CHAPTER X

Tuning the Cutting Conditions on a Machining Transfer Line Using Fuzzy Constraint Propagation

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Abstract

The attractiveness of fuzzy set-based methods, and especially fuzzy arithmetic is shown on a technological problem: that of tuning the feed and cutting speed of a machine-tool for the proper performance of metal-cutting operations. A method for simultaneous tuning of machine-tools in a synchronized transfer line is described. Fuzzy arithmetic makes it possible to delay as long as necessary the assignment of precise values to parameters. The feasibility analysis is carried out once for all, from fuzzy-valued cutting parameters reflecting subjective preferences. The procedure eventually converges to a trade-off between the quality of cutting and the consistency of the *tool-replacement policy, under the synchronization constraint. The advantages of the proposed methodology are its computational efficiency and its ability for handling different sources of knowledge.*

X.1 INTRODUCTION

Fuzzy sets were first suggested by Zadeh [11] as a convenient mathematical representation of incompletely or imprecisely defined quantities or concepts. The aim is to enable computers to process information pervaded with vagueness. The kind of imprecision which is modelled by fuzzy sets is typically that which any designer of complex systems encounters in the first stages of his task: he can only supply a rough description of what he is planning, an approximate appraisal of expected performance and characteristic parameter values. But such vague notions expressing preference are guiding the decisions of designers, until imprecision has entirely vanished. Proceeding further in the design process requires some verifications to be made in order to check the consistency of decisions. The corresponding computations are performed by arbitrarily assigning precise values to significant parameters, and modifying them until satisfactory tuning is obtained. Such a procedure is very often tedious. On the contrary, modelling the system as a fuzzy one, by integrating in the model the actual imprecision pervading specifications, obviates this blind search because it is possible to make computations out of fuzzy data. The designer is no longer obliged to supply precise values when he is unable to.

Clearly the kind of fuzziness which is dealt with here in is not related to random phenomena, and refers to preference. However the system under design may be prone to random phenomena such as failures, wear, …

To illustrate such considerations we consider a manufacturing system made of a machine-tool or a synchronized line of machine-tools, on which cutting conditions must be tuned. This problem has been studied at length in the literature, especially the single-machine case, for which optimization algorithms have been proposed. The transfer-line problem has been scarcely dealt with in the past. It is hard to solve because all the machine-tools must be simultaneously tuned under many constraints such as prescribed cycle times, synchronization of all stages, quality of cutting and consistent replacement policy for worn tools. This task is very common in the industry, and is carried out by trained people using rules of thumb through a —sometimes long— interative procedure. As indicated in this paper, fuzzy arithmetic leads to speeding up this procedure which no longer has to be initialized by precise (and often infeasible) *a priori* specification of cutting parameters. The necessary background in fuzzy set theory and fuzzy arithmetic can be found in the books of Dubois and Prade [5], [7] or in the survey [6]. This paper is a short version of a previously published article [3].

X.2 RESULTS IN FUZZY ARITHMETICS

Let U be a set taken as a reference. A fuzzy set F on U is a subset of U whose elements are weighted by numbers conventionally in the unit interval [0,1]. The weight of element u is denoted $\mu_F(u)$ and called grade of membership of u in F. Crisp sets are obtained by restricting the range of membership grades to {0,1}. Grades of membership can be interpreted in various ways according to the context, e.g., preference grades, similarity indices, degrees of compatibility with a subjective category, upper probabilities, etc. In this paper membership functions will express preference or feasibility, not uncertainty.

The membership function μ_F is interpreted as a possibility distribution restricting the values of a variable. Let S be a crisp subset of U. The *possibility* of selecting in S *some* element in accordance with (= belonging to) F is assessed by the quantity $Pos_F(S)$ such that [12]:

$$Pos_F(S) = \sup_{u \in S} \mu_F(u), \tag{1}$$

i.e., we search for the element in S which is most in accordance with F. When μ_F expresses preference, $Pos_F(S)$ is the degree of consistency of a constraint-satisfaction problem involving the fuzzy constraints F and the crisp constraint S. It presupposes that the variable x whose value is constrained by F and S is controllable and can be chosen as u* such that $\mu_F(u) = Pos_F(S)$.

Similarly, the extent to which S includes *all* feasible elements in the sense of F is viewed as the grade of impossibility of finding in the complement $\bar{S}$ some element belonging to F. The grade of *necessity* for all elements consistent with F to be in S is thus defined by [5], [13]

$$Nec_F(S) = 1 - Pos_A(\bar{S}) = \inf_{u \in S} 1 - \mu_F(u). \tag{2}$$

If S is a fuzzy event, i.e., is a fuzzy set, equations (1) and (2) can be generalized. $Nec_F(S)$ is also the degree of certainty of satisfying a constraint S when all that is known about an uncontrollable variable x is that "x is F". Then μ_F expresses uncertainty and not preference. This situation will not be met here (see Dubois, Fargier and Prade [4] for fuzzy constraint satisfaction under uncertainty).

