Acceptability semantics accounting for strength of attacks in argumentation

Claudette CAYROL
Caroline DEVRED
Marie-Christine LAGASQUIE-SCHIEX

Rapport conjoint IRIT–LERIA

numéro IRIT/RR–2010-13–FR

Abstract

We consider argumentation systems taking into account several attack relations of different strength. We focus on the impact of various strength attacks on the semantics of such systems. First, we refine the classical notion of defence, by comparing the strength of an attack with the strength of a counter-attack: an argument C will be a defender of A against B if the attack from B to A is not stronger than the attack from C to B. Then, we propose different ways to compare defenders, and sets of defenders. That enables us to define admissible sets offering a best defence for their elements.

keywords: Argumentation Systems, Acceptability semantics, Strength of attacks

Contents

1 Introduction

Argumentation has become an influential approach in Artificial Intelligence to model cognitive tasks such as inconsistency handling and defeasible reasoning (e.g. [KAEF95, PV02, AC02a]), decision making (e.g. [KM03]), or negotiation between agents (e.g. [ADM07]).

Argumentation is based on the evaluation of interacting arguments, which support opinions, claims, decisions,... Usually, the interaction takes the form of conflicts between arguments, and the fundamental issue is the selection of acceptable sets of arguments, based on the way they interact. Most of the argumentation-based proposals are instantiations of the abstract system proposed by Dung ([Dun95]), which is reduced to a set of arguments (completely abstract entities) and a binary relation, called attack, which captures the conflicts between arguments. The increasing interest for the argumentation formalism has led to numerous extensions of the basic abstract system which are more appropriate to the applications.

A first extension of Dung's system has included a preference relation between arguments, which models their relative strength. For instance, an argument built from certain knowledge is stronger than an argument reying upon default knowledge (see e.g. [AC02b, BC03, WBC07, Mod09]). Another kind of extension is necessary to make a distinction between various types of conflict. For instance, when arguments are built from logical rules and knowledge, rebut and undercut conflict have been distinguished ([EGH95]). More generally, symmetric attacks may be considered as weaker than non symmetric attacks. [MGS08b] has distinguished between blocking attacks and proper attacks, as a consequence of preference between arguments. In a multi-agent setting, various attack relations over a common set of arguments represent different criteria and different contexts according to which the conflicts are perceived ([TBS08]). Moreover, it is natural to consider that not all attacks are equal in strength. [DHM+09] has proposed weighted argument systems, in which attacks are associated with a numeric weight, indicating how reluctant one would be to disregard the attack. Behind these proposals, there is a common idea that attacks may have different strength and can be compared according to their relative strength. However, there is so far no consensus about how it should be used to define extensional semantics, according to which acceptable sets of arguments are selected. A first promising work towards that direction has been proposed in [MGS08a], where an abstract argumentation system with varied-strength attacks has been defined. The nature and structure of every attack is not specified and it is sufficient to state an order of strength between these abstract attacks. In that novel system, the classical concepts of defence and admissibility are revisited, in different directions, leading to several different refinements.

Our work takes place in that abstract system with attacks of various strength. As in the work of [MGS08a], our motivation is to compare attacks and not to consider weighted attacks so that a collection of weaker attacks should be able to overrule a stronger attack. Our purpose is to focus on one restriction of the notion of defense, and to come to define extensional semantics accounting for the strength of defence. In Section 2, we present the fundamental notions. First, we propose a restricted notion of defence, by requiring that the counter-attack is not weaker then the attack. Then, following Dung's classical construction, we define a restricted admissibility which is used, in Section 3, for revisiting the classical preferred and stable semantics. The next step is to compare the defence collectively offered by sets of arguments; so, in Section 4, different comparisons are proposed, at different levels, enabling us to define two new semantics according to which the selected extensions offer a best defence for their elements. Section 5 is devoted to concluding remarks and some related works.

2 The fundamental

We consider the abstract system defined in [MGS08a]:

Definition 1 ([MGS08a] Argumentation system with attacks of various strength) *An* argumentation system with attacks of various strength *is a triple* $\langle \mathbf{A}, \mathbf{ATT}, \succeq \rangle$, *denoted by* $\mathbf{AS_{vs}}$*, where*

- $\mathbf{A}$ *is a finite*[1] *set of arguments,*
- $\mathbf{ATT}$ *is a finite set of attack relations* $\langle \xrightarrow{1}, \ldots, \xrightarrow{n} \rangle$ *on* $\mathbf{A}$ *and*
- $\succeq$ *is a binary relation on* $\mathbf{ATT}$.

