A Recommender System Based on Multi-Criteria Aggregation[1]

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ABSTRACT

Recommender systems aim to support decision-makers by providing decision advice. We review briefly tools of Multi-Criteria Decision Analysis (MCDA), including aggregation operators, that could be the basis for a recommender system. Then we develop a multi-criteria recommender system, STROMa (SysTem of RecOmmendation Multi-criteria), to support decisions by aggregating measures of performance contained in a performance matrix. The system makes inferences about preferences using a partial order on criteria input by the decision-maker. To determine a total ordering of the alternatives, STROMa uses a multi-criteria aggregation operator, the Choquet integral of a fuzzy measure. Thus, recommendations are calculated using partial preferences provided by the decision maker and updated by the system. An integrated web platform is under development.
Keywords: Recommender System, Choquet Integral, MCDA

1. INTRODUCTION

Research in the field of multi-criteria decision aid (MCDA) [1] has provided models and principles for decision problems that are both flexible and robust. In particular, a decision on multiple criteria must take synergy into account: positive synergy among criteria means that the criteria are related in that they all tend to have large or small values at the same time; a negative synergy means that one criterion has a negative influence on the others. Indeed, the use of fuzzy measures [2] provided both flexibility and robustness, and their use in Choquet

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integrals [3,4,5] effectively models the preferences of the decision-maker while taking into account both positive or negative synergies among criteria.

The quantitative approach forces a decision maker to think carefully about preferences over criteria. In addition, the decision maker must determine values for all alternatives on all criteria, which is often causes the greatest difficulties. These problems can be avoided by using a fuzzy measure to support decision-making based on a numerical representation that reflects the decision-maker's preference input.

The main objective of this article is the setting up of a Multicriteria recommendation system called STROMa. This system allows, for a decision problem and from a partial order established by the decision maker on a subset of alternatives, to determine the fuzzy measure digitally representing the decision maker's preferences.

Implementation of the Choquet integral as an aggregation operator is also discussed. This operator allows us to calculate the overall score of each alternative in order to establish a final ranking.

This paper is organized as follows. In the next section, we talk about related work on recommendation systems. The notation required to describe several aggregation operators and some axioms is introduced in the third section. The next section then describes the application of the Choquet integral to the evaluation of a fuzzy measure and its subsequent use to rank alternatives. In the fourth section, we present the web interface recommender system, STROMa, that we have developed and its limitations. In the final section, we offer some conclusions and an outlook.

2. Recommendation systems

Recommendation systems are as interactive decision support systems to take into account evolving preferences of users with a view to make recommendations. There are three main families of recommendation systems:

Content-based recommendation: Uses only object characteristics and user preferences to issue recommendations. This type of system is very effective when detailed information is available on the objects. Several such recommendation systems have emerged. For example, Martin et al. [6] used a content-based referral system to help select a provider for a user based on their profile. Also, Eureka is a system available on CanalSat TV channels. It analyzes the programs watched by the user to find out what type of program he enjoys. The system of recommendation that we put in place called STROMa is also of this type. But this method also has disadvantages. To make recommendations in relation to user preferences, the user must be familiar with the system. Thus during the initialization step of the preferences of the user, the system will not be able to make recommendations or these will be irrelevant.

Collaborative Recommendation: Uses the preferences of all available users to make proposals [7]. The basic idea of this method is that if a user has tastes similar to other users, then he should appreciate the objects chosen by them. Unlike the content-based recommendation, the system does not need to have much information to offer objects to the user. But the collaborative recommendation also has drawbacks. In the case of a system with few users or if the user has atypical preferences, there may be no user with a similar behavior, in which case the recommendations are not relevant. Very large systems use the collaborative recommendation. For example, Twitter suggests to its users a list of people to follow.

Hybrid Recommendation: Hybrid recommendation is a combination of content-based recommendation and collaborative recommendation. The aim is to eliminate the disadvantages of both approaches. The best-known hybrid recommendation system is the one used by Amazon [8]. Thus, in [9] Lakiotaki, K., Matsatsinis, N.F. and Tsoukias have set up a recommender system for movies of the high-performance hybrid type, based on the disaggregation-aggregation approach. Nevertheless, an aggregation approach is interesting in order avoid to too much frequently ask questions to the user.

3. NOTATION and AGGREGATION OPERATORS

We start by introducing some concepts. Let $X = \{a, b, \ldots\}$ be the set of alternatives (solutions), and let $N = \{1, \ldots, n\}$ be an index set representing the criteria.

Let $\succeq$ be a relation on X representing the decision-maker's preference. ($\succeq$ is usually pronounced "at least as good as".) As a binary relation, $\succeq$is usually assumed reflexive. For alternatives a and b, we use both the prefix notation $\succeq(a, b)$ and the infix notation $a \succeq b$ to mean that a is preferred to b. As the prefix notation indicates, $\succeq$ is considered to be a function on X^2. If the relation is binary, then $\succeq$ takes the values 0 and 1 only; if the relation is fuzzy (blurred or valued [10]), $\succeq$ takes values in [0, 1]. Note that $\sim$ is the symmetric part of the $\succeq$ relation, i.e., $a \sim b$ iff $a \succeq b$ and $b \succeq a$ (and is pronounced "a is indifferent to b").

