Introduction: Uncertainty Issues in Spatial Information

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Abstract. This introductory chapter serves two purposes. First, it provides a brief overview of research trends in different areas of information processing for the handling of uncertain spatial information. The discussion focuses on the diversity of spatial information, and the different challenges that may arise. Second, an overview of the contents of this edited volume is presented. We also point out the novelty of the book, which goes beyond geographical information systems and considers different forms of quantitative and qualitative uncertainty.

1 The Nature of Spatial Information

Variety of Spatial Information

The term *spatial information* refers to pieces of information that are associated with locations, which typically refer to points or regions in some two- or three-dimensional space. Many applications deal with geographic information [8, 35], in which case the space under consideration is the surface of the Earth. Other

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applications, however, deal with spatial information of a quite different kind, ranging from medical images (e.g. MRI scans [11]) to industrial product specifications (e.g. computer aided design and manufacturing [56]), or the layout of buildings or campuses [31, 32, 3]. In addition to describing aspects of the real world, spatial information may also describe virtual environments [26]. Beyond virtual environments, we may even consider space in a metaphorical way for describing or reasoning about the meaning of concepts, viewed as regions in a multi-dimensional space (e.g. the conceptual spaces of Gardenförs [25]).

Spatial information may have various origins. Geographic information may be derived from satellite images or other types of remote sensing, it may be collected using various types of sensors, it may result from the collection of census data, or from textual descriptions, among others. Other types of spatial information may be explicitly created by a human user (e.g. in the case of computer aided design), although it may also be derived e.g. from scanning devices, from a robot mapping its environment, or again from textual descriptions.

Given this diversity, it should not come as a surprise that spatial information has been studied from different angles in different research communities, as described in more detail in Section 2.

Dealing with Spatial Uncertainty

Appropriate abstractions are needed to deal with the complexity of spatial configurations. A first observation is that some information is naturally associated with points, e.g. the altitude of a location, while other types of information pertain only to regions, e.g. the population of a city. A common approach to deal with the former type is to discretize a bounded fragment of space into a finite number of cells. This allows us to quantify the value of the parameters associated with each cell, leading to the so-called field-based models, which are also referred to as grids or matrices. To deal with spatial information that is not tied to particular points, we may introduce a mechanism to refer to spatial entities of interest, e.g. using names or coordinates, and to describe relations between them.

Both the qualitative and quantitative spatial information available to applications usually contains uncertainty. Common to other applications that rely on measurement of observed phenomena, sensor data may be only imprecisely known (e.g. in the form of an interval); see e.g. [28]. Similarly, for instance, the classification of land coverage based on remote sensing images may be given in the form of a probability distribution [47]. In addition to these problems related to data acquisition, the way applications use spatial information may, by itself, introduce uncertainty. For instance, as it is impossible to make measurements at every point on Earth, interpolation or extrapolation techniques are required to estimate parameters for points where measured values are missing [9]. Information fusion is another process which may introduce uncertainty [16]. Indeed, in the face of conflicting sources, inconsistency may cast doubts on previously held beliefs, thus introducing uncertainty about them. A final cause for uncertainty is related to the use of labels to refer to places, their properties, and the relations between them. For instance, in fusion problems, apart from the uncertainty affecting the pieces of information, the fact that different sources may use different partitions of space (e.g. electoral wards vs. parishes) is another reason for imprecision or uncertainty. Also, the labels used to describe properties of regions may belong to different ontologies (e.g. agricultural vs. botanical terminology). While the use of labels is paramount for interactions between a system and human users, they give rise to many practical problems. For instance, how do you say Genève in Italian (Genova, Geneva)? Ghent in French? Lille in Dutch? Even worse, the same label is often used to refer to different places (e.g. Paris, France vs. Paris, Texas [33]), and even when a label unambiguously refers to one place, the spatial extent of this place may be ill-defined (e.g. downtown Toulouse [40]). Similar considerations apply to spatial relations (e.g. near, North-West of, etc.) and properties of places (e.g. densely populated). Note that labels may be used both for stating information, and for expressing queries. An example of the former case may be spatial representations that are derived from textual information (e.g. the web). In the case of queries, the uncertainty in the meaning of the labels may suggest some flexibility regarding what solutions are acceptable.

