Report: Bipolar Layered Frameworks (BLFs)

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1 Context

This report consists of an outline of the research I will carry out at Laboratoire d'Économie Mathématique et de Microéconomie Appliquée (LEMMA), Université Paris 2 as a PhD student. My research builds upon the work of Florence Bannay and Romain Guillaume at Institute Recherche de Informatique Toulouse (IRIT), Universite Paul Sabatier III, Toulouse. We present an outline of the work that I have carried out this year under the supervision of Florence Bannay and Romaine Guillaume at IRIT, Toulouse. Our work at IRIT forms an integral part of our overall research work.

2 Introduction

The decision analysis process involves an agent (the decision maker) taking account of the decision situation and then evaluating different courses of action (decision alternatives). In order to be able to do so, the decision maker must characterize the decision-making situation with respect to two distinct components: a formulation of the decision goals and a characterization of the decision alternatives [23]. Usually, the evaluation is based on an associated utility function (see e.g. the introductory book of [20]) which encodes the degree of satisfaction achieved by choosing every decision alternative. Despite a lot of works on decision theory, two issues are often not easy to solve. The information of the agent about the decision situation if often uncertain, incomplete and distributed. Hence the first issue is to deal with imperfect information (uncertainty, incomplete and distributed knowledge). The second issue is to be able to explain and justify the decisions that are made. Another kind of issue is related to explanation and justification, it concerns the clarity of the decision making system. It is also a desirable goal to enable the decision makers to have a broader view of the principles that governs decision and to enable them to participate to their elaboration.

The three standard approaches [19] of optimisation (here maximize a utility or minimze a cost) under uncertainty are: *expected cost minimization*, *risk minimization* and *chance constraints*. The first one consists in choosing the decision that minimizes the mathematical expectation of the cost. In the discrete case, it amounts to compute a weighted sum of the costs weighted with their respective probabilities. However, the probabilities may not be known precisely and also, applying this method on only one situation can lead to have an effective cost far from the expectation. The two other approaches handle probability and utility separately. *Risk minimization* [19] aims at minimizing the probability that a cost is greater than a threshold and *Chance constraints* approach [13] consists in minimizing the cost that can appear with a given probability level. In this paper, we use another kind of uncertainty representation based on possibility distributions in a qualitative context that is close to the *Chance constraints* by the use of defeasible reasoning. Indeed, when dealing with uncertainty, it is convenient to use default rules [21, 22] that allow us to express general principles concisely (i.e., without making explicit all the possible exceptions) and to derive a conclusion in presence of incomplete knowledge with the ability to revise it when more precise information is known (specificity principle [22]).

[11, 10, 9] have formalized a framework for handling this kind of decision making scenario, it is called Bipolar Layered Framework (BLF). A BLF is an intuitive visual tool for analysis and explanation of decisions since it presents the principles that governs the decision (called decision principles) directly to the decision maker, with a clear view of the goals and features that are involved. Moreover a BLF enables the user to represent defeasible information (like decision principles), and to handle incomplete knowledge (about the candidates). A BLF combines an agent's information about the decision-making situation. The decision-making situation is characterized in the bigger picture by two concepts: features and goals. Features are used to represent the environment in which the agent operates and to describe the decision alternatives. Goals are what can be achieved by selecting the alternative. The agent characterizes the decision alternatives (candidates) by a propositional knowledge base. From a decision-making perspective, we think that four kinds of information are of relevance here:

- the goals that are involved in the decision making process and their division into desirable and undesirable goals;
- the relative importance of the goals;
- the decision principles (DP): The agent knows that it is plausible that a goal (say, to swim) will be achieved when a feature holds (say, the hotel has a swimming pool) but does not know for certain that it will always hold. This uncertain and incomplete intuitive information of the agent is encoded as a decision principle. Decision principles are defeasible rules that represent an agent's a priori knowledge about a goal being achieved when a feature holds.
- the exceptions to the decision principles. DPs being default rules, the agent can have intuitive information about exceptions. If a feature holds (say, it is raining), the default rule allowing the goal swim to be achieved will not be triggered. These exceptions are called inhibitors.

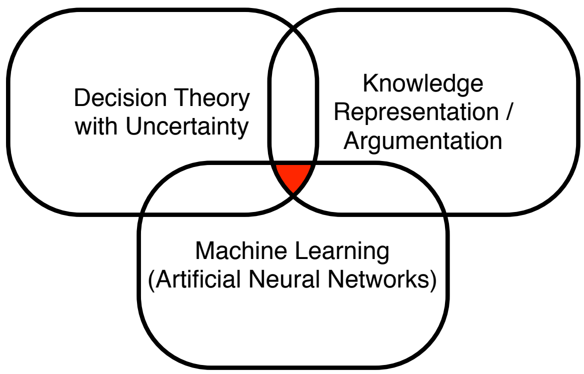
Once a BLF is constructed out of the information of the agent, it can be used to evaluate and analyze decision alternatives.

Classical BLFs only admit inhibitors to decision principles. As explained above, the idea behind inhibitors is that the agent can know that some feature, if it holds, could cancel out his defeasible intuition contained in the decision principle. In natural, real life decision making situation, it is more often the case that the agent, in addition to the inhibitor, knows that some feature, if it holds, can support it's decision principle. This support increases the agent's trust in the initial causal link established in the decision principle. It may happen that a decision principle have more than one inhibitor and support. Consequently, in order to be able to compare the inhibitors and supports that a decision principle admits, the agent needs additional information.Adding strengths/weights to these components seemed the logical step to take. The weights attached to inhibitors and supports denote the strength with which they prohibit/support the agent's intuitive causal link expressed in the decision principle. This updated and enhanced framework is called Bipolar Layered Frameworks with Supports and Weights (BLFSW).

An important problem in many domains is to explain and compute the weights associated to some items, it is the case for instance in Weighted Argumentation [63]. The ability to explain how the weights on inhibitors and supports of our decision principles are computed is one of the main result of this paper. This result is based on possibility theory which proposes a measure of uncertainty which differs from probablity theory by its ability to express ignorance. In possibility theory one may distinguishing what is the normal course of things from what is not, what is surprising from what is expected. [15, 16]. In this work, we use possibility theory to characterize the various components of a BLF. For instance, a decision principle of the form (φ, g) correspond to a default rule which in possibilistic terms translates that it is more plausible that the goal g is achieved when the feature φ holds than that the goal g is not achieved when the feature φ holds. This report is organized as: We first introduce and provide essential background for the main topics of my PhD research: Decision Theory with Uncertainty, Argumentation and Machine Learning. Then we present the work we accomplished during my stay at IRIT, Toulouse as a guest student. The second part outlines the future research that I will undertake at LEMMA, Paris in collaboration with IRIT, Toulouse. That part will focus on Decision Theory, Argumentation and Machine Learning.

3 Background

In this section, we will present the theoretical background of our main topics of research: Decision Theory, Uncertainty and Argumentation.



3.1 Decision Theory

Decision theory is concerned with the reasoning underlying the possible choices of an agent (the decision maker). It provides a formalism for modeling the decision process of an agent with different decision alternatives (candidates). In decision-making environments, the information of the agent is often uncertain or incomplete. As mentioned in [22], a decision analysis process can be thought of as consisting of three stages: (1) characterization of the decision alternatives (candidates) that are to be evaluated; (2) formulation of decision goals; (3) the decision making itself. To evaluate decision alternatives, a utility function assigning a value to every decision alternative (the degree of satisfaction) is used [19]. Given this information, the decision-maker can evaluate decision alternatives, by ranking them and / or comparing them based on specific criteria. The maximum expected utility principle enjoins the decision maker to choose the alternative with the highest expected utility. In quantitative decision theory, both the probabilities (dealing with uncertainty of outcomes) and utilities (of outcomes or goals) are explicitly required. In qualitative decision theory, on the other hand, preferences over outcomes and likelihood of outcomes are expressed in a qualitative way [13, 58, 59, 60]. This allows an agent to reason and decide in a situation where he has knowledge of his preferences and intuition about achievement of goals without having to quantify this knowledge. Human reasoning is often uncertain. The unsettled nature of the information source or the less than perfect ability of a human agent to read his environment can cause uncertainty. The very nature of the relevant information can have inherent uncertainty, the source of which can be incompleteness, inconsistencies and change in the available information. Besides, uncertainty is also inherent in the decision making process [16, 19, 23]. The decision-maker (agent) may have only partial cognizance of the environment. He may also only have an uncertain intuition of what outcomes different features of candidates lead to. And he may only have partial and uncertain information about which features of the candidate holds in the decision-making environment. Finally, uncertainty is also a part of other knowledge representation systems. Dealing with uncertainty in information is an essential part of AI theories and applications [23]. Decision Theory deals with the question: ”Given my preferences and my goals, what is the most rational decision alternative”.

3.2 Uncertainty

Uncertainty is an inherent part of the decision making process. Decision makers often (almost always) operate in uncertain environments. Additionally, human capabilities are less than perfect. Possibility theory provides a formalism that is of specific relevance for the modeling of uncertain situations [25, 14, 16, 24]. It was introduced to model reasoning processes dealing with imprecise or vague knowledge involving uncertainty. Possibility theory is qualitative and represents an alternative to probability theory from which it differs by its ability to express complete ignorance. In possibility theory one may distinguish what is the normal course of things from what is not, what is surprising from what is expected [15].

3.3 Argumentation

Human reasoning is based on argumentation. This view applies both to a single rational actor and to a group of rational actors. In the first case, the scenario consists of a rational actor deliberating over conflicting arguments and trying to arrive at a rational outcome. In the second case, the scenario involves a number of rational actors each asserting a different view point. The aim of this exercise is to solve the inherent conflicts between the arguments of various actors. Argumentation Theory is a knowledge representation and reasoning formalism used for the modeling of human argumentation and dialog processes. The objective of the formalism is to model an argumentation process with conflicting truth claims, to reason over them and arrive at a coherent conclusion regarding their outcome. It can be seen both as a mechanism for solving conflicts between truth claims of multiple agents and as a mechanism for constructing justification / explanation for a particular decision / stance. In computational argumentation, two kinds of argumentation are generally identified: Monological argumentation: the argumentation scenario is given and the objective is to evaluate it and reach a conclusion about its outcome [42]. Dialogical argumentation: this is a more interactive scenario where agents can engage in negotiation and persuasion through “exchange of arguments and counterarguments” [42, 61, 62]. In Abstract Argumentation, the argumentation process is modeled by means of three features: (1) a number of arguments, each stating a fact or a belief to be true. These arguments are considered as atomic entities in the sense that we abstract away from their internal structures. (2) a conflict relation between these arguments. (3) a set of rules (semantics) specifying how conflicts between the arguments (i.e., disagreements between facts / beliefs asserted by arguments) are to be resolved. The application of these rules leads to acceptance of some arguments and rejection of others. In abstract argumentation, the arguments are considered to be atomic entities and the content of these arguments are not considered. Formally, an argumentation framework (AF) consists of a directed graph where the nodes represent the arguments and the edges represent the binary attack relation between them [30]. Severals semantics have been defined which are rules for solving the conflicts between the arguments of the AF [31]. A basic argumentation framework is illustrated in Figure 1. In Structured Argumentation, on the other hand, an argument is not an abstract entity but is conceptualized as a formally defined premises/claim structure expressed in a logical language. Both the claim/assertion by the agent and the reasoning behind that truth claim is evident. In this way, structured reasoning "builds" arguments by providing a justification and explanation for positions held by an agent [42, 48].