In decision support, an aggregation operator is usually used to determine an overall score for an alternative from its local performance on the criteria and the user's preferences over criteria, in order to compare it to other alternatives. With the overall score, a ranking can be established that will guide the decision-maker's decision. In this section, we review several popular aggregation operators.

Weighted sum

The weighted sum is often used as an aggregation operator because of its simplicity. It requires a weight for each criterion to reflect its degree of importance in the decision problem. Weighted sum aggregation is defined by

$$\psi(a) = \psi(a_1, \ldots, a_n) = \sum_{i=1}^{n} w_i a_i \qquad (1.1)$$

where $a = (a_1, \ldots, a_n)$ represents the vector of (normalized) performance scores for alternative a and, for $i = 1, 2, \ldots, n$, $w_i \in [0, 1]$ is the weight assigned to criterion i, where

$$\sum_{i=1}^{n} w_i = 1 \tag{1.2}$$

The weighted sum is a very limited operator because it does not take into account any dependencies or relationships among criteria. Moreover, the preferences of the decision maker are included in a simplistic way using the fixed weight assigned to each criterion.

Ordered weighted sum

Ordered weighted sum [11] refers to a class of aggregation operators (called OWA) that determines the weight of a criterion for an alternative based on performance of that alternative relative to others. (In contrast, in weighted sum aggregation, the weight of a criterion depends only on the nature of the criterion.) OWA is defined by

$$OWA_w(a_1, \ldots, a_n) = \sum_{i=1}^{n} w_i a_{(i)} \tag{1.3}$$

where $W = (w_1, \ldots, w_n)$ is a weight vector such that $w_i \in [0, 1]$ for all i and (1.2) holds. The subscript (i) refers to the i^{th} component of the (normalized) performance scores after they have been arranged in ascending order, that is $\{a_1, \ldots, a_n\} = \{a_{(1)}, \ldots, a_{(n)}\}$ and $a_{(1)} \leq \ldots \leq a_{(n)}$. It is possible to use OWA to model many basic functions such as:

- The « Max » function, with weight vector $W = (0,0, \ldots, 1)$
- The « Min » function, with weight vector $W = (1, \ldots, 0, 0)$
- The « average», or mean, over the n criteria, with weight vector $W = (\frac{1}{n}, \ldots, \frac{1}{n})$.

Both the weighted sum and the ordered weighted sum aggregation operators effectively assume that criteria are independent, a feature that is unfortunately rare in practical cases. As an example, consider a decision maker about to buy a cell phone, whose criteria are price and design. The price criterion is to be minimized and the design criterion maximized (we generally want to have the most beautiful phone at the lowest price). But we know that the most beautiful phones are generally the most expensive, so the two criteria price and design are dependent and have negative synergy.

For a robust decision process that reflects reality, it is essential to take into account the interactions between criteria. Fortunately, other aggregation operators that do so are available. One of them is the Choquet integral, which is described in the next section.

The Choquet Integral

Aggregation operators such as weighted sum and OWA are unable to model interactions because they depend on weight vectors. What is needed is a non-additive function that defines a weight, not only for each criterion, but also for each subset of criteria. These non-additive functions can thus model both the importance of criteria and the positive and negative synergies between them. A suitable aggregation operator can be based on the Choquet integral that uses non-additive functions that Sugeno proposed be called fuzzy measures [2].

Definition: *A fuzzy measure μ on N is a function μ: $2^N \rightarrow [0, 1]$ that is monotonic in the sense* *that $\mu(S) \leq \mu(T)$ whenever $S \subseteq T$, and that satisfies limit conditions* $\mu(\emptyset) = 0$ *and* $\mu(N) = 1$.

The measure of a subset $S \subseteq N$ of criteria, $\mu(S)$, reflects the weight or importance of the criteria in S (compared to all others). To determine a fuzzy measure means to determine 2^n weights, corresponding to the 2^n subsets of N. The measure μ may be additive, that is, $\mu(S \cup T) = \mu(S) + \mu(T)$ for all subsets S and T, in which case, the weights of the n criteria are sufficient to calculate the fuzzy measure.

Fuzzy measures are appropriate as aggregation operators, as the interactions within a subset of criteria are represented by the weight of that subset in comparison to the weights of subsets of that subset. In fact, there are several classes of fuzzy integrals, of which one of the most representative and simplest is the Choquet integral [2, 3].

The Choquet integral is defined as follows: Let μ be a fuzzy measure on N. The Choquet integral of $x \in \mathbb{R}^n$ with respect to μ is defined by:

$$C\mu(x) := \sum_{i=1}^{n} x_{(i)}[\mu(A_{(i)}) - \mu(A_{(i+1)})] \tag{1.4}$$

where (.) denotes the permutation of the components of $x = (x_1, \ldots, x_n)$ such that $x_{(1)} \leq \ldots \leq x_{(n)}$. As well, $A_{(i)}=\{(i), \ldots, (n)\}$ and $A_{(n+1)} = \emptyset$.