Spatial Information Processing

In practice, information processing is often divided into several subproblems. At a high level, we may consider three main steps [15], although they may not all be present in every application. The first one aims at clarifying raw information, e.g. by cleaning sensor information, or by synthesizing and structuring it. In the spatial domain, this may involve removing noise from remote sensing images, as well as analyzing and interpreting them. When collecting census information, for instance, this step also involves the assessment of the confidence levels in different sources. The second step obtains the information needed to address the problem at hand. This involves retrieving and combining information from one or more data repositories, and reasoning about it. Apart from querying geographic information systems (GIS) in general, this step may refer to the aforementioned information fusion and interpolation/extrapolation problems, or to a diagnosis based on the interpretation of medical images. The third and last step uses the retrieved information for decision making, or solving design or optimization problems: finding the best location for building a nuclear power plant, managing air traffic control, or deriving a strategy for avoiding the future flooding of a river.

The three steps outlined above are usually studied in different research communities, as briefly described in the next section, but emphasizing only the issue of handling uncertainty. As we shall see in the last section, providing an overview of the book, most of the chapters address problems related to the second step.

2 Research Areas in Spatial Information

The causes and remedies of spatial uncertainty have been studied for a long time. Spatial uncertainty is related mostly to spatial reasoning, to decision making involving space, and to the difficulty, sometimes the impossibility, of building a deterministic approach. The research about spatial uncertainty in the last decade has focused on the following domains:

- uncertainty in spatial cognition, artificial intelligence, and vision, which is coupled to spatial reasoning;
- uncertainty in geographic information systems, which is coupled to spatial data quality and spatial statistics.

Uncertainty in Spatial Cognition, Artificial Intelligence, and Vision

Spatial uncertainty has been extensively studied in the domain of cognitive science, from psychology to artificial intelligence. What does it mean to say "I've got no sense of direction"? What kind of information must a robot keep in memory to find its path? Choosing the right representation of space for a particular application is always an issue: every time you have to explain a route by phone, you must face a new problem. Is it relevant to use East-West indications? Or rather use "towards downtown", "towards the river", etc.? It always depends on what is the easiest to grasp, the most secure information that you will not mishandle, and not mismatch with a similar but wrong reference.

Names are not the only source of spatial uncertainty; visual perception is one too. Optical illusions have been noticed and investigated for centuries. They are sometimes used in architecture, in the Greek Parthenon, in art works — noticeably in the Italian and French Renaissance — and more recently by the artist M.C. Escher. Interestingly enough, artificial intelligence constraint satisfaction approaches may be successfully applied for deciding if a line drawing of a three-dimensional object is actually realizable in physical space [12]. Optical illusions have been related to unconscious inferences, an idea first suggested in the mid 19th century by H. von Helmholtz, and to inhibition-influenced vision, by E. Mach. Experimental psychology has also addressed this problem, for instance, focusing on the question of the possible existence of a specific spatial working memory. Experiments made with 3D rotated figures seem to demonstrate that the difficulty of recognizing shapes depends on the rotation angle [38]. Based on such experiments D. Marr developed his computational theory in the late 1970s, collected in his posthumous book "Vision" [37]. At about the same time, the book "Mental Models" by P. Johnson-Laird [36] was published. It makes intensive use of relational reasoning, including spatial and temporal reasoning, as well as defeasible reasoning which allows for a change in one's beliefs in the face of new observations. When communicating such a relational description, one of the arguments is used as a reference object. The proper choice of the reference objects is important when communicating a series of relational statements, as it may make the overall message easier to understand [20].

In artificial intelligence, leaving aside the early work on computer vision [4], the issue of reasoning about time, space, and uncertainty were considered independently at first. But only once temporal reasoning had been sufficiently mastered, after the introduction of Allen's temporal interval relations [1], research on spatial reasoning has blossomed [19, 45, 6, 46, 10]. In parallel, studies in uncertainty representation have led to the development of various settings beyond probability theory, namely Zadeh's fuzzy sets [57, 30] and possibility theory [14], Shafer's theory of belief functions [51], and imprecise probability [54]. The most prominent artificial intelligence domain where uncertainty and space have been jointly addressed — already since the late 1980s — is robotics, for automated planning purposes [31]. In particular, simultaneous localization and mapping (SLAM) [53, 41, 17] is a technique used in (mobile) robotics to build up a map within an unknown environment. Estimating uncertain spatial relationships is then one of the key issues. In spite of preliminary early attempts [27, 22] there have been few applications of fuzzy set based methods in robot navigation, with [58, 43, 49] being some of the exceptions. However, outside robotics, some works have focused on representing and reasoning about fuzzy spatial relations [18, 21, 34, 7, 50].

Uncertainty in Geographic Information Systems

From artificial intelligence and robotics, we go to another domain where spatial uncertainty is definitely a big issue: cartographic mapping. Cartographic representations, land surveys, remote sensing, geographic information systems, and global positioning are providing data for the two main subdomains of mapping: the mapping of administrative and man-made spatial features, on the one hand, and the mapping of natural resources on the other.