3.4 Machine Learning / Artificial Neural Networks

Machine Learning is a subfield of Artificial Intelligence that seeks to endow a computer system with the ability to "learn" patterns from data. The term learning refers to the capability of the system to use experience in order to progressively improve its performances on a specific task (or class of tasks). Machine learning is employed in a range of computing tasks where designing and programming explicit algorithms with good performance is difficult or infeasible (like in computer vision or speech recognition for instance). Machine learning algorithms are typically classified into two broad categories: *supervised* and *unsupervised* learning.

Artificial Neural Networks (ANN) is a class of Machine Learning algorithms loosely inspired by biological neurons. The network are made of input, hidden and output *artificial neurons* related together by weighted *synaptic connections*. Information is processed from the input to the output neurons by traveling through the synaptic connections. The training process of artificial neural networks consists of an adjustment of their synaptic weights according to some algorithmic task that is to be achieved. Usually, the weights are updated by means of the *backpropagation* algorithm, which is a gradient descent-based minimization process of the error function of the network. Nowadays, artificial neural networks are among the best Machine Learning techniques, thanks to their highly efficient training capabilities as well as to the tremendous success of deep learning methods (see [50] for a brilliant survey and the references therein). An artificial neural network is illustrated in Figure 1.

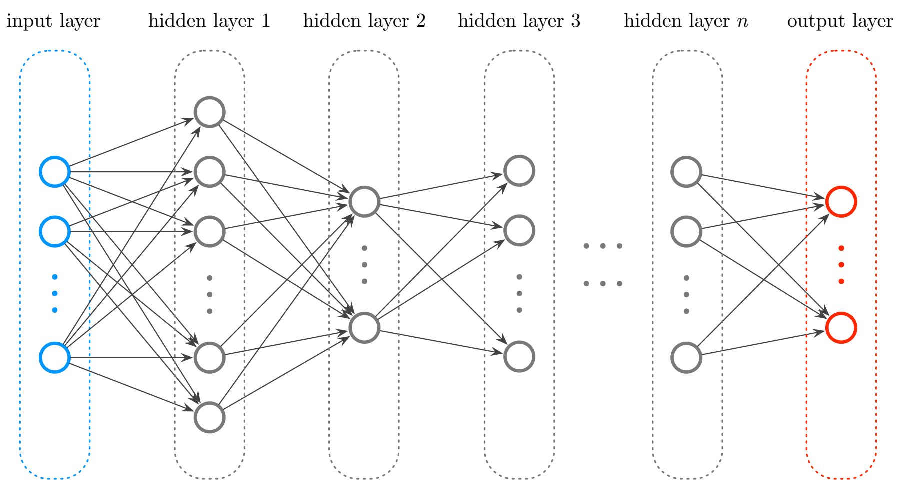


Figure 1: An artificial neural network (ANN). The nodes and edges represent the neurons and synaptic connections between them. In this case, information is processed in a feedforward manner, i.e., from one layer to the next without any recurrent connection. The training of the neural network consist of an adjustment of its synaptic weights according to some algorithmic task that is to be achieved.

4 BLF: a structure encoding decision criteria

4.1 Preliminaries

here go the preliminaries of the first orde We consider a set $\mathscr{C}$ of candidates[1] about which some information is available and two languages $\mathscr{L}_F$ (a propositional language based on a vocabulary $\mathcal{V}_F$) representing information about some features that are believed to hold for a candidate and $\mathscr{L}_G$ (another propositional language based on a distinct vocabulary $\mathcal{V}_G$) representing information about the achievement of some goals when a candidate is selected. In the propositional languages used here, the logical connectors "or", "and", "not" are denoted respectively by $\vee$, $\wedge$, and $\neg$. A *literal* is a propositional symbol x or its negation $\neg x$, the set of literals of $\mathscr{L}_G$ are denoted by LIT_G. Classical inference, logical equivalence and contradiction are denoted respectively by $\models$, $\equiv$, $\perp$. The reason why we propose two distinct languages is to clearly differentiate beliefs (coming from observations) from desires (goals to be achieved when selecting a candidate). In the following we denote by K a set of formulas representing features that are believed to hold: hence $K \subseteq \mathscr{L}_F$ is the available information. Using the inference operator $\models$, the fact that a formula $\varphi \in \mathscr{L}_F$ holds[2] in K is written $K \models \varphi$.

The BLF is a structure that contains two kinds of information: decision principles

[1] Candidates are also called alternatives in the literature.

[2] The agent's knowledge K being considered to be certain, we write "φ holds" instead of "φ is believed to hold".

and inhibitors. A decision principle can be viewed as a defeasible reason enabling to reach a conclusion about the achievement of a goal. More precisely, a decision principle is a pair (φ, g), it represents the default rule meaning that "if the formula φ is believed to hold for a candidate then the goal g is a priori believed to be achieved by selecting this candidate":

Definition 1 (decision principle (DP)) *A* decision principle *p is a pair $(\varphi, g) \in \mathscr{L}_F \times LIT_G$, where φ is the reason denoted $reas(p)$ and g the conclusion of p denoted $concl(p)$.*

$\mathcal{P}$ denotes the set of decision principles.

We illustrate the BLF notions on a toy example concerning the choice of an hotel.

Example 1 *Let us imagine an agent who wants to find a hotel which is not expensive (e) and in which he can swim (s). This agent prefers to avoid crowded hotels (c). The possible pieces of information concern the following attributes: $\mathcal{V}_F = \{p,\ f,\ w,\ o\}$ that describes the respective features of the hotel "to have a pool", "to be a four star hotel", "to be in a place where the weather is fine", "to propose special offers". The agent may consider the following principles: $\mathcal{P} = \{p_1 = (p, s),\ p_2 = (f, e),\ p_3 = (w, c)\}$. p_1 expresses that "a priori when there is a pool the agent can swim", p_2 encodes that "a priori if the hotel is four star then it is expensive", p_3 says that "if the weather is fine in this area then the hotel is a priori crowded".*

Depending on whether the achievement of its goal is wished or dreaded, a decision principle may have either a positive or a negative polarity. Moreover some decision principles are more important than others because their goal is more important. The decision principles are totally ordered accordingly.

Definition 2 (polarity and importance) *A function $pol : \mathcal{V}_G \to \{\oplus, \ominus\}$ gives the polarity of a goal $g \in \mathcal{V}_G$, this function is extended to goal literals by $pol(\neg g) = -pol(g)$ with $-\oplus = \ominus$ and $-\ominus = \oplus$. Decision principles are polarized accordingly: $pol(\varphi, g) = pol(g)$. The set of positive and negative goals are abbreviated $\overline{\oplus}$ and $\overline{\ominus}$ respectively: $\overline{\oplus} = \{g \in LIT_G : pol(g) = \oplus\}$ and $\overline{\ominus} = \{g \in LIT_G : pol(g) = \ominus\}$.*

LIT_G is totally ordered by the relation $\preceq$ ("less or equally important than").

Decision principles are ordered accordingly:$\forall \varphi, \psi \in \mathscr{L}_G, \forall g \in LIT_G$,

$$(\varphi, g) \preceq (\psi, g') \quad \textit{iff} \quad g \preceq g'$$

The polarities and the relative importances of the goals in $\mathcal{V}_G$ are supposed to be given by the decision maker.

Example 1 (continued): *In our example, the decision maker (our agent) may want to avoid crowded and expensive hotels (hence c and e are negative goal), while being able to swim (s) in the hotel is a positive goal. Moreover the agent gives more importance to the possibility to swim and the expensiveness than to the fact that the hotel is crowded. It means that the set of possible goals is $\mathcal{V}_G = \{s,\ e,\ c\}$, with $pol(s) = \oplus$ and $pol(e) = pol(c) = \ominus$ and $s \simeq e \succ c$.*

A decision principle (φ, g) is a defeasible piece of information because sometimes there may exist some reason φ_1 to believe that it does not apply in the situation, this reason is called an *inhibitor*.

The fact that φ_1 inhibits a decision principle (φ, g) is interpreted as follows: when the decision maker knows $\varphi \wedge \varphi_1$ she has no more information about the achievement of the goal g. In that case, the inhibition is represented with an arc towards the decision principle.

Definition 3 (inhibitor) *Let $\varphi_1 \in \mathcal{L}_F$ and $(\varphi, g) \in \mathcal{P}$. An inhibitor is a pair $(\varphi_1, (\varphi, g))$. The set of inhibitors is denoted by $\mathcal{I}$ where $\mathcal{I} \subseteq \mathcal{L}_F \times \mathcal{P}$.*

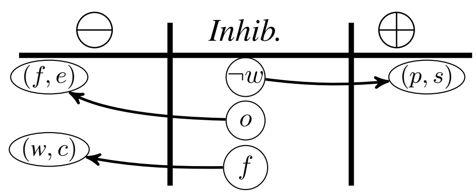
The decision principles and their inhibitors are supposed to be given by the decision maker. We are now in position to define the BLF structure.

Definition 4 *A BLF is a tuple $(\mathcal{P}, \mathcal{I}, pol, \preceq)$ where: $\mathcal{P}$ is a set of decision principles ordered accordingly to their goals by $\preceq$ and with a polarity built on pol, and $\mathcal{I} \subseteq (\mathcal{L}_F \times \mathcal{P})$ is a set of inhibitors.*

The elements of the BLF are supposed to be available prior to the decision and to be settled for future decisions as if it was a kind of utility function. A graphical representation of a BLF is given below, it is a tripartite graph represented in three columns, the DPs with a positive level are situated on the left column, the inhibitors are in the middle, and the DPs with a negative polarity are situated on the right. The more important (positive and negative) DPs are in the higher part of the graph, equally important DPs are drawn at the same horizontal level. Hence the highest positive level is at the top left of the figure, the bottom right contains DPs with negative goals of low importance. The height of the inhibitors is not significant only their existence is meaningful.

Example 1 (continued): *When the hotel is not in a place where the weather is usually fine then the fact that there is a pool is not sufficient to ensure that the agent can swim, it means that there is an inhibition on p_1 by $\neg w$, and the DP p_3 that expresses that "if the weather is fine the hotel will be crowded" is inhibited when its a four stars hotel, and the DP p_2 is inhibited when the hotel proposes a special offer, i.e. $\mathcal{I} = \{(\neg w, p_1)$, (f, p_3), $(o, p_2)\}$.*

Below is the picture of the BLF where the only positive DP, p_1, is on the right, and the two negative DPs, p_2 and p_3 are on the left. The height of the DPs represents the importance of their goal, hence p_1 and p_2 are at the same height while p_3 is below. The arrows are representing inhibitions.