A fuzzy number M is a fuzzy set of real numbers clustered around some value. More specifically, the membership function μ_M is normalized and unimodal. Any level set of M, $M_\lambda = \{u \mid \mu_M(u) \geq \lambda\}$, $\forall\ \lambda \in\ ]0,1]$ is a closed interval. The modal value can be extended to a closed interval $[m, \overline{m}]$. M is then called a fuzzy interval (see Figure X.1). Any arithmetic real operation $*$, i.e., any two-place mapping can be extended to interval arguments

$$[x_1,x_2] * [y_1,y_2] = \{x * y \mid x \in [x_1,x_2], y \in [y_1,y_1]\}; \tag{3}$$

and, consistently, to fuzzy numbers or intervals as follows: if M and N are fuzzy intervals, $M * M$ has the membership function

$$\forall\ w \in \mathbb{R},\ \mu_{M*N}(w) = \sup_{u,v} \min(\mu_M(u), \mu_N(v)) \tag{4}$$

under the constraint $u * v = w$.

It can be seen that $\mu_{M*N}(w) = \mathrm{Pos}_{M\times N}(\{u, v \mid u * v = w\})$, where $M \times N$ is the Cartesian product of M and N (translated via the minimum operator). In other words (4) can be viewed as computing the degree of consistency of a constraint satisfaction problem involving variables x and y. It operates the propagation of preference from x, y to the range of variable $z = x * y$.

As soon as the supremum is reached in equation (4) (e.g., $*$ is continuous, μ_M and μ_N upper semi-continuous), the λ-level set of $M * N$, say $(M * N)_\lambda$, can be obtained by combining the level sets M_λ and N_λ via equation (3). Consequently, as soon as equation (3) is easy to perform (e.g., $*$ = addition, product), equation (4) becomes simple. It can become even simpler if we adopt the following representation of fuzzy intervals: M is said to be a LR-type fuzzy interval if it defined by

– a range $[\underline{m}, \overline{m}]$ of values with membership grade 1,
– a support $\{u \mid \mu_M(u) > 0\} =]\underline{m} - \alpha, \overline{m} + \beta[$
– shape functions L and R $[0, +\infty) \to [0,1]$ with $L(0) = R(0) = 1$; $R(1) = L(1) = 0$, L and R are non-increasing upper semi-continuous mappings.

μ_M is then defined by (see Figure X.1)

$$\begin{aligned}
\mu_M(u) &= L\left(\frac{\underline{m} - u}{\alpha}\right);\ u \le \underline{m} \\
&= 1;\ u \in [\underline{m}, \overline{m}] \\
&= R\left(\frac{u - \overline{m}}{\beta}\right);\ u \le \overline{m}.
\end{aligned}$$

M is denoted $(\overline{m}, \underline{m}, \alpha, \beta)_{LR}$ or $(m, \alpha, \beta)_{LR}$ if it is a fuzzy number; a and b are called left and right spreads, respectively. The following results hold [5], [6]:

(a) if $M = (m, \alpha, \beta)_{LR}$, $N = (n, \gamma, \delta)_{LR}$, then
$$M \oplus N = (m + n, \alpha + \gamma, \beta + \delta)_{LR} \quad \text{(extended addition);}$$

(b) 1/M is defined by $\mu_{1/M}(w) = \mu_M\left(\frac{1}{w}\right)$;

(c) $\forall\, a \in R,\ a \cdot M = (am, a\alpha, a\beta)_{LR} \quad (a > 0)$
$$= (am, -a\beta, -a\alpha)_{LR} \quad (a > 0)$$
(scalar multiplication);

(d) if $M = (m, \alpha, \beta)_{LR}$; $N = (n, \gamma, \delta)_{RL}$ (N is of the opposite type), then $M - N = (m - n, \alpha + \delta, \beta + \gamma)_{LR}$ (extended subtraction);

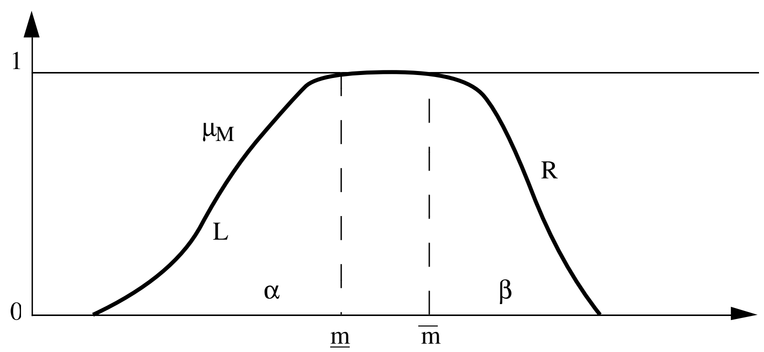

Figure X.1

(e) if M and N have supports in $[0, +\infty)$, and both L – R typed, then

$$\mu_{M \cdot N}(w) = L\left(\frac{n\alpha + m\gamma - \sqrt{(m\gamma - n\alpha)^2 + 4\alpha\gamma w}}{2\alpha\gamma}\right), \; w \le mn,$$
$$= R\left(\frac{n\beta + m\delta + \sqrt{(m\delta - n\beta)^2 + 4\beta\delta w}}{2\alpha\gamma}\right), \; w \le mn$$

(extended product).

A second-order approximation (small spreads) of the product formula is $M \cdot N \simeq (mn, m\gamma + n\alpha, m\delta + n\beta)_{LR}$.

