Characterizing change in abstract argumentation systems

Pierre Bisquert,
Claudette Cayrol,
Florence Dupin de Saint-Cyr,
M-Christine Lagasquie-Schiex

IRIT, Université Paul Sabatier,
118 route de Narbonne, 31062 Toulouse, France
{bisquert,ccayrol,bannay,lagasq}@irit.fr

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Abstract

An argumentation system can undergo changes (addition/removal of arguments/interactions). At an abstract level, we propose a typology to classify the different properties describing a change operation. This typology reflects the evolution of three features:

- the set of extensions in Dung's sense (*e.g.*, the set of extensions is empty before the change and not empty after the change),
- the sets of accepted arguments (*e.g.*, all the arguments skeptically accepted before the change are still skeptically accepted after the change) and
- the status of some given argument (*e.g.*, an accepted argument may become rejected after the change).

Then, an important issue is to provide characterizations for these properties: *i.e.* conditions on the argumentation system and on the change operation that are necessary or sufficient to guarantee that the properties are satisfied.

So, in this paper, we present this typology and the characterization results obtained either directly or by using an approach based on a notion of duality. Our results are twofold, they can be considered as a guide for selecting the change operation to perform in order to obtain a desired property on an argumentation system and they may also be used as a tool for predicting the result of a change operation in a given context.

Contents

1 Introduction

The main feature of argumentation framework is the ability to deal with incomplete and / or contradictory information, especially for reasoning (Dung 1995; Amgoud and Cayrol 2002). Moreover, argumentation can be used to formalize dialogues between several agents by modeling the exchange of arguments in, *e.g.*, negotiation between agents (Amgoud *et al.* 2000). An argumentation system (AS) consists of a collection of arguments interacting with each other through a relation reflecting conflicts between them, called *attack*. The issue of argumentation is then to determine "acceptable" sets of arguments (*i.e.*, sets able to defend themselves collectively while avoiding internal attacks), called *"extensions"*, and thus to reach a coherent conclusion. Another form of analysis of an AS is the study of the particular status of each argument, this status is based on membership (or non-membership) of the extensions. Formal frameworks have greatly eased the modeling and study of AS. In particular, the framework of (Dung 1995) allows to completely abstract the "concrete" meaning of the arguments and relies only on binary interactions that may exist between them. This approach enables the user to focus on other aspects of argumentation, including its dynamic side. Indeed, in the course of a discussion or due to the acquisition of new pieces of information, an AS can undergo changes such as the addition of a new argument or the removal of an argument considered as illegal. Thus, it is interesting to study these changes and to characterize them by giving properties describing a change operation and by providing conditions under which these properties hold. Moreover, the study of the links between addition and removal through the concept of duality is a way to complete the characterization of removal through the work previously done on addition, and conversely. The following example shows that some knowledge about duality could help to benefit from known results:

> *Mr Pink knows that one simple argument could defeat Mr White's argumentation, but this argument is lacking. Another way to win could be to remove one of Mr White's arguments (e.g. by doing an objection). Unfortunately, Mr Pink does not know the consequences of this removal.*

Although the research on dynamics of AS is growing (Boella *et al.* 2009a; 2009b; Baumann and Brewka 2010; Moguillansky *et al.* 2010; Liao *et al.* 2011), the removal of argument has so far been little considered. A realistic example of the use of removal may nevertheless be found in (Bisquert *et al.* 2011) and shows that studying argument re-

moval is at least as important as studying argument addition. *A fortiori*, the relationship between addition and removal of argument has not, to our knowledge, been treated so far.
This paper presents a synthesis about change characterization based on already published papers (Cayrol *et al.* 2010; Bisquert *et al.* 2012b; 2012a) and including new results. A brief background is given in Section 2. Section 3 displays properties of a change operation reflecting possible modifications of an AS. A direct characterization of these properties is given in Section 4 to 6. Then various notions of duality are presented in Section 7 and are used for enriching the characterization (see Section 8 to 9). Section 10.1 describes the possible use of these characterization results. Finally, Section 10.2 concludes and suggests perspectives of our work. In Appendix A, the reader will find tables synthesizing all characterization results and the proofs of the new direct results are given in Appendix B.

2 Background

We give here some background concerning argumentation systems (Section 2.1) as well as change operations (Section 2.2).

2.1 Dung's abstract argumentation system

The work presented in this paper uses the framework of (Dung 1995):

Def. 1 (Argumentation System) *An* argumentation system (AS) *is a pair* $\langle \mathbf{A}, \mathbf{R} \rangle$*, where* $\mathbf{A}$ *is a finite nonempty set of arguments and* $\mathbf{R}$ *is a binary relation on* $\mathbf{A}$*, called* attack relation*. Let* $A, B \in \mathbf{A}$*,* $A\mathbf{R}B$ *means that* A *attacks* B*.* $\langle \mathbf{A}, \mathbf{R} \rangle$ *will be represented by an* argumentation graph $\mathcal{G}$ *whose vertices are the arguments and whose edges correspond to* $\mathbf{R}$[1]*.*

Let $A \in \mathbf{A}, B \in \mathbf{A}$, A *indirectly attacks* B iff[2] there exists an odd-length path from A to B in $\mathcal{G}$. In this paper, we also use the following notions based on the attack relation, namely the attack of an argument to - and from - a set:

Def. 2 (Attack from and to a set) *Let* $A \in \mathbf{A}$ *and* $\mathcal{S} \subseteq \mathbf{A}$*.* $\mathcal{S}$ *attacks* A *iff* $\exists X \in \mathcal{S}$ *such that* $X\mathbf{R}A$*.* A *attacks* $\mathcal{S}$ *iff* $\exists X \in \mathcal{S}$ *such that* $A\mathbf{R}X$*.*

[1] In this paper, we use freely $\langle \mathbf{A}, \mathbf{R} \rangle$ or $\mathcal{G}$ to refer to an AS. Similarly, if there is no ambiguity, we use without distinction $\mathbf{A}$ and $\mathcal{G}$.

[2] iff = if and only if.

The acceptable sets of arguments ("extensions") are determined according to a given semantics which is usually based on the following concepts:

Def. 3 (Conflict-freeness, defense, admissibility) *Let $A \in \mathbf{A}$ and $\mathcal{S} \subseteq \mathbf{A}$. $\mathcal{S}$ is* conflict-free *iff there does not exist $A, B \in \mathcal{S}$ such that $A\mathbf{R}B$. $\mathcal{S}$* defends *an argument A iff each attacker of A is attacked by an argument of $\mathcal{S}$. The set of the arguments defended by $\mathcal{S}$ is denoted by $\mathcal{F}(\mathcal{S})$; $\mathcal{F}$ is called the* characteristic function *of $\langle \mathbf{A}, \mathbf{R} \rangle$. More generally, $\mathcal{S}$* indirectly defends *A iff $A \in \bigcup_{i \geq 1} \mathcal{F}^i(\mathcal{S})$. $\mathcal{S}$ is an* admissible *set iff it is conflict-free and it defends all its elements.*

The set of extensions of $\langle \mathbf{A}, \mathbf{R} \rangle$ is denoted by $\mathbf{E}$ (with $\mathcal{E}_1, \ldots, \mathcal{E}_n$ standing for the extensions). In this article, we restrict our study to the most traditional semantics proposed by (Dung 1995):

Def. 4 (Acceptability semantics) *Let $\mathcal{E} \subseteq \mathbf{A}$, $\mathcal{E}$ is a* preferred extension *iff $\mathcal{E}$ is a maximal admissible set (with respect to set inclusion $\subseteq$). $\mathcal{E}$ is* the only grounded extension *iff $\mathcal{E}$ is the least fixed point (with respect to $\subseteq$) of the characteristic function $\mathcal{F}$. $\mathcal{E}$ is a* stable extension *iff $\mathcal{E}$ is conflict-free and attacks any argument not belonging to $\mathcal{E}$.*

The status of an argument is determined by its membership of the extensions of the selected semantics: *e.g.*, an argument can be "skeptically accepted" (resp. "credulously") if it belongs to all the extensions (resp. at least to one extension) and be "rejected" if it does not belong to any extension.

Prop. 1 *(Dung 1995)*

1. *There is at least one preferred extension, always a unique grounded extension, while there may be zero, one or several stable extensions.*
2. *Each admissible set is included in a preferred extension.*
3. *Each stable extension is a preferred extension, the converse is false.*
4. *The grounded extension is included in each preferred extension.*
5. *Each argument which is not attacked belongs to the grounded extension (hence to each preferred and to each stable extension).*
6. *If $\mathbf{R}$ is finite, then the grounded extension can be computed by iteratively applying the function $\mathcal{F}$ from the empty set.*
7. *If $\mathbf{A}$ is non empty, then a stable extension is always non empty.*

Prop. 2 *(Dunne and Bench-Capon 2001; 2002)*

1. *If $\mathcal{G}$ contains no cycle, then $\langle \mathbf{A}, \mathbf{R} \rangle$ has a unique preferred extension, which is also the grounded extension and the unique stable extension.*
2. *If $\{\}$ is the unique preferred extension of $\langle \mathbf{A}, \mathbf{R} \rangle$, then $\mathcal{G}$ contains an odd-length cycle.*

3. *If $\langle \mathbf{A}, \mathbf{R} \rangle$ has no stable extension, then $\mathcal{G}$ contains an odd-length cycle.*
4. *If $\mathcal{G}$ contains no odd-length cycle, then preferred and stable extensions coincide.*
5. *If $\mathcal{G}$ contains no even-length cycle, then $\langle \mathbf{A}, \mathbf{R} \rangle$ has a unique preferred extension.*

2.2 Dynamics in argumentation systems

We rely on the work of (Cayrol *et al.* 2010) which have distinguished four change operations; in this paper, we only use the operations of addition and removal of an argument and its interactions:

Def. 5 (Change operations) *Let $\langle \mathbf{A}, \mathbf{R} \rangle$ be an AS, Z be an argument and $\mathcal{I}_z$ be a set of interactions concerning Z*

- Adding $Z \notin \mathbf{A}$ *and* $\mathcal{I}_z \not\subseteq \mathbf{R}$ *is a change operation, denoted by* $\oplus$, *providing a new AS such that:* $\langle \mathbf{A}, \mathbf{R} \rangle \oplus (Z, \mathcal{I}_z) = \langle \mathbf{A} \cup \{Z\}, \mathbf{R} \cup \mathcal{I}_z \rangle$.
- Removing $Z \in \mathbf{A}$ *and* $\mathcal{I}_z \subseteq \mathbf{R}$ *is a change operation, denoted by* $\ominus$, *providing a new AS such that:* $\langle \mathbf{A}, \mathbf{R} \rangle \ominus (Z, \mathcal{I}_z) = \langle \mathbf{A} \setminus \{Z\}, \mathbf{R} \setminus \mathcal{I}_z \rangle$.