In the following, the BLF $(\mathcal{P}, \mathcal{I}, pol, \preceq)$ is set and we show how it can be used for analyzing the acceptability of a candidate. First, we present the available information and the notion of instantiated BLF, called valid-BLF.

Given a candidate $c \in \mathscr{C}$, we consider that the knowledge of the decision maker about c has been gathered in a knowledge base K_c with $K_c \subseteq \mathscr{L}_F$. Given a formula φ describing a configuration of features ($\varphi \in \mathscr{L}_F$), the decision maker can have three kinds of knowledge about c: φ holds for candidate c (i.e., $K_c \models \varphi$), or not ($K_c \models \neg\varphi$) or the feature φ is unknown for c ($K_c \not\models \varphi$ and $K_c \not\models \neg\varphi$). When there is no ambiguity about the candidate c, K_c is denoted K.

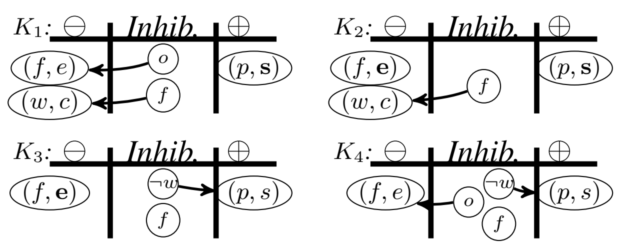
Definition 5 (K-valid-BLF) *Given a* consistent *knowledge base K, a K-valid-BLF associated to a BLF $B = (P, \mathcal{I}, pol, \preceq)$ is a quadruplet ($\mathcal{P}_K$, $\mathcal{I}_K$, pol, $\preceq$) where*

- $\mathcal{P}_K = \{(\varphi, g) \in \mathcal{P}$, *s.t.* $K \models \varphi\}$ *is the set of DPs in $\mathcal{P}$ whose reason φ holds in K, called* K-valid-DP*s.*
- $\mathcal{I}_K = \{(\varphi, p) \in \mathcal{I}$, *s.t.* $K \models \varphi$ *and* $p \in \mathcal{I}_K\}$ *is the set of* valid inhibitions *according to K.*

When there is no ambiguity, we simply use "valid-BLF" instead of "K-valid-BLF". The validity of a DP only depends on the fact whether the features that constitute its reason φ hold or not, it does not depend on its goal g since the link between the reasons and the goal is given in the BLF (hence it is no longer questionable).

Example 1 (continued): *Let us consider 4 candidates that are the hotels described by the following knowledge bases*[3]. $K_1 = \{p \wedge w \wedge f \wedge o\}$, $K_2 = \{p \wedge w \wedge f\}$, $K_3 = \{p \wedge \neg w \wedge f\}$, $K_4 = \{p \wedge \neg w \wedge f \wedge o\}$.

The four corresponding valid-BLFs are:



Now in the K-valid-BLF the principles that are not inhibited are the ones that are going to be trusted, which leads us to define the realized goals wrt a candidate described by the knowledge base K.

Definition 6 *A goal $g \in LIT_G$ is said to be* realized *wrt a K-valid-BLF ($\mathcal{P}_K$, $\mathcal{I}_K$, pol, $\preceq$) if there is a K-valid-DP in $\mathcal{P}_K$ concluding g that is not inhibited in $\mathcal{I}_K$.*

Example 1 (continued): *The candidate described by K_1 achieves only one goal: swim s, (the DPs concerning the two other goals are inhibited); the hotel described by K_2*

[3]The knowledge bases can be given under the form of sets of propositional formulas, interpreted as their conjunction, namely $K_1 = \{p \wedge w, o \vee \neg f, f\}$ which is equivalent to $p \wedge w \wedge f \wedge o$. In the example we show them in their conjunctive normal form.

achieves two goals swim s (which is positive) but also expensive e (which is negative), the one described by K_3 is only expensive e, the last hotel described by K_4 achieves no goal (neither positive nor negative since the two DPs are inhibited).

Once we are able to say what goals are realized wrt a BLF instanciated for a given candidate described by K (i.e, wrt a given K-valid BLF), then we are in position to compare candidates according to the goals they achieve. In [12], the authors have introduced three decision rules called Pareto, Bipolar Possibility and Bipolar Leximin dominance relations. We have chosen to only translate the Bipolar Leximin dominance relation in order to compare two candidates, the definition has slightly changed wrt [9][4].

Definition 7 (BiLexi decision rule) *Given a BLF $B = (\mathcal{P}, \mathcal{I}, pol, \preceq)$ and two candidates described respectively by K and K' with their associated realized goals $\mathtt{R} = Real(B, K)$ and $\mathtt{R}' = Real(B, K)$, the Bipolar Leximin dominance relation denoted $\succeq_{BiLexi}$ (which stands for "is $BiLexi$-preferred to") is defined by:*

$$\begin{array}{lll} K \succ_{BiLexi} K' & \textit{iff} & \left| \begin{array}{l} M \textit{ exists and} \\ |\mathtt{R}_M^{\oplus}| \geq |\mathtt{R}_M'^{\oplus}| \textit{ and } |\mathtt{R}_M^{\ominus}| \leq |\mathtt{R}_M'^{\ominus}| \end{array} \right. \\ K \simeq_{BiLexi} K' & \textit{iff} & M \textit{ does not exist} \end{array}$$

where $X_g = \{g' \in X \text{ s.t. } g \simeq g'\}$ is the set of goals in a set X that have the same level of importance than g and $M = \max(\{g \in \mathtt{R} \cup \mathtt{R}' \text{ s.t. } |\mathtt{R}_g^{\oplus}| \neq |\mathtt{R}_g'^{\oplus}| \text{ or } |\mathtt{R}_g^{\ominus}| \neq |\mathtt{R}_g'^{\ominus}|\}, \preceq)$ is the highest goal s.t. the number of positive or negative sets of achieved goals for K and K' differs.

In other words, a candidate described by K is $BiLexi$-preferred to another described by K' if there is a goal M such that the number of realized positive and negative goals at levels strictly more important than M are the same, but at the level M either the number of positive goals of K is greater than those of K' or the number of negative goals of K is lower than those of K'.

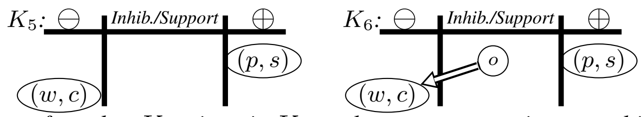
Example 1 (continued): *We can rank the candidates according to the goals that they achieve: $K_1 \succ_{BiLexi} (K_2 \simeq_{BiLexi} K_4) \succ_{BiLexi} K_3$. Indeed, the best candidate is K_1 since it achieves the best positive goal, then K_2 and K_4 are incomparable since one achieves a negative and a positive goal of same importance and the other does not achieve any goal (so no negative nor positive ones), the last candidate is K_3 since it achieves only a negative goal.*

4.2 BLF with supports and weights

In some cases, some candidates can be incomparable while the information concerning them is not the same. In order to refine our ranking, the notion of support may be introduced. Indeed sometimes some information may increase the strength of the DP, since in the new context the goal is more likely to be achieved. This is the case in the example below.

[4] This allows us to avoid to associate numeric levels to the goals.

Example 2 *When there is a special offer then the fine weather hotels are even more likely to be crowded than with no special offer. If we consider two hotels with $K_5 = \{p \wedge w\}$ and $K_6 = \{p \wedge w \wedge o\}$. Their K_5-valid BLF and K_6-valid BLF can be represented as follows:*



In that case, K_5 should be preferred to K_6, since in K_5 we have more certainty to achieve the negative goal in the presence of the support o.

If a designer of a BLF wants to express that some features are supporting some DPs, then this can raise some ambiguities in the following situations:

- when a DP has at least one support and one inhibitor: what happens when they are both valid wrt a candidate? the designer should precise which one is stronger than the other, and more generally give their relative strength.
- when a DP has several supports: if one support is valid for a candidate and a distinct support is valid for the other candidate, how to decide which candidate has the most supported DP? again the designer should precise if a support is stronger than the other

A simple and expressive way for a designer to give these precisions is to extend the BLF with supports and weights. The weights are associated to supports and inhibitors in order to resolve the possible ambiguities that can occur in all possible valid-BLF. Their exact value has no importance.

More formally, we propose the following definitions of a BLFwith supports and weights:

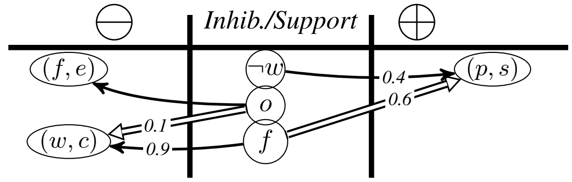
Definition 8 (BLFSW) *: A Bipolar Leveled Framework with Supports and Weights is a tuple $(\mathcal{P}, \mathcal{I}, \mathcal{S}, pol, \preceq, w)$ where: $\mathcal{P}$ is a set of decision principles ordered accordingly to their goals by $\preceq$ and with a polarity built on pol, $\mathcal{I} \subseteq (\mathcal{L}_F \times \mathcal{P})$ is a set of inhibitors, $\mathcal{S} \subseteq (\mathcal{L}_F \times \mathcal{P})$ is a set of supports, and $w : \mathcal{I} \cup \mathcal{S} \rightarrow]0, 1]$ is a weight function.*

where $\mathcal{I}(p) = (\mathscr{L}_F \times \{p\}) \cap \mathcal{I}$ and $\mathcal{S}(p) = (\mathscr{L}_F \times \{p\}) \cap \mathcal{S}$ are the set of inhibitors and supports attached to the DP p.

For a simpler representation, the drawing of a BLFSW obeys the convention that **if no weight is given for a set of supports and inhibitors concerning the same DP then all weights are equal to** 1. Note that the possibility to express that all supports and attacks have the same weight is a way to preserve ambiguity when it is desired by the designer. The weight are expressing a certainty degree that the goal is going to be achieved when the premise hold. We do not allow for supports or inhibitors of weight 0, since this weight would mean that there is no information about the supporting/inhibiting effects of those features, hence it is not useful to mention them.