In the Choquet integral, fuzzy measures represent the dependencies between (and among) criteria, as well as the relative weight of each criterion. The importance index [12] or Shapley value for criterion i with respect to μ is defined by:

$$\boldsymbol{\Phi}(\mu, i) := \sum_{\boldsymbol{T \subseteq N \setminus i}} \frac{(n-t-1)!\,t!}{n!}[\mu(T \cup i) - \mu(T)] \tag{1.5}$$

If μ is additive, we have $\mu(T \cup i) - \mu(T) = \mu(i)$ for all T; otherwise, this equality is false for some T and the criteria are dependent. To assess the degree of interaction between criteria i and j with respect to μ, the index of interaction [13] can be employed:

$$I(\mu, i, j) = \sum_{\boldsymbol{T \subseteq N \setminus ij}} \frac{(n-t-2)!\,t!}{(n-1)!}(\Delta_{ij}\mu)(T) \tag{1.6}$$

$$(\Delta_{ij}\mu)(T) := \mu(T \cup ij) - \mu(T \cup i) - \mu(T \cup j) + \mu(T). \tag{1.7}$$

The index of interaction $I(\mu,i,j)$ is in the range [-1, 1] for all $i, j \in N$. If the index is positive, then there is a positive synergy between these two criteria. If it is negative, the criteria are called redundant or substitutive, and one criterion may be chosen to represent the two.

Models that are 2-additive are economical because only interactions between two criteria need be considered. In this case, the Choquet integral is defined for all $a \in \mathbb{R}^n$ by

$$C_\mu(a) = \sum_{I_{ij}>0}(a_i \wedge a_j)I_{ij} + \sum_{I_{ij}<0}(a_i \vee a_j)|I_{ij}| + \sum_{i=1}^{n} a_i(\Phi(i) - \frac{1}{2}\sum_{j\neq i}|I_{ij}|) \qquad (1.8)$$

Properties of an aggregation operator :

Anonymity : Often called symmetry or commutativity : The order of the arguments has no influence on the result. This property is compulsory when the aggregation is made of arguments having the same importance or arises from anonymous experts or sources.
For every permutation σof {1,2, … , *n* } the operator satisfies :

$$\text{Aggreg}\ (x_{\sigma(1)}\ ,\ x_{\sigma(2)}\ ,\dots\ ,\ x_{\sigma(n)}\ ) = \text{Aggreg}\ (x_1,\ x_2,\dots,\ x_n) \qquad (1.9)$$

Continuity : The function Aggregation is continuous with respect to each of its variables. This property is a guaranty for certain robustness, for a certain consistency and for a non chaotic behavior.

Neutral Element
If the operator of aggregation has a neutral element *e,* then it can be used to be associated to an argument that should not have any influence on the aggregation :

$$\text{Aggreg}^{[n]}\ (x_1\ ,\dots,e,\dots\ ,\ x_{n-1}\ ) = \text{Aggreg}^{[n-1]}\ (x_1,\dots,\ x_{n-1}) \qquad (1.10)$$

Positive responsiveness: In the social choice, for any admissible profile <v_1, v_2, …, v_n>, if some voters change their votes in favour of one alternative (say the first) and all other votes remain the same, the social decision does not change in the opposite direction. In multi-criterion decision support, this means that adding a new criterion in a combination of criteria can not decrease its importance.

Independence of Irrelevant Alternatives: Let A and B be two alternatives. If A $\succeq$ B in the given choice {A, B} then the existence of a third solution X, which transforms the choice into {A, B, X}, must not make B preferable to A. In other words, the pre-existing choice between A and B can not be influenced by X, which is not relevant for the choice between A and B.

Completeness : The family of criteria must cover all the points of view expressed by the the stakeholders in a comprehensive manner. This means that no decisive criterion in the evaluation should not be forgotten. Formally, for any Pair of alternatives a and b whose performances on all the criteria are, they must be indifferent to the decision-maker.

4. DETERMINATION OF A FUZZY MEASURE

To develop a decision support system, it is crucial to find a fuzzy measure (or *capacity*) on the criteria so that the Choquet integral with respect to this capacity represents the preferences of the decision-maker. In practice, the decision-maker can usually indicate a low-cardinality subset $O \subseteq X$ of alternatives of interest on which the decision-maker has definite preferences. Alternatively, the decision-maker may be willing to specify partial preferences on the set of all criteria. Therefore the input from which the capacity is to be determined may consist of, for example:

- A partial preorder $\succeq_O$ on the subset $O \subseteq X$;
- A total preorder $\succeq_N$ on the set of all criteria, N;

In the present context, it seems natural to translate the partial preorder $\succeq_O$ using the following rules:

- $a \succeq_O b$ is equivalent to $C_\mu(a) > C_\mu(b)$
- $a \sim_O b$ is equivalent to $C_\mu(a) = C_\mu(b)$

where μ is the capacity to be determined. Similarly, $i \succeq_N j$ can be taken to be equivalent to $\Phi(\mu, i) \geq \Phi(\mu, j)$ on the set of criteria N and $i \sim_N j$ to $\Phi(\mu, i) = \Phi(\mu, j)$ on the same set.