We denote by $\mathcal{O}$ a change operation ($\oplus$ or $\ominus$)[3]*. The new AS $\langle \mathbf{A}', \mathbf{R}' \rangle$ obtained by the application of $\mathcal{O}$ will be represented by the argumentation graph $\mathcal{G}' = \mathcal{O}(\mathcal{G})$. Moreover, we assume that Z does not attack itself and $\forall (X, Y) \in \mathcal{I}_z$, we have either ($X = Z$ and $Y \neq Z$, $Y \in \mathbf{A}$) or ($Y = Z$ and $X \neq Z$, $X \in \mathbf{A}$). In case of removing, let us note that $\mathcal{I}_z$ is the set of all the interactions concerning Z in $\langle \mathbf{A}, \mathbf{R} \rangle$.*

The set of extensions of $\langle \mathbf{A}', \mathbf{R}' \rangle$ is denoted by $\mathbf{E}'$ (with $\mathcal{E}'_1, \ldots, \mathcal{E}'_n$ standing for the extensions). In this chapter, we assume that the semantics remains the same before and after any change operation. Note that a change operation is a non injective application (thanks to Def. 5, we know that $\forall \mathcal{G}, \mathcal{G}' = \mathcal{O}(\mathcal{G})$ is unique; however, for a given $\mathcal{G}'$, there may be several $\mathcal{G}$):

Ex. 1 *With $\mathcal{O} = \ominus$, three systems can be changed into $\mathcal{G}'$, such that $\mathcal{O}(\mathcal{G}_1) = \mathcal{O}(\mathcal{G}_2) = \mathcal{O}(\mathcal{G}_3) = \mathcal{G}'$ (see Table 1 which also gives the grounded extension of each system).*

The impact of a change operation will be studied through the notion of change property. A change property $\mathcal{P}$ can be seen as a set of pairs $(\mathcal{G}, \mathcal{G}')$, where $\mathcal{G}$ and $\mathcal{G}'$ are argumentation graphs:

Ex. 1 (cont'd) *Let $\mathcal{P}$ be the property defined by "$\mathcal{P}(\mathcal{G}, \mathcal{G}')$ holds iff any extension of $\mathcal{G}'$ is included in at least one extension of $\mathcal{G}$". Thus, $\mathcal{P}(\mathcal{G}_1, \mathcal{G}')$ does not hold while $\mathcal{P}(\mathcal{G}_2, \mathcal{G}')$ and $\mathcal{P}(\mathcal{G}_3, \mathcal{G}')$ hold.*

[3] The symbols $\oplus$ and $\ominus$ used here correspond to the symbols $\oplus_I^a$ and $\ominus_I^a$ of (Cayrol *et al.* 2010), where a stands for "argument" and I for "interactions", meaning that the operation concerns an argument and its interactions.

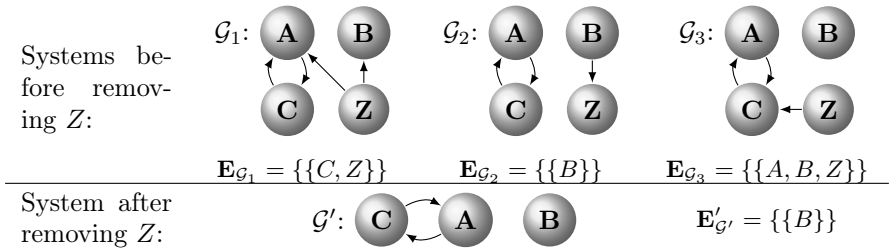

Table 1: On the non injective nature of the removal operation.

Def. 6 (Operation satisfying a property) *A change operation* $\mathcal{O}$ satisfies *a property* $\mathcal{P}$ *iff* $\forall \mathcal{G},\ \mathcal{P}(\mathcal{G}, \mathcal{O}(\mathcal{G}))$ *holds.*

Ex. 1 (cont'd) $\mathcal{P}(\mathcal{G}_1, \mathcal{G}')$ *does not hold. Thus,* $\mathcal{O} = \ominus$ *does not satisfy* $\mathcal{P}$*.*

3 Properties of change operations

For a change operation, there exist three classes of properties concerning:
- either the evolution of the set of extensions,
- or the evolution of the acceptability of a set of arguments,
- or the evolution of the status of a given argument.

3.1 Properties about the set of extensions

Change properties express structural modifications of an AS that are caused by a change operation. In this section, we focus on these modifications in order to obtain a clear and accurate classification. For that purpose, a new partition, inspired by the work of (Cayrol *et al.* 2010) and based on three possible cases of evolution of the set of extensions, has been defined:
- the *extensive* case, in which the number of extensions increases,
- the *restrictive* case, in which the number of extensions decreases,
- the *constant* case, in which the number of extensions remains the same.

For each case, numerous sub-cases are proposed and denoted by a letter (e for the *extensive* case, r for the *restrictive* case and c for the *constant* case) subscripted by the expression $\gamma - \gamma'$, where γ (resp. γ') describes the set of extensions before (resp. after) the change. Thus γ and γ' can be:
- $\varnothing$: the set of extensions is empty,
- $1e$: the set of extensions is reduced to one empty extension,

- $1ne$: the set of extensions is reduced to one non-empty extension,
- k (resp. j): the set of extensions contains k (resp. j) extensions such that $1 < k$ (resp. $1 < j < k$: note that the symbol j is used only if the symbol k belongs also to the expression $\gamma - \gamma'$).

For instance, the notation $e_{\varnothing-1ne}$ means that the change increases the number of extensions (so it is an *extensive* case), with no initial extension ($\varnothing$) and one non-empty final extension ($1ne$).

Nevertheless, some special sub-cases of the *constant* case are denoted by another method since they are based on notions distinct from the emptiness or the number of the extensions; for these sub-cases, the subscript is replaced by a qualifier. For instance, the *c-conservative* case describes the case where the extensions remain unchanged after the change.

Note also that for the sake of clarity, we say that a change satisfying a property $\mathcal{P}$ is a "$\mathcal{P}$ *change*"; for example, a change that satisfies the *constant* property is said *constant* change.

Here is the formal definition of these changes. First, we study the case in which a change increases the number of extensions, called *extensive* change.

Def. 7 (Extensive change) *The change from $\mathcal{G}$ to $\mathcal{G}'$ is* extensive *iff* $|\mathbf{E}| < |\mathbf{E}'|$. *The sub-cases of* extensive *changes from $\mathcal{G}$ to $\mathcal{G}'$ are:*
1. $e_{\varnothing-1ne}$ *iff* $|\mathbf{E}| = 0$ *and* $|\mathbf{E}'| = 1$, *with* $\mathcal{E}' \neq \varnothing$.
2. $e_{\varnothing-k}$ *iff* $|\mathbf{E}| < |\mathbf{E}'|$, $|\mathbf{E}| = 0$ *and* $|\mathbf{E}'| > 1$.
3. e_{1e-k} *iff* $|\mathbf{E}| < |\mathbf{E}'|$ *and* $|\mathbf{E}| = 1$, *with* $\mathcal{E} = \varnothing$.
4. e_{1ne-k} *iff* $|\mathbf{E}| < |\mathbf{E}'|$ *and* $|\mathbf{E}| = 1$, *with* $\mathcal{E} \neq \varnothing$.
5. e_{j-k} *iff* $1 < |\mathbf{E}| < |\mathbf{E}'|$.

The *restrictive* change, in which a change decreases the number of extensions, is defined symmetrically to the *extensive* change.

Def. 8 (Restrictive change) *The change from $\mathcal{G}$ to $\mathcal{G}'$ is* restrictive *iff* $|\mathbf{E}| > |\mathbf{E}'|$. *The sub-cases of* restrictive *changes from $\mathcal{G}$ to $\mathcal{G}'$ are:*
1. $r_{1ne-\varnothing}$ *iff* $|\mathbf{E}| = 1$, *with* $\mathcal{E} \neq \varnothing$, *and* $|\mathbf{E}'| = 0$.
2. $r_{k-\varnothing}$ *iff* $|\mathbf{E}| > |\mathbf{E}'|$, $|\mathbf{E}| > 1$ *and* $|\mathbf{E}'| = 0$.
3. r_{k-1e} *iff* $|\mathbf{E}| > |\mathbf{E}'|$ *and* $|\mathbf{E}'| = 1$, *with* $\mathcal{E}' = \varnothing$.
4. r_{k-1ne} *iff* $|\mathbf{E}| > |\mathbf{E}'|$ *and* $|\mathbf{E}'| = 1$, *with* $\mathcal{E}' \neq \varnothing$.
5. r_{k-j} *iff* $1 < |\mathbf{E}'| < |\mathbf{E}|$.

The *constant* change corresponds to the case where the number of extensions remains unchanged and its sub-cases depend on the inclusions between the various possible extensions ($\mathcal{G}$ to $\mathcal{G}'$ and vice versa), emptiness of these extensions, ...

Def. 9 (Constant change) *The change from $\mathcal{G}$ to $\mathcal{G}'$ is* constant *iff* $|\mathbf{E}| = |\mathbf{E}'|$. *The sub-cases of* constant *changes from $\mathcal{G}$ to $\mathcal{G}'$ are:*

1. **c-conservative** *iff* $\mathbf{E} = \mathbf{E}'$.
2. c_{1e-1ne} *iff* $\mathbf{E} = \{\{\}\}$ *and* $\mathbf{E}' = \{\mathcal{E}'\}$, *with* $\mathcal{E}' \neq \varnothing$.
3. c_{1ne-1e} *iff* $\mathbf{E} = \{\mathcal{E}\}$, *with* $\mathcal{E} \neq \varnothing$ *and* $\mathbf{E}' = \{\{\}\}$.
4. **c-expansive** *iff* $\mathbf{E} \neq \varnothing$ *and* $|\mathbf{E}| = |\mathbf{E}'|$ *and* $\forall \mathcal{E}_i \in \mathbf{E}, \exists \mathcal{E}'_j \in \mathbf{E}', \varnothing \neq \mathcal{E}_i \subset \mathcal{E}'_j$ *and* $\forall \mathcal{E}'_j \in \mathbf{E}', \exists \mathcal{E}_i \in \mathbf{E}, \varnothing \neq \mathcal{E}_i \subset \mathcal{E}'_j$.
5. **c-narrowing** *iff* $\mathbf{E} \neq \varnothing$ *and* $|\mathbf{E}| = |\mathbf{E}'|$ *and* $\forall \mathcal{E}_i \in \mathbf{E}, \exists \mathcal{E}'_j \in \mathbf{E}', \varnothing \neq \mathcal{E}'_j \subset \mathcal{E}_i$ *and* $\forall \mathcal{E}'_j \in \mathbf{E}', \exists \mathcal{E}_i \in \mathbf{E}, \varnothing \neq \mathcal{E}'_j \subset \mathcal{E}_i$.
6. **c-altering** *iff* $|\mathbf{E}| = |\mathbf{E}'|$ *and it is neither* c-conservative, *nor* c_{1e-1ne}, *nor* c_{1ne-1e}, *nor* c-expansive, *nor* c-narrowing.

Def. 9.1, 9.2, 9.3 and 9.6 are fairly straightforward. Def. 9.4 states that a *c-expansive* change is a change where all the extensions of $\mathcal{G}$, which are not initially empty, are increased by some arguments. A *c-narrowing* change, according to Def. 9.5, is a change where all the extensions of $\mathcal{G}$ are reduced by some arguments without becoming empty.

3.2 Properties about the acceptability of a set of arguments

A change can also have an impact on the acceptability of sets of arguments. For instance, in a dialog, it would be interesting to know if the addition or the removal of an argument modifies the acceptability of the arguments previously accepted. We speak of "monotony from $\mathcal{G}$ to $\mathcal{G}'$" when every argument accepted *before* the change is still accepted *after* the change, *i.e.*, no accepted argument is lost and there is a (not necessarily strict) *expansion* of acceptability. A second case, referred as "monotony from $\mathcal{G}'$ to $\mathcal{G}$", occurs when every argument accepted *after* the change was already accepted *before* the change, *i.e.*, no new accepted argument appears and there is a (not necessarily strict) *restriction* of acceptability.

Def. 10 (Simple monotony)

1. *The change from* $\mathcal{G}$ *to* $\mathcal{G}'$ *satisfies the property of* simple expansive monotony *iff* $\forall \mathcal{E}_i \in \mathbf{E}, \exists \mathcal{E}'_j \in \mathbf{E}',\ \mathcal{E}_i \subseteq \mathcal{E}'_j$.
2. *The change from* $\mathcal{G}$ *to* $\mathcal{G}'$ *satisfies the property of* simple restrictive monotony *iff* $\forall \mathcal{E}'_j \in \mathbf{E}', \exists \mathcal{E}_i \in \mathbf{E},\ \mathcal{E}'_j \subseteq \mathcal{E}_i$.