Example 3 *Let us come back to the hotel example. We have described the inhibitions, but we may also express supports: a special offer will encourage people to go to an hotel hence it may increase the certainty to have a crowded hotel, moreover a four star hotel is more likely to offer the possibility to swim when it has a pool i.e.* $\mathcal{S} = \{(o, p_3), (f, p_1)\}$

The double arrows are representing the supports. The arcs are labeled by numbers that represents the weight of the supports/inhibitions. Two DPs have possible ambiguities: (w, c) *and* (p, s)*. Concerning the DP* p_3*, we have chosen to give a higher weight to the inhibitor* f *than to the support* o *since we consider that even when there are special offers, four star hotels are not crowded (since they remain very expensive). Concerning the DP* p_1*, we consider that the support is stronger than the inhibitor, since four star hotel that have pool have often also an indoor pool. The weights that are given are following these considerations, the exact values of these weights are meaningful only for the comparison.*



Given a consistent knowledge base K, a K-valid-BLFSW is a tuple $(\mathcal{P}_K, \mathcal{I}_K, \mathcal{S}_K, pol, \preceq, w)$ whose definition is similar to a K-valid-BLF, the only change is that the arcs can be inhibitors or supports and they have weights. More formally,

Definition 9 (K-valid-BLFSW) *Given a consistent knowledge base* K *and a BLFSW* $B = (\mathcal{P}, \mathcal{I}, \mathcal{S}, pol, w)$*, a* K*-valid-BLFSW associated to* B *is a tuple* $(\mathcal{P}_K, \mathcal{I}_K, \mathcal{S}_K, pol, \preceq, w_K)$ *where*

- $\mathcal{P}_K = \{(\varphi, g) \in \mathcal{P}$, *s.t.* $K \models \varphi\}$ *is the set of DPs in* $\mathcal{P}$ *whose reason* φ *holds in* K*, called* valid-DP*s.*
- $\mathcal{I}_K = \{(\varphi, p) \in \mathcal{I}$, *s.t.* $K \models \varphi$ *and* $p \in \mathcal{P}_K\}$ *is the set of* valid inhibitions *according to* K*.*
- $\mathcal{S}_K = \{(\varphi, p) \in \mathcal{S}$, *s.t.* $K \models \varphi$ *and* $p \in \mathcal{P}_K\}$ *is the set of* valid supports *according to* K*.*
- w_K *is the restriction of* w *on* $\mathcal{I}_K \cup \mathcal{S}_K$*.*

Similarly as with classical BLFs, the DPs that are not inhibited in the K-valid BLFSW are the ones that are trusted, in order to know if a DP is inhibited, we may have to compare the weights of its inhibitor and supports. Moreover the notion of realized goal can be enriched with the pieces of information coming from the weights of its supports.

Definition 10 *Given a K-valid BLFSW ($\mathcal{P}_K$, $\mathcal{I}_K$, $\mathcal{S}_K$, pol, $\preceq$, w_K), we define the activation level of $p \in \mathcal{P}_K$ as follows:*

$$\alpha(p) = \left(\sum_{s \in \mathcal{S}_K(p)} w_K(s,p) - \sum_{i \in \mathcal{I}_K(p)} w_K(i,p) \right)$$

where $\mathcal{S}_K(p) = \{\varphi \in \mathscr{L}_F | (\varphi, p) \in \mathcal{S}_K\}$ and $\mathcal{I}_K(p) = \{\varphi \in \mathscr{L}_F | (\varphi, p) \in \mathcal{I}_K\}$ are the set of supports and inhibitors of p respectively.

According to $\alpha(p)$, the DP p is either

- *inhibited iff $\alpha(p) < 0$*
- *supported iff $\alpha(p) > 0$*
- *unaffected iff $\alpha(p) = 0$*

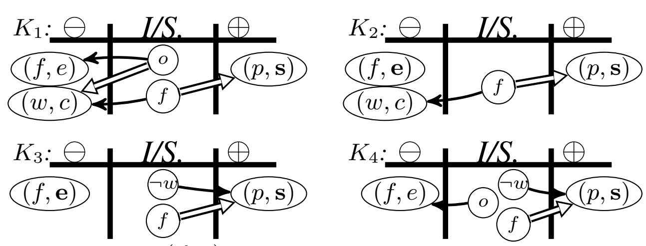
In other words, the weights of supports and inhibitors that concerns a given DP p are used to determine whether p is globally supported or inhibited or unaffected. The DP is considered as supported when the supports are stronger than the inhibitors, it is considered as inhibited in the opposite case, when inhibitors and supports are balanced they cancel each other.

Definition 11 (realized goals) *Given a K-valid BLFSW $B_K = (\mathcal{P}_K, \mathcal{I}_K, \mathcal{S}_K$, pol, $\preceq$, w_K), a goal g in LIT_G is said to be* realized *wrt B_K if there is a K-valid-DP in $\mathcal{P}_K$ that concludes g and that is not inhibited.*

The set of goals that are realized by a candidate described by K given a BLFSW B is denoted Real(B, K).

Example 3 (continued): *Let us come back to our 4 candidates $K_1 = \{p \wedge w \wedge f \wedge o\}$, $K_2 = \{p \wedge w \wedge f\}$, $K_3 = \{p \wedge \neg w \wedge f\}$, $K_4 = \{p \wedge \neg w \wedge f \wedge o\}$.*

The four corresponding valid-BLFSWs are:



In K_1, $p_2 = (f, e)$ is inhibited (since it has only one inhibitor with a default weight of 1), $p_3 = (w, c)$ is inhibited (since it has an inhibitor of weight 0.9 which is heavier than the weight 0.1 of its support) and $p_1 = (p, s)$ is supported. Hence the only realized goals of K_1 is swim s. Similarly, we compute the realized goals of the other hotels: K_2 and K_3 have the same realized goals: e and s, K_4 has only one realized goal: s (since the DP (p, s) is supported by a support that is heavier than its inhibitor).

Similarly as what was done with BLFs, the definition of realized wrt a BLFSW instanciated for a given candidate described by K allows us to compare candidates according to the goals they achieve.

Example 3 (continued): *With the BLFSW, the hotels can be ranked as follows:*

$$(K_1 \simeq_{BiLexi} K_4) \succ_{BiLexi} (K_2 \simeq_{BiLexi} K_3)$$

Note that in case of equality between two candidates, the activation levels of the DPs that are justifying the goals achieved by the candidates, can be used to choose between them as in Example 2. This means that goals are going to be associated with two evaluations, one concerning their importance (that can be called utility when the goal is positive and disutility when it is negative, in our framework it is characterized by $\preceq$ and pol) and one concerning the certainty about their realization for a given candidate K (the weight w_K defined below).

Definition 12 (Weight of a realized goal) *Given a BLFSW $B = (\mathcal{P}, \mathcal{I}, \mathcal{S}, pol, \preceq, w)$ and a candidate described by K, for all $g \in Real(B, K)$, the weight associated to a realized goal is:*

$$w_K(g) = \max_{p \in P_K, concl(p) = g} \alpha(p)$$

The weight associated to a goal g corresponds to the maximum certainty that this goal is achieved by a condidate, i.e., the maximum weight of a DP concluding g.

Definition 13 (BW decision rule) *Given a BLFSW $B = (\mathcal{P}, \mathcal{I}, \mathcal{S}, pol, \preceq, w)$ and two candidates described respectively by K and K' the BW dominance relation denoted $\succeq_{BW}$ (which stands for "is $BiLexi\&Weight$-preferred to") is defined by:*
$K \succ_{BW} K'$ iff $K \succ_{BiLexi} K'$ or ($K \simeq_{BiLexi} K'$ and $\exists M = \max(\{g \in \mathtt{R} \cup \mathtt{R}'$ s.t.
$\max_{g_1 \in \mathtt{R}_g^{\oplus}} w_K(g_1) > \max_{g_2 \in \mathtt{R}_g'^{\oplus}} w_{K'}(g_2)$ or
$\max_{g_1 \in \mathtt{R}_g^{\ominus}} w_K(g_1) < \max_{g_2 \in \mathtt{R}_g'^{\ominus}} w_{K'}(g_2) \quad \}, \preceq)$).

In the previous definition, M is the highest important goal s.t. the maximum weight of a positive or a negative achieved goal of same priority for K and K' differs (in favor of K i.e., either the maximum weight of positive achieved goals for K is strictly greater than the one for K' or the maximum weight of negative achieved goals for K is strictly lower than the one for K').

Example 3 (continued): *This refinement applied to our running example gives:*

$$K_1 \succ_{BW} K_4 \succ_{BW} K_2 \succ_{BW} K_3$$

Since in K_1, p_1 is supported and not attacked hence the activation level of p_1 is 0.6, while in K_4, p_1 has an activation level of $0.6 - 0.4 = 0.2$ which means that the DP p_1 is more certain about the achievement of swim in the situation described by K_1 than in the situation described by K_4. The same refinement is done to differenciate K_2 and K_3, their negative goal e is achieved with the same certainty while the positive goal s is more certainly achieved in K_2 than in K_3.

In the next section we are going to show that BLFSWs are compact representation of classical BLFs with more information.

From BLFSW to BLF and Return

In order to translate a BLFSW into a BLF we are going to consider that a support $s = (\psi, p)$ of a DP $p = (\varphi, g)$ can be viewed as a new DP $p_S = (\psi \wedge \varphi, g)$. Indeed, according to Definitions 10 and 11, the interest of a support is to protect a DP against an inhibitor, hence the translation of a support should make it possible to achieve the goal in presence of an inhibitor, the new DP p_S should be created only if the weight of the support s is greater than at least one inhibitor i of p. In that case p_S and p are both DPs of the translated BLF, but only p is attacked by i in this new BLF.

The tricky part of the translation concerns DPs that are supported by several features. For such a DP p we have to create a new DP for each combination of its supports (conjunction of the features of any subset of $S(p)$). This is due to the fact that the BLF is going to be instantiated with different knowledge bases in which some features may appear or not, the behavior has to be the same as with the BLFSW where each support can be instantiated individually. In conclusion, the set of DPs $\mathcal{P}_S(p)$ that should be constructed in order to capture all the support of a DP $p = (\varphi, g)$ in a BLFWS are those based on the power set of $\mathcal{S}(p)$ that outweigh at least one inhibitor, i.e.,:

$$P_S(p) = \Big\{ \ (\bigwedge_{\varphi_i \in E} \varphi_i \wedge \varphi, g) \Big| E \subseteq \mathcal{S}(p) \text{ and } \exists \psi \in \mathcal{I}(p), \sum_{\varphi_i \in E} w(\varphi_i, p) - w(\psi, p) \geq 0 \Big\}$$

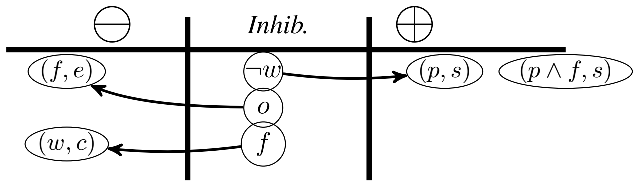
Moreover we are going to create inhibitor relations between an existing inhibitor and a new DP only if the existing inhibitor outweigh the supports that belong to this new DP, hence the new inhibitions $\mathcal{I}_S(p)$ associated to a DP p are:

$$I_S(p) = \Big\{ \ (\psi, p_S) \Big| p_S = (\bigwedge_{\varphi_i \in E} \varphi_i \wedge \varphi, g) \in \mathcal{P}_S(p), \text{ and } \psi \in \mathcal{I}(p), \sum_{\varphi_i \in E} w(\varphi_i, p) - w(\psi, p) < 0 \Big\}$$

The translation of a BLFSW $B = (\mathcal{P}, \mathcal{I}, \mathcal{S}, pol, \preceq, w)$ into a BLF is thus a tuple $(\mathcal{P}', \mathcal{I}', pol', \preceq')$ such that

- $\mathcal{P}' = \mathcal{P} \cup \bigcup_{p \in \mathcal{P}} \mathcal{P}_S(p)$
- $\mathcal{I}' = \mathcal{I} \cup \bigcup_{p \in \mathcal{P}} \mathcal{I}_S(p)$
- $pol' = pol$ and $\preceq' = \preceq$

Example 3 (continued): *Here is a picture of the translation of the BLFSW into a BLF: only one DP has been created $p_{1S} = (p \wedge f, s)$. This DP replaces the support of f to p_1, since when the candidate is a four star hotel (f holds), then the inhibitor of the DP is not strong enough (0.4) to withdraw the belief in the possibility to swim from the existence of a pool. Hence this new DP is not attacked by $\neg w$. Concerning the other support (o, p_3), we can see that it has not enough weight (0.1) to outweigh the inhibitor (f, p_3) (0.9), since there is only one inhibitor of p_3, the creation of a new DP based on p_3 is not useful.*



The reader can check that the four candidates described by $K_1, \dots K_4$ have the same realized goals in the K_i-valid BLFs associated to this BLF. Note also that this BLF is different from the initial BLF of Example 1, indeed the information about the existence of a strong support for p_1 has changed the goals that can be realized, hence the initial BLF was not faithful to represent it while the translation above would have been a way to encode it in a BLF with no support framework.