Each $\xrightarrow{i} \subseteq \mathbf{A} X \mathbf{A}$ represents a conflict relation, and $\succeq$ represents a relative strength between these conflict relations. The relation $\succeq$ is only assumed reflexive (it may be partial, and transitive or not). The corresponding strict relation will be denoted by $\succ$. If the relation $\succeq$ is empty, a classical system (in Dung's sense) is recovered with the single attack relation obtained as the union of the attack relations $\xrightarrow{i}$. In the following of this paper, $\mathbf{AS}$ will denote the classical system $\langle \mathbf{A}, \bigcup_i \xrightarrow{i} \rangle$ associated with the argumentation system with attacks of various strength $\mathbf{AS_{vs}} = \langle \mathbf{A}, \mathbf{ATT}, \succeq \rangle$.

This kind of argumentation system can be illustrated using the following example:

Example 1 *Consider the* $\mathbf{AS_{vs}}$ *defined by* $\mathbf{A} = \{A, B, C_1, C_2\}$, $\mathbf{ATT} = \langle \xrightarrow{i}, \xrightarrow{j}, \xrightarrow{k} \rangle$ *with* $\xrightarrow{i} = \{(B, A)\}$, $\xrightarrow{j} = \{(C_1, B)\}$, $\xrightarrow{k} = \{(C_2, B)\}$, *and* $\succeq$ *defined by* $\xrightarrow{j} \succeq \xrightarrow{i}$.

$\mathbf{AS_{vs}}$ *can be depicted by the graph:*

$$\begin{tikzcd} C_1 \arrow[r, "j"] & B \arrow[r, "i"'] & A \\ C_2 \arrow[ur, "k"'] & & \end{tikzcd}$$

An intuitive counterpart to this system can be provided by the following dialogue between the prosecutor and the counsel for the defence during a criminal trial:

Argument A *(Prosecutor): Tom is a suspect since Bob has seen Tom leaving the scene of crime.*

Argument B *(Counsel): Bob is myopic, and was too far from the scene of crime; so, he couldn't see Tom.*

[1] We assume that $\mathbf{A}$ represents the set of arguments proposed by rational agents at a given time; so it makes sense to assume that $\mathbf{A}$ is finite.

Argument C_1 *(Prosecutor's witness number 1, Bob's ophthalmologist): Bob has achieved very good results for eye tests; he is not myopic.*

Argument C_2 *(Prosecutor's witness number 2, Bob's girl friend): Bob does not wear glasses, so he is not myopic.*

As regards Bob's shortsightedness, the opinion of Bob's ophthalmologist is more reliable than the opinion of Bob's girl friend. That may lead to state that $\xrightarrow{j} \succ \xrightarrow{k}$.

Using the framework proposed in [MGS08a], our purpose is to study the impact of these attacks of various strength on the notion of defence, which is a key concept in argumentation. The first level to be considered is the individual defence level. In classical systems, an argument A attacked by an argument B is said defended (against B) as soon as there exists an argument C attacking B. Indeed, any attack on B is relevant for inhibiting the attack from B to A. Now, if attacks may have different strengths, it is natural to compare the attack on B with the attack from B to A. The idea is that some attacks on B will be too weak to inhibit the attack on A and thus will not be relevant for reinstating A.

Let $\mathbf{AS_{vs}} = \langle \mathbf{A}, \mathbf{ATT}, \succeq \rangle$. We propose the following definition to capture the idea of relevant defender:

Definition 2 (vs-defence – vs means "various-strength") *Let* A, B, $C \in \mathbf{A}$ *such that* $C \xrightarrow{j} B$ *and* $B \xrightarrow{i} A$. C vs-defends A *against* B *(or* C *is a* vs-defender *of* A *against* B*) iff* $\xrightarrow{i} \not\succ \xrightarrow{j}$ *(i.e. the attack from* B *to* A *is not strictly stronger than the one from* C *to* B*).*

Note that the same kind of definition is encountered in works about preference-based argumentation, for combining attack relation and preference relation (see [AC02b, WBC07]): the idea is that an attack from B to A is relevant if A is not strictly preferred to B; otherwise, it can be considered that A defends itself against B.

So, in a similar way, Definition 2 states that the attack from B to A is overruled by the attack from C to B if the attack from B to A is not strictly stronger than the attack from C to B.

At that point, it is interesting to restate Definition 2 in the system proposed by [MGS08a], where defenders are classified in four categories. Let A, B, $C \in \mathbf{A}$ such that $C \xrightarrow{j} B$ and $B \xrightarrow{i} A$. According to [MGS08a]:

- C is a *strong defender* of A against B iff $\xrightarrow{j} \succ \xrightarrow{i}$;
- C is a *weak defender* of A against B iff $\xrightarrow{i} \succ \xrightarrow{j}$;
- C is a *normal defender* of A against B iff $\xrightarrow{j} \succeq \xrightarrow{i}$ and $\xrightarrow{i} \succeq \xrightarrow{j}$;
- C is an *unqualified defender* of A against B iff $\xrightarrow{i}$ and $\xrightarrow{j}$ are uncomparable.

