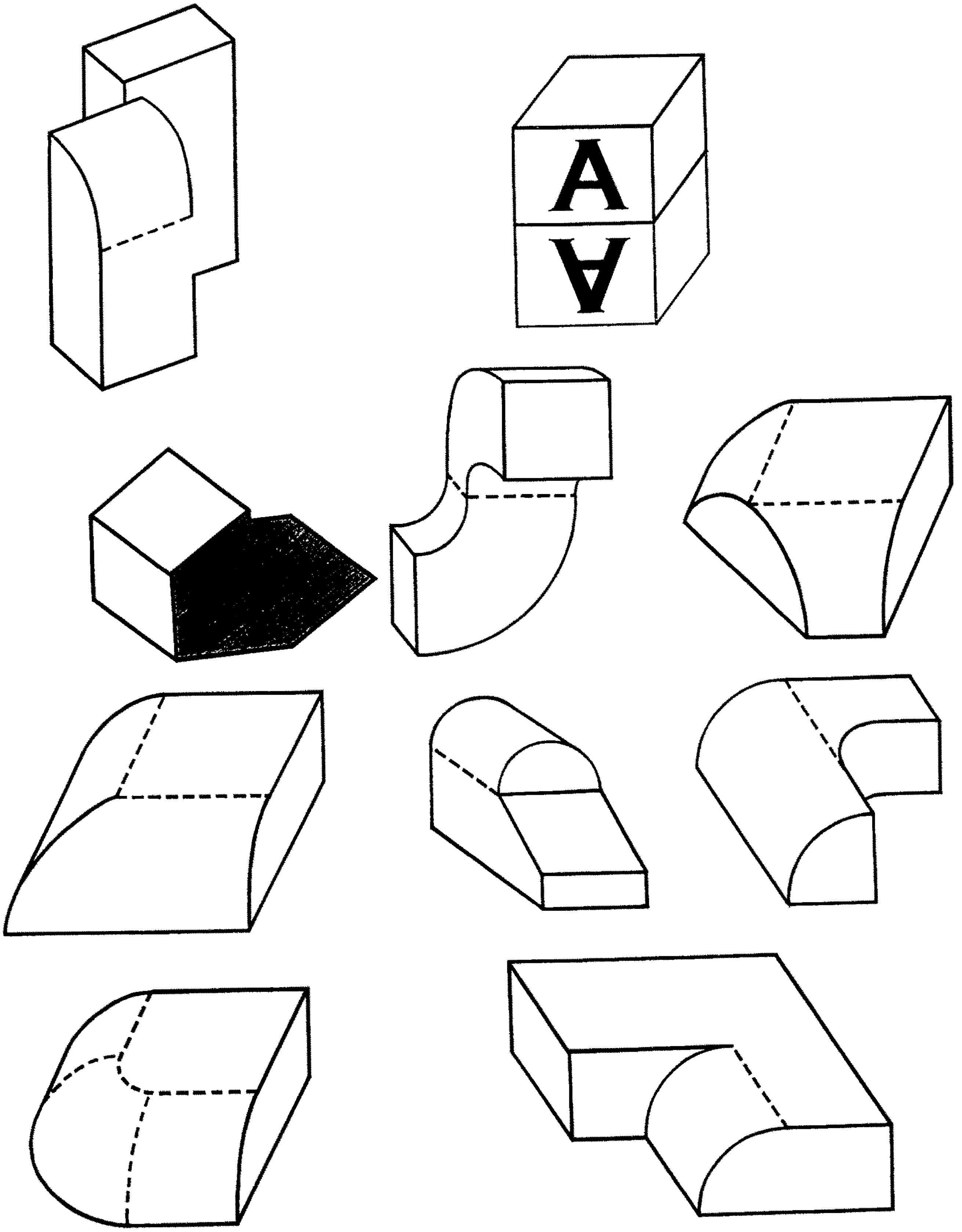


Figure 13. Line drawings that do not satisfy all the assumptions

on both sides of the junction, as do the two broken lines in junction e. For typographical reasons, the label for the projection of an extremal edge is shown as $\Leftarrow$ instead of the usual double-headed arrow.

The new catalogue of junction labellings is the combination of the sets of labellings in Figures 9 and 11. The labellings in Figure 11a must be added to the list of legal labellings of a curvature-L junction. This therefore implies greater ambiguity, since two more labellings are now possible for curvature-L junctions. However, this is counter-balanced by the fact that junctions of type b–d f and g are all new sources of information. For example, junctions of type f and g uniquely identify the label of the unbroken line as the projection of an extremal edge E. (In fact, as pointed out by Nalwa[22], there is a tangent discontinuity in E at the point in 3D which projects into the junction). We note that the labellings given in Figure 11f and g are unchanged if the ramp line approaches the junction from the right rather than the left. Junction types e and h provide no information, since all combinations of labels are physically possible.