These properties are refined into *credulous expansive monotony* and *credulous restrictive monotony* when acceptability is restricted to credulous acceptability, and into *skeptical expansive monotony* and *skeptical restrictive monotony* when skeptical acceptability is considered:

Def. 11 (Credulous and skeptical monotonies)

1. *The change from* $\mathcal{G}$ *to* $\mathcal{G}'$ *satisfies the property of* credulous expansive monotony *iff :* $\bigcup_{1 \leq i \leq |\mathbf{E}|} \mathcal{E}_i \subseteq \bigcup_{1 \leq j \leq |\mathbf{E}'|} \mathcal{E}'_j$.

2. *The change from $\mathcal{G}$ to $\mathcal{G}'$ satisfies the property of* credulous restrictive monotony *iff :* $\bigcup_{1\leq j\leq|\mathbf{E}'|} \mathcal{E}'_j \subseteq \bigcup_{1\leq i\leq|\mathbf{E}|} \mathcal{E}_i$.
3. *The change from $\mathcal{G}$ to $\mathcal{G}'$ satisfies the property of* skeptical expansive monotony *iff :* $\bigcap_{1\leq i\leq|\mathbf{E}|} \mathcal{E}_i \subseteq \bigcap_{1\leq j\leq|\mathbf{E}'|} \mathcal{E}'_j$.
4. *The change from $\mathcal{G}$ to $\mathcal{G}'$ satisfies the property of* skeptical restrictive monotony *iff* $\bigcap_{1\leq j\leq|\mathbf{E}'|} \mathcal{E}'_j \subseteq \bigcap_{1\leq i\leq|\mathbf{E}|} \mathcal{E}_i$.

Some links between these properties are given in Prop 6, 7, Section 4.

3.3 Properties about the status of a given argument

A change operation about an argument Z can of course have an influence on Z, but also on other given arguments (particularly arguments that are attacked or defended by Z). Considering the influence on Z, there are only three possible cases that concern the establishment of the acceptability of Z (credulous, skeptical or not established at all) and they occur only when there is an *addition* of Z (if we remove Z, its acceptability is of course modified but it is obvious and irrelevant):

Def. 12 (Priority to recency) *The change from $\mathcal{G}$ to $\mathcal{G}'$ which adds Z satisfies* priority to recency *iff*

- *(*credulous-only priority to recency*)* $\exists \mathcal{E}'_j, \mathcal{E}'_l \in \mathbf{E}'$, $Z \in \mathcal{E}'_j$ *and* $Z \notin \mathcal{E}'_l$.
- *(*skeptical priority to recency*)* $\forall \mathcal{E}'_j \in \mathbf{E}', Z \in \mathcal{E}'_j$.

Considering the influence on a given argument X distinct from Z (so $X \in \mathcal{G} \cap \mathcal{G}'$), we identify several properties expressing the modification of the status of X when a change operation is done on the AS.

Def. 13 (Acceptability establisment) *Let $X \in \mathcal{G} \cap \mathcal{G}'$. The change from $\mathcal{G}$ to $\mathcal{G}'$* establishes acceptability *for X iff $\forall \mathcal{E}_i \in \mathbf{E}, X \notin \mathcal{E}_i$ and*

- *(*credulous-only acceptability establisment*)* $\exists \mathcal{E}'_j, \mathcal{E}'_l \in \mathbf{E}'$, $X \in \mathcal{E}'_j$ *and* $X \notin \mathcal{E}'_l$.
- *(*skeptical acceptability establisment*)* $\forall \mathcal{E}'_j \in \mathbf{E}', X \in \mathcal{E}'_j$.

Def. 14 (Acceptability removal) *Let $X \in \mathcal{G} \cap \mathcal{G}'$. The change from $\mathcal{G}$ to $\mathcal{G}'$* removes acceptability *for X iff $\forall \mathcal{E}'_j \in \mathbf{E}', X \notin \mathcal{E}'_j$ and*

- *(*credulous-only acceptability removal*)* $\exists \mathcal{E}_i, \mathcal{E}_k \in \mathbf{E}$, $X \in \mathcal{E}_i$ *and* $X \notin \mathcal{E}_k$.
- *(*skeptical acceptability removal*)* $\forall \mathcal{E}_i \in \mathbf{E}, X \in \mathcal{E}_i$.

The modification of the status of X can be less "drastic", *e.g.*, an argument can belong to some extension before the change, and become member of every extension after it:

Def. 15 (General diffusion of acceptability) *Let $X \in \mathcal{G} \cap \mathcal{G}'$. The change from $\mathcal{G}$ to $\mathcal{G}'$ is a* general diffusion of acceptability *for X iff $\exists \mathcal{E}_i \in \mathbf{E}, X \in \mathcal{E}_i$, $\exists \mathcal{E}_k \in \mathbf{E}, X \notin \mathcal{E}_k$ and $\forall \mathcal{E}'_j \in \mathbf{E}', X \in \mathcal{E}'_j$.*

Def. 16 (Partial degradation of acceptability) *Let $X \in \mathcal{G} \cap \mathcal{G}'$. The change from $\mathcal{G}$ to $\mathcal{G}'$* partially degrades acceptability *for X iff $\forall \mathcal{E}_i \in \mathbf{E}, X \in \mathcal{E}_i$, $\exists \mathcal{E}'_j \in \mathbf{E}', X \in \mathcal{E}'_j$ and $\exists \mathcal{E}'_l \in \mathbf{E}', X \notin \mathcal{E}'_l$.*

And lastly, the acceptability of X can also remain unchanged (note that the following definition refines the *partial monotony* property defined by (Cayrol *et al.* 2010)).

Def. 17 (Status preservation) *Let $X \in \mathcal{G} \cap \mathcal{G}'$. The change from $\mathcal{G}$ to $\mathcal{G}'$* preserves the status *of X iff*

- *(*preserves credulous-only acceptability*) $\exists \mathcal{E}_i, \mathcal{E}_k \in \mathbf{E}$, $X \in \mathcal{E}_i$, $X \notin \mathcal{E}_k$, $\exists \mathcal{E}'_j, \mathcal{E}'_l \in \mathbf{E}'$, $X \in \mathcal{E}'_j$ and $X \notin \mathcal{E}'_l$.*
- *(*preserves skeptical acceptability*) $\forall \mathcal{E}_i \in \mathbf{E}$, $X \in \mathcal{E}_i$ and $\forall \mathcal{E}'_j \in \mathbf{E}'$, $X \in \mathcal{E}'_j$.*
- *(*preserves the rejected status*) $\forall \mathcal{E}_i \in \mathbf{E}, X \notin \mathcal{E}_i$ and $\forall \mathcal{E}'_j \in \mathbf{E}', X \notin \mathcal{E}'_j$.*

Note that (credulous-only or skeptical) *preservation of acceptability* for X does not mean that arguments that were accepted together with X remain accepted after the change (this differs from the monotony property presented in Section 3.2). Nevertheless, some links exist (see Prop. 7, Section 4).

4 Characterizing change operations: Preliminary results

In this section, we give some general results about the characterization of addition and removal in argumentation under some semantics (some of them are taken from (Cayrol *et al.* 2010; Bisquert *et al.* 2012b)). The first result is due to the uniqueness of the grounded extension (Prop. 1.1).

Prop. 3 (Prop. 12 of (Cayrol *et al.* 2010), extended)
Under the grounded semantics, a change (addition or suppression) is never $e_{\varnothing-1ne}$, nor $e_{\varnothing-k}$, nor $r_{k-\varnothing}$, nor $r_{1ne-\varnothing}$, nor e_{1e-k}, nor e_{1ne-k}, nor e_{j-k}, nor r_{k-j}, nor r_{k-1e}, nor r_{k-1ne}.

The second result is due to the fact that there always exists a preferred extension (Prop. 1.1):

Prop. 4 *Under the preferred semantics, a change (addition or suppression) is never $e_{\varnothing-1ne}$, nor $e_{\varnothing-k}$, nor $r_{k-\varnothing}$, nor $r_{1ne-\varnothing}$.*

The stable semantics is taken into account in the third result, due to Prop. 1.7 and to the assumption that the set of arguments is not empty:

Prop. 5 *Under the stable semantics, a change (addition or suppression) is never* c_{1e-1ne}*, nor* c_{1ne-1e}*, nor* e_{1e-k}*, nor* r_{k-1e}*.*

The following proposition is due to the uniqueness of the grounded extension:

Prop. 6

- *Under the grounded semantics,* skeptical expansive monotony *and* credulous expansive monotony *both correspond to* simple expansive monotony*.*
- *Under the grounded semantics,* skeptical restrictive monotony *and* credulous restrictive monotony *both correspond to* simple restrictive monotony*.*

According to monotony definitions given in Section 3.2, the following proposition holds for each semantics studied in this paper:

Prop. 7

- *Simple expansive monotony implies* credulous expansive monotony*.*
- *Simple restrictive monotony implies* credulous restrictive monotony*.*
- *Simple expansive monotony implies* preservation of acceptability *and* preservation of credulous-only acceptability*.*
- *Skeptical expansive monotony implies* preservation of skeptical acceptability*.*

The next propositions and notations will be used for establishing propositions given in Section 5 to 9.

Nota. 1 *Let* $Z \in \mathcal{G}$*,* $\mathcal{U}_Z = \{X \in \mathcal{G}$ *s.t.* X *is not attacked by* $\mathcal{G} \setminus \{Z\}\}$*.*

Lem. 1 ((Bisquert *et al.* 2012b)) *When removing an argument* Z *under the grounded semantics,* Z *does not attack* $\mathcal{E}'$ *in* $\mathcal{G}$ *iff* $\forall X \in \mathcal{G}'$*, if* Z *attacks* X *in* $\mathcal{G}$ *then (*X *is attacked by* $\mathcal{G} \setminus \{Z\}$ *and* X *is not indirectly defended by* $\mathcal{U}_Z$ *in* $\mathcal{G} \setminus \{Z\}$*).*

Lem. 2 ((Bisquert *et al.* 2012b)) *When removing an argument* Z *under the grounded semantics, if* Z *does not attack* $\mathcal{E}'$*, then the following equivalence holds:* $Z \in \bigcup_{i \geq 1} \mathcal{F}^i(\mathcal{U}_Z)$ *iff* $\mathcal{E}'$ *defends* Z *in* $\mathcal{G}$*.*

The following lemma uses the fact that the argument Z is the only one argument which is added or removed; moreover, by assumption, Z is not a self-attacking argument:

Lem. 3

1. *When adding an argument Z, Z does not attack $\mathcal{G}'$ (resp. is not attacked by $\mathcal{G}'$) iff Z does not attack $\mathcal{G}$ (resp. is not attacked by $\mathcal{G}$) in $\mathcal{G}'$.*
2. *When removing an argument Z, Z does not attack $\mathcal{G}'$ (resp. is not attacked by $\mathcal{G}'$) in $\mathcal{G}$ iff Z does not attack $\mathcal{G}$ (resp. is not attacked by $\mathcal{G}$).*

The following new lemma is straightforward:

Lem. 4

- *When removing an argument Z from $\mathcal{G}$, let $X \neq Z$ be an argument of $\mathcal{G}$. If X is attacked in $\mathcal{G}'$ then X is also attacked in $\mathcal{G}$ by an argument distinct from Z. If X is attacked in $\mathcal{G}$ then X is either attacked in $\mathcal{G}'$ or X is attacked only by Z in $\mathcal{G}$.*
- *When adding an argument Z to $\mathcal{G}$, let $X \neq Z$ be an argument of $\mathcal{G}$. If X is attacked in $\mathcal{G}$ then X is also attacked in $\mathcal{G}'$ by an argument distinct from Z. If X is attacked in $\mathcal{G}'$ then X is either attacked in $\mathcal{G}$ or X is attacked only by Z in $\mathcal{G}$.*

5 Characterizing argument addition: direct results

Among the results given here, one is taken from (Bisquert *et al.* 2012b), two others are new and the others are taken from (Cayrol *et al.* 2010) (sometimes simplified: useless conditions have been removed). They only concern two semantics (the grounded and the preferred one). The proofs of the new propositions are given in Appendix B.