Other results, and proofs are given by Dubois and Prade [5], [6]. Such simple formulae indicate that it is not too difficult to perform calculations with ill-defined quantities modelled by fuzzy numbers.

X.3 THE CUTTING CONDITIONS TUNING PROBLEM ON A TRANSFER LINE

Very often the tuning of cutting conditions must be carried out on a group of machines rather than on an isolated one. It is sometimes crucial to achieve simultaneous tuning, i.e., take into account the dependence between several cutting operations. This is especially true on a synchronized transfer line. The methodology presented in this section has been developed in collaboration with a machine-tool company [1]. It was first outlined in [2].

X.3.1 Statement of the Problem

The transfer line is supposed to be made of a sequence of M work-stations. At each station i a sequence of K_i operations is performed. One operation may be the performance of several identical tasks in parallel (e.g., drills). Each operation j at station i is characterized by a cutting speed v_{ij} and a feed rate f_{ij}, limited by technological bounds such as the following constraints:

$$\underline{v} \le v \le \overline{v}, \tag{5}$$
$$\underline{f} \le f \le \overline{f}, \tag{6}$$
$$v \cdot f^{bf} \le B, \tag{7}$$
$$f \cdot v^{\lambda v} \ge C_S, \tag{8}$$

where β_f and γ_v are technological coefficients. The upper bound on v is due to the maximum available spindlespeed and temperature limitations; the upper bound on f is due to cutting force restrictions or surface finish requirements. Lower bounds on v and f, together with inequality (8) define the stable cutting region. Constraint (7) stems from the maximal available horsepower. Constant B also depends upon the depth of cut. Objectives are usually the minimization of machining cost and the maximization of production rate, metal removal rate, tool life, production between tool changes etc. When technological coefficients are assumed to be well-known, many techniques exist for the single objective optimization of the cutting operation. A classical survey is the one by Philipson and Ravindran [10], who also discuss explicit solving through differential calculus methods. Cutting speeds at each station are related via the spindle speed w_i:

$$\forall j,\ v_{ij} = \pi D_{ij} w_i, \tag{9}$$

where D_{ij} is the work-piece diameter. Hence, decision variables at each station are K_i feed rates and one spindle speed. The transfer line is synchronized in the sense that each station has the same cycle time t_c. If t_{ij} denotes the cutting time for operation j of stage i, then one must have

$$t_c \geq \Sigma_{j=1,K_i} t_{ij} + \Delta t_i,\ \forall\ i = 1, M, \tag{10}$$

where Δt_i is a non-productive set-up time. Goals to be achieved are:

– Suitable machining times so that the cycle time, which defines the production rate is respected.

– A good quality of cutting, expressed in terms of preferred speeds and feed rate ranges for each operation.

– A reasonable worn-tool replacement policy which allows a single replacement frequency and specifies a number N of produced parts without changing tools. For each operation j of stage i, the number of parts machined by the same tool, say N_j should be of the form

$$\begin{cases} N_{ij} = q_{ij}N + r_{ij} \\ r_{ij} << N_{ij}, \end{cases} \tag{11}$$

where q_{ij} = integer part of N_{ij} / N, i.e., tool (i,j) is changed every q_{ij} replacement stops of the linear; r_{ij} is the number of parts which could still have been safely machined by the replaced tool.

Optimizing the whole system at once may be quite challenging and leads to rigid procedures (e.g., maximizing N, minimizing t_c). Another approach consists in suboptimizing each operation. This has been done by Iwata et al. [9] and Hitomi [8] with single objectives (minimization of cost, maximization of production rate), and Philipson and Ravindran [10] with several objectives, by goal programming. Here the optimization will be performed at each station.

The spirit of the procedure described below is to provide an interactive design tool where consequence of cycle time constraint and cutting quality requirements are evaluated, but the final choice of cutting conditions is the designer's responsibility. The procedure will only guide the designer's choice so that goals are attained, including the tool-replacement policy. Basically, the methodology is based on fuzzy constraint propagation.

X.3.2 Outline of the Tuning Procedure

Usually, designers are compelled to resort to experience-guided search for good cutting conditions, i.e., try, check, then modify, check again, etc. until a reasonable solution is reached. The main idea of the procedure described here is to avoid assigning precise values to cutting parameters immediately, but to perform suitable evaluations using as operands fuzzy numbers whose shapes account for the cutting quality requirements. Here the role of fuzzy arithmetics is to propagate preference.

More specifically, for each operation, the designer specifies ranges of permitted values for the cutting speed $[v, \bar{v}]$ and feed rate $[\underline{f}, \bar{f}]$ (subscripts are dropped for simplicity); i.e., $v \notin [\underline{v}, \bar{v}]$, $f \notin [\underline{f}, \bar{f}]$ is forbidden. Ranges of preferred values $[v_*, v^*]$, $[f_*, f^*]$ are also requested. Fuzzy numbers are thus obtained as shown in Figures X.2 and X.3, i.e.,

$$\begin{cases} \text{a fuzzy cutting speed } \tilde{v} = (v_*, v^*, v_* - \underline{v}, \bar{v} - v^*)_{LR} & (12) \\ \text{a fuzzy feed rate } \tilde{f} = (f_*, f^*, f_* - \underline{f}, \bar{f} - f^*)_{LR}; & (13) \end{cases}$$