Translating all the preferences expressed by the decision-maker using the rules above produces an optimization problem whose solution is the fuzzy measure μ on N. This optimization problem is expressed as follows:

Minimize or Maximize $F(. . .)$

$$\text{Subject to} \begin{cases} \mu(S \cup i) - \mu(S) \geq 0, \forall i \in N, \; \forall S \subseteq N \setminus i, \\ \mu(\emptyset) = 0, \mu(N) = 1, \\ C_{\mu(a)} - C_{\mu(b)} \geq \boldsymbol{\delta} c, \\ \dots \\ \Phi(\mu, i) - \Phi(\mu, j) \geq \boldsymbol{\delta} sh \\ \dots \end{cases} \tag{1.11}$$

where F is an objective function that depends on the method of identification chosen. Among the main methods are:

- Approaches based on least squares [14];
- Approaches based on linear programming [15];
- Method of minimum variance [16], which can also be interpreted as a method of maximum entropy. We will detail this last method below.

We use the Kappalab package [17] (Non-Additive Measure and Integral Manipulation Functions) to determine the capacity using the minimum variance method [16]. Other tools such as JRI (Java \ R Interface for using a Java program within R), JDK (Java Development Kit), and J2EE application libraries, could also be used. The capacity determination process can be summarized in the following steps:

1. Define the set of criteria used for the decision problem, entered by the user.

2. Determine the performance of each alternative on each criterion. This is the performance matrix, entered by the user.
3. Establish a partial order on the subset of alternatives specified by the user. Preferences are defined for a pair of alternatives by preference value, which must be one of the following:
    - 1, if the first alternative is preferred to the second,
    - −1, if the second alternative is preferred to the first,
    - 0, if both alternatives are indifferent or equivalent.
4. Using the preference table, create an R matrix containing all preferential information.
5. Use the function mini.var.capa.ident of package Kappalab to determine the capacity corresponding to the preferential information. This function, executed on the R platform using components JRI, uses the minimum variance identification method [15] to determine a capacity.
6. Extract the resulting capacity from the R platform.

After this step of identifying the fuzzy measure from the decision maker's preferences, came the aggregation phase using the Choquet integral. But before that, we will briefly see the minimum variance method used when identifying capacity in the next part.

The Minimum Variance Method

The idea of the minimum variance method, detailed by Kojadinovic in [18], is to promote the ability of "less specific" compatible with the initial preferences of the decision maker, if it exists. The objective function is the variance of the capacity.

$$F_{MV}(m_v) := \frac{1}{n}\sum_{i\in N}\sum_{S\subseteq N\backslash i}\gamma_s(n)\left(\sum_{T\subseteq S} m_{\mu}(T\cup i)-\frac{1}{n}\right)^2 \tag{1.12}$$

The optimization problem takes the form of a convex quadratic program:

Min $F_{MV}(m_v)$

$$\text{Subject to}\quad \begin{cases} \sum_{T\subseteq S} m_v(T\cup i)\geq 0, \forall i\in N, \forall\, S\ \in N\backslash i \\ \sum_{T\subseteq N} m_v(T)=1 \\ C_{m_v(u(x))}-C_{m_v}(u(x'))\geq \delta\mathrm{c} \\ \ldots. \\ \Phi_{\mathrm{m_v}}(i)-\ \Phi_{\mathrm{m_v}}(j)\geq \boldsymbol{\delta}\mathrm{sh} \\ \ldots.. \end{cases} \tag{1.11}$$

As discussed in [18], the Choquet integral with respect to the minimum variance capacity compatible with the preferences of the decision maker, if it exists, is one that will use the most on average arguments.

One advantage of this approach is that the solution, if it exists at all, must be unique, because of the strict convexity of the objective function. In addition, when the decisión-maker's preferences contain few synergies, this unique solution will not be extreme; in other words, it will not exhibit strong interactions between criteria that imply a disproportionate importance in the decision.

Choquet Integral Implementation

After determining the fuzzy measure using the Kappalab package [17], we can now aggregate the performance of each alternative using the Choquet integral. This allows us to obtain an overall score for each alternative in order to establish a final ranking. We proceed to calculate the full Choquet integral as follows:

1. Calculate the indices of interactions between all the subsets of criteria. We use the Boolean algebra to determine the different subsets of criteria. The number of subsets is 2^n, where n is the number of criteria.
2. Calculate the importance index, also called Shapley value, for each criterion.

3. For each alternative, calculate the value of the Choquet integral as follows:
   - Swap the vector $x = (x_1, \ldots, x_n)$ containing the performance of the alternative in order to have the following order: $x_{(1)} \leq \ldots \leq x_{(n)}$. The criteria must be arranged in the same order for the rest of the calculation .
   - Initialize valueG to 0, which will contain the Choquet value of the alternative..
   - For i=1 to n do //(n : the number of criterion)

     Determine the subset of criteria a(i) using the expression a(i) = {(i), ... (n)} and a (n + 1) = O on the permuted criteria.

     Similarly, determine the criterion subset a(i + 1).

     valueG+=x(i)*(μ(a(i))-μ(a(i+1))

   End For.

4. Sort the alternatives in descending order of overall score.

Note that Step 3 can be simplified if the capacity determined above is 2-additive, i.e., if the interaction index of all subsets of more than two criteria is 0, which can be determined by first finding the index of interaction of all 2^n subsets of criteria and then the Shapley values of all alternatives. Using 2-additivity, the Choquet integral value for each alternative can be obtained by applying the simpler formula (1.8) for the Choquet integral using the indices of interaction and Shapley values.