5.1 Results for the grounded semantics

Prop. 8 (Prop. 7 of (Cayrol *et al.* 2010)) *Under the grounded semantics, if X belongs to $\mathcal{E}$, and Z does not indirectly attack X, then $\oplus$ preserves the acceptability status for X (i.e. X belongs to $\mathcal{E}'$).*

Prop. 9 (Prop. 8 of (Cayrol *et al.* 2010)) *Under the grounded semantics, if Z is not attacked by $\mathcal{G}$, then $\oplus$ satisfies priority to recency (i.e. Z belongs to $\mathcal{E}'$).*

Prop. 10 (Prop. 9 of (Cayrol *et al.* 2010)) *Under the grounded semantics, in the case of addition,*

1. *if $\mathcal{E} = \{\}$ then it holds that: $\mathcal{E}' = \{\}$ iff Z is attacked by $\mathcal{G}$, moreover,*
2. *if $\mathcal{E} = \{\}$ and Z is not attacked by $\mathcal{G}$, then $\mathcal{E}' = \{Z\} \cup \bigcup_{i \geq 1} \mathcal{F}'^i(\{Z\})$.*

Note that Prop. 10.1 implies that "if Z is attacked by $\mathcal{G}$ and $\mathcal{E} = \{\}$ then $\mathcal{E}' = \{\}$". Thus "Z is attacked by $\mathcal{G}$ and $\mathcal{E} = \{\}$" is a sufficient condition for having a *c-conservative* change. The following two propositions are extensions of propositions of (Cayrol *et al.* 2010) to cover the case $\mathcal{E} = \varnothing$.

Prop. 11 (Prop. 10 of (Cayrol *et al.* 2010) extended)
Under the grounded semantics, if Z does not attack $\mathcal{E}$, then $\bigoplus$ satisfies simple expansive monotony *(i.e. $\mathcal{E} \subseteq \mathcal{E}'$).*

Prop. 12 (Prop. 11 of (Cayrol *et al.* 2010) extended)
Under the grounded semantics, if Z does not attack $\mathcal{E}$, we have:
1. *if $\mathcal{E}$ does not defend Z, then $\mathcal{E}' = \mathcal{E}$. (The change $\bigoplus$ is* c-conservative*).*
2. *if $\mathcal{E}$ defends Z, then $\mathcal{E}' = \mathcal{E} \cup \{Z\} \cup \bigcup_{i \geq 1} \mathcal{F}'^i(\{Z\})$.*
3. *Moreover, if $\mathcal{E}$ defends Z and Z does not attack $\mathcal{G}$ then $\mathcal{E}'$ reduces to $\mathcal{E} \cup \{Z\}$. (The change $\bigoplus$ is c-expansive if $\mathcal{E} \neq \{\}$, otherwise it is c_{1e-1ne}).*

Let $X \in \mathcal{G} \cap \mathcal{G}'$, a consequence of Prop. 12 gives a characterization (sufficient condition) of *acceptability establisment* for X:

Conseq. 1 *In case of addition under the grounded semantics, if Z does not attack $\mathcal{E}$ and $\mathcal{E}$ defends Z and Z indirectly defends X and $X \notin \mathcal{E}$ then $X \in \mathcal{E}'$.*

Prop. 13 (Prop. 13 of (Cayrol *et al.* 2010)) *Under the grounded semantics, if $\mathcal{E} \neq \varnothing$ and Z attacks each unattacked argument of $\mathcal{G}$ and Z is attacked by $\mathcal{G}$ then the change $\bigoplus$ is c_{1ne-1e}; the converse also holds.*

Let $X \in \mathcal{G} \cap \mathcal{G}'$, the *preservation of the rejected status* for X is characterized by:

Prop. 14 (Prop. 3 of (Bisquert *et al.* 2012b)) *When adding an argument Z under the grounded semantics, $\forall X \in \mathcal{G}$, if $X \notin \mathcal{E}$ and Z does not indirectly defend X, then $X \notin \mathcal{E}'$.*

Let $X \in \mathcal{G} \cap \mathcal{G}'$, the following results are new and give respectively a characterization of *acceptability establisment* for X, then a characterization of *acceptability removal* for X:

Prop. 15 *When adding an argument Z under the grounded semantics, if Z is not attacked by $\mathcal{G}$ and Z indirectly defends X and $X \notin \mathcal{E}$ then $X \in \mathcal{E}'$.*

Prop. 16 *When adding an argument Z under the grounded semantics, if $\mathcal{E} \setminus \{X\}$ does not attack Z and Z attacks X and $X \in \mathcal{E}$ then $X \notin \mathcal{E}'$.*

5.2 Results for the preferred semantics

Prop. 17 (Prop. 14 of (Cayrol *et al.* 2010)) *Under the preferred semantics, if Z is not attacked by $\mathcal{G}$, then $\bigoplus$ satisfies skeptical priority to recency (i.e. Z belongs to each $\mathcal{E}'_i$).*

Prop. 18 (Prop. 15 of (Cayrol *et al.* 2010)) *When adding an argument Z under the preferred semantics,*
1. *if Z does not attack $\mathcal{E}_i$, then $\mathcal{E}_i$ remains admissible in $\mathcal{G}'$;*
2. *if Z does not attack $\mathcal{E}_i$ and $\mathcal{E}_i$ defends Z in $\mathcal{G}'$, then $\mathcal{E}_i \cup \{Z\}$ is admissible in $\mathcal{G}'$.*

Prop. 19 (Prop. 16 of (Cayrol *et al.* 2010)) *When adding an argument Z under the preferred semantics, if $\mathbf{E} = \{\{\}\}$ and Z is not attacked by $\mathcal{G}$ and there is no even-length cycle in $\mathcal{G}$ then $\mathbf{E}' = \{\mathcal{E}'\}$ and Z belongs to $\mathcal{E}'$ (so, $\bigoplus$ is c_{1e-1ne}).*

Prop. 20 (Prop. 17 of (Cayrol *et al.* 2010)) *When adding an argument Z under the preferred semantics, if Z attacks no argument of $\mathcal{G}$ and $\mathbf{E} = \{\{\}\}$, then $\mathbf{E}' = \{\{\}\}$; or equivalently, if $\mathbf{E} = \{\{\}\}$ the change $\bigoplus$ by Z is c_{1e-1ne} only if Z attacks $\mathcal{G}$.*

Note that the previous proposition gives a sufficient condition for having a *c-conservative* change: "Z attacks no argument of $\mathcal{G}$ and $\mathbf{E} = \{\{\}\}$".

Prop. 21 (Prop. 18 of (Cayrol *et al.* 2010)) *When adding an argument Z under the preferred semantics, if Z attacks no argument of $\mathcal{G}$ and $\mathbf{E} \neq \{\{\}\}$, then for each i:*
1. *if $\mathcal{E}_i$ defends Z, then $\mathcal{E}_i \cup \{Z\}$ is an extension of $\mathcal{G}'$;*
2. *if $\mathcal{E}_i$ does not defend Z, then $\mathcal{E}_i$ is an extension of $\mathcal{G}'$;*

moreover, $\mathcal{G}$ and $\mathcal{G}'$ have the same number of extensions (so the change is constant).

Note again that Prop. 21.2 gives a sufficient condition for having a *c-conservative* change: "Z attacks no argument of $\mathcal{G}$ and $\mathbf{E} \neq \{\{\}\}$ and for each i, $\mathcal{E}_i$ does not defend Z".

Prop. 22 (Prop. 19 of (Cayrol *et al.* 2010)) *Under the preferred semantics, if Z attacks no extension of $\mathcal{G}$ then the change $\bigoplus$ satisfies* simple expansive monotony.

Prop. 23 (Prop. 20 of (Cayrol *et al.* 2010)) *Under the preferred semantics, assume that $\mathcal{G}$ contains no controversial argument*[4]*. If Z does*

[4]An argument A is controversial iff there exists at least an argument B such that A is a defender (direct or indirect) and an attacker (direct or indirect) of B.

not attack $\bigcap_{i\geq 1} \mathcal{E}_i$*, then the change* $\bigoplus$ *satisfies* skeptical expansive monotony*, that is* $\bigcap_{i\geq 1} \mathcal{E}_i \subseteq \bigcap_{i\geq 1} \mathcal{E}'_i$.

Prop. 24 (Prop. 21 of (Cayrol *et al.* 2010)) *Under the preferred semantics, if* $\mathbf{E} \neq \{\{\}\}$ *and there is no even-length cycle in* $\mathcal{G}'$ *and each unattacked argument of* $\mathcal{G}$ *is attacked in* $\mathcal{G}'$ *and* Z *is attacked in* $\mathcal{G}'$ *then the change* $\bigoplus$ *is* c_{1ne-1e}.

6 Characterizing argument removal: direct results

6.1 Results concerning the three semantics

These results can be found in (Bisquert *et al.* 2012a) and concern the three well-known semantics (grounded, stable and preferred).

Prop. 25 (Prop. 1 of (Bisquert *et al.* 2012a)) *When removing an argument* Z*,*

1. *If* $\mathcal{E}$ *is a preferred extension of* $\mathcal{G}$ *and* $Z \notin \mathcal{E}$ *then* $\mathcal{E}$ *is admissible in* $\mathcal{G}'$ *and so there exists a preferred extension* $\mathcal{E}'$ *of* $\mathcal{G}'$ *such that* $\mathcal{E} \subseteq \mathcal{E}'$.
2. *If* $\mathcal{E}$ *is a stable extension of* $\mathcal{G}$ *and* $Z \notin \mathcal{E}$ *then* $\mathcal{E}$ *is stable in* $\mathcal{G}'$.
3. *If* $\mathcal{E}$ *is the grounded extension of* $\mathcal{G}$ *and* $Z \notin \mathcal{E}$ *then* $\mathcal{E} \subseteq \mathcal{E}'$ *where* $\mathcal{E}'$ *is the grounded extension of* $\mathcal{G}'$.

The following result gives a necessary and sufficient condition for *simple expansive monotony.*

Prop. 26 (Prop. 2 of (Bisquert *et al.* 2012a)) *When removing an argument* Z *under preferred, stable or grounded semantics, it holds that:*

$$(\forall \mathcal{E} \in \mathbf{E},\ Z \notin \mathcal{E}) \text{ iff } (\forall \mathcal{E} \in \mathbf{E}, \exists \mathcal{E}' \in \mathbf{E}' \text{ such that } \mathcal{E} \subseteq \mathcal{E}').$$

The following proposition concerns the notion of "weak" *simple expansive monotony* (*i.e.* a kind of monotony in which Z is not taken into account):

Prop. 27 (Prop. 3 of (Bisquert *et al.* 2012a)) *When removing an argument* Z*, if* Z *attacks no argument in* $\mathcal{G}$*, then*

1. $\forall \mathcal{E}$ *preferred extension of* $\mathcal{G}$*,* $\mathcal{E} \setminus \{Z\}$ *is admissible in* $\mathcal{G}'$ *and so* $\exists \mathcal{E}'$ *preferred extension of* $\mathcal{G}'$ *such that* $\mathcal{E} \setminus \{Z\} \subseteq \mathcal{E}'$.
2. $\forall \mathcal{E}$ *stable extension of* $\mathcal{G}$*,* $\mathcal{E} \setminus \{Z\}$ *is a stable extension of* $\mathcal{G}'$.
3. *If* $\mathcal{E}$ *is the grounded extension of* $\mathcal{G}$*,* $\mathcal{E} \setminus \{Z\}$ *is the grounded extension of* $\mathcal{G}'$.