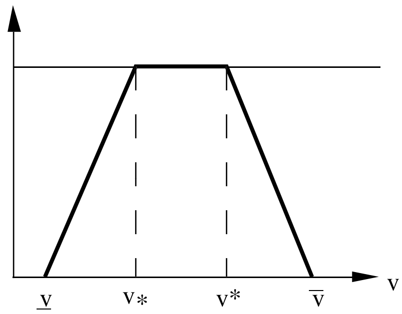

Figure X.2

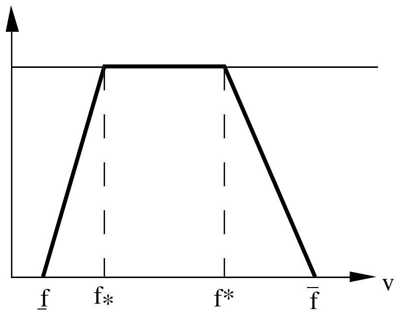

Figure X.3

according to the representation proposed in Section 2. Here L(u) = R(u) = max(0, 1 – u), i.e., linear membership functions are assumed. $\tilde{v}$ and $\tilde{f}$ may significantly vary according to the nature of the operation (e.g., rough or final cutting), of the tool or the processed part.

At a given station i, from the knowledge of $\{\tilde{v}_{ij}, \tilde{f}_{ij,}\ j = 1, K_i\}$ the following quantities can be computed, owing to the results presented in Section 2 and expressions (9)-(11):

(a) Fuzzy specifications $\tilde{w}_{ij}$ of spindle speeds whose consistency must be checked.

(b) A fuzzy processing time at station i, to be compared with the required cycle time t_c.

(c) A fuzzy specification $\tilde{N}_{ij}$ of the number of processed parts per tool. This fuzzy number will encompass the cycle time constraint and will also reflect the cutting quality specifications. $\tilde{N}_{ij}$ clearly pictures for the designer a range of possible choices regarding the operation under concern.

Once the set of $\tilde{N}_{ij}$ values is known, the designer chooses crisp numbers of processed parts N_{ij} consistent with these fuzzy numbers [i.e., $\mu_{\tilde{N}_{ij}}(N_{ij})$ close enough to 1] so that the tool-replacement condition (11) is met. The computer is then able to determine the best corresponding cutting conditions $\hat{v}_{ij}$, $\hat{f}_{ij}$, i.e., maximizing $\min(\mu_{\tilde{v}_{ij}}, \mu_{\tilde{f}_{ij}})$, in accordance with the cycle time constraint. This is the final defuzzification step. The fuzzy Cartesian product $\tilde{f}_{ij} \times \tilde{v}_{ij}$ is thus supposed to be a reasonable approximation of the feasibility region described by constraints (5)-(8). In the following the necessary calculations are detailed for single- and multi-operation stations.

X.3.3 Single-Operation Station

The cutting time at single-operation station i is

$$t_i = \frac{l_i \pi D_i}{f_i v_i}, \tag{14}$$

where l_i = distance travelled by the tool in a pass. Let $G_i \triangleq l_i \pi D_i$ express the geometrical data.

The fuzzy cutting time $\tilde{t}_i$ is now obtained by calculating the fuzzy product $\tilde{f}_i \cdot \tilde{v}_i$, using the expressions provided in Section 2 and the fuzzy intervals defined by equations (12) and (13), and we have

$$\mu_{\tilde{t}_i}(\theta) = \mu_{\tilde{f}_i \cdot \tilde{v}_i}\left(\frac{G_i}{\theta}\right). \tag{15}$$

In the above, $\tilde{f}_i \cdot \tilde{v}_i$ is obtained by calculating a product of fuzzy intervals (see Section X.2) and $\tilde{t}_i$ is a fuzzy interval whose support is

$$\left]\frac{G_i}{\overline{f}_i \overline{v}_i}, \frac{G_i}{\underline{f}_i \underline{v}_i}\right[\triangleq]\underline{t}, \overline{t}[$$

and

$$\forall\ \theta \in \left[\frac{G_i}{f^*_i v^*_i}, \frac{G_i}{f_{i*} v_{i*}}\right] \triangleq [t_*, t^*],\ \mu_{\tilde{t}_i}(\theta) = 1.$$

$\tilde{t}_i$ must now be compared with the cutting time $\hat{t}$ induced from the cycle time constraints, i.e.,

$$\hat{t} = t_c - \Delta t_i. \tag{16}$$

The following situations may occur:

(a) $\hat{t} > t^*$ and $\mu_{\tilde{t}_i}(\hat{t})$ is low. The station is underloaded whatever the cutting condition. It may be useful to add another operation to this station. Otherwise the cycle time constraint can be dropped.