The following proposition shows that any BLFSW can be translated into a BLF such that the Bilexi-dominance relation gives the same ranking for the BLFSW and for its translation.

Proposition 1 *Let $B = (\mathcal{P}, \mathcal{I}, \mathcal{S}, pol, \preceq, w)$ be a BLFSW, and $Tr(B) = (\mathcal{P}', \mathcal{I}', pol', \preceq'$) its translation as described above, $\forall K_1, K_2 \subseteq \mathscr{L}_F$ s.t. K_1 and K_2 are consistent, it holds that:*

$$K_1 \preceq_{BiLexi} K_2 \text{ iff } Tr(K_1) \preceq_{BiLexi} Tr(K_2)$$

where $\forall i \in \{1, 2\}, Tr(K_i)$ is the K_i-valid BLF associated to $Tr(B)$

Proof: Due to our definition of the translation that are made in accordance to the definitions 10 and 11, the realized goals in any K-valid-BLFSW are the same as the realized goals of the K-valid translated BLF. Hence the result. □

Remark 1

- *Due to the presence of weights the BLFSW allows to use a more refined dominance relation on candidates, namely $\preceq_{BW}$. Hence the BLFSW is more expressive than the BLF.*
- *A BLF is a particular case of a BLFSW with no support nor weights. Hence it is easy to translate a BLF $B = (\mathcal{P}, \mathcal{I}, pol, \preceq$ into a BLFSW $B' = (\mathcal{P}, \mathcal{I}, \varnothing, pol, \preceq, w)$ where w is the weight function s.t. $\forall i \in \mathcal{I}, w(i) = 1$.*

Proposition 2 *The translation of a BLFSW to a BLF is exponential in the worse case situation.*

Proof: This result is simply due to the necessity in the translation to compute the power set of the supports for each DP. □

The previous propositions and remarks allows us to conclude that it is interesting to allow for supports and weights in the BLF framework, since a BLFSW is more expressive and more compact. The next section answers the question of the validity of this formalism by relating it to a standard qualitative decision theory, namely possibility theory.

4.3 Possiblistic Interpretation of a BLFSW

In this section we show how the decision principles, inhibitors and weights of a BLFSW can be interpreted in terms of possibility theory. We follow up the work of [9] in order to build DPs from uncertain knowledge expressed under the form of a possibility distribution on worlds and from preferences expressed as a utility of some goals. In this process, a DP (φ, g) is viewed as a defeasible rule saying that "if φ hols then a priori g is achieved" , and we explain how weighted inhibitions and supports can be defined according to this view.

Background on Possibility Theory and Defaults

In [17], possibility theory is introduced as a basis for qualitative decision theory. The author relate the expected pay-off $u(x)$ of a situation x to a preference relation $\preceq$ over situations s.t. $x \preceq y$ iff $u(x) \geq u(y)$. In presence of uncertainty, i.e., when situations are not precisely known, the belief state about what is the actual situation is represented by a possibility distribution π. The theory of possibility is a qualitative setting first introduced by Zadeh [26] and further developed by Dubois and Prade in [15]. It is qualitative in the sense that the only operations required are $\max$, $\min$ and order-reversing operations. However, numbers in the scale [0,1] are often used for convenience but the exact values of the numbers are not meaningful, it is only their order in the scale that is taken into account.

Two measures π and N are defined for representing the plausibility of a situation: $\pi(x) \leq \pi(x')$ means that it is at least as plausible for x' to be the actual situation as for x to be it. $\pi(x) = 0$ means impossibility, $\pi(x) = 1$ means that x is unsurprising or normal. The state of total ignorance is represented by a possibility distribution where any situation is totally possible ($\forall x, \pi(x) = 1$). In order to reason on formula, the possibility measure Π evaluates how unsurprising a formula is, hence $\Pi(\varphi) = 0$ means that φ is bound to be false. The necessity measure is its dual defined by $N(\varphi) = 1 - \Pi(\neg\varphi)$: $N(\varphi) = 1$ means that φ is bound to be true. The definition of N from a possibility distribution π is given by: $N(\varphi) = min_{\omega \models \neg\varphi}(1 - \pi(\omega))$, it expresses that a formula is all the more necessary as its counter models are less plausible.

In [18], the authors show that the utility of a decision d can be evaluated by combining the plausibilities $\pi(x)$ of the states x in which d is made and the utility $u(d(x))$ of the possible resulting state $d(x)$ after d, where $u(d(x))$ represents the satisfaction to be in the precise situation $d(x)$ (it is equal to the membership degree to the fuzzy set of preferred situations). The pessimistic criterion has been first introduced by Whalen [24] and leads to a pessimistic utility level of a decision d defined as follows: $u_{pes}(d) = \inf_{x \in X} \max(1 - \pi(x), u(d(x)))$. The optimistic criterion has been first proposed by Yager [25] and is defined by: $u_{op}(d) = \sup_{x \in X} \min(\pi(x), u(d(x)))$.

In possibilistic decision theory, the scales for possibilities and utilities are the same, hence, commensurable. In our proposal the commensurability of the two scales is not required: we do not aggregate possibilities and utilities, we rather use a kind of chance constraint approach in which they are dealt with separately.

Since a decision principle (DP) represents a defeasible reason to believe that some goal is achieved, we also need to recall some basics about handling defeasible rules in a possibilistic setting. A defeasible rule is a compact way to express a general rule without mentioning every exception to it. In a BLF the exceptions to a decision principle are its inhibitors. The conditional possibility measure denoted $\Pi(\varphi|\psi)$ is the possibility that φ holds in the worlds where ψ holds. It is related to the conditional possibility distribution as follows: $\Pi(\varphi|\psi) = \max_{\omega \models \varphi} \pi(\omega|\psi)$. A default rule $a \rightsquigarrow b$ translates, in the possibility theory framework, into the constraint $\Pi(a \wedge b) > \Pi(a \wedge \neg b)$ which expresses that having b true is strictly more possible than having it false when a is true [22]. Note that the constraint $\Pi(a \wedge b) > \Pi(a \wedge \neg b)$ is equivalent to $N(a \wedge b) > 0$. Hence, if we know a and we search for a conclusion which satisfies the constraint $N() > 0$ then a solution is b. In this sense, decision principles are related to chance constraints in quantitative optimization problem. In this article, we will use the min conditioning ($|_{min}$) since we are interested in qualitative decision problem.

Interpreting a BLFSW in Possibility Theory

This section is devoted to give an interpretation of Support/Inhibitor and strength of a DP in a possibilistic setting. This will allow the designer of a Decision System to move from one formalism to another in order to check the accuracy of his proposed model. In addition, the theory of possibilities is recognized as a theory taking into account uncertainty and qualitative reasoning, so showing that there is a translation from a possibilistic representation of uncertainty and preferences to a BLFSW increases the validity of this framework. The BLFSW is able to take into account the degree of certainty of a DP which is not possible in a classical BLF. Nevertheless the possibilistic meaning of a DP and an inhibitor in a BLFSW are the same than those found for a BLF in [9]. First, we restate these two definitions, we have modified the DP interpretation in order to enforce a DP to be informative, i.e., the DP (φ, g) is well defined if the necessity of the goal increases when φ holds, this mean that $N(g|\varphi)$ should be strictly greater than $N(g)$.

Definition 14 (Π-DP) *Given a possibility measure Π, a Π-DP $p = (\varphi, g)$ is s.t.*

$$N(g|\varphi) > N(g) \geq 0$$

In other words, the DP is the piece of knowledge which increases the certainty that the goal is realized. In the same way we can interpret the notion of inhibitor and support in possibility theory: an inhibitor ψ makes the default rules $\varphi \rightsquigarrow g$ no more valid in such a way that we are no more sure that g will be realized when φ and ψ hold together.

Definition 15 (Π-Inhibitor) *[[9]] Given a possibility measure Π, the pair (ψ, p) is a Π-Inhibitor of the DP $p = (\varphi, g)$ if*

$$N(g|\varphi \wedge \psi) = 0$$

In contrast, the support increases the certainty of the default rules. So when the support ψ holds, we are more sure that g will be realized.

Definition 16 (Π-Support) *Given a possibility measure Π, the pair (ψ, p) is a Π-Support of the DP $p = (\varphi, g)$ if*

$$N(g|\varphi \wedge \psi) > N(g|\varphi)$$

Moreover to complete the interpretation of a BLFSW in possibility theory we need to define the global strength $\alpha(p)$ of a DP p in possibilistic terms.

Definition 17 (Π-weight) *Given a possibility measure Π and a weight function w. w is Π-weight function iff for all possible K-valid BLFSW $(\mathcal{P}_K,\ \mathcal{I}_K,\ \mathcal{S}_K,\ pol,\ \preceq,\ w_K\ )$ and $\forall p = (\varphi, g), p' = (\varphi', g') \in \mathcal{P}_K$*

- $\alpha(p) < 0$ *iff* $N(g \,|\, \varphi \bigwedge_{\psi \in \mathcal{I}_K(p) \cup \mathcal{S}_K(p)} \psi) = 0$
- $\alpha(p) \geq 0$ *iff* $N(g \,|\, \varphi \bigwedge_{\psi \in \mathcal{I}_K(p) \cup \mathcal{S}_K(p)} \psi) > 0$
- $\alpha(p) \geq \alpha(p') \geq 0$ *iff* $N(g \,|\, \varphi \bigwedge_{\psi \in \mathcal{I}_K(p) \cup \mathcal{S}_K(p)} \psi) \geq N(g' \,|\, \varphi' \bigwedge_{\psi \in \mathcal{I}_K(p') \cup \mathcal{S}_K(p')} \psi) > 0$

In other words, the activation level $\alpha(p)$ of $p = (\varphi, g)$ defined in Definition 10 should reflect the certainty about the rule $\varphi \rightsquigarrow g$ and should behave as stated in Definition 10: a negative activation level means that the default rule does not hold in presence of all its supports and inhibitors, a strictly positive one that means that the goal is all the more likely to be achieved that the level is high, two distinct positive activation levels should be ranked accordingly to the two necessities of the DPs. This last point will allow us to rank order candidates more precisely. Using the definitions above we are now in position to define a Π-BLFSW.