Prop. 28 (Prop. 4 of (Bisquert *et al.* 2012a)) *When removing an argument Z under preferred, stable or grounded semantics, if Z attacks no argument in $\mathcal{G}$, then for any extension $\mathcal{E}$ of $\mathcal{G}$ such that $Z \notin \mathcal{E}$, $\mathcal{E}$ is an extension of $\mathcal{G}'$.*

Prop. 29 (Prop. 8 of (Bisquert *et al.* 2012a)) *When removing an argument Z under preferred, stable or grounded semantics, if the change is* c-narrowing*, then there exists an extension $\mathcal{E}$ of $\mathcal{G}$ such that $Z \in \mathcal{E}$.*

6.2 Results concerning only some semantics

Three new results are given here (proofs in Appendix B), the other propositions being taken from (Bisquert *et al.* 2012a).

Prop. 30 (Prop. 5 of (Bisquert *et al.* 2012a)) *When removing an argument Z under the preferred semantics, if Z attacks no argument in $\mathcal{G}$ then, for each extension $\mathcal{E}_i$ of $\mathcal{G}$,*
1. If $Z \notin \mathcal{E}_i$ then $\mathcal{E}_i$ is a preferred extension of $\mathcal{G}'$.
2. If $Z \in \mathcal{E}_i$ then $\mathcal{E}_i \backslash \{Z\}$ is a preferred extension of $\mathcal{G}'$.
Moreover, $|\mathbf{E}| = |\mathbf{E}'|$ (so the change is constant).

Prop. 31 (Prop. 6 of (Bisquert *et al.* 2012a)) *When removing an argument under the stable semantics, a change cannot be* c-expansive*.*

Prop. 32 (Prop. 7 of (Bisquert *et al.* 2012a)) *When removing an argument Z under the preferred or the grounded semantics, if this change is* c-expansive*, then*
1. Z belongs to no extension of $\mathcal{G}$ and
2. Z attacks at least one element of $\mathcal{G}$.

Prop. 33 (Prop. 9 of (Bisquert *et al.* 2012a)) *When removing an argument Z under the preferred or the grounded semantics, if Z attacks no argument of $\mathcal{G}$ and $\forall \mathcal{E}, Z \in \mathcal{E}$, then the change is* c-narrowing*.*

Let $X \in \mathcal{G} \cap \mathcal{G}'$, the three following propositions are new. The first one gives a characterization of *acceptability establisment* for X:

Prop. 34 *When removing an argument Z under the grounded semantics, if $X \notin \mathcal{E}$ and Z is the unique attacker of X then $X \in \mathcal{E}'$.*

The previous proposition is weak, but many examples can be easily found for illustrating the fact that an argument X attacked by several arguments and such that $X \notin \mathcal{E}$ cannot be reinstated by the removal of only one argument. For instance, with the attack relation $\{(a, b),$

(b, c), (c, b), (c, d), (d, c), $(b, e)\}$, for reinstating b, both a and c must be removed, and for reinstating c, both b and d must be removed.

Let $X \in \mathcal{G} \cap \mathcal{G}'$, a characterization of *acceptability removal* for X is given by the two following propositions:

Prop. 35 *When removing an argument Z under the grounded semantics, if $X \neq Z$ and $X \in \mathcal{E}$ and $Z \in \mathcal{E}$ and $X \in \bigcup_{i \geq 1} \mathcal{F}^i(\{Z\})$ and (there exists Y attacker of X such that each odd-length path from $\mathcal{F}(\varnothing)$ to Y contains Z at an even place) and (there exists no odd-length path from Z to X), then $X \notin \mathcal{E}'$.*

Prop. 36 *When removing an argument Z under the grounded semantics, if $X \in \mathcal{E}$ and X is attacked in $\mathcal{G}$ and $\forall S$ s.t. $X \in \mathcal{F}(S), Z \in S$ then $X \notin \mathcal{E}'$.*

7 Duality

As far as we know, the problem of removing an argument and, *a fortiori*, the link between addition and removal of an argument have been little discussed. However, it can be worthy to use the links between these operations in order to study the properties characterizing the changes that may impact an AS. For that purpose, the notion of duality seems pertinent.

7.1 Two definitions of duality

We focus on two concepts of duality: first, duality at the level of change operations, *based on the notion of inverse*, expressing the opposite nature of two operations, then duality at the level of change properties, *based on the notion of symmetry*, conveying a correspondence between two properties.

Def. 18 (Duality based on the notion of inverse) *Two change operations $\mathcal{O}$ and $\mathcal{O}'$ are the* inverse *of each other iff: "$\forall \mathcal{G}, \forall \mathcal{G}'$, $\mathcal{O}(\mathcal{G}) = \mathcal{G}'$ iff $\mathcal{O}'(\mathcal{G}') = \mathcal{G}$".*

Obviously, following Def. 18, it is clear that addition and removal operations defined in Section 2.2 are the inverse of each other.

Def. 19 (Duality based on the notion of symmetry) *Two properties $\mathcal{P}$ and $\mathcal{P}'$ are* symmetric *iff: "$\forall \mathcal{G}$, $\forall \mathcal{G}'$, $\mathcal{P}'(\mathcal{G}', \mathcal{G})$ holds iff $\mathcal{P}(\mathcal{G}, \mathcal{G}')$ holds".*

From these definitions, we can draw a condition for the satisfaction of a property by a change operation:

Prop. 37 *Let $\mathcal{O}$ and $\mathcal{O}'$ be two inverse change operations and $\mathcal{P}$ and $\mathcal{P}'$ be two symmetric properties. $\mathcal{O}$ satisfies $\mathcal{P}$ iff $\mathcal{O}'$ satisfies $\mathcal{P}'$.*

Both concepts of duality can be used for linking the change properties:

Prop. 38

1. *A change is* restrictive *iff the inverse change is* extensive.
2. *A change is $r_{1ne-\varnothing}$ iff the inverse change is $e_{\varnothing-1ne}$.*
3. *A change is $r_{k-\varnothing}$ iff the inverse change is $e_{\varnothing-k}$.*
4. *A change is r_{k-1e} iff the inverse change is e_{1e-k}.*
5. *A change is r_{k-1ne} iff the inverse change is e_{1ne-k}.*
6. *A change is r_{k-j} iff the inverse change is e_{j-k}.*

Prop. 39

1. *A change is* constant *iff the inverse change is also* constant.
2. *A change is c_{1ne-1e} iff the inverse change is c_{1e-1ne}.*
3. *A change is* c-conservative *iff the inverse change is also* c-conservative.
4. *A change is* c-narrowing *iff the inverse change is* c-expansive.
5. *A change is* c-altering *iff the inverse change is also* c-altering.

Prop. 40

1. *A change satisfies* simple restrictive monotony *iff the inverse change satisfies* simple expansive monotony.
2. *A change satisfies* credulous restrictive monotony *iff the inverse change satisfies* credulous expansive monotony.
3. *A change satisfies* skeptical restrictive monotony *iff the inverse change satisfies* skeptical expansive monotony.

Prop. 41 *Let $X \in \mathcal{G} \cap \mathcal{G}'$.*

1. *A change* establishes credulous-only acceptability *for X iff the inverse change* removes credulous-only acceptability *of X.*
2. *A change* establishes skeptical acceptability *for X iff the inverse change* removes skeptical acceptability *of X.*

Prop. 42 *Let $X \in \mathcal{G} \cap \mathcal{G}'$. A change is a* general diffusion of acceptability *for X iff the inverse change* partially degrades acceptability *for X.*

Prop. 43 *Let $X \in \mathcal{G} \cap \mathcal{G}'$.*

1. *A change* preserves credulous-only acceptability *for X iff the inverse change* preserves credulous-only acceptability *for X.*
2. *A change* preserves skeptical acceptability *for X iff the inverse change* preserves skeptical acceptability *for X.*

3. A change preserves the rejected status *for X iff the inverse change* preserves the rejected status *for X.*

7.2 Methodology for Using Duality

This part describes how to use duality in order to obtain new propositions for the operation of removal, starting from propositions concerning addition[5]. Note first that we restrict our study to the grounded semantics. Let us describe this methodology using Prop. 14. In order to clarify the presentation, the graphs and the extensions are renamed by adding two capital letters in subscripts - IA, OA, IR and OR - representing respectively the **I**nput system for **A**ddition, the **O**utput system for **A**ddition, the **I**nput system for **R**emoval and the **O**utput system for **R**emoval. Thus, Prop. 14 can be rewritten as follows:

Prop. 14.1 *When adding an argument Z under the grounded semantics, if $X \notin \mathcal{E}_{IA}$ and Z does not indirectly defend X, then $X \notin \mathcal{E}_{OA}$.*

Let $\mathcal{P}$ be a property and $\mathcal{P}^{-1}$ its symmetric. Due to Prop. 37, it holds that: $\oplus$ *satisfies* $\mathcal{P}$ *iff* $\ominus$ *satisfies* $\mathcal{P}^{-1}$.

And due to Def. 6, we know that a change operation $\mathcal{O}$ satisfies $\mathcal{P}$ iff $\forall \mathcal{G}$, it holds that $\mathcal{P}(\mathcal{G}, \mathcal{O}(\mathcal{G}))$. Hence:

$\forall \mathcal{G}_{IA}, \mathcal{P}(\mathcal{G}_{IA}, \oplus(\mathcal{G}_{IA}))$ *holds iff* $\forall \mathcal{G}_{IR},\ \mathcal{P}^{-1}(\mathcal{G}_{IR}, \ominus(\mathcal{G}_{IR}))$ *holds.*

Moreover, due to Def. 19, we have:

$\forall \mathcal{G}_{IR}, \mathcal{P}^{-1}(\mathcal{G}_{IR}, \ominus(\mathcal{G}_{IR}))$ *holds iff* $\mathcal{P}(\ominus(\mathcal{G}_{IR}), \mathcal{G}_{IR})$ *holds.*

And so, we have:

$\forall \mathcal{G}_{IA},\ \forall \mathcal{G}_{IR},\ \mathcal{P}(\mathcal{G}_{IA}, \oplus(\mathcal{G}_{IA}))$ *holds iff* $\mathcal{P}(\ominus(\mathcal{G}_{IR}), \mathcal{G}_{IR})$ *holds.*

Let $\mathcal{G}_{OA} = \oplus(\mathcal{G}_{IA})$ and $\mathcal{G}_{OR} = \ominus(\mathcal{G}_{IR})$. Since we know that $\mathcal{P}$ holds for the operation of addition, we can rewrite it for removal:

Prop. 14.2 *When removing an argument Z under the grounded semantics, if $X \notin \mathcal{E}_{OR}$ and Z does not indirectly defend X, then $X \notin \mathcal{E}_{IR}$.*

Which is equivalent to:

Prop. 14.3 *When removing an argument Z under the grounded semantics, if $X \in \mathcal{E}_{IR}$ and Z does not indirectly defend X, then $X \in \mathcal{E}_{OR}$.*

Thus, for the operation of removal, we obtain a proposition analogous to Prop. 14 denoted by Prop. $14^{\ominus}$; in the remainder of this article, the exponent ($^{\oplus}$ or $^{\ominus}$) will represent the correspondence between a proposition and the one obtained by applying the duality methodology:

Prop. $14^{\ominus}$ *When removing an argument Z under the grounded semantics, if $X \in \mathcal{E}$ and Z does not indirectly defend X, then $X \in \mathcal{E}'$.*

[5]This methodology can also be used the other way round from removal to addition.

In the next sections, we use this methodology on the propositions given in Section 5 and 6 together with the lemmas given in Section 2 in order to obtain new results.