(b) $\hat{t} > t_*$ and $\mu_{\tilde{t}_i}(\hat{t})$ is 1 or close to 1. The stage is adapted to the cycle time constraint; cutting conditions can be chosen in the preferred ranges. If $\hat{t} < t^*$, then, for instance,

$$v_i \in [v_{i*}, v^*_i] \cap \left[\frac{G_i}{\hat{t}f^*_i}, \frac{G_i}{\hat{t}f_{i*}}\right]. \tag{17}$$

(c) $\hat{t} < t^*$ and $\mu_{\tilde{t}_i}(\hat{t})$ close to 1. The best cutting conditions $(\hat{f}_i, \hat{f}_i)$ maximizing $\min(\mu_{\tilde{f}_i}(f_i), \mu_{\tilde{v}_i}(v_i))$ under the cycle time constraint are such that

$$\mu_{\tilde{f}_i}(\hat{f}_i) = \mu_{\tilde{v}_i}(\hat{v}_i) = \mu_{\tilde{t}_i}(\hat{t}) \triangleq \alpha_i. \tag{18}$$

Since f_i and v_i are L-R fuzzy intervals, it can be checked that

$$\left.\begin{aligned} \hat{f}_i &= f^*_i + (\bar{f}_i - f^*_i)R^{-1}(\alpha_i) \\ \hat{v}_i &= v^*_i + (\bar{v}_i - v^*_i)R^{-1}(\alpha_i). \end{aligned}\right\} \tag{19}$$

(d) $\hat{t} < t^*$ and $\mu_{\tilde{t}_i}(\hat{t})$ is 0 or low. The station is overloaded and needs some modification.

The consequence of the cycle time constraint on the choice of cutting conditions can be calculated more generally, so as to encompass all cases (a)-(d). This is achieved via fuzzy constraint propagation. Consider the feed rate: it is restricted not only by $\tilde{f}_i$, but also by the fuzzy range

$$\mu_{\tilde{t}_i}(\theta) = \mu_{\tilde{f}_i \cdot \tilde{v}_i}\left(\frac{G_i}{\theta}\right). \tag{20}$$

i.e.,

$$\tilde{f}'_i = \frac{G_i}{[0,\hat{t}] \cdot \tilde{v}_i} \tag{21}$$

where $[0,\hat{t}]$ is viewed as a fuzzy interval with null spreads, and the fuzzy product

$$[0,\hat{t}] \cdot \tilde{v}_i = (0,\ \hat{t}v^*,\ 0,\ \hat{t}(\bar{v} - v^*))_{LR}. \tag{22}$$

In other words, including the cycle time constraints leads to reduction of the fuzzy specification $\tilde{f}_i$ on the feed rate, to $\tilde{f}_i \cap \tilde{f}'_i$.

It can be checked that when the station is very much underloaded ($\hat{t} > \bar{t}$), $\tilde{f}'_i \cap \tilde{f}_i = \tilde{f}_i$ [case (a)] or if the station is too much overloaded ($\hat{t} < \bar{t}$), $\tilde{f}_i \cap \tilde{f}'_i = \emptyset$ [case (d)]. Otherwise $\tilde{f}'_i \cap \tilde{f}_i$ is as shown in Figure X.5.

Note that $\sup\mu_{\tilde{f}_i \cap \tilde{f}'_i} = \alpha_i$ is attained for $f_i = \hat{f}_i$, as calculated in case (c). Similarly, $\tilde{v}_i$ can be changed into

$$\tilde{v}_i \cap \tilde{v}'_i = \frac{G_i}{[0,\hat{t}] \cdot \tilde{f}_i}.$$

The fuzzy restriction on the number N_i of processed parts per tool can now be obtained from the restricted fuzzy cutting conditions, via Taylor's formula:

$$N_i = \frac{T}{t_i}, \text{ where } T = \frac{K}{v^\alpha f^\beta} \text{ is the tool life,}$$

$$= \frac{K}{G_i} \cdot v_i^{1-\alpha} \cdot f_i^{1-\beta}, \tag{23}$$

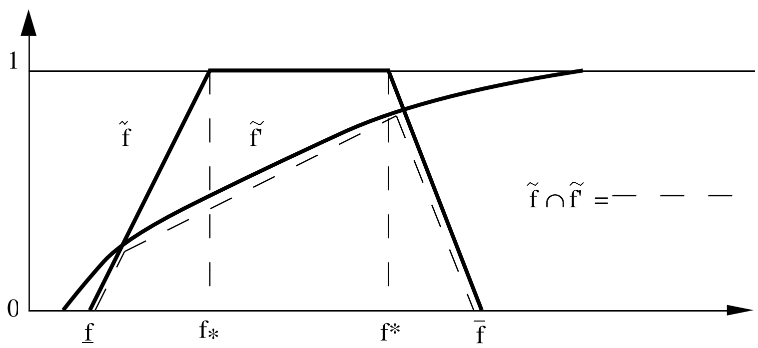

Figure X.4 Fuzzy constraints on the feed where K depends upon the depth of cut and α and β are Taylor coefficients. When stage i is fully utilized ($t_i = \hat{t}$), N_i depends only upon, say the feed rate:

$$N_i = \frac{K}{G_i^{\alpha}} \cdot \frac{1}{\hat{t}^{1-\alpha}} f_i^{1-\beta}. \tag{24}$$

The fuzzy interval $\tilde{N}_i$ is obtained by substituting $\tilde{f}_i \cap \tilde{f}'_i$ in equation (24). $\tilde{N}_i$ is of the form $a \cdot M^p$, whose membership function is simply

$$\mu_{a \cdot M^p}(n) = \mu_M\left(\frac{n^{1/p}}{a}\right).$$

When the station is definitely underloaded $\tilde{N}_i$ has to be obtained via equation (23) and $\tilde{f}_i \cap \tilde{f}'_i$, $\tilde{v}_i \cap \tilde{v}'_i$; the calculation must then be performed through a level-set combination, or by means of an approximate formula, as given in Section 2 or in Dubois and Prade [6], [7].