It is easy to see that the notion of vs-defender exactly corresponds to "not weak defender", or equivalently to "strong or normal or unqualified defender". In the particular case of a complete relation on $\mathbf{ATT}$, $\xrightarrow{i} \not\succeq \xrightarrow{j}$ means that $\xrightarrow{j} \succeq \xrightarrow{i}$. Then, "$C$ vs-defends A against B" exactly corresponds to "C is a strong defender or a normal defender of A against B".

As required above, the notion of vs-defence refines the classical notion of defence.

Proposition 1 *Let A, B, $C \in \mathbf{A}$. If C vs-defends A against B then C defends A against B in Dung's sense.*

Then, following the classical construction of acceptability (or collective defence), we propose to refine the classical notions of acceptability and admissibility introduced by Dung in [Dun95]: a set $S \subseteq \mathbf{A}$ is *conflict-free* if $\forall A, B \in S$, B does not attack A; S is *admissible* if it is conflict-free and defends all its elements against all attacks.

So, using the vs-defence leads to the new notion of vs-acceptability:

Definition 3 (vs-acceptability) *Let $S \subseteq \mathbf{A}$ and $A \in \mathbf{A}$. A is* vs-acceptable *with regard to (wrt) S (or S* collectively vs-defends *A against any attack) iff $\forall B \in \mathbf{A}$, if B attacks A then $\exists C \in S$ such that C vs-defends A against B.*

The notion of vs-acceptability is a particular case of the notion of "constrained acceptability" of [MGS08a]. Indeed, it is sufficient to consider the defence profile containing the three levels strong, normal and unqualified. Note also that vs-acceptability refines classical acceptability (this is a direct consequence of Proposition 1).

We define a vs-admissible set, which is a set proposing a valuable defence for each of its elements:

Definition 4 (conflict-free set and vs-admissibility)
S is conflict-free *in $\mathbf{AS_{vs}}$ iff $\forall A, B \in S$, $\nexists \xrightarrow{i} \in \mathbf{ATT}$, s.t. $B \xrightarrow{i} A$ (iff S is conflict-free in the associated $\mathbf{AS}$ in Dung's sense).*
S is vs-admissible *iff S is conflict-free in $\mathbf{AS_{vs}}$, and $\forall A \in S$, A is vs-acceptable wrt S.*

Note that the vs-admissibility requires the classical strict notion of conflict-free. That ensures that for any vs-admissible set S, no attack may occur between elements of S. Note also that the empty set is vs-admissible.

From Proposition 1, it follows directly that vs-admissibility refines classical admissibility:

Proposition 2 *If $S \subseteq \mathbf{A}$ is vs-admissible, then S is admissible in Dung's sense.*

The converse is false: Let $C \xrightarrow{j} B \xrightarrow{i} A$, with $\xrightarrow{i} \succ \xrightarrow{j}$; $\{C, A\}$ is admissible but not vs-admissible.

Moreover, it is interesting to say that there does not exist a formal compilation of an $\mathbf{AS_{vs}}$ into a classical Dung's argumentation framework $\mathbf{AR} = \langle \mathbf{A}, \rightarrow \rangle$ (over the same arguments) which would be equivalent, *i.e.* such that the vs-admissible sets of $\mathbf{AS_{vs}}$ would coincide with the admissible sets of $\langle \mathbf{A}, \rightarrow \rangle$. The following example illustrates the impossibility to obtain such a translation:

Example 2 *Consider the* $\mathbf{AS_{vs}}$ *represented by*

$$\begin{tikzcd} C_1 \arrow[dr, "j"] & & A_1 \\ & B \arrow[ur, "i"'] \arrow[dr, "k"'] & \\ C_2 \arrow[ur, "h"'] & & A_2 \end{tikzcd}$$

with:

- $\xrightarrow{i} \succ \xrightarrow{j}$ *(C_1 is not a vs-defender of A_1),*
- $\xrightarrow{j} \succeq \xrightarrow{k}$ *(C_1 is a vs-defender of A_2),*
- $\xrightarrow{h} \succeq \xrightarrow{i}$ *(C_2 is a vs-defender of A_1),*
- $\xrightarrow{h} \succeq \xrightarrow{k}$ *(C_2 is a vs-defender of A_2).*

Assuming that $\mathbf{AS_{vs}}$ *can be compiled into an equivalent classical system* $\mathbf{AR} = \langle \mathbf{A}, \rightarrow \rangle$*, the following constraints hold:*