8 Characterizing argument addition thanks to duality

The propositions given here are new (proofs are given in Appendix B).

8.1 Results for the three semantics

Prop. 25$^{\oplus}$ *When adding an argument Z,*

1. *If $\mathcal{E}'$ is a preferred extension of $\mathcal{G}'$ and $Z \notin \mathcal{E}'$ then $\mathcal{E}'$ is admissible in $\mathcal{G}$, hence there exists a preferred extension $\mathcal{E}$ of $\mathcal{G}$ such that $\mathcal{E}' \subseteq \mathcal{E}$.*
2. *If $\mathcal{E}'$ is a stable extension of $\mathcal{G}'$ and $Z \notin \mathcal{E}'$ then $\mathcal{E}'$ is stable in $\mathcal{G}$.*
3. *If $\mathcal{E}'$ is the grounded extension of $\mathcal{G}'$ and $Z \notin \mathcal{E}'$ then $\mathcal{E}' \subseteq \mathcal{E}$ where $\mathcal{E}$ is the grounded extension of $\mathcal{G}$.*

Prop. 26$^{\oplus}$ *When adding an argument Z under the preferred, grounded or stable semantics, ($\forall \mathcal{E}' \in \mathbf{E}'$, $Z \notin \mathcal{E}'$) iff ($\forall \mathcal{E}' \in \mathbf{E}', \exists \mathcal{E} \in \mathbf{E}$ such that $\mathcal{E}' \subseteq \mathcal{E}$).*

Prop. 27$^{\oplus}$ *When adding an argument Z, if Z attacks no argument in $\mathcal{G}$, then*

1. *$\forall \mathcal{E}'$ preferred extension of $\mathcal{G}'$, $\mathcal{E}' \setminus \{Z\}$ is admissible in $\mathcal{G}$ and so $\exists \mathcal{E}$ preferred extension of $\mathcal{G}$ such that $\mathcal{E}' \setminus \{Z\} \subseteq \mathcal{E}$.*
2. *$\forall \mathcal{E}'$ stable extension of $\mathcal{G}'$, $\mathcal{E}' \setminus \{Z\}$ is a stable extension of $\mathcal{G}$.*
3. *If $\mathcal{E}'$ is the grounded extension of $\mathcal{G}'$, $\mathcal{E}' \setminus \{Z\}$ is the grounded extension of $\mathcal{G}$.*

Prop. **27**$^{\oplus}$ completes the results given by Prop. 20, 21 (for the preferred semantics) and 12 (for the grounded semantics).

Prop. 28$^{\oplus}$ *When adding an argument Z under the preferred, grounded or stable semantics, if Z attacks no argument in $\mathcal{G}$, then for any extension $\mathcal{E}'$ of $\mathcal{G}'$ such that $Z \notin \mathcal{E}'$, $\mathcal{E}'$ is an extension of $\mathcal{G}$.*

Prop. 29$^{\oplus}$ *When adding an argument Z under the preferred, grounded or stable semantics, if the change is* c-expansive, *then there exists an extension $\mathcal{E}'$ of $\mathcal{G}'$ such that $Z \in \mathcal{E}'$.*

8.2 Results concerning only some semantics

Prop. 30$^{\oplus}$ *When adding an argument Z under the preferred semantics, if Z attacks no argument in $\mathcal{G}$ then, for each extension $\mathcal{E}'_i$ of $\mathcal{G}'$,*

1. If $Z \notin \mathcal{E}'_i$ then $\mathcal{E}'_i$ is a preferred extension of $\mathcal{G}$.
2. If $Z \in \mathcal{E}'_i$ then $\mathcal{E}'_i \backslash \{Z\}$ is a preferred extension of $\mathcal{G}$.
Moreover, $|\mathbf{E}'| = |\mathbf{E}|$.

Prop. **30**$^{\oplus}$ completes the results given by Prop. 21.

Prop. 31$^{\oplus}$ *When adding an argument under the stable semantics, a change cannot be* c-narrowing.

Prop. 32$^{\oplus}$ *When adding an argument Z under preferred or grounded semantics, if the change is* c-narrowing, *then*
1. Z belongs to no extension of $\mathcal{G}'$ and
2. Z attacks at least one element of $\mathcal{G}'$.

Prop. 33$^{\oplus}$ *When adding an argument Z under preferred or grounded semantics, if Z attacks no argument in $\mathcal{G}'$ and belongs to each extension of $\mathcal{G}'$, then the change is* c-expansive.

Note that the duality-based translation of Prop. 34 is not interesting because the resulting proposition produces a contradictory condition ($X \notin \mathcal{E}$ and Z is the only attacker of X, Z being the added argument).

Similarly, Prop. 35 is not translated since the resulting condition is contradictory ($X \in \mathcal{E}$ and there exists an attacker Y of X such that each odd-length path from $\mathcal{F}'(\varnothing)$ to Y contains Z at an even place and there exists no odd-length path from Z to X).

Let $X \in \mathcal{G} \cap \mathcal{G}'$, the following proposition gives a characterization of *acceptability removal* for X. However, even if the proposed condition is not contradictory, it is not easy to check.

Prop. 36$^{\oplus}$ *When adding an argument Z under the grounded semantics, if $X \in \mathcal{E}$, X is attacked in $\mathcal{G}'$, and $\forall S$ s.t. $X \in \mathcal{F}'(S), Z \in S$, then $X \notin \mathcal{E}'$.*

9 Characterizing removal thanks to duality

9.1 Results for the grounded semantics

These results can be found in (Bisquert *et al.* 2012b). Let $X \in \mathcal{G} \cap \mathcal{G}'$, the first proposition characterizes the *preservation of the rejected status* for X:

Prop. 8$^{\ominus}$ **(Prop. 1.1**$^{\ominus}$ **of (Bisquert *et al.* 2012b))** *When removing an argument Z under the grounded semantics, if $X \notin \mathcal{E}$ and Z does not indirectly attack X, then $X \notin \mathcal{E}'$.*

Note that the duality-based translation of Prop. 9 would give a trifling result under the grounded semantics ("$Z \notin \mathcal{E}$ implies Z is attacked by

$\mathcal{G}$").

Prop. 10.1$^{\ominus}$ (Prop. 1.2$^{\ominus}$ of (Bisquert *et al.* 2012b)) *When removing an argument Z under the grounded semantics, if $\mathcal{E} \neq \varnothing$ and Z is attacked by $\mathcal{G}$, then $\mathcal{E}' \neq \varnothing$.*

Prop. 10.2$^{\ominus}$ (Prop. 1.3$^{\ominus}$ of (Bisquert *et al.* 2012b)) *When removing an argument Z under the grounded semantics, if $\mathcal{E} \neq \{Z\} \cup \bigcup_{i\geq 1} \mathcal{F}^i(\{Z\})$ and Z is not attacked by $\mathcal{G}$, then $\mathcal{E}' \neq \varnothing$.*

Corol. 1 (Corol. 1 of (Bisquert *et al.* 2012b)) *When removing an argument Z under the grounded semantics, if the change is c_{1ne-1e}, then Z is not attacked by $\mathcal{G}$ and $\mathcal{E} = \{Z\} \cup \bigcup_{i\geq 1} \mathcal{F}^i(\{Z\})$.*

Prop. 11$^{\ominus}$ (Prop 2.1$^{\ominus}$ (v2) of (Bisquert *et al.* 2012b)) *When removing an argument Z under the grounded semantics, if $\forall X \in \mathcal{G}$, if Z attacks X then (X is attacked by $\mathcal{G} \setminus \{Z\}$ and X is not indirectly defended by $\mathcal{U}_Z$ in $\mathcal{G} \setminus \{Z\}$), then $\mathcal{E}' \subseteq \mathcal{E}$.*

Prop. 12.1$^{\ominus}$ (Prop. 2.2$^{\ominus}$ (v2) of (Bisquert *et al.* 2012b)) *When removing an argument Z under the grounded semantics, if*

- *$\forall X \in \mathcal{G}$, if Z attacks X then (X is attacked by $\mathcal{G} \setminus \{Z\}$ and X is not indirectly defended by $\mathcal{U}_Z$ in $\mathcal{G} \setminus \{Z\}$) and*
- *$Z \notin \bigcup_{i\geq 1} \mathcal{F}^i(\mathcal{U}_Z)$,*

then $\mathcal{E} = \mathcal{E}'$.

The following proposition is a translation by duality of Prop.12.2, but this result is not easy to exploit in a prescriptive purpose since the condition concerns the system after the change.

Prop. 12.2$^{\ominus}$ *When removing an argument Z under the grounded semantics, if Z does not attack $\mathcal{E}'$ and $\mathcal{E}'$ defends Z, then $\mathcal{E} = \mathcal{E}' \cup \{Z\} \cup \bigcup_{i\geq 1} \mathcal{F}^i(\{Z\})$.*

Prop. 12.3$^{\ominus}$ (Prop. 2.4$^{\ominus}$ (v2) of (Bisquert *et al.* 2012b)) *When removing an argument Z under the grounded semantics, if*

- *$Z \in \bigcup_{i\geq 1} \mathcal{F}^i(\mathcal{U}_Z)$ and*
- *Z does not attack $\mathcal{G} \setminus \{Z\}$,*

then $\mathcal{E}' = \mathcal{E} \setminus \{Z\}$.

The next proposition gives a sufficient and necessary condition for a c_{1e-1ne} change:

Prop. 13$^{\ominus}$ (Prop. 2.5$^{\ominus}$ of (Bisquert *et al.* 2012b) restricted) *When removing an argument Z under the grounded semantics, if $\mathcal{E}' \neq \varnothing$ and Z attacks each unattacked argument of $\mathcal{G} \setminus \{Z\}$ and Z is attacked by $\mathcal{G} \setminus \{Z\}$ then $\mathcal{E} = \varnothing$. And the converse also holds.*

Let $X \in \mathcal{G} \cap \mathcal{G}'$, the following proposition (used for illustrating our methodology in Section 7.2) characterizes *preservation of acceptability* for X:

Prop. 14$^{\ominus}$ (Prop. 3$^{\ominus}$ of (Bisquert *et al.* 2012b)) *When removing an argument Z under the grounded semantics, if $X \in \mathcal{E}$ and Z does not indirectly defend X, then $X \in \mathcal{E}'$.*

Note that the application of duality on Prop. 15 and 16 gives impossible conditions (for Prop. 15: "Z is not attacked by $\mathcal{G}$, Z indirectly defends X and $X \notin \mathcal{E}$", and for Prop. 16: "$\mathcal{E}' \setminus \{X\}$ does not attack Z, Z attacks X and $X \in \mathcal{E}$").

9.2 Results for the preferred semantics

The propositions given in this section are new. Note that the duality-based translation of Prop. 17 would give a trifling result under the preferred semantics ("Z not attacked by $\mathcal{G}$ implies Z belongs to each $\mathcal{E}_i$").

Prop. 18$^{\ominus}$ *When removing Z under the preferred semantics,*

1. *if Z does not attack $\mathcal{E}'_i$, then $\mathcal{E}'_i$ is admissible in $\mathcal{G}$;*
2. *if Z does not attack $\mathcal{E}'_i$ and $\mathcal{E}'_i$ defends Z, then $\mathcal{E}'_i \cup \{Z\}$ is admissible in $\mathcal{G}$.*

This result is related to Prop. 27 to 30 which are specific cases of this proposition when we restrict to preferred semantics.