X.3.4 Multi-Operation Station

If K_i operations must be performed at station i, K_i fuzzy cutting speeds $\tilde{v}_{ij}$ are provided. $\tilde{v}_{ij}$ induces a fuzzy range of spindle speeds $\tilde{w}_{ij}$, such that

$$\forall j, \forall w, \mu_{\tilde{w}_{ij}}(w) = \mu_{\tilde{v}_{ij}}(\pi D_i w). \tag{25}$$

Since only one spindle speed w_i need be determined, its corresponding range is the intersection $\bigcap_{j=1,K_i} \tilde{w}_{ij}$. Especially when the D_i values are somewhat different, this range may be very narrow. Moreover, the set of possible spindle speeds, say Ω, is discrete. Thus the (fuzzy) set of spindle speeds compatible with the fuzzy cutting speeds is usually very small. It is reasonable to choose w_i, which maximizes $\min_j \mu_{\tilde{w}_{ij}}(w)$. If no spindle speed can be found which is consistent with the quality of cutting, the station must be modified.

The cutting time at station i is now of the form

$$\tilde{t}_i = \frac{1}{w_i} \Sigma_{j=1,K_i} \frac{l_{ij}}{\tilde{f}_{ij}}. \tag{26}$$

This fuzzy sum of inverses can be calculated by means of level sets, by L-R approximation formulae or exactly, if $\tilde{f}_{ij}$ is defined such that $1/\tilde{f}_{ij}$ is of LR type. If the $\tilde{f}_{ij}$ are all equal to $\tilde{f}_i$, equation (26) becomes

$$\tilde{t}_i = \frac{1}{w_i \cdot \tilde{f}_i} \Sigma_{j=1,K_i} l_{ij}. \tag{27}$$

The same argument for matching $\tilde{t}_i$ and $\hat{t}_i$ as in the previous section may be used. The calculation of the range of feed rates induced from the cycle time constraint is carried out as follows: if t_i is the cutting time, then f_{ij} satisfies

$$\frac{l_j}{f_{ij}} = w_i t_i - \Sigma_{k \neq j} \frac{l_k}{f_{ik}}. \tag{28}$$

The induced fuzzy range $\tilde{f}'_{ij}$ is obtained by changing t_i into $[0, \hat{t}_i]$ and f_{ik} into $\tilde{f}_{ik}$ $(k \neq j)$ in equation (40), similarly to equation (38).

For each operation j, the fuzzy number of processed parts per tool is, from equation (35),

$$\tilde{N}_{ij} = \frac{K}{l_i (w_i)^{\alpha - 1} \cdot (\pi D_i)^{\alpha}} (\tilde{f}_{ij} \cap \tilde{f}'_{ij})^{1 - \beta}. \tag{29}$$

X.3.5 The Tool-Replacement Policy

The fuzzy ranges $\tilde{N}_{ij}$ visualize the possible tool-replacement policies, and bottleneck tools are located, i.e., tools (i,j) with smallest $\tilde{N}_{ij}$. Note that when optimum cutting conditions exist in the sense of equation (28), they correspond to a number $\hat{N}_{ij}$ which maximizes the membership function of $\tilde{N}_{ij}$.

The tool-replacement policy problem is now of the form: find an integer N maximizing the feasibility level $\alpha(N)$ under the constraints

$$\begin{cases} N_{ij} = q_{ij}N & q_{ij} \text{ positive integer} \\ N = \min_{i,j} N_{ij} & \\ \text{and} & \\ \mu_{\tilde{N}_{ij}}(N_{ij}) \geq \alpha(N) & \forall i,j. \end{cases} \tag{30}$$

In other words, the transfer line stops every N produced parts, and tool (i,j) is replaced every q_{ij} stops. System (30) can be solved interactively, or a solution can be obtained by the computer. To ensure no $(N_{ij})_\alpha$ is empty, the threshold $\alpha(N)$ is obviously such that

$$\alpha(N) \leq \hat{\alpha} = \max_{ij} \sup \mu_{\tilde{N}_{ij}}. \tag{31}$$

If system (30) has no solution for $\alpha = \hat{\alpha}$, then α must be decreased until a solution is found. The cutting conditions can then be retrieved in the following way:

– if $N_{ij} < \hat{N}_{ij}$, the optimal cutting conditions (28) are kept, provided tool (i,j) is not under-utilized too much ($\hat{N}_{ij} - N_{ij}$ remain small);

– otherwise, the knowledge of N_i and $t_i = \hat{t}_i$ on single-operation stages completely determines the cutting conditions. On multi-operation stages the knowledge of N_{ij} and w_i determines f_{ij}.

An algorithm for the calculation of the values N_{ij} solutions to system (30) can be the following. Let $A = \{\alpha \leq \hat{\alpha} \mid \exists i,j,n \text{ s.t. } \mu_{\tilde{N}_{ij}}(n) = \alpha\} \triangleq \{\alpha^1 \geq \alpha^2 \geq \ldots \geq \alpha^p\}$, be the set of attained membership values. Let ε be a subutilization rate, such that a tool is allowed to be replaced when it has cut $N_{ij}(1 - \varepsilon)$ parts instead of N_{ij}. Let $(\tilde{N}_{ij})_\alpha = \{n \mid \mu_{\tilde{N}_{ij}}(n) \geq \alpha\}$. $a \leftarrow b$ means: assign to a the value of b.