Figure 12 illustrates two line drawings labelled according to this labelling scheme. These drawings have a unique labelling, except for the usual ambiguity between the labels $-$ and $\leftarrow$ (due to the fact that the objects could be floating in air, be resting on a horizontal surface, or be attached to a vertical wall). At the points X and Y, in the second line drawing, the ramp lines change label from $<$ to $>$. This is exactly equivalent to the transition from a + label to a $-$ label, illustrated in Figure 4. The ramp lines fade out as they approach X and Y from both the left and the right; hence each of X and Y is really two junctions of type h in Figure 11 placed back-to-back. As with the + to $-$ transition in Figure 4, the $<$ to $>$ transition is detectable in the intensity image.

It will rarely be the case that ramp edges will be detected with 100% reliability. However, when a junction of type d, f or g is detected with high confidence, this provides considerable probabilistic information about the labels of the lines meeting at the junction.

It is well known that a concave line ($-$) may occur in the intensity image as a ramp edge. However, since junctions of type d and h are the only junctions in which the ramp line could be replaced by a concave line to give a legal junction labelling, confusion between ramp lines and concave lines will be rare.

It should be noted that, as with step edges, ramp edges may be caused by shadows (for example, the boundaries of shadows cast by a non-point light source).

OTHER LINES AND JUNCTIONS

The catalogue of junction labellings in Figures 9 and 11 have been deduced under certain implicit assumptions about the formation of line drawings. A complete list of these assumptions is given below:

1 Objects are opaque, have Lambertian surfaces and no surface markings.
2 Objects are regular (i.e. an object is the closure of its interior). Therefore, no part of any object is wafer-thin or string-like.
3 Object surfaces are C^3 patches separated by surface-normal discontinuity edges and curvature discontinuity edges (smooth edges).
4 The only vertices involving smooth edges are:
   (a) the termination of a smooth edge at a discontinuity edge,
   (b) the intersection of two smooth edges.
5 No two object surface patches are tangential; no two object edges are tangential; no edge is tangential to a surface.
6 A maximum of three edges meet at a vertex.
7 Each vertex satisfies the simple circuit condition.
8 General viewpoint and light source positions, i.e. a small change in their positions does not change the configurations of the line drawing.
9 General positions of obejcts, i.e. no vertex of an object is in contact with an edge or vertex (of another object) and no two edges are colinear or tangential.
10 There are no shadows
11 The line drawing is a perfect projection. For example, there are no missing lines due to contrast failure.

For the catalogue in Figure 9 it is necessary to add the condition that there are no smooth edges.

Figure 13 illustrates some line drawings which do not satisfy all of these conditions. It can be seen that new junction labellings are possible if any of the above conditions are relaxed. The condition of piecewise C^3 surfaces excludes tangent developable surfaces and apices of cones[22]. Since two extremal edges meet at the apex A of a cone, A would project into a L-junction whose labelling is not in the catalogue of Figure 9 and 11. All the lines drawings in Figure 13 have non-zero probability, since each of the above conditions is only true with a probability p which is less than 1. To overcome the fact that a line drawing provides only probabilistic information, it is important to be able to use all possible sources of information, such as ramp lines, shape-from-shading analysis of the intensity image, multiple drawings, or *a priori* knowledge about the objects in the scene. For example, the system of Gamble *et al.*[23] uses the output from stereo, motion and colour modules to differentiate between discontinuities of surface-normal, depth, albedo, specularity and illumination.

CONCLUSION

It is clear that the utility of the machine interpretation of line drawings depends critically on the assumptions that can be made about the class of objects observed. In order to analyse line drawing of realistic objects it is essential to allow the possibility of T-junctions not caused by occlusion (Figure 2a), transitions from a convex edge to a concave edge (Figure 4), and smooth edges.

Slackening the assumptions to allow smooth edge increases the number of legal junction labellings. However, the resulting increase in ambiguity at certain junctions can be reduced by detecting and labelling smooth edges. An analogy can be drawn with shadow edges, which Waltz[2] found to be a useful source of information. Similarly, convex-concave transitions can be detected in the intensity image, and probabilistic information about the shape of objects can also help to reduce ambiguity at C, curvature-L and three-tangent junctions.

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