It has been a long time before men were able to use maps in the way we know them today. Cadastres are as old as tax collection, invented by the first Babylonian monarchs or Egyptian pharaohs. Areas and relative positions were all they needed to compute taxes, but errors and quarrels fed the courts, as they continue to feed them today. Geometers gained importance as land surveyors, as "arpenteurs", and often, turning around spatial uncertainty, their statements became the reality. But this does not erase uncertainty, it merely adds other constraints to the system, which sometimes helps, and sometimes hampers the decision making. Spatial uncertainty analysis, in this context, mostly relates to geographic data quality, in the sense that the role of data quality management is to reduce the uncertainty in making decisions. Spatial uncertainty analysis has been, since the 1990s, an acknowledged discipline that integrates expertise from geographical sciences, remote sensing, spatial statistics and many others. Several international organizations have working groups on these issues: ISPRS (quality of spatio-temporal data and models), the European AGILE (spatial data usability), the International Cartographic Association (spatial data uncertainty and map quality). Standardisation bodies are concerned too: ISO 9000, which addresses the production and distribution of goods and services, and those in the field of geographic information, e.g. FGDC, OGC, CEN. There is a community of engineers in spatial uncertainty in the various national mapping agencies and in large private companies such as Microsoft and Google, as indeed the quality of geographic information is clearly an issue for them. In addition, several books have been published on this topic. In particular, [29] presents reflections by members of the International Cartographic Association, while [28, 52, 13] present research breakthroughs on issues related to the quality and the uncertainty of geographic information, while [59, 2] focus on uncertainty in geographic data.

Cartographic mapping is not the only area related to GIS in which uncertainty appears. In the early years of image processing [48], following the release of the first digital cameras on Earth and above (with the release of Landsat Imagery in the early 1970s [55]), there has been a big boost in the automated acquisition of geographical information. Indeed, the automated processing of remote sensing images has proven to be an invaluable tool for feeding field-based models. Clearly, handling uncertainty is a central issue when interpreting remote sensing images in this way. Besides, spatial uncertainty is also prevalent in natural resources assessment [42], and must be estimated as it propagates in ecological models [8]. Geostatistics is a branch of statistics developed originally to predict distributions for mining operations [39]. It is currently applied in diverse branches of hydrology, meteorology, landscape ecology, soil science and agriculture (especially in precision farming), and branches of geography, such as epidemiology, and planning (logistics).

3 Overview

Generations of children, reading "Hop o' My Thumb" by Charles Perrault, also known under its original French title "Le Petit Poucet", have been scared by spatial uncertainty. They were afraid to lose their path back, if being forced to decide their way: turn right or left? The intent of this book is not to scare children, nor to scare scholars. Still, we may give the reader some small white pebbles to keep him or her from getting lost in the forest of the literature on spatial uncertainty handling, or in the bush of the chapters that follow.

To the best of our knowledge, this is the first book which is devoted to spatial uncertainty handling outside the traditional GIS setting. Uncertainty issues are especially addressed from a representation and reasoning point of view. In this sense, we only consider the second step of the information processing chain as described at the end of the first section. As such, the book does not discuss uncertainty issues in the interpretation of remote sensing images at the acquisition stage, and does not exclusively focus on geographical applications. Similarly, the book does not encompass planning and reasoning about actions in spatial environments, which is why uncertainty issues in robotics are not considered. The concept of uncertainty by itself is understood in a broad sense, including both quantitative and more qualitative approaches, dealing with variability, epistemic uncertainty, as well as with vagueness of terms.

The contributions of this book are clustered around three general issues: i) description of spatial configurations, ii) symbolic reasoning and information merging, and iii) prediction and interpolation. Part 1 especially focuses on modeling uncertainty in the meaning (vagueness) of linguistic constructs that are used to describe spatial relations and predicates. The first chapter in this part, by B. Bennett, presents a tutorial overview of diverse methods to deal with the problem of spatial vagueness, before focusing in more detail on a new method called *standpoint semantics*. This latter approach refines the supervaluation semantics of Fine [23], by adding structure on the set of possible precisifications. In particular, in the spatial domain, precisifications of a vague predicate or relation may depend on the standpoint one takes regarding the most appropriate thresholding of some parameter.