Prop. 19$^{\ominus}$ *When removing Z under the preferred semantics, if $\mathbf{E}' = \{\{\}\}$ and Z is not attacked by $\mathcal{G}$ and there is no even-length cycle in $\mathcal{G}'$ then $\mathbf{E} = \{\mathcal{E}\}$ and Z belongs to $\mathcal{E}$ (so, $\ominus$ is c_{1ne-1e}).*

Prop. 20$^{\ominus}$ *When removing Z under the preferred semantics, if Z attacks no argument of $\mathcal{G}$ and $\mathbf{E}' = \{\{\}\}$, then $\mathbf{E} = \{\{\}\}$; or equivalently, if $\mathbf{E}' = \{\{\}\}$ the change $\ominus$ by Z is c_{1ne-1e} only if Z attacks $\mathcal{G}'$.*

The following proposition completes Prop. 30:

Prop. 21$^{\ominus}$ *When removing Z under the preferred semantics, if Z attacks no argument of $\mathcal{G}$, and $\mathbf{E}' \neq \{\{\}\}$, then for each i:*

1. *if $\mathcal{E}'_i$ defends Z, then $\mathcal{E}'_i \cup \{Z\}$ is an extension of $\mathcal{G}$;*
2. *if $\mathcal{E}'_i$ does not defend Z, then $\mathcal{E}'_i$ is an extension of $\mathcal{G}$;*

moreover, $\mathcal{G}'$ and $\mathcal{G}$ have the same number of extensions.

Prop. 22$^{\ominus}$ *Under the preferred semantics, if Z attacks no extension of $\mathcal{G}'$ then the change $\ominus$ satisfies* simple restrictive monotony.

Prop. 23$^{\ominus}$ *Under the preferred semantics, assume that $\mathcal{G}'$ contains no controversial argument. If Z does not attack $\bigcap_{i\geq 1} \mathcal{E}'_i$, then the change $\ominus$ satisfies* skeptical restrictive monotony, *that is $\bigcap_{i\geq 1} \mathcal{E}'_i \subseteq \bigcap_{i\geq 1} \mathcal{E}_i$.*

Note that the following proposition is not easy to exploit in a prescriptive purpose since the condition concerns the system after the change:

Prop. 24$^{\ominus}$ *When removing Z under the preferred semantics, if $\mathbf{E}' \neq \{\{\}\}$, and there is no even-length cycle in $\mathcal{G}$ and each unattacked argument of $\mathcal{G}'$ is attacked in $\mathcal{G}$ and Z is attacked in $\mathcal{G}$ then $\mathbf{E} = \{\{\}\}$ (and so the change $\ominus$ is c_{1e-1ne}).*

10 Discussion

10.1 How to use these change properties? A road-map

Among these properties, the user may wonder how to select the useful properties. For this purpose, three criteria may be taken into account:

- the kind of change concerned: some changes may be considered as useful according to the role of the user, *e.g.* a debate moderator may be interested in focusing or enlarging the dialog depending on the remaining time, while an orator may have dialog strategies and may want to focus on particular arguments. Let us review some of these properties:
  - A "decisive" (*e.g.* c_{1e-1ne}) change is useful to lower ignorance since after this change one and only one extension remains. It can be used by a moderator for concluding the debate.
  - An "expansive" (*e.g.* *c-expansive*) change increases the accepted arguments while conserving those already accepted, it can also be used by a moderator or by an orator in order to convince a larger audience about the current view of the debate
  - A "conservative" (*e.g.* *c-conservative*) change may be a more neutral attitude that can be adopted by a moderator or an orator that does not want to deliver new information but wants to participate (very useful political waffle)
  - "Monotony" and "priority to recency" allow to focus on some particular arguments and may be used strategically by an orator
  - "Questioning" (*e.g.* e_{j-k}) and "destructive" (*e.g.* r_{k-1e}) changes are increasing ignorance either by augmenting the possible views or by destroying any coherent view, they may be used desperately by a strategical orator or by a manager that wants to forbid any decision to be made.
  - An "altering" (*e.g.* *c-altering*) change allows to completely change the point of view, it may also be done to reverse the course of the debate.
- The nature of the characterization obtained in terms of computational time required to check its condition (*e.g.*, checking if an argument attacks no other argument is easier than checking if it belongs to an extension).

- The nature of the characterization obtained in terms of typicality: is the condition often realized in usual AS or is this condition scarcely encountered?

10.2 Conclusion and future works

In this paper, we have presented a comprehensive study of change in argumentation (addition or removal of an argument and its interactions). The first step of this study is the definition of change properties that describe the impact of change on argumentation systems. The second step is to characterize these properties by giving (sufficient or necessary) conditions under which they hold. Some of the characterization results are obtained by using duality between addition and removal of an argument.

Let us come back to the example given in Section 1. For Mister Pink, adding a new argument attacking a specific argument of Mister White without threatening his own accepted arguments corresponds to Prop. 8. Prop. $14^{\ominus}$, on the other hand, allows him to ensure that the removal of his opponent's argument achieves the same result if this argument is not giving assistance to any of his own accepted arguments. Thereby, instead of using Prop. 8, Mister Pink can benefit from Prop. $14^{\ominus}$ thanks to our methodology (see "*preserves acceptability*" lines of the tables in Appendix A).

Our work deals with a facet of the argumentation theory that has not been studied so far. Hence, many points are to be deepened or explored further; here are some issues that seem to be of short-term importance:

- In this work, we have studied only two of the four operations of (Cayrol *et al.* 2010). A first issue is to extend our work to the two missing operations, addition and removal of an interaction.
- Moreover, we could consider the addition or removal of a set of arguments. These special operations may be seen as a sequence of change operations and their study seems essential in order to approach minimal change problems.

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A Synthesis

All the characterization results given in this paper are synthesized in the following tables. It should be noted that some CS (Sufficient Condition) or CN (Necessary Condition) obtained by the application of duality are not really useful because they relate in general to the output system and it is very difficult to translate them in terms of conditions on the input system, whether it is for the addition or the removal (these CS and CN are denoted with *CS* and *CN* in the tables).

For each change operation ($\ominus$ and $\oplus$), two tables are given, the first long table giving the CS and CN found for the properties defined in Section 3 and the second short one giving some additional propositions.

Properties of the change $\oplus$	Grounded semantics	Preferred semantics	Stable semantics
$e_{\varnothing-1ne}$, $e_{\varnothing-k}$, $r_{k-\varnothing}$, $r_{1ne-\varnothing}$	Never (Prop. 3)	Never (Prop. 4)	
c_{1e-1ne}	CNS: Prop. 10.2	CS: Prop. 19 CN: Prop. 20	Never (Prop. 5)
r_{k-1ne}	Never (Prop. 3)		
r_{k-j}	Never (Prop. 3)	CN: $\exists$ even-length cycle in $\mathcal{G}$ + Prop. 18	
e_{1ne-k}	Never (Prop. 3)	CN: $\exists$ even-length cycle in $\mathcal{G}'$ + Prop. 20, Prop. 21	
e_{1e-k}	Never (Prop. 3)	CN: $\exists$ even-length cycle in $\mathcal{G}'$ + Prop. 20, Prop. 21	Never (Prop. 5)
e_{j-k}	Never (Prop. 3)	CN: $\exists$ even-length cycle in $\mathcal{G}'$ + Prop. 20, Prop. 21	
c_{1ne-1e}	CNS: Prop. 13	CS: Prop. 24 CN: Prop. 1.5, 2.2	Never (Prop. 5)
r_{k-1e}	Never (Prop. 3)	CN: Prop. 1.5, 2.2	Never (Prop. 5)
c-expansive	CS: Prop. 12.2 + 12.3 *CN* : Prop. $29^{\oplus}$ *CS* : Prop. $33^{\oplus}$	CS: Prop. 21.1 *CN* : Prop. $29^{\oplus}$ *CS* : Prop. $33^{\oplus}$	*CN* : Prop. $29^{\oplus}$
c-conservative	CS: Prop. 10.1 CS: Prop. 12.1	CS: Prop. 20 CS: Prop. 21.2	
c-narrowing	CN: Prop. 11 *CN* : Prop. $32^{\oplus}$	CN: Prop. 18 *CN* : Prop. $32^{\oplus}$	Never (Prop. $31^{\oplus}$)
c-altering	CN: Prop. 11	CN: Prop. 18	
simple expansive monotony	CS: $\mathcal{E} = \{\}$ CS: Prop. 11	CS: Prop. 22	

simple restrictive monotony	*CNS* : Prop. $26^{\oplus}$	*CNS* : Prop. $26^{\oplus}$	*CNS* : Prop. $26^{\oplus}$
weak expansive monotony			
weak restrictive monotony	*CNS* : Prop. $27.3^{\oplus}$	*CNS* : Prop. $27.1^{\oplus}$	*CNS* : Prop. $27.2^{\oplus}$
priority to recency	CS: Prop. 9 CS: Prop. 12	CS: Prop. 17 CS: Prop. 19	
skeptical expansive monotony	*cf. simple expansive monotony*	CS: Prop. 23	
skeptical restrictive monotony	*cf. simple restrictive monotony*		
credulous expansive monotony	*cf. simple expansive monotony*	*cf. simple expansive monotony*	*cf. simple expansive monotony*
credulous restrictive monotony	*cf. simple restrictive monotony*	*cf. simple restrictive monotony*	*cf. simple restrictive monotony*
preserves the rejected status	CS: Prop. 14		
preserves acceptability	CS: Prop. 8 and also *cf. simple expansive monotony*	*cf. simple expansive monotony*	*cf. simple expansive monotony*
preserves skeptical acceptability	CS: Prop. 8	*cf. skeptical expansive monotony*	*cf. skeptical expansive monotony*
preserves credulous-only acceptability	CS: Prop. 8	*cf. simple expansive monotony*	*cf. simple expansive monotony*
establishes acceptability	CS: Conseq.1 CS: Prop. 15		
removes acceptability	CS: Prop. 16 *CS*: Prop. $36^{\oplus}$		

Prop. number	For $\bigoplus$, corresponds to	Semantics
Prop. $28^{\oplus}$	*CS* for the "preservation of a final extension" (*i.e.* in which case a final extension was already an initial extension)	S, P, G
Prop. $30^{\oplus}$	*CS* for the preservation (eventually weak) of a final extension (*i.e.* in which case a final extension without Z was already an initial extension)	P

Properties of the change $\ominus$	Grounded semantics	Preferred semantics	Stable semantics
$e_{\varnothing-1ne}$, $e_{\varnothing-k}$, $r_{k-\varnothing}$, $r_{1ne-\varnothing}$	Never (Prop. 3)	Never (Prop. 4)	
c_{1e-1ne}	CNS: Prop. 13$^{\ominus}$	*CS*: Prop. 24$^{\ominus}$ CN: Prop. 1.5 + Prop. 2.2	Never (Prop. 5)
r_{k-1ne}	Never (Prop. 3)	*CN*: $\exists$ even-length cycle in $\mathcal{G}$ + Prop. 20$^{\ominus}$, Prop. 21$^{\ominus}$	
r_{k-j}	Never (Prop. 3)	*CN*: $\exists$ even-length cycle in $\mathcal{G}$ + Prop. 20$^{\ominus}$, Prop. 21$^{\ominus}$	
e_{1ne-k}	Never (Prop. 3)		
e_{1e-k}	Never (Prop. 3)	CN: Prop. 1.5 + Prop. 2.2	Never (Prop. 5)
e_{j-k}	Never (Prop. 3)	*CN*: $\exists$ even-length cycle in $\mathcal{G}'$ + Prop. 18$^{\ominus}$	
c_{1ne-1e}	CN : Prop. 10.1$^{\ominus}$ CN : Prop. 10.2$^{\ominus}$ CN : Corollary 1	*CS*: Prop. 19$^{\ominus}$ *CN*: Prop. 20$^{\ominus}$	Never (Prop. 5)
r_{k-1e}	Never (Prop. 3)	*CN*: $\exists$ even-length cycle in $\mathcal{G}$ + Prop. 20$^{\ominus}$, Prop. 21$^{\ominus}$	Never (Prop. 5)
c-expansive	CN : Prop. 32	CN : Prop. 32 *CN*: Prop. 18$^{\ominus}$	Never (Prop. 31)
c-conservative	CS : Prop. 12.1$^{\ominus}$	*CS*: Prop. 20$^{\ominus}$ *CS*: Prop. 21.2$^{\ominus}$	
c-narrowing	CN : Prop. 29 CS : Prop. 33 *CS* : Prop. 12.2$^{\ominus}$ CS : Prop. 12.3$^{\ominus}$	CN : Prop. 29 CS : Prop. 33 *CS*: Prop. 21.1$^{\ominus}$	CN : Prop. 29
c-altering	CN: Prop. 11$^{\ominus}$	*CN*: Prop. 18$^{\ominus}$	
simple expansive monotony	CNS : Prop. 26	CNS : Prop. 26	CNS : Prop. 26
simple restrictive monotony	CS: $\mathcal{E}' = \{\}$ CS: Prop. 11$^{\ominus}$	*CS*: Prop. 22$^{\ominus}$	
weak expansive monotony	CNS : Prop. 27.3	CNS : Prop. 27.1	CNS : Prop. 27.2