- $\{C_1, C_2, A_1, A_2\}$ *is vs-admissible, so* $\{C_1, C_2, A_1, A_2\}$ *must be admissible in* $\mathbf{AR}$*, and so* $\{C_1, C_2, A_1, A_2\}$ *must be conflict-free in* $\mathbf{AR}$*.*
- $\{A_2\}$ *is not vs-admissible, so* $\{A_2\}$ *should not be admissible in* $\mathbf{AR}$*. So,* A_2 *must be attacked and since* $\{C_1, C_2, A_1, A_2\}$ *is conflict-free,* A_2 *is attacked by* B *in* $\mathbf{AR}$*, and* B *is the only attacker of* A_2 *in* $\mathbf{AR}$*.*
- *And similarly,* B *is the only attacker of* A_1 *in* $\mathbf{AR}$*.*
- $\{C_1, A_2\}$ *is vs-admissible, so* $\{C_1, A_2\}$ *must be admissible in* $\mathbf{AR}$*. So,* C_1 *must attack* B *in* $\mathbf{AR}$*.*
- $\{C_1, A_1\}$ *is not a vs-admissible set, so* $\{C_1, A_1\}$ *should not be admissible in* $\mathbf{AR}$*.*

As C_1 *attacks* B*, which is the only attacker of* A_1*, and due to the fact that* $\{C_1, C_2, A_1, A_2\}$ *is conflict-free, it must be the case that* $\{C_1, A_1\}$ *does not defend* C_1*. The only possibility is that* C_1 *is attacked by* B *(since* $\{C_1, C_2, A_1, A_2\}$ *is conflict-free). But then, the set* $\{C_1, A_1\}$ *would still be admissible (since* C_1 *attacks* B*).*

So, in conclusion, it is not possible to translate the above $\mathbf{AS_{vs}}$ *into an equivalent classical system.*

3 Classical semantics revisited

Admissibility is the basis of most of the classical semantics, in Dung's abstract system. So, it is straightforward to revisit some classical semantics, using the notion of vs-admissibility.

For instance, let us consider the preferred semantics which produces maximal (for set-inclusion) admissible sets of arguments.

Let $\mathbf{AS_{vs}}$ = $\langle \mathbf{A}, \mathbf{ATT}, \succeq \rangle$ be an argumentation system with attacks of various strength.

Definition 5 (preferred vs-extension) *Let $S \subseteq \mathbf{A}$ be a vs-admissible set. S is a* preferred vs-extension *of $\mathbf{AS_{vs}}$ iff $\nexists S' \subseteq \mathbf{A}$ such that $S \subset S'$ and S' is vs-admissible.*

Example 1 (cont'd) *If $\xrightarrow{i} \succ \xrightarrow{k}$, C_1 is the only vs-defender of A, otherwise C_1 and C_2 are vs-defenders of A. In both cases, $\{A, C_1, C_2\}$ is the only preferred vs-extension, since C_2 is not attacked.*

Preferred vs-extensions have nice properties:

Proposition 3 *Preferred vs-extensions satisfy the 3 following points:*

1. *Let $S \subseteq \mathbf{A}$ be a vs-admissible set, there exists a preferred vs-extension E of $\mathbf{AS_{vs}}$ such that $S \subseteq E$.*
2. *There always exists at least one preferred vs-extension of $\mathbf{AS_{vs}}$.*
3. *For each preferred vs-extension E of $\mathbf{AS_{vs}}$, there exists a preferred extension E' of $\mathbf{AS}$ such that $E \subseteq E'$.*

Proof:

1. It follows directly from Definition 5 since $\mathbf{A}$ is finite.
2. It follows directly from the previous point, since the empty set is vs-admissible.
3. And for the third point, we have: let E be a preferred vs-extension; E is vs-admissible, so E is admissible for $\mathbf{AS}$ (due to Proposition 2); so, using a result of [Dun95], E is included in a preferred extension of $\mathbf{AS}$.

$\square$

Now, we consider the stable semantics.
Classically, a conflict-free set S is a stable extension iff S attacks each argument which does not belong to S. Note that this definition does not use the notion of defence. In the classical system, it can be proved that a stable extension is admissible. That is the reason why the following equivalent definition has sometimes been given: S is a stable extension iff S is admissible and attacks each argument which does not belong to S (see for instance [Cam06]).

Following this line, we propose:

Definition 6 *Let $S \subseteq \mathbf{A}$ be a vs-admissible set. S is a* stable vs-extension *of $\mathbf{AS_{vs}}$ iff S attacks each argument which does not belong to S.*

From this definition, it follows directly that a stable vs-extension of $\mathbf{AS_{vs}}$ is a stable extension of $\mathbf{AS}$. Moreover, it is easy to prove that a stable vs-extension is also a preferred vs-extension. The converse does not hold as shown by a variant of Example 1.