The second chapter, by P. Matsakis, L. Wendling and J. Ni, is concerned with describing relative positions between objects, and spatial relations to reference objects, where an object is a crisp or fuzzy region, in raster or vector form, of 2D or 3D space. The models presented may be used, e.g. to identify the most salient spatial relation between two given objects, or to identify the object that best satisfies a given relation to a reference object. Subsequently, the chapter by I. Bloch explores the idea of bipolarity in the modeling of spatial information. The idea is to distinguish between locations considered as being *really possible* for a given object, and locations which are *only not impossible*. This bipolar view is then embedded in the framework of fuzzy mathematical morphology, and finally illustrated on a medical application. The last chapter of the first part, by T. Beaubouef and F. E. Petry, provides a broad overview of the possible uses of fuzzy and rough sets [44] in geographical information systems. Rough sets naturally allow for a granular view of space and of the description of land coverage, while the use of fuzzy sets and relations applies to the modeling of linguistic terms. Special attention is paid to the modeling of rough spatial relations, to the use of spatial indexing techniques, such as R-trees, for fuzzy regions, and to rough object-oriented spatial databases.

Part 2 of the book deals with applications in artificial intelligence, and in particular with the problem of reasoning about spatial relations, and dealing with inconsistency in information merging. The first chapter by F. Dupin de Saint-Cyr, O. Papini and H. Prade provides an extensive survey of propositional and modal logics for describing mereo-topological or geometrical relations between regions, and for handling properties associated with regions. The handling of uncertainty in such frameworks is discussed. In particular, one may be uncertain whether a property holds in a region, and if it does, whether it holds everywhere in the region, or only in some part(s) of it. The next chapter, by O. Doukari, R. Jeansoulin and E. Würbel, presents a particular approach to the revision of propositional knowledge bases when receiving new information, which is well-suited for geographical information. This approach is centered around an assumption of locality, where conflicts related to one region of space do not affect what is known about regions that are far away. The third chapter of the second part, by O. Curé, addresses the problem of merging spatial ontologies that are used to describe properties of regions. To deal with the problem of heterogeneous vocabulary usage, an approach based on formal concept analysis [24, 5] is proposed which enables the creation of concepts that are not encountered in any of the given ontologies, and manages the resulting uncertainty. The last chapter, by S. Schockaert and P. D. Smart, deals with generating spatial scenarios which are compatible with available, and possibly conflicting, spatial constraints, using a genetic algorithm. To handle potential conflicts in a more flexible way, the approach may result in fuzzy regions, which are represented as a finite collection of nested polygons.

Part 3 gathers chapters about interpolation and prediction of spatial phenomena. The first three chapters are methodologically oriented, while the two others are directly motivated by real-world case studies. The chapter by A. Stein presents a decision making approach based on the use of remote sensing images. In an image mining approach it discusses how such images are obtained and interpreted, how the resulting information may be used to identify objects, how these objects are tracked over time, and how this may lead to meaningful predictions for the future. Special consideration is given to issues of data quality, and solutions are provided based on fuzzy set theory and spatial statistics. The approach is illustrated with a case study on the flooding of Tongle Sap Lake in Cambodia. Next, the chapter by K. Loquin and D. Dubois deals with the kriging methodology used in geostatistics for the interpolation and extrapolation of parameters which are known only at a finite number of points in space. The kriging methodology is supposed to account for the uncertainty induced by the variability of the considered parameter over space. However, the chapter emphasizes the fact that the epistemic uncertainty appearing both in data specification and random function estimation is not properly taken into account by the standard approach. Subsequently, the merits and limitations of fuzzy and interval based extensions of kriging for handling epistemic uncertainty are discussed. The third chapter, by F. Parisi, A. Parker, J. Grant and V. S. Subrahmanian is concerned with spatial probabilistic temporal databases. The goal is to provide efficient support for queries asking for all pairs of objects and time points such that the object is in a specified region at that time, with a probability that is within a given interval. Solutions to such queries may be represented as convex polytopes in some high-dimensional space, and methods are provided for approximating such polytopes in an efficient way. The approach is evaluated on both synthetic data and real-world data about ship locations. The next chapter, by C. de Runz and E. Desjardin, presents a way to deal with scarce pieces of evidence, obtained from archaeological data. The goal is to reconstruct plausible spatial configurations (e.g. the layout of the streets in an ancient city), and to visualize them. The proposed method takes advantage of a fuzzy extension of the Hough transform from image processing, which may be applied to fuzzy pieces of information. Finally, the chapter by G. Fusco uses a Bayesian network methodology for predicting the evolution of spatial networks derived from data about flows through a dominant flows approach. Such networks may be as diverse as representing people commuting between their home and work, money transfers between different areas, or the migration of people between different suburbs. The presented case study is based on commuter trips in a region of Southeastern France.

As a whole, the book intends to illustrate the different circumstances where spatial uncertainty may be encountered, and the different approaches that may be considered to cope with it.

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