weak restrictive monotony			
priority to recency	Never (def. priority to recency)	Never (def. priority to recency)	Never (def. priority to recency)
skeptical expansive monotony	(*cf. simple expansive monotony*		
skeptical restrictive monotony	(*cf. simple restrictive monotony*	*CS*: Prop. $23^{\ominus}$	
credulous expansive monotony	(*cf. simple expansive monotony*)	(*cf. simple expansive monotony*)	(*cf. simple expansive monotony*)
credulous restrictive monotony	(*cf. simple restrictive monotony*)	(*cf. simple restrictive monotony*)	(*cf. simple restrictive monotony*)
preserves the rejected status	CS: Prop. $8^{\ominus}$		
preserves acceptability	CS: Prop. $14^{\ominus}$ and also *cf. simple expansive monotony*	*cf. simple expansive monotony*	*cf. simple expansive monotony*
preserves skeptical acceptability	*cf. skeptical expansive monotony*	*cf. skeptical expansive monotony*	*cf. skeptical expansive monotony*
preserves credulous-only acceptability	*cf. simple expansive monotony*	*cf. simple expansive monotony*	*cf. simple expansive monotony*
establishes acceptability	CS: Prop. 34		
removes acceptability	CS: Prop. 35 CS: Prop. 36		

Prop. number	For $\ominus$, corresponds to	Semantics
Prop. 28	CS for the "preservation of an initial extension" (*i.e.* in which case an initial extension is always a final extension)	S, P, G
Prop. 30	CS for the preservation (eventually weak) of an initial extension (*i.e.* in which case an initial extension with eventually the adding of Z is always a final extension)	P

B Proofs

Proof of Prop. 15: If Z is not attacked in $\mathcal{G}$, then $Z \in \mathcal{E}'$. So $\bigcup_{i \geq 1} \mathcal{F}'^{i}(\{Z\} \subseteq$

$\mathcal{E}'$ since the function $\mathcal{F}'$ is monotonic. Since Z indirectly defends X, $X \in \bigcup_{i\geq 1} \mathcal{F}'^i(\{Z\}$, so $X \in \mathcal{E}'$. □

Proof of Prop. 16: For proving this proposition, the following lemmas are used:

Lem. 5 *If $S \subseteq \mathcal{G}$ and $\mathcal{F}'(S) \subseteq \mathcal{G}$, then $\mathcal{F}'(S) \subseteq \mathcal{F}(S)$.*

> **Proof of Lem. 5:** Let $X \in \mathcal{F}'(S)$. By assumption, $\mathcal{F}'(S) \subseteq \mathcal{G}$ so $X \in \mathcal{G}$, and by definition S defends X in $\mathcal{G}'$. Let assume that X is attacked by Y in $\mathcal{G}$. Then Y also attacks X in $\mathcal{G}'$. So, S attacks Y in $\mathcal{G}'$. But, $S \subseteq \mathcal{G}$ and $Y \in \mathcal{G}$, so S also attacks Y in $\mathcal{G}$. Thus S defends X in $\mathcal{G}$, *i.e.* $X \in \mathcal{F}(S)$. □

Lem. 6 *If $Z \notin \mathcal{E}'$, then $\forall i \geq 1, \mathcal{F}'^i(\varnothing) \subseteq \mathcal{F}^i(\varnothing)$.*

> **Proof of Lem. 6:** by induction on i and using Lemma 5.
> First note that, if $Z \notin \mathcal{E}'$, then $\mathcal{E}' \subseteq \mathcal{G}$ and so $\forall i \geq 1, \mathcal{F}'^i(\varnothing) \subseteq \mathcal{G}$.
> Case $i = 1$: since $Z \notin \mathcal{E}'$, Z is attacked by $\mathcal{G}$ in $\mathcal{G}'$. The elements of $\mathcal{F}'(\varnothing)$ are not equal to Z and by definition they are unattacked in $\mathcal{G}'$. Since no attack is removed, they are also unattacked arguments in $\mathcal{G}$. So $\mathcal{F}'(\varnothing) \subseteq \mathcal{F}(\varnothing)$. Let assume that $\mathcal{F}'^k(\varnothing) \subseteq \mathcal{F}^k(\varnothing)$ and let consider $\mathcal{F}'^{(k+1)}(\varnothing) = \mathcal{F}'(\mathcal{F}'^k(\varnothing)$. Let $S = \mathcal{F}'^k(\varnothing)$. $S \subseteq \mathcal{G}$ and $\mathcal{F}'(S) \subseteq \mathcal{G}$. Using Lemma 5, it holds that $\mathcal{F}'^{(k+1)}(\varnothing) \subseteq \mathcal{F}(\mathcal{F}'^k(\varnothing)$. Using the induction assumption, $\mathcal{F}'^k(\varnothing) \subseteq \mathcal{F}^k(\varnothing)$. And using the monotony of $\mathcal{F}$, and the transitivity of the set-inclusion, it holds that $\mathcal{F}'^{(k+1)}(\varnothing) \subseteq \mathcal{F}(\mathcal{F}^k(\varnothing)$, *i.e.* $\mathcal{F}'^{(k+1)}(\varnothing) \subseteq \mathcal{F}^{(k+1)}(\varnothing)$. □

Let $X \in \mathcal{E}$ such that Z attacks X and $\mathcal{E} \setminus \{X\}$ does not attack Z. Assume that $X \in \mathcal{E}' = \bigcup_{i\geq 1} \mathcal{F}'^i(\varnothing)$. As Z attacks X, $X \notin \mathcal{F}'(\varnothing)$. Let i be the smallest index ≥ 2 such that $X \in \mathcal{F}'^i(\varnothing)$. So $X \in \mathcal{F}'(\mathcal{F}'^{(i-1)}(\varnothing))$ and $X \notin \mathcal{F}'^{(i-1)}(\varnothing)$. Z attacks X and so there exists $Y \in \mathcal{F}'^{(i-1)}(\varnothing)$ such that Y attacks Z in $\mathcal{G}'$. Since $X \notin \mathcal{F}'^{(i-1)}(\varnothing)$, it holds that $Y \neq X$. Moreover, Z attacks X and we assume that $X \in \mathcal{E}'$, so $Z \notin \mathcal{E}'$ and thus $\mathcal{E}' \subseteq \mathcal{G}$. Using Lemma 6, we can infer that $\mathcal{F}'^{(i-1)}(\varnothing) \subseteq \mathcal{F}^{(i-1)}(\varnothing)$. Thus $Y \in \mathcal{E}$. So we have found $Y \in \mathcal{E} \setminus \{X\}$ which attacks Z, that is contradictory with the assumption. □

Proof of Prop. 34: X is never attacked in $\mathcal{G}'$. □

Proof of Prop. 35: The proof uses the following lemma:

Lem. 7 *Let $\mathcal{G}$ be an argumentation graph and $\mathcal{F}$ be the associated characteristic function. If $X \in \bigcup_{i\geq 1} \mathcal{F}^i(\varnothing)$, then for each T attacker of X, there exists an odd-length path from $\mathcal{F}(\varnothing)$ to T.*

> **Proof of Lem. 7:** By induction on $i \geq 1$, one proves that, if $X \in \mathcal{F}^i(\varnothing)$, then for each T attacker of X, there exists an odd-length path from $\mathcal{F}(\varnothing)$ to T. For the case $i = 1$, the result obviously holds ($X \in \mathcal{F}(\varnothing)$ implies that X is unattacked). For the other cases, assume that the property holds at the rank $p \geq 1$ and that

$X \in \mathcal{F}^{p+1}(\varnothing)$. Let T be an attacker of X. Thus T is attacked by an argument $W \in \mathcal{F}^{p}(\varnothing)$. If W is unattacked in $\mathcal{G}$, then $W \in \mathcal{F}(\varnothing)$, so there exists a path whose length equals 1 from $\mathcal{F}(\varnothing)$ to T. Otherwise, there exists U attacker of W. Using the assumption of the induction on W, there exists an odd-length path from $\mathcal{F}(\varnothing)$ to U. Then this path is augmented with the attack from U to W and the attack from W to T. So we obtain a new odd-length path from $\mathcal{F}(\varnothing)$ to T. □

First, note that $Y \neq Z$. Indeed, by assumption, $X \in \mathcal{E}$ and $Z \in \mathcal{E}$, so Z cannot attack X. So $Y \in \mathcal{G}'$. Moreover, $X \neq Z$ is also an assumption of the proposition. Using a reductio ad absurdum, assume that $X \in \mathcal{E}'$, *i.e.* $X \in \bigcup_{i \geq 1} \mathcal{F}'^{i}(\varnothing)$. Following the assumption on X, there exists Y attacker of X in $\mathcal{G}$ such that each odd-length path from $\mathcal{F}(\varnothing)$ to Y in $\mathcal{G}$ contains Z at an even place. Note that $X \in \mathcal{G}'$ and $Y \in \mathcal{G}'$. So, Lemma 7 can be applied on $\mathcal{G}'$ and $\mathcal{F}'$. Y attacking X in $\mathcal{G}'$, there exists an odd-length path from $\mathcal{F}'(\varnothing)$ to Y. Let $T \in \mathcal{F}'(\varnothing)$ be the root argument of this path to Y. This path only contains arguments of $\mathcal{G}'$, so it cannot contain Z so T cannot belong to $\mathcal{F}(\varnothing)$. This means that T is attacked in $\mathcal{G}$ and not attacked in $\mathcal{G}'$, so T is only attacked by Z. Thus there exists an even-length from Z to Y (via T) and so there exists an odd-length path from Z to X, that contradicts the last assumption of the proposition. □

Proof of Prop. 36: First note that $Z \in \mathcal{E}$. Indeed, $\mathcal{E} = \mathcal{F}(\mathcal{E})$ so $X \in \mathcal{F}(\mathcal{E})$ and so, with the assumption, $Z \in \mathcal{E}$. So there is no attack between Z and X, and the attackers of X in $\mathcal{G}$ are also the attackers of X in $\mathcal{G}'$. Using a reductio ad absurdum, assume that $X \in \mathcal{E}'$. We will show that $\mathcal{E}'$ defends X in $\mathcal{G}$. Let Y be an attacker of X in $\mathcal{G}$. Then Y attacks X in $\mathcal{G}'$. As $X \in \mathcal{E}'$ and $\mathcal{E}' = \mathcal{F}'(\mathcal{E}')$, it holds that $\mathcal{E}'$ defends X in $\mathcal{G}'$. So $\mathcal{E}'$ attacks Y in $\mathcal{G}'$ and also in $\mathcal{G}$. Thus $X \in \mathcal{F}(\mathcal{E}')$. Using the assumption given the proposition, it holds that $Z \in \mathcal{E}'$ that is impossible since Z has been removed. □