Algorithm
For k = 1 to p
begin
(1) $\alpha \leftarrow \alpha^k$; $\underline{N}_{ij} \leftarrow (1 - \varepsilon) \cdot \min(\tilde{N}_{ij})_\alpha$; $\bar{N}_{ij} \leftarrow \max(\tilde{N}_{ij})_\alpha$; $\forall ij$.
(2) $\underline{N} \leftarrow \min_{ij} \underline{N}_{ij}$; $\bar{N} \leftarrow \bar{N}_{ij}$ s.t. $\underline{N}ij = N$.
(3) For $N = \bar{N}$ down to $\underline{N}$
begin $q_{ij} \leftarrow \bar{N}_{ij}/N$ $\forall i,j$ (Euclidean division)
$N_{ij} \leftarrow q_{ij}N$ $\forall i,j$

If $\forall i,j,\ N_{ij} \geq \underline{N}_{ij}$ stops with success

end

end; stop with failure.

X.3.6 Example

To illustrate the procedure, we consider a transfer line comprising three work stations:

Station 1: rough turning l_1 = 200 mm; D_1 = 100 mm; single-edge tool.
Station 2: final turning l_2 = 200 mm; D_2 = 100 mm; single-edge tool.
Station 3: turning l_{31} = 200 mm; D_{31} = 210 mm; two-edge tool levelling l_{32} = 17 mm; $D_{32} \in$ [166, 210]mm; one-edge tool.

The cycle time is 1 min. $\Delta t_1 = \Delta t_2 = \Delta t_3 = 0.1$ min, i.e., $\hat{t} = 0.9$ min.

(a) *Station 1*

Quality specifications: $\tilde{v}_1$ = (80, 90, 10, 30) m/min (L(x) = R(x) = max(0, 1 – x)); $\tilde{f}_1$ = (0.4, 0.8, 0.2, 0.2) mm.

Cutting time:

$$\mu_{\hat{t}_1}(\theta) = \begin{cases} 1 - \frac{1}{4}\left(20 - \sqrt{144 + \frac{160\pi}{\theta}}\right), & \theta \geq 1.96 \text{ min;} \\ 1 - \frac{1}{12}\left(42 - \sqrt{36 + \frac{480\pi}{\theta}}\right), & \theta \geq 0.87 \text{ min.} \end{cases}$$

It can be checked that $\mu_{\tilde{t}_1}(\hat{t}) = 1$ ($\hat{t} = 0.9$ min), i.e., this stage fits the cycle time.

Fuzzy feed rate *(including cycle time constraint)* $f_1 \cap f'_1$: see Figure X.6a. Fuzzy number of processed parts per tool $\tilde{N}_1$ [using equation (36)]:

n	14	15	16	17	18	19	20	21	22	23
μ(n)	0.2	0.4	0.62	0.82	1	0.9	0.69	0.47	0.25	0.03

(b) *Station 2*

Quality specifications: $\tilde{v}_2$ = (110, 120, 20, 10) m/min; $\tilde{f}_2$ = (0.5, 0.5, 0.3, 0.1) mm.

Cutting time:

$$\mu_{\hat{t}_2}(\theta) = \begin{cases} 1 - \frac{1}{2}\left(17 - \sqrt{49 + \frac{80\pi}{\theta}}\right), & \theta \geq 1.05 \text{ min;} \\ 1 - \frac{1}{12}\left(43 - \sqrt{549 + \frac{480\pi}{\theta}}\right), & \theta \geq 1.14 \text{ min.} \end{cases}$$

It can be checked that $\mu_{\tilde{t}_2}(\hat{t}) = \mu_{\tilde{t}_2}(0.9) = 0.44$. This stage is slightly overloaded with respect to the best cutting conditions.

Fuzzy feed rate (including cycle time constraint) $\tilde{f}_2 \cap \tilde{f}'_2$: as shown in Figure X.6b the range of possible values has become quite narrow.