Definition 18 (Π-BLFSW) *A BLFSW $B = (\mathcal{P},\ \mathcal{I},\ \mathcal{S},\ pol,\ \preceq,\ w)$ is a Π-BLFSW iff there exists a possibility distribution π over Ω and a utility function u on the set of goals literals LIT_G, such that*

- $\forall p = (\varphi, g) \in \mathcal{P}$, $u(g) \neq 0$
- $\forall g \in LIT_G$, $pol(g) = \oplus$ *iff* $u(g) > 0$
- $\forall g, g' \in LIT_G$, $u(g) \leq u(g')$ *iff* $g \preceq g'$
- *for all consistent knowledge base K, $\forall g \in Lit_G$ s.t. $u(g) \neq 0$, $\forall \omega \in \Omega$, $\pi(g|\omega)$ satisfies the constraints of definitions 14, 15, 16 and 17*

Intuitively, in a Π-BLFSW the polarities and importances of the goals are based on a utility function and the weights on supports and inhibitors of DPs are consistent with the necessities of the default rules associated to DPs.

Thanks to this last definition, the designer can check whether her BLFSW is a Π-BLFSW hence whether it is coherent wrt a classical qualitative theory of uncertainty. If the possibility distribution does not seem realistic to the designer, she should modify it and convert it into a modified BLFSW (which summarizes it).

Remark 2 *It may happen that a designer wants to distinguish between the strengths of two DPs $p_1 = (\varphi_1, g_1)$ and $p_2 = (\varphi_2, g_2)$ because he knows that $N(g_1|\varphi_1) > N(g_2|\varphi_2)$. In order to do that, she may use the notion of support, by adding a support $s_1 = (\varphi_1, p_1)$. In that case, $\alpha_{p_1} = w(s_1)$ is necessarily greater than $\alpha_{p_2} = 0$.*

The following proposition shows the relation between the weight associated to a goal in a K-valid BLFSW and its necessity to hold wrt this knowledge base K. It enables us to say that the ranking[5] of a candidate described by K based on the goals it achieves will be the same in a Π-BLFSW than the one obtained by computing the necessities of these goals knowing the information K about the candidate.

Proposition 3 *Given a possibility distribution π on the set of worlds Ω and a Π-BLFSW $B = (\mathcal{P}, \mathcal{I}, \mathcal{S}, pol, \preceq, w)$ built on π, for all consistent knowledge bases K and K':*

- *for all $g \in LIT_G$, $g \notin Real(B, K)$ iff $N(g|K) = 0$*
- *for all $g, g' \in LIT_G$ s.t. $g \in Real(B, K)$ and $g' \in Real(B, K')$ $w_K(g) > w_{K'}(g')$ iff $N(g|K) > N(g'|K')$*

From Possibility theory to BLFSW: Example

Let us suppose that the designer is able to give a possibility distribution over all possible worlds $\omega \in \Omega$ and to give the possibility that the goal is true or not in each world (Table.1). From Definition 14 and Tables 1 we can build the DPs of a BLFSW. For each goal, we check if we can generate a DP concluding it by checking Definition 14, on all the conjunctive formulas that can be built starting from a conjunction with only one litteral. Let us consider the goal "swim" s, if we know p hence $N(s|p) = 1 - \Pi(\neg s|p) = 1 - 0.4 = 0.6 > 0$ hence $(p, s) = P_1$ is a DP. If we suppose that we know w, $N(s|w) = 1 - \Pi(\neg s|w) = 1 - 1 = 0$ due to the world ω_{12} hence (s, w) is not a DP. Let us look for supports and inhibitors, we have $N(s|p \wedge f) = 0.8 > N(s|p)$, so due to definition 16, $S_1 = (f, P_1)$ is a support. P_1 have also an inhibitor since adding $\neg w$ we get $N(s|p \wedge \neg w) = 0$. $N(s|p \wedge f \wedge \neg w) = 0.7$ thus $S_2 = (f \wedge \neg w, P_1)$ is a support. Simply doing the same for the other two goals, we obtain $P_2 = (w, c), N(c|w) = 0.6$, $I_2 = (f, P_2), N(c|w \wedge f \wedge o) = 0$, $S_3 = (o, P_2), N(c|w \wedge o) = 0.7$,$I_3 = (\wedge f \wedge o, P_2), N(c|w \wedge f \wedge o) = 0$ $P_3 = (f, e), N(e|f) = 0.6$ and $I_4 = (o, P_3), N(e|f \wedge o) = 0$.

Let us now focus on the attribution of weights. The weights must satisfy all the constraints entailed by Definition 17. For instance, $w_{S_1} > w_{S_1} + w_{S_2} - w_{I_1} > 0$ and $0 \geq -w_{I_1}$. Note that if $N(s|p \wedge f \wedge \neg w) = N(s|p \wedge f) = 0.6$ then $w(f \wedge \neg w, P_1) = w(s|p \wedge \neg w)$. In that case the inhibitor $\neg w$ is cancelled by the support $f \wedge \neg w$.

There is an infinity of possible ways to settle the weights, for instance the ones given in example 3: $w_{p_1} = 0.4$, $w_{S_1} = 0.6$, $w_{S_2} = 0$, $w_{I_1} = 0.4$ satisfy the constraints.

5 Future work

Beginning in October 2018, I will work at LEMMA, Paris. Building upon the work done at IRIT, our research program centers around two major research phases. In the first phase, we plan to further enhance the uncertainty modeling capabilities of BLFs

[5] This ranking can be obtained with the relations $\preceq_{BiLexi}$ or $\preceq_{BB}$. A similar definition of dominance relation can be built in the possibilistic setting which would give the same results.

ω:					$\pi(\omega)$
ω_1:	p	w	f	o	0.3
ω_2:	p	w	f	$\neg o$	0.3
ω_3:	p	w	$\neg f$	o	1
ω_4:	p	w	$\neg f$	$\neg o$	1
ω_5:	p	$\neg w$	f	o	0.2
ω_6:	p	$\neg w$	f	$\neg o$	0.2
ω_7:	p	$\neg w$	$\neg f$	o	0.4
ω_8:	p	$\neg w$	$\neg f$	$\neg o$	0.4
ω_9:	$\neg p$	w	f	o	0.3
ω_{10}:	$\neg p$	w	f	$\neg o$	0.3
ω_{11}:	$\neg p$	w	$\neg f$	o	1
ω_{12}:	$\neg p$	w	$\neg f$	$\neg o$	1
ω_{13}:	$\neg p$	$\neg w$	f	o	0.3
ω_{14}:	$\neg p$	$\neg w$	f	$\neg o$	1
ω_{15}:	$\neg p$	$\neg w$	$\neg f$	o	1
ω_{16}:	$\neg p$	$\neg w$	$\neg f$	$\neg o$	0.4

ω	$(\pi(s\|\omega)$, $\pi(\neg s\|\omega))$	$(\pi(c\|\omega)$, $\pi(\neg c\|\omega))$	$(\pi(e\|\omega)$, $\pi(\neg e\|\omega))$
ω_1:	(1,0)	(1,1)	(0.2,1)
ω_2:	(1,0)	(0.8,1)	(1,0.4)
ω_3:	(1,0)	(1,0.3)	(0,1)
ω_4:	(1,0)	(1,0.4)	(0,1)
ω_5:	(1,0.3)	(0.2,1)	(0.2,1)
ω_6:	(1,0.3)	(0.1,1)	(1,0.4)
ω_7:	(1,1)	(0.8,1)	(0,1)
ω_8:	(1,1)	(0,1)	(0,1)
ω_9:	(0,1)	(1,1)	(0,1)
ω_{10}:	(0,1)	(0.8,1)	(1,0.4)
ω_{11}:	(0,1)	(1,0.3)	(0,1)
ω_{12}:	(0,1)	(1,0.4)	(0,1)
ω_{13}:	(0,1)	(0.2,1)	(0,1)
ω_{14}:	(0,1)	(0.1,1)	(1,0.4)
ω_{15}:	(0,1)	(0.2,1)	(0,1)
ω_{16}:	(0,1)	(0,1)	(0,1)

Table 1: Possibility distribution on worlds and goals

by incorporating into BLFs other sources of uncertainty in the decision making process. Then, we intend to integrate BLFs with important knowledge representation and reasoning formalisms involving conflict, dynamic interactions, aggregated preferences and self-interested agents. In the second phase, we will develop a Machine Learning implementation of BLFs via Artificial Neural Networks. We intend to build BLFs underlying the process of decision-making under uncertainty, by means of a neural network training procedure. We will use domain specific (such as economics or medicine) data sets. Beginning with BLFs as structures for qualitative decision-making under uncertainty, we will develop and further study BLFs in both theoretical and application / implementation directions. To that end, we have planned and ordered my PhD research into two inter-related and closely interacting components: the Decision-making and Reasoning Component and the Cognitive Component. We now describe these research components in detail.

5.1 Decision-Making and Reasoning Component

The over-arching theme of this research component is to seek interface and establish common dynamics and insights between BLFs and important knowledge representation formalisms. This research component is ordered into five inter-related research activities. Each activity is further divided into specific research tasks. First, we will further investigate uncertainty in the BLFs. Then, we will study and compare how BLFs in connection with the Argumentation Theory, Social Choice Theory, Game Theory and Belief Change. This plan of action is summarized in Figure 2.

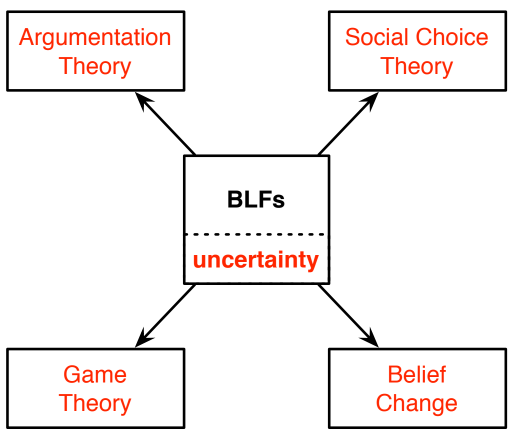


Figure 2: First component of the research project.