Example 1 (cont'd) *With $\xrightarrow{i} \succ \xrightarrow{j}$ and $\xrightarrow{i} \succ \xrightarrow{k}$, C_1 and C_2 are not vs-defenders of A. So, the set $\{C_1, C_2\}$ is a preferred vs-extension but not a stable vs-extension.*

$\{A, C_1, C_2\}$ is a stable extension of $\mathbf{AS}$, but is not vs-admissible so is not a stable vs-extension.

And finally, the classical grounded and complete semantics can be revisited in an analogous way, using Dung's methodology and the fixpoint of the characteristic function. For lack of space, we just mention the following fundamental lemma, according to which the construction is sound.

Lemma 1 (Fundamental lemma) *Let $S \subseteq \mathbf{A}$ be a vs-admissible set, and $A, B \in \mathbf{A}$ be two arguments vs-acceptable wrt S. We have:*

1. *$S' = S \cup \{A\}$ is vs-admissible, and*
2. *B is vs-acceptable wrt S'.*

Proof:

1. S is vs-admissible, so S is conflict-free. A is vs-acceptable wrt S, so due to Proposition 1, A is also acceptable wrt S. From Dung's fundamental lemma, it follows that $S \cup \{A\}$ is conflict-free. Moreover, as S is vs-admissible and A is vs-acceptable wrt S, each argument in $S \cup \{A\}$ is vs-acceptable wrt S, and then also vs-acceptable wrt $S \cup \{A\}$. So, $S \cup \{A\}$ is vs-admissible.
2. We have: B is vs-acceptable wrt S means that S vs-defends B. So, $S \cup \{A\}$ also vs-defends B. So, B is vs-acceptable wrt $S \cup \{A\}$.

$\square$

4 Semantics accounting for the quality of defence

As stated in Section 2, a vs-admissible set proposes a valuable defence for each of its elements. This is a basic requirement. The next step is to take into account the existence of attacks of various strength for evaluating the quality of a valuable defence, and for selecting vs-admissible sets which offer a best defence.
Clearly, if an argument B attacks an argument A, comparing two attacks against B enables to compare two defenders of A against B. Starting from that remark, we propose to compare defences at different levels:

- comparing two vs-defenders of a same argument
- comparing two sets which collectively vs-defend a same argument
- comparing two vs-admissible sets

4.1 Comparing defences of an argument

As said above, two vs-defenders of A can be compared by considering the relative strength of the attacks they put on an attacker of A.

Let $\mathbf{AS_{vs}}$ be an argumentation system with attacks of various strength.

Definition 7 (Comparison of vs-defenders) *Let $A, B, C_1, C_2 \in \mathbf{A}$ such that both C_1 and C_2 are vs-defenders of A against B with $C_1 \xrightarrow{j} B$ and $C_2 \xrightarrow{k} B$.*

$$C_1 \text{ is better than } C_2 \textit{ iff } \xrightarrow{j} \succeq \xrightarrow{k}$$

$$C_1 \textit{ is } \text{strictly better than } C_2 \textit{ iff } \xrightarrow{j} \succ \xrightarrow{k}$$

Now, let S_1 and S_2 be two sets which collectively vs-defend A (that is A vs-acceptable wrt S_1 and wrt S_2). In order to determine whether S_1 offers a stronger defence for A than S_2, it is sufficient to compare the subset of S_1 containing the vs-defenders of A with the subset of S_2 containing the vs-defenders of A.

So, we need the following definition:

Definition 8 (Set of defenders) *Let $S \subseteq \mathbf{A}$ and $A \in \mathbf{A}$ such that A is vs-acceptable wrt S. $\mathbf{Def}(A, S)$ is the set of the vs-defenders of A which belong to S.*

Note that, if $C \in \mathbf{Def}(A, S)$, there exists B attacking A such that C attacks B. So, if A is not attacked, $\mathbf{Def}(A, S) = \varnothing$.

Example 1 (cont'd) *Assuming that $\xrightarrow{j} \succ \xrightarrow{i}$ and $\xrightarrow{k} \succ \xrightarrow{i}$ implies $\mathbf{Def}(A, \{C_1, C_2, A\})$ $= \{C_1, C_2\}$. Assuming that $\xrightarrow{j} \succ \xrightarrow{i} \succ \xrightarrow{k}$ implies $\mathbf{Def}(A, \{C_1, C_2, A\}) = \{C_1\}$.*

Given S_1 and S_2 two sets of arguments which collectively vs-defend A, we have to compare $\mathbf{Def}(A, S_1)$ and $\mathbf{Def}(A, S_2)$.