5. STROMa: A WEB PLATFORM RECOMMENDER SYSTEM:

We illustrate the STROMa system using two examples.
Example 1: Four Chefs, a problem proposed by Marichal & Rubens **[**15].
We want to evaluate the chefs based on their ability to prepare three dishes:

- Frog legs (FL)
- Steak tartare (ST)
- Scallops (SC).

The evaluation of the 4 chefs A, B, C, and D for each dish is given on a scale 0 to 20 in the following performance matrix:

	FL	**ST**	**SC**
A	18	15	19
B	15	18	19
C	15	18	11
D	18	15	11

Reasoning of the decision maker:

- When a chef is known for his preparation of Scallops, it is better that he prepares Frog Legs well, as compared to Steak Tartare;
- Conversely, when a chef does not do a good job preparing Scallops, it is better that he prepares Steak Tartare well, as compared to Frog Legs.

From the performance matrix, we can see that C and D are very good for FL but not for SC. Therefore, they should be good for ST; both are, but C is better than D. Chef B's marks are the same as C's except that he is better than C for SC. We conclude that B is better than C. Similarly, A is better than D. Both A and B prepare SC well, so according to the reasoning the score on FL is more important than the score of ST. We conclude that A should rank ahead of B. Similarly, C should rank ahead of D. Thus we can conclude than the decision-maker's ordering is A $\succeq$ B $\succeq$ C $\succeq$ D.

Now we use the STROMa system on this problem

Step 1: Definition of the criteria.

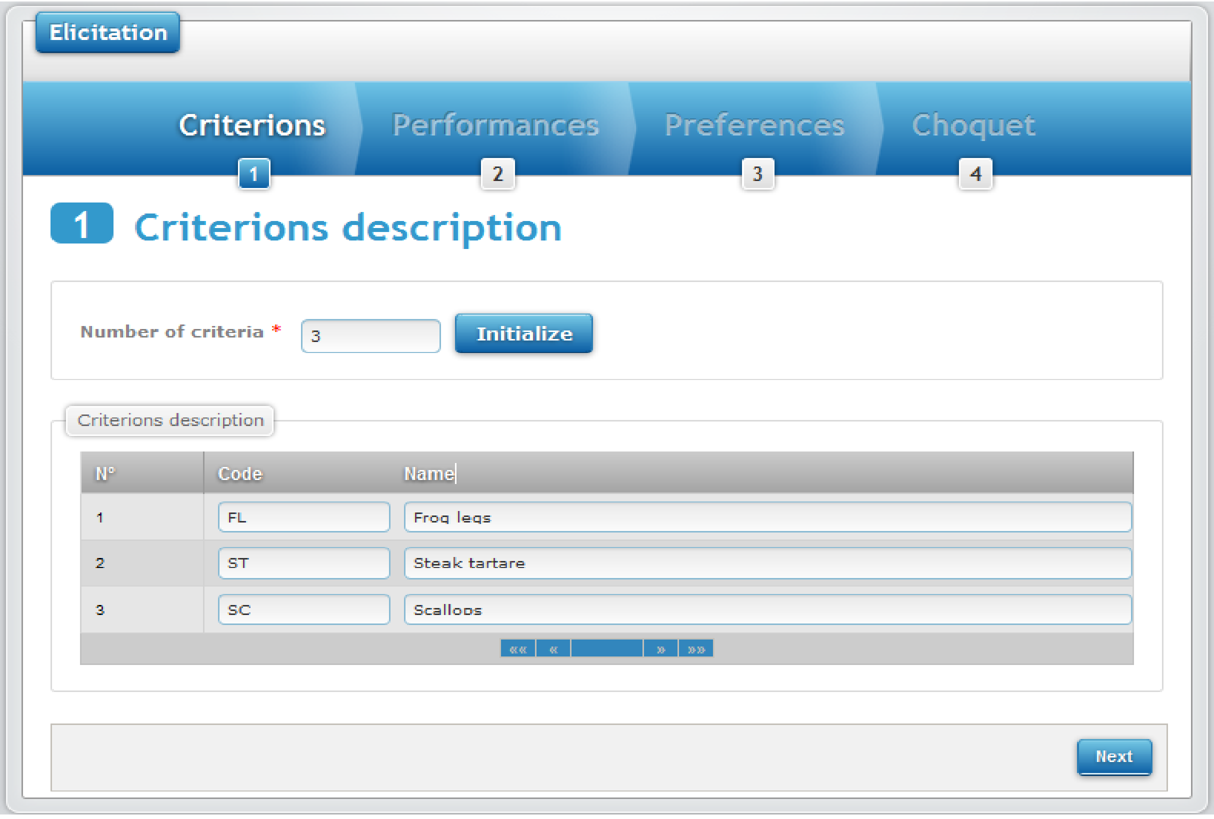


Figure 1: Definition of the criteria

Then the four alternatives are defined using a screen similar to step1. For the step2, we define the performance matrix.