Uncertainty in features in BLFs

BLFs deal with qualitative decision-making under uncertainty. Currently in BLFs, the uncertainty of the agent about the decision situation is contained/encoded as default rules in decision principles [10, 9]. It arises out of the uncertainty in the agent's information about the achievement of a goal when a feature holds. But another important source of uncertainty is of specific relevance: the agent's information about the the features of individual decision alternatives (candidates) can itself be incomplete and uncertain.

Research Outcome: Characterize and integrate uncertainty about features of decision alternatives into the BLF structure.

BLFs and Argumentation Theory

This research activity is focused on exploring the common dynamics and connections between BLFs and argumentation theory. We have planned the following specific research tasks:

- *Argumentation and Decision Justification/Explanation:* In this task, we will investigate the decision justification/explanation dimension of argumentation as discussed, for example, in [49] and apply it to the decision-explanation function of BLFs where decision principles can be thought of as providing explanations for decisions.

- *BLFs as structured argumentation*: This research task is in line with the structured argumentation approach and connected to the previous task. Decision principles in BLFs can be thought of as "arguments" for believing that a goal is achieved when a feature holds. Structured argumentation deals with how arguments are constructed as a premise/conclusion "structure" using sound logical reasoning [43]. We will study the relationship between decision principles of BLFs and defeasible structured arguments [33].

- *BLFs as Dung AFs/ADFs/BADFs:* This research tasks is in line with the abstract argumentation approach. The use of supports and attacks as internal elements within an argumentation model is a basic feature of argumentation. Similarly, BLFs also model interactions between components as inhibitors and supports. In addition to Dung style frameworks, more richer and expressive argumentation frameworks have been developed, for example *Abstract Dialectical Frameworks (ADFs)* [44, 45] and *Bipolar Abstract Dialectical Frameworks (BADFs)* [44]. Specifically, in Bipolar Abstract Dialectical Frameworks (BADFs) all the links between arguments are either "attacking" or "supporting". We will investigate how BLFs and their internal dynamics compare with these abstract argumentation formalisms.

- *BLFs with weights, weighted argumentation frameworks and Probabilistic Argumentation Frameworks:* We have introduced notions of strengths/weights being associated with inhibitors and supports in a BLF. Weighted argumentation frameworks have also been proposed which give arguments internal strengths [46] and allow weighted attacks [48, 47]. Furthermore, Probabilistic argumentation frameworks have been defined where probabilities are assigned to arguments and attacks. These probabilities "represent the likelihood of existence of a specific argument or defeat, and thus capture the uncertainties inherent in the argument system" [34]. We will study and compare the strengths/weights dynamic in BLFs with internal weights/attack weights concept in argumentation frameworks.

Research Outcome: This research activity explores the common dynamics of BLFs and argumentation theory so as to arrive at a fuller understanding of the connection between decision making under uncertainty and argumentation theory.

BLFs and Social Choice Theory

Social Choice Theory is the study of collective decision processes and procedures [51, 52, 53, 54, 58]. It deals with preference aggregation, collective decisions and judgments and voting mechanisms [56, 57, 55]. We will model a multi-agent, dynamic interaction scenario where multiple agents have their own individual BLFs representing their preferences etc. The goal is to build an aggregate / cumulative BLF which represents the information, aggregated preferences, interests and uncertainties of the multi-agent environment. This opens up the following specific research tasks:

- *Aggregate BLFs and modeling competition among agents*: every agent wants to have his uncertain decision principles included in the cumulative BLF? Every agent would likely want to include the most preferred and most certain of his decision principles to be included in the cumulative BLF.
- *Aggregate BLFs and modeling cooperation and coalitions among agents*: First, how can agents cooperate so that the cumulative BLF realizes their preferences to a higher degree than would be the case if they negotiated alone? Secondly, can we model a situation where two or more agents form a coalition in order to prevent a third agent's decision principles/preferences from being included in the cumulative BLF?

Research Outcome: This research activity extends the aggregate/cumulative BLF building and reasoning process to a multi-agent and dynamic setting with competition, cooperation and coalitions among agents.

BLFs and Game Theory

Game Theory has nowadays become a major research topic. The field finds its main scope of applications in economics, towards the modeling of competing agents, but is likewise extensively used in computer science, logic, biology, political science and psychology (see the books series [1, 2, 3, 4] for an outstanding survey of the field). Game theory concerns the study of conflict and cooperation between rational decision-making agents. Within this general framework, the field of *Interactive Epistemology* provides a formal approach to the general logic of knowledge and beliefs of multiple agents involved in interactive situations [5, 6]. *Epistemic Game Theory* studies the behavioral implications of epistemic hypotheses (e.g., various kinds of knowledge or beliefs of rationalities) in the specific context of games (see [7] and the references there). For instance, it is well-known that if the players have common knowledge of their respective rationalities, then they will iteratively remove the strategies that are strictly dominated, and play the game according to the strategies that survive to this process (common knowledge of rationality implies iterated strict dominance).

In epistemic game theory, the connection between the interactive epistemology and the game is made by means of *choice functions* for the players. For each player, its choice functions specifies the strategy that he or she will play in any of the possible worlds of the epistemic model. As things stand, the choice functions are atomic entities of the epistemic model. But BLFs could provide an explicit modeling of these latter.

Indeed, the *features* of BLFs would correspond to the different characteristic that are taken into consideration to model the possible worlds. The *goals* would correspond to the various strategies of the players. The *decision principles* would then correspond to the diverse strategies adopted by the players, depending on the possible world in which they are. The *ordering* of the decision principles would represent a preference relation between these epistemic-strategic features, and the *inhibitors* would model additional exception rules between these latter.

We intend to investigate this BLF approach to epistemic game theory. Within this enhanced framework, the choice functions are not anymore atomic components of the epistemic model, but are rather built upon the characteristics and goals of the possible worlds and agents, respectively. These considerations enables a more fined-grained modeling of the epistemic game-theoretic situations. We envision to formalize this enriched epistemic-game-theoretic framework, and subsequently study the strategic implications of epistemic hypotheses in this context.

Research Outcome: This research activity investigates a BLF approach to epistemic game theory by studying the strategic implications of epistemic hypotheses in this enriched context.

BLFs and Belief Change

Belief Change refers to the process of changing beliefs when a new piece of information is encountered [29, 30]. An agent's belief about the state of the world is represented by a set of sentences. Two important belief change operations can be considered:

- *revision* where the agent's belief about the existing state of the world changes.
- *update* where the agent's belief is "updated" with new information about a new state of the world.

This activity explores the dynamics of how a BLF behaves when encountered with new information. An important principle of the belief change process is *minimal information loss*: the agent gives up the minimal information required to incorporate the new information and maintain consistency. Belief change also involves the idea of *epistemic entrenchment* whereby the least entrenched of the beliefs (least important or least consequential) are given up in the revision process. We plan the following specific research tasks:

- Revising the decision principles of a BLF in presence of new information. The information loss must be minimal.
- Exploring the relationship between *epistemic entrenchment* and the notion of "vitality" of a decision principle in terms of how vital it is to the final outcome of the BLF. More particularly, causality can be studied inside a BLF in order to determine which argument was the most important for the decision outcome. This amounts to update the BLF with the fact that the argument is not there.
- This study may also us allow to compute the potential impact of any argument in order to know whether an information should be disclosed.

5.2 Cognitive Component: Learning Agent's Information

The theme of this component is to study and investigate the interface between Machine Learning and Decision making under uncertainty as encoded in BLFs. This component is divided into two research activities. First, we will develop a learning mechanism using Artificial Neural Networks where an agent will learn the information about the decision environment from data and use it to build the corresponding BLF. Secondly, we will implement and apply the model to specific real life situations like decision making in economic or medical environments.

Training and Learning

As things stand, we are assuming that the information contained in the BLF could either be one agent's information or consensus information of a number of agents. But we do not mention where the agent learns this information, we just assume that he has this information. We plan to investigate how the agent "learns" this information from which it constructs a decision scenario. Could this information arise from some coherent neural network computation? This would complete the decision-making scenario whereby the agent "learns" some information from a neural networks component, organizes that information into a BLF and then makes the decision using the BLF.

As explained previously, uncertainty is an inherent part of the decision making process. We will investigate how this uncertainty can be expressed in ANNs. As output, we expect the cognitive component to be able learn the decision making environment and build the corresponding BLF from this information. To that end, the information the agent should be able to learn should include:

- the features of the decision environment that characterize a decision alternative.
- the rules that help define the decision alternatives and their possibilities.
- the set of goals involved in the decision process.
- information that helps define the defeasible rules, inhibitors and supports.
- information that defines the importance of decision goals.
- information that defines the polarity (desirable/undesirable) of decision goals.

Application and Implementation

We will train our learning model using domain specific data sets. We want to do a DATA $\rightarrow$ BLF implementation. The data sets we will use would be ones which describe a decision situation confronting an agent and the decision the agent makes in a specific decision making domain. For example:

- Data sets from the financial / economic domain: We envision this data set to be a complex structure containing the information of an economic agent in, say, a decision involving whether to increase or decrease the production of an item given the relevant information. This information would reflect the complexity of the situation and could include, for example: volume being produced now, volume being produced by other economic actors, the demand from the market for the item, effect on production cost of increasing / decreasing the volume, the effect of increase / decrease in production cost on the profit margin, the profit margins for competitors and so forth. The data set that encodes this complex decision making situation would give rise to a similarly complex BLF which would be "learned" from the data set. A data set consisting of similar decision situations encoding consumer behavior is another possibility.
- Data sets from the medical domain: A data set which similarly encodes the decision making situation in a medical environment.

We will study extending this implementation and application component to include complex, multi-agent and interactive scenarios in light of theoretical insights from the Decision-making and Reasoning Component.

References

[1] R. Aumann and S. Hart, editors. Handbook of Game Theory with Economic Applications, volume 1. Elsevier, 1st edition, 1992.

[2] R. Aumann and S. Hart, editors. Handbook of Game Theory with Economic Applications, volume 2. Elsevier, 1st edition, 1994.

[3] R. Aumann and S. Hart, editors. Handbook of Game Theory with Economic Applications, volume 3. Elsevier, 1st edition, 2002.

[4] H. P. Young and S. Zamir, editors. Handbook of Game Theory with Economic Applications, volume 4. Elsevier, 1st edition, 2015.

[5] R. Aumann. Interactive epistemology I: Knowledge. International Journal of Game Theory, 28(3):263–300, 1999.

[6] R. Aumann. Interactive epistemology II: Probability. International Journal of Game Theory, 28(3):301–314, 1999.

[7] E. Dekel and M. Siniscalchi. Epistemic game theory. In Handbook of Game Theory with Economic Applications, volume 4, chapter 12, pages 619–702. Elsevier, 2015.

[8] Florence Dupin De Saint Cyr, Romain Guillaume. Towards a transparent deliberation protocol inspired from supply chain collaborative planning. In : International Conference on Information Processing and Management of Uncertainty in Knowledge-based Systems (IPMU 2014), Montpellier, France, Springer, CCIS 443, pp. 335–344, 2014.