The idea is the following: given E_1 and E_2 two sets of vs-defenders of A, E_2 is at least as strong as E_1 if E_2 improves the defence of A offered by E_1 on at least one defender. That idea can be formalized using the following *general scheme of set-comparison*:

Definition 9 (Scheme of set-comparison) *Let $\mathcal{E}$ be a set of elements. Let $E_1 \subseteq \mathcal{E}$ and $E_2 \subseteq \mathcal{E}$. Let* `is-strictly-better-than` *be a strict binary relation on $\mathcal{E}$. E_2* strictly better than E_1 *will be denoted by $E_2 \sqsupset E_1$ and defined by:*

- *$E_2 \sqsupseteq E_1$ iff there exist $C_1 \in E_1$ and $C_2 \in E_2 \setminus E_1$ such that C_2* `is-strictly-better-than` *C_1.*
- *$E_2 \sqsupset E_1$ iff ($E_2 \sqsupseteq E_1$ and not $E_1 \sqsupseteq E_2$).*

Note that there exist many schemes for defining a comparison of subsets from the comparison of their elements (see for instance [CRS93, Hal97]). For instance, a more careful definition, discussed in [Hal97], would consider that $E_2 \sqsupset E_1$ iff for each $C_1 \in E_1$ there exists $C_2 \in E_2$ such that C_2 `is-strictly-better-than` C_1. This latter definition is very restrictive since it comes to prefer E_2 only if E_2 strictly improves each element of E_1. As stated above, our motivation is different. We consider that if E_2 improves the defence of A offered by E_1 on at least one defender, and the converse is false, it is sufficient to say that E_2 is stronger than E_1. That is the reason why we choose Definition 9.

Note that Definition 9 supports *a priori* neither the minimal sets for set-inclusion, nor the maximal sets for set-inclusion. It is also a weak alternative of the following definition given in [MGS08a]: $E_2 \sqsupset E_1$ iff there exist $C_1 \in E_1$ and $C_2 \in E_2 \setminus E_1$ such that C_2 `is-strictly-better-than` C_1 and there exist no $C_1 \in E_1$ and no $C_2 \in E_2$ such that C_1 `is-strictly-better-than` C_2.

The scheme of Definition 9 obviously satisfies:

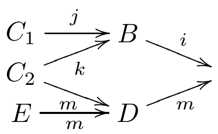
Proposition 4

1. *If $E_2 \subseteq E_1$, either E_1 and E_2 are uncomparable, or $E_1 \sqsupset E_2$.*
2. *If $E_2 \subseteq E_3$ and $E_2 \sqsupseteq E_1$ then $E_3 \sqsupseteq E_1$.*

Our aim is to use the scheme of Definition 9 for comparing sets of vs-defenders of a given argument A. So, in the following, we instantiate this scheme by replacing the relation `is-strictly-betterthan` by the strict relation given in Definition 7.

Example 1 (cont'd) *Assume that C_1 and C_2 are two vs-defenders of A against B with $\xrightarrow{j}\succ\xrightarrow{k}$ (so C_1 is strictly better than C_2). $\{C_1, C_2\} \sqsupset \{C_2\}$, $\{C_1\} \sqsupset \{C_2\}$, but $\{C_1, C_2\}$ and $\{C_1\}$ are uncomparable.*
Indeed, although C_1 is strictly better than C_2, we cannot consider that $\{C_1\}$ offers a stronger defence than $\{C_1, C_2\}$ since C_1 already belongs to $\{C_1, C_2\}$.

Example 3 *Consider the $\mathbf{AS_{vs}}$ depicted by the following graph (note that A is not vs-acceptable wrt $\{C_1\}$ in this $\mathbf{AS_{vs}}$):*



with $\xrightarrow{i}\not\succ\xrightarrow{k}$, $\xrightarrow{i}\not\succ\xrightarrow{j}$ and $\xrightarrow{j}\succ\xrightarrow{k}$

Due to the constraints $\xrightarrow{i}\not\succ\xrightarrow{k}$ and $\xrightarrow{i}\not\succ\xrightarrow{j}$, C_1 and C_2 are vs-defenders of A. Note that no constraint on m is mandatory to ensure that E is a vs-defender of A. So, the following set-comparisons hold: $\{C_1, C_2\} \sqsupset \{C_2\}$, $\{C_1, E\} \sqsupset \{C_2\}$ and $\{C_1, E\} \sqsupset \{C_2, E\}$. Note that these set-comparisons are all based on the only comparison concerning C_1 and C_2 as vs-defenders of A against B.