Step 2: Fill in the performance matrix

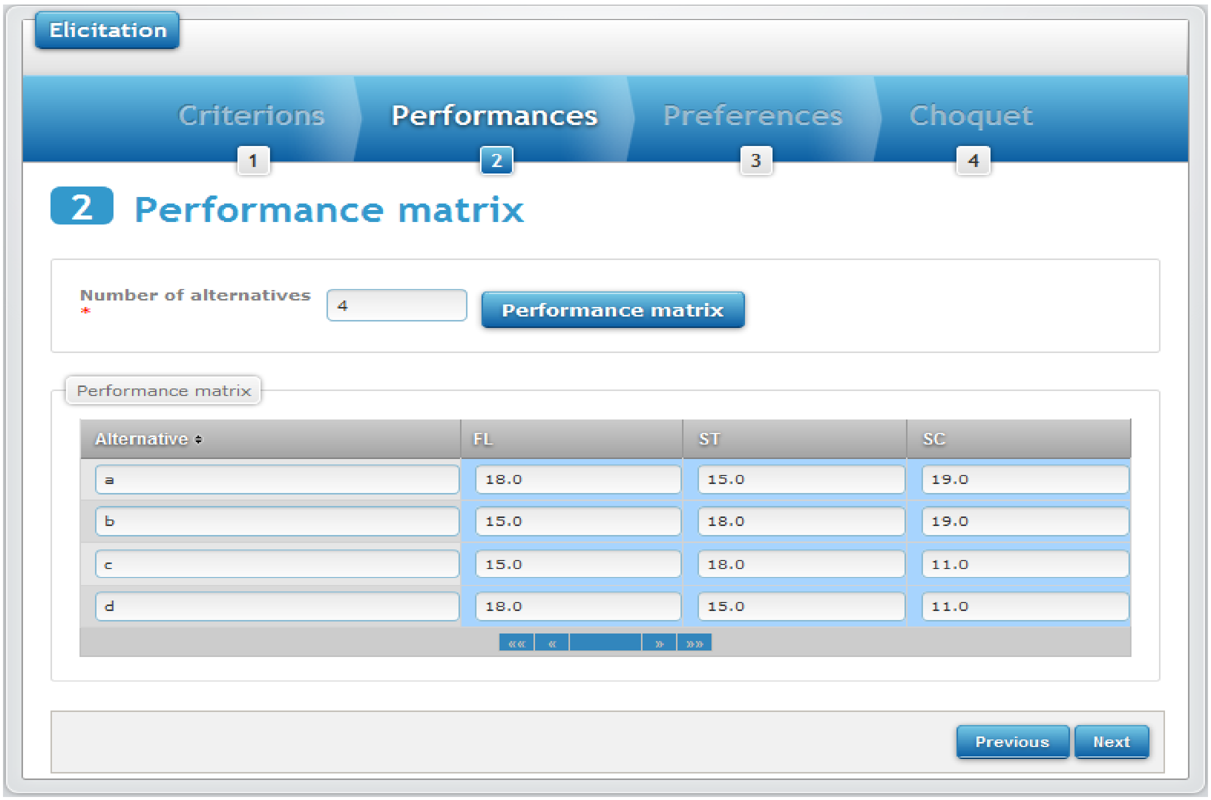


Figure 2: Performance matrix

Fort he step3, the user enters a preference partial order.
Step 3: Preference partial order
In this decision problem, we have the following preferences following directly from the reasoning: A ≥ B and C ≥ D.

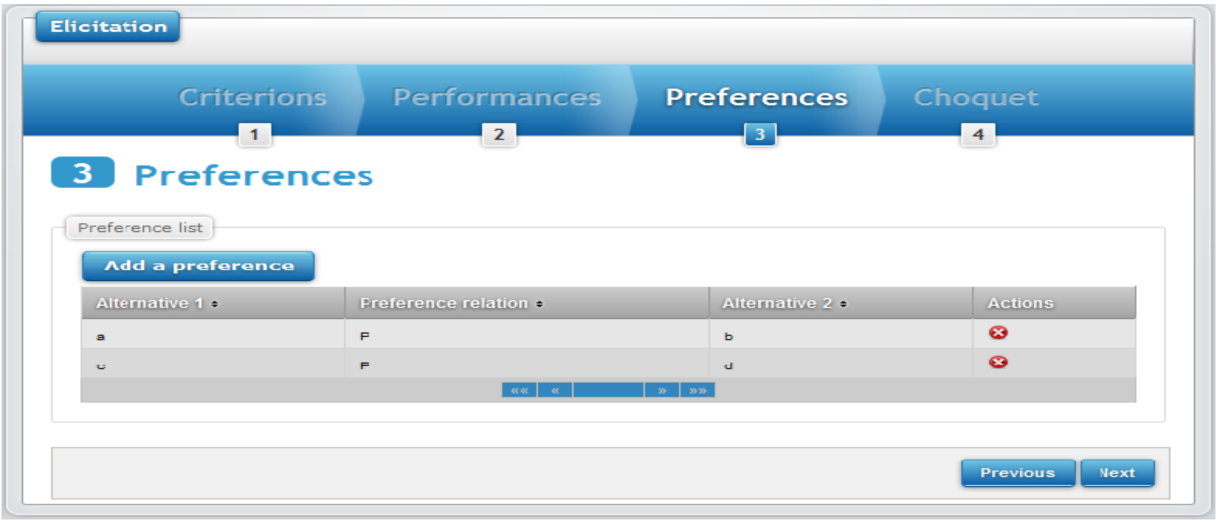


Figure 3: Partial order of decision maker preferences of

For the preference relation:

-P: expresses a preference in the broad sense of the alternative A versus B.