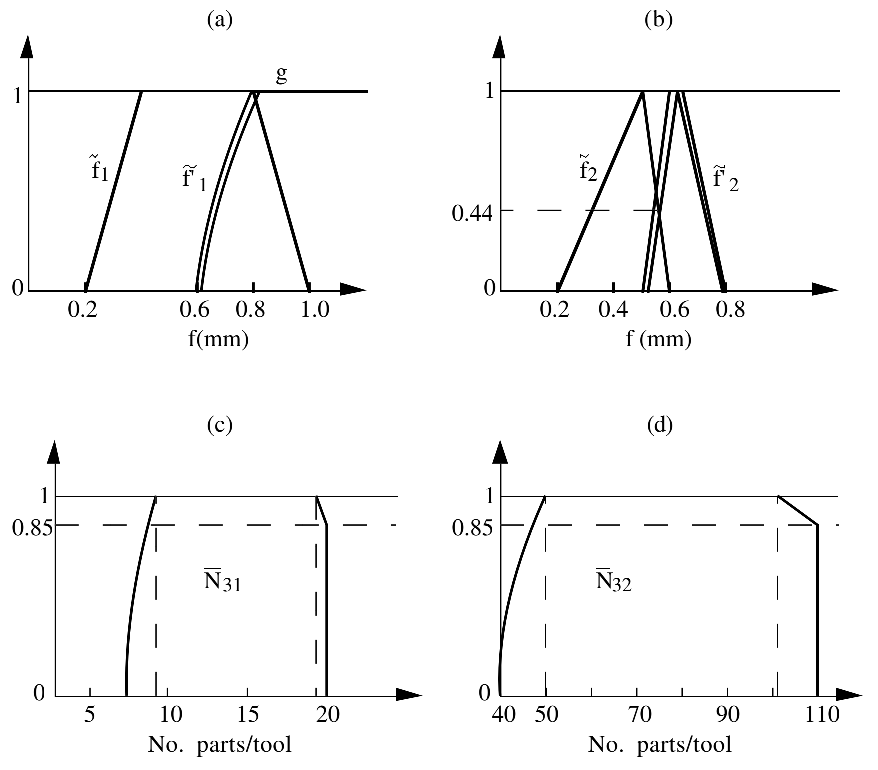

Figure X.5 Fuzzy sets of solutions

The optimal cutting conditions are $\tilde{f}_2 = 0.555$ mm; $v_2 = 125{,}5$ m/min. $\tilde{N}_2$ is a fuzzy singleton whose support is {13} and membership grade = 0.39.

(c) *Station 3*

Quality specification: $\tilde{v}_3 = (80, 90, 10, 30)$ m/min for both operations; $\tilde{f}_3 = (0.4, 0.8, 0.2, 0.2)$ mm for both operations.

Spindle speed: $D_{31} = 210$ mm $\Rightarrow \tilde{w}_{31} = (121.3, 136.4, 15, 45)$ rev/min; $(D_{32})_{min} = 166$ mm $\Rightarrow \tilde{w}_{32} = (153.40, 172.5, 19, 57)$ rev/min; $(D_{32})_{max} = 210$ mm $\Rightarrow \tilde{w}'_{32} = \tilde{w}_{31}$. Range of spindle speeds $\Omega = \{50, 100, 150, 200\ldots)$ rev/min. The support of $\tilde{w}_{31} \cap \tilde{w}_{32}$ is included in [134, 181]; the highest membership grade = 0.73, for a spindle speed $w_3 = 148.3$ rev/min. Hence $w_3 = 150$ rev/min.

Cutting time: $\tilde{t}_3 = \dfrac{l_{31} + 2l_{32}}{2w_3\tilde{a}_3}$ (coefficient 2 comes from a two-edge tool). It can be checked that $\mu_{\tilde{t}_3}(\hat{t}) = \mu_{\tilde{t}_3}(0.9) = 0.85$, and the stage is slightly underloaded (the one-level cut is $(\tilde{t}_3)_1 = [0.42, 0.85]$min).

Fuzzy feed rates: $\tilde{f}'_{31} = \tilde{f}'_{32} = \left[\dfrac{l_{31} + 2l_{32}}{2\hat{t}w_3}, +\infty\right) = [0, 37, +\infty)$ mm, i.e., $\tilde{f}_{31} = \tilde{f}_{32} = \tilde{f}_3$ is crisply truncated from below. $\tilde{N}_{31}$ and $\tilde{N}_{32}$ are shown in Figures X.6c and X.6d. $\tilde{N}_{32}$ is calculated from an average diameter (188 mm).

(d) *Tool-replacement policy*: $\hat{\alpha} = 0.39$ due to $\tilde{N}_2$. For $\alpha = 0.39$, $\varepsilon = 0.25$ we get the values shown in Table X.1.

Note that the algorithm finds $\bar{N} = 13$, $N = 10$, and finds nothing for $N = 13, 12, 11$. Station 2 tool is under-utilized since v_2 and f_2 are the optimal cutting conditions, i.e., the membership value is $\mu = 0.39$,

Table X.1

Station	Operation	$\min(\tilde{N}_{ij})_\alpha$	$\bar{N}_{ij}$	$\underline{N}_{ij}$	N_{ij}	μ	f_{ij}(mm)	v_{ij}(m/min)	t_{ij}(min)	t_i(min)
1	1	15	21	12	20	0.69	0.861	81	0.9	0.9
2	2	13	13	10	10	0.39	0.555	125,5	0.9	0.9
3	3.1	8	19	6	10	1	0.739	99	0.3	0.58
3	3.2	43	109	33	100	1	0.406	[78,99]	0.28	

μ is the grade of membership of the cutting conditions which corresponds to N_2 = 13. The cutting conditions at Station 1 are revised in order to properly use the tool (μ = 0.69).

X.4 CONCLUSION

The attractiveness of fuzzy sets for computer-aided design has been stressed in the transfer-line case. We have described a procedure where, starting from fuzzy information describing subjective preferences, physical constraints are introduced little by little, in order to reduce fuzziness, until final decisions are made. This procedure may be useful for choosing the stages of a cutting process, i.e., the clustering of machining operations into production stages corresponding to work-stations.

X.5 REFERENCES

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