Example 4 *Assume that C_1, C_2 and C_3 are three vs-defenders of A against B with C_3 strictly better than C_2 and C_2 strictly better than C_1. We have $\{C_1, C_2, C_3\} \sqsupset \{C_1, C_3\}$ since C_2 is strictly better than C_1 and C_2 does not belong to $\{C_1, C_3\}$. In contrast, according to the definition proposed in [MGS08a], these sets are uncomparable.*

To sum up the above discussion: S_1 and S_2 being two sets which collectively vs-defend A (that is A vs-acceptable wrt S_1 and wrt S_2), we are able to determine whether S_1 offers a stronger defence for A than S_2, using the comparison of $\mathbf{Def}(A, S_1)$ and $\mathbf{Def}(A, S_2)$ by the appropriate instantiation of Definition 9:

Definition 10 (Set-comparison wrt the defence of an argument) *Let S_1 and S_2 be two sets of arguments such that A is vs-acceptable wrt S_1 and wrt S_2. S_2* stronger than *S_1* wrt the defence of *A will be denoted by $S_2 \gg_A S_1$, and is defined by:*

$$S_2 \gg_A S_1 \text{ iff } \mathbf{Def}(A, S_2) \sqsupset \mathbf{Def}(A, S_1)$$

Definition 10 enables to compare two sets of vs-defenders of a given argument, wrt the defence of that argument. However, this comparison is sometimes useless, as shown by the following example.

Example 5 *This example has been taken from [MGS08b].*

$$
\begin{tikzcd}
 & F \arrow[r, "j"] & C \arrow[dr, "i"] & \\
G \arrow[r] & D \arrow[r, "j"] & B \arrow[r, "i"'] & A \\
 & E \arrow[ur, "k"'] & &
\end{tikzcd}
$$

with $\xrightarrow{i} \not\succ \xrightarrow{j}$, $\xrightarrow{i} \not\succ \xrightarrow{k}$ *and* $\xrightarrow{j} \succ \xrightarrow{k}$

Let S_1 = $\{F, E\}$ *and* S_2 = $\{F, D\}$. A *is vs-acceptable wrt* S_1 *and wrt* S_2. *And* $S_2 \gg_A S_1$. *However, no vs-admissible set contains* S_2.

So, it seems important that the set-comparison wrt the defence of an argument is restricted to vs-admissible sets.

4.2 Comparing admissible sets

The third step towards the evaluation of the quality of the defence consists in the comparison of two vs-admissible sets. We want to determine whether the defence proposed by one set is globally as strong as the defence proposed by the other set. By definition, if S is vs-admissible, S vs-defends each of its elements. So, it makes sense to compare two vs-admissible sets wrt one or several elements of their intersection.

An interesting situation occurs when we have compare two vs-admissible sets S_1 and S_2 such that: consider A and B, both in $S_1 \cap S_2$; S_1 is stronger than S_2 wrt the defence of A; S_2 is stronger than S_1 wrt the defence of B. In that case, it seems reasonable to conclude that no set is strictly preferred to the other one. They may rather be considered as equivalent.

That remark leads to consider that the defence proposed by S_2 is globally as strong as the defence proposed by S_1 when S_2 offers a stronger defence than S_1 for at least one common element. This will be denoted by $S_2 \gg= S_1$, and defined as follows:

Definition 11 (Set-comparison of vs-admissible sets) *Consider* S_1 *and* S_2 *two vs-admissible sets.*

- $S_2 \gg= S_1$ *iff there exists an argument* A *in* $S_1 \cap S_2$ *such that* $S_2 \gg_A S_1$ *(i.e. such that* $\mathbf{Def}(A, S_2) \sqsupset \mathbf{Def}(A, S_1)$*).*
- $S_2 \gg S_1$ *iff (*$S_2 \gg= S_1$ *and not* $S_1 \gg= S_2$*).*

Example 6 *Consider the* $\mathbf{AS_{vs}}$ *depicted by the following graph:*

$$
\begin{tikzcd}
C_1 \arrow[dr, "j"] \arrow[dddddd, bend right, leftrightarrow, "m"'] & & \\
& B_1 \arrow[ddr, "i"] & \\
C_3 \arrow[ur, "k"'] \arrow[dd, bend right, leftrightarrow, "m"] & & \\
& & A \\
C_2 \arrow[dr, "m"] & & \\
& B_2 \arrow[uur, "i"'] & \\
C_4 \arrow[ur, "m"'] & &
\end{tikzcd}
\qquad \textit{with } \xrightarrow{i} \not\succ \xrightarrow{j},\ \xrightarrow{i} \not\succ \xrightarrow{k},\ \xrightarrow{i} \not\succ \xrightarrow{m} \textit{ and } \xrightarrow{k} \succ \xrightarrow{j}.
$$

Let $S_1 = \{A, C_1, C_2\}$ and $S_2 = \{A, C_3, C_4\}$. S_1 and S_2 are vs-admissible. Due to $\xrightarrow{k} \succ \xrightarrow{j}$, we have $S_2 \gg_A S_1$. We have also $S_2 \gg S_1$.