-I: expresses a indifference or equivalence between the two alternatives.

After this third stage, the capacity if it exists is determined and we proceed to the different calculations: importance index, the Shapley value and finally the value of Choquet integral. These results are visible on the next step.

Step 4: Results

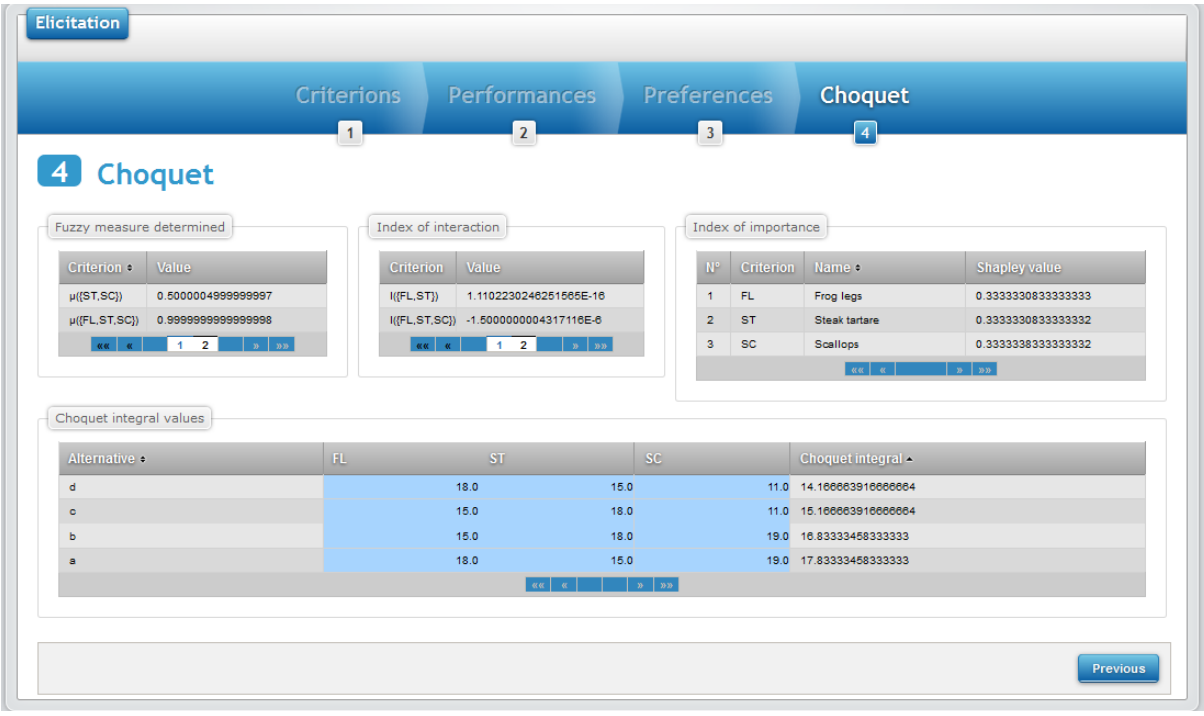


Figure 4: Results of the example of the four chefs

Results: one can easily check that the decision maker's preferences were taken into account. The system we implemented obtained the same final ranking, A ≥ B ≥ C ≥ D.

Example 2: Choosing a father's car, an example used by Roy [1]

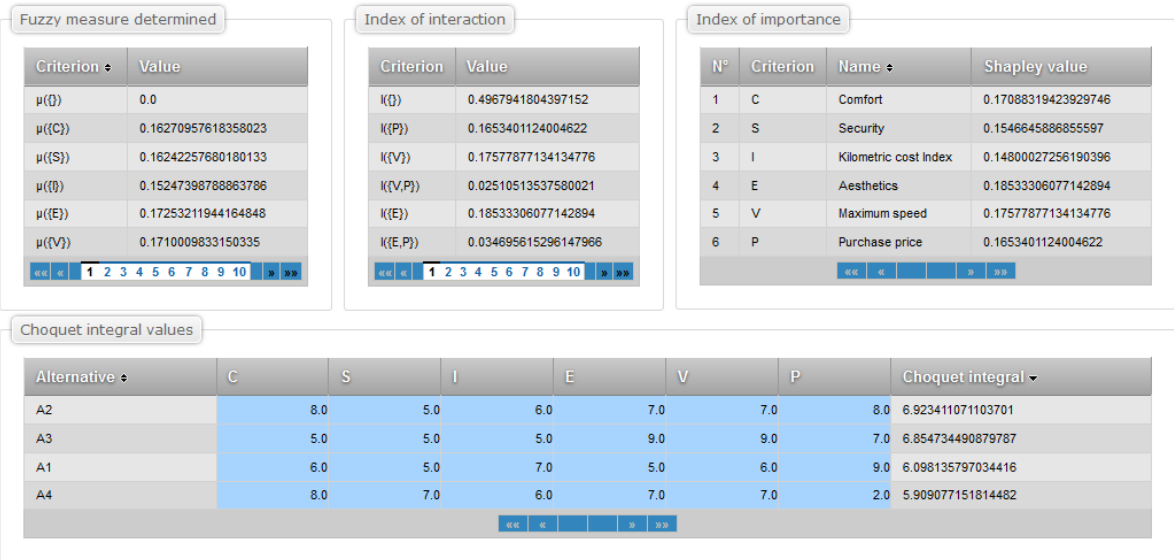
A father wants to buy a car taking account the following criteria:
- Comfort (C)
- Security (S)
- Index of cost per kilometre (I)
- Aesthetics (E)
- Maximum speed (V)
- Purchase price (P)

He has to choose one of the four cars (alternatives): a1, a2, a3, a4.