[9] Florence Dupin de Saint-Cyr, Romain Guillaume. Analyzing a Bipolar Decision Structure through Qualitative Decision Theory. In : KI - Künstliche Intelligenz, Springer, Vol. 31 N. 1, pp. 53–62, March 2017.

[10] Florence Dupin de Saint-Cyr, Romain Guillaume. Group Decision Making in a Bipolar Leveled Framework. In. Principles and Practice of Multi-Agent Systems (PRIMA 2017), Springer, Lecture Notes in Computer Science, pp. 34–52, 2017.

[11] de Saint-Cyr F.D., Guillaume R. (2018) How Potential BLFs Can Help to Decide Under Incomplete Knowledge. In: Medina J., Ojeda-Aciego M., Verdegay J., Perfilieva I., Bouchon-Meunier B., Yager R. (eds) Information Processing and Management of Uncertainty in Knowledge-Based Systems. Applications. IPMU 2018. Communications in Computer and Information Science, vol 855. Springer, Cham

[12] JF. Bonnefon, D. Dubois, and H. Fargier. An overview of bipolar qualitative decision rules. In Preferences and Similarities, vol 504, CISM, pages 47–73. Springer, 2008.

[13] A. Charnes and W. Cooper. Chance-constrained programming. Management science, 6(1):73–79, 1959.

[14] Craig Boutilier, Toward a Logic for Qualitative Decision Theory, In The Morgan Kaufmann Series in Representation and Reasoning, edited by Jon Doyle, Erik Sandewall and Pietro Torasso, Morgan Kaufmann, 1994, Pages 75-86, ISSN 10469567, Principles of Knowledge Representation and Reasoning, ISBN 9781483214528.

[15] D. Dubois and H. Prade. Possibility theory. Plenum Press, New York, 1988.

[16] D. Dubois, J. L., and Prade, H. 1994. Possibilistic logic.Handbook of Logic for Artificial Intelligence and Logic Pro-gramming3:439?513.

[17] D. Dubois and H. Prade. Possibility theory: qualitative and quantitative aspects. In Quantified Representation of Uncertainty and Imprecision, pages 169–226. Kluwer Academic Publishers, 1998.

[18] D. Dubois, H. Prade, and R. Sabbadin. Decision-theoretic foundations of qualitative possibility theory. Euro. J. of Operational Research, 128(3):459–478, 2001.

[19] B. Liu. Theory and practice of uncertain programming. Springer, 2002.

[20] H. Raiffa. Decision Analysis: Introductory lectures on choices under uncertainty. AddisonWesley, Reading, Massachusetts, 1970.

[21] R. Reiter. On reasoning by default. In Proc. of the 1978 workshop on Theoret. issues in NLP, pages 210–218. Association for Computational Linguistics, 1978.

[22] H. Prade S. Benferhat, D. Dubois. Representing default rules in possibilistic logic. In 3rd Int. Conf. Principles Knowl. Rep. and Reasoning (KR'92), pages 673–684, 1992.

[23] A. Tchangani, Y. Bouzarour-Amokrane, and F. Pérès. Evaluation model in decision analysis: Bipolar approach. Informatica, 23(3):461–485, 2012.

[24] T. Whalen. Decision making under uncertainty with various assumptions about available information. IEEE Trans. on Systems, Man and Cybern., 14(6):888–900, 1984.

[25] R. Yager. Possibilistic decision making. IEEE Transactions on Systems, Man and Cybernetics, 9:388–392, 1979.

[26] L.A. Zadeh. Fuzzy Sets as a Basis for a Theory of Possibility. Memorandum: Electronics Research Laboratory. College of Eng., University of California, 1977.

[27] Parul Agarwal, Dr. H.S. Nayal, Possibility Theory versus Probability Theory in Fuzzy Measure Theory, Int. Journal of Engineering Research and Applications, ISSN : 2248-9622, Vol. 5, Issue 5, (Part -2) May 2015, pp.37-43.

[28] Klir G.J. and Parviz B. (1992), Probability-possibility transformations: a comparison, International Journal of General Systems, 21 (3), pp. 291-310.

[29] Alchourrón, C.E., P. Gärdenfors, and D. Makinson, "On the Logic of Theory Change: Partial Meet Contraction and Revision Functions", Journal of Symbolic Logic, 50: 510–530, 1985.

[30] Katsuno, H. and A. O. Mendelzon, On the Difference between Updating a Knowledge Base and Revising it, In Belief Revision. Cambridge University Press. pp. 183-203 (1992).

[31] P. M. Dung, "On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games", Artificial Intelligence, vol. 77, no. 2, pp. 321–357, 1995.

[32] Baroni, P, Caminada, M, Giacomin, M. An introduction to argumentation semantics. The Knowledge Engineering Review, Vol. 00:0, 1?24, 2004, Cambridge University Press.

[33] Henry Prakken (2010) An abstract framework for argumentation with structured arguments, Argument and Computation, 1:2, 93-124, DOI: 10.1080/19462160903564592

[34] H. Li, N. Oren, and T. J. Norman, "Probabilistic argumentation frameworks", in Theories and Applications of Formal Argumentation, ser. Lecture Notes in Computer Science, S. Modgil, N. Oren, and F. Toni, Eds. Springer, vol. 7132, pp. 1–16, 2012.

[35] T. Bench-Capon and P. Dunne, "Argumentation in artificial intelligence", Artificial Intelligence, vol. 171, pp. 701–729, 2007.

[36] P. Besnard and A. Hunter, Elements of Argumentation, 1st ed. Cambridge, Massachusetts: The MIT Press, 2008.

[37] L. Amgoud and H. Prade, "Using arguments for making and explaining decisions", Artificial Intelligence, vol. 173, pp. 413–436, 2009.

[38] W. Ouerdane, Y. Dimopoulos, K. Liapis, and P. Moraitis, "Towards automating decision aiding through argumentation", Journal of Multi-Criteria Decision Analysis, vol. 18, pp. 289–309, 2011.

[39] D. Walton, "Justification of argument schemes", Australasian Journal of Logic, vol. 3, pp. 1–13, 2005.

[40] Baroni, P., Caminada, M., Giacomin, M.: An introduction to argumentation semantics. Knowl. Eng. Rev. 26(4), 365-410, (2011).

[41] Caminada, M.: Semi-stable semantics. In: Dunne, P.E., Bench-Capon, T.J.M. (eds.) Proceedings of the 1st Conference on Computational Models of Argument (COMMA). Frontiers in Artificial Intelligence and Applications, vol. 144, pp. 121-130. IOS Press (2006)

[42] Caminada, M., Gabbay, D.: A logical account of formal argumentation. Stud. Log. (Spec. Issue: New Ideas Argumentation Theory) 93 (2-3):109-145 (2009).

[43] Besnard, Philippe; Garcia, Alejandro; Hunter, Anthony; Modgil, Sanjay; Prakken, Henry; Simari, Guillermo; Toni, Francesca. (2014). Introduction to structured argumentation. Argument and Computation.

[44] Gerhard Brewka and Stefan Woltran. Abstract dialectical frameworks. In Proc. KR 2010, pages 102-111. AAAI Press, 2010.

[45] Gerhard Brewka and Stefan Ellmauthaler and Hannes Strass, Johannes Peter Wallner and Stefan Woltran,Abstract Dialectical Frameworks Revisited, Proceedings of the Twenty-Third international joint conference on Artificial Intelligence, 978-1-57735-633-2, Beijing, China, pp. 803-809, 2013, AAAI Press.

[46] Leila Amgoud , Jonathan Ben-Naim , Dragan Doder , Srdjan Vesic. Acceptability Semantics for Weighted Argumentation Frameworks. Proceedings of the Twenty-Sixth International Joint Conference on Artificial Intelligence (IJCAI-17).

[47] Sylvie Coste-Marquis, Sebastien Konieczny, Pierre Marquis, Mohand Akli Ouali. Weighted Attacks in Argumentation Frameworks, Principles of Knowledge Representation and Reasoning: Proceedings of the Thirteenth International Conference, KR 2012, Rome, Italy, June 10-14, 2012.

[48] Dunne, P.; Hunter, A.; McBurney, P.; Parsons, S.; and M. Woolridge, M. 2011. Weighted Argumentation Systems: Basic Definitions, Algorithms and Complexity Results. Artificial Intelligence 175(2):457?486.

[49] Leila Amgoud, Henri Prade, Using arguments for making and explaining decisions, Artificial Intelligence, Volume 173, Issues 3?4, 2009, Pages 413-436, ISSN 0004-3702.

[50] J. Schmidhuber. Deep learning in neural networks: An overview. Neural Networks, 61:85?117, 2015

[51] Arrow, K., 1951/1963, Social Choice and Individual Values. New York: Wiley.

[52] Austen-Smith, D. and J. S. Banks. Information Aggregation, Rationality, and the Condorcet Jury Theorem. The American Political Science Review Vol. 90, No. 1 (Mar., 1996), pp. 34-45.

[53] Black, D., 1948. On the Rationale of Group Decision-Making. Journal of Political Economy, 56: 23-34, 1948.

[54] Bossert W., Weymark J.A. (2004) Utility in Social Choice. In: Barberà S., Hammond P.J., Seidl C. (eds) Handbook of Utility Theory. Springer, Boston, MA.

[55] Brandt, F., V. Conitzer, and U. Endriss. Computational Social Choice. Multiagent Systems. G. Weiss (ed.). Cambridge, MA: MIT Press, pp. 213-283, 2013.

[56] Dietrich, F. Judgment Aggregation: (Im)Possibility Theorems. Journal of Economic Theory, 126: 286-298, 2006.

[57] Gilboa, I., D. Samet, and D. Schmeidler. Utilitarian Aggregation of Beliefs and Tastes. Journal of Political Economy, 112: 932-938, 2004.

[58] McLean, I. and A. B. Urken (eds.), 1995, Classics of Social Choice. Ann Arbor: University of Michigan Press.

[59] K. Arrow, L. Hurwicz, An optimality criterion for decision-making under ignorance, in: J.L. Ford, C.F. Carter (Eds.), Uncertainty and Expectations in Economics, Basil Blacwell and Mott Ltd, 1972.

[60] R. Brafman, M. Tennenholtz, On the foundations of qualitative decision theory, in: Proc. AAAI-96, Portland, OR, 1996, pp. 1291-1296.

[61] D. Dubois, H. Fargier, P. Perny, H. Prade, Qualitative decision theory: From Savage's axioms to non-monotonic reasoning, J. ACM 49 (4) (2002) 455-495.

[62] Besnard, P., Hunter, A.: Elements of Argumentation. MIT Press (2008)

[63] Amgoud, L., and Ben-Naim, J. 2018. Evaluation of Argu-ments in Weighted Bipolar Graphs.International Journal ofApproximate Reasoning.

[64] Amgoud, L., Maudet, N., Parsons, S.: Arguments, dialogue and negotiation. In: European Conf. on Artificial Intelligence (ECAI 2000), IOS Press (2000) 338-342.