The entries in the performance matrix are given on a scale from 0 to 10:

Car model	Comfort	security	kilometer cost index	Aesthetic	Top speed	Buying price
a1	6	5	7	5	6	9
a2	8	5	6	7	7	8
a3	5	5	5	9	9	7
a4	8	7	6	7	7	2

After internal investigations, it emerged that the father prefers model a2 to a1 and model a3 to a4. This constitutes the partial order given by the decision maker. These preferences are then input to the system.



Fuzzy measure determined

Criterion	Value
μ({})	0.0
μ({C})	0.16270957618358023
μ({S})	0.16242257680180133
μ({I})	0.15247398788863786
μ({E})	0.17253211944164848
μ({V})	0.1710009833150335

Index of interaction

Criterion	Value
I({})	0.4967941804397152
I({P})	0.1653401124004622
I({V})	0.17577877134134776
I({V,P})	0.02510513537580021
I({E})	0.18533306077142894
I({E,P})	0.034695615296147966

Index of importance

N°	Criterion	Name	Shapley value
1	C	Comfort	0.17088319423929746
2	S	Security	0.1546645886855597
3	I	Kilometric cost Index	0.14800027256190396
4	E	Aesthetics	0.18533306077142894
5	V	Maximum speed	0.17577877134134776
6	P	Purchase price	0.1653401124004622

Choquet integral values

Alternative	C	S	I	E	V	P	Choquet integral
A2	8.0	5.0	6.0	7.0	7.0	8.0	6.923411071103701
A3	5.0	5.0	5.0	9.0	9.0	7.0	6.854734490879787
A1	6.0	5.0	7.0	5.0	6.0	9.0	6.098135797034416
A4	8.0	7.0	6.0	7.0	7.0	2.0	5.909077151814482

Results: We can see that the preferences of the decision maker have been taken into account. a2 has a better overall score than a1 and a3 also scores higher than a4. The final ranking obtained by the system is a2 ≥ a3 ≥ a1 ≥a4.

6. Limitations of the proposed model.

For the case of a boundary exists in the scoring scale where a neutral level separates the bad from the good marks, the Choquet integral as we use it, is unable to answer this issue. In this case, we should go to more general models such as the bipolar models [20] of the Choquet integral. These models use the notion of bi-capacity [20] to respond to these types of problems.

The example of student evaluation can be considered.

A principal of a school wants to evaluate his students from their marks in mathematics (M), statistics (S) and languages (L). Rather than fixing importance ratios for each subject and using a weighted sum, he believes that the importance of the subjects actually depends on the profile of the student in question, particularly his mathematical abilities. It expresses the following two rules:

(R1) For a good student in mathematics, languages are more important than statistics. (R2) For a bad student in mathematics, statistics are more important than languages. The underlying reason for these two rules is that, since the school is of a scientific type, it is impossible to admit a student who is weak in both fields of science and, if possible, students who are equally good at languages.

He considers the two students A and B.

Students	**Mathematics(M)**	**Statistics (S)**	**Language (L)**
A	14	16	7
B	14	15	8

By the rule (R1), we have unambiguously A $\prec$ B. If we want to represent this preference by a Choquet integral, this implies μ ({M, S}) + μ ({S}) <1 , As can easily be verified. The Director now considers students C and D.

Students	**Mathematics(M)**	**Statistics (S)**	**Language (L)**
C	9	16	7
D	9	15	8

This time, according to rule (R2), we have C$\succ$D, which implies that μ ({M, S}) + μ ({S})> 1. But this leads to a contradiction! Because we had already established that μ ({M, S}) + μ ({S})< 1. It is therefore impossible to represent these preferences by an integral of Choquet. It is appropriate in this case to go to the bipolar models of the integral of Choquet, which use the notion of bi-capacity.

7. CONCLUSIONS

We reviewed some aggregation operators useful for MCDA, and showed how a fuzzy measure could be used to address a problem of decision aid when a partial order is an input of the decision maker in using Choquet integral. A recommender system called STROMa is implemented in an integrated web platform, allowing, from a partial order defined by the decision-maker, to determine the fuzzy measure in order to establish a final ranking. In the future, it will be strengthened by incorporating other aggregation operators and other decision aid concepts, such as bicapacity [20] concepts and the bipolar Choquet integral [20]. Another possible development would be the automatic adjustment of aggregation techniques, based on the context of decision making the situation and the profile of the user. It would also be interesting to introduce new faster and robust capacity identification algorithms.

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