Note that no element of S_2 is a strictly better vs-defender of A than C_2. So, S_2 does not improve S_1 on each vs-defender of A belonging to S_1.

This example illustrates the difference between the chosen scheme of set-comparison (cf. Definition 9) and some other schemes given in the literature.

Example 7 *This example has been taken from [MGS08a].*

$$
\begin{tikzcd}
& & C \arrow[dl, "k"'] \arrow[dr, "j"] \arrow[dd, bend left, leftrightarrow, "m"] & & \\
F & E \arrow[l, "i"'] & & B \arrow[r, "i"'] & A \\
& & D \arrow[ul, "j"] \arrow[ur, "k"'] & &
\end{tikzcd}
$$

Assume that C and D are two vs-defenders of A and F. Let $S_1 = \{A, C, F\}$ and $S_2 = \{A, D, F\}$. S_1 and S_2 are vs-admissible. If we assume that $\xrightarrow{j} \succ \xrightarrow{k}$, we have $S_1 \gg_A S_2$ and $S_2 \gg_F S_1$. So, S_1 and S_2 are equivalent wrt Definition 11.

As a direct consequence of Proposition 4, we have:

Proposition 5 *If $S_2 \subseteq S_1$, either S_2 and S_1 are uncomparable, or $S_1 \gg S_2$.*

4.3 Towards the strongest extensions

In this section, we will define new semantics accounting for the quality of the defence, taking advantage of the material presented above.

What seems to be relevant is to choose vs-admissible sets which offer a strongest defence (from a global point of view) for their elements, or in other words, a defence which cannot be improved wrt an argument without being damaged wrt another argument. However, the empty set trivially fulfils this first requirement. And clearly, we do not expect the empty set as an output for the argumentation system. So, it will be more interesting to select vs-admissible sets which fulfil two requirements: offering a strongest defence, and being maximal for set-inclusion. This will lead to two proposals depending on the priority between these two requirements.

Let us first give a formal definition for the vs-admissible sets offering a strongest defence.

Definition 12 (Strong-admissibility) *Let $S \subseteq \mathbf{A}$. S is a* strong-admissible set *iff S is vs-admissible and S is maximal for the relation $\gg$ (see Definition 11) among the vs-admissible sets.*

In other words, a vs-admissible set S is strong-admissible iff $\forall A \in S, \forall S'$ vs-admissible containing A, if $S' \gg_A S$, then there exists $B \in S \cap S'$ such that $S \gg_B S'$.

Note that strong-admissibility encompasses two refinements of classical admissibility: from a local point of view, defence between two arguments is restricted to vs-defence, and from a collective point of view, defence is at strongest.

The first semantics we propose aims at selecting strong-admissible sets which are maximal for set-inclusion, called max-strong-admissible sets.

Definition 13 (Max-strong-admissible set) *Let $S \subseteq \mathbf{A}$ be a strong-admissible set. S is a* max-strong-admissible set *iff $\nexists S' \subseteq \mathbf{A}$ such that $S \subset S'$ and S' is strong-admissible.*

Note that the empty set is strong-admissible, so there always exists at least one max-strong-admissible set.

Example 1 (cont'd) *C_1 and C_2 are vs-defenders of A against B. Assume that C_1 is strictly better than C_2. The vs-admissible sets are: $\varnothing$, $\{C_1\}$, $\{C_2\}$, $\{A, C_1\}$, $\{A, C_2\}$, $\{C_1, C_2\}$ et $\{A, C_1, C_2\}$. All of them, except $\{A, C_2\}$, are also strong-admissible. $\{A, C_1, C_2\}$ is the unique max-strong-admissible set.*

Example 8 *It is a variant of Example 7, replacing $D \xrightarrow{k} B$ by $D \xrightarrow{j} B$.*

$$\begin{tikzcd}
 & & C \arrow[dl, "k"'] \arrow[dr, "j"] \arrow[dd, bend left, "m"] & & \\
F & E \arrow[l, "i"'] & & B \arrow[r, "i"'] & A \\
 & & D \arrow[ul, "j"] \arrow[ur, "j"'] \arrow[uu, bend left, "m"] & &
\end{tikzcd}$$

Assume that $\xrightarrow{j} \succ \xrightarrow{k}$. $\{A, C\}$ and $\{A, D\}$ are strong-admissible, the defence is at strongest for A. $\{A, D, F\}$ is also strong-admissible, the defence is at strongest for A and for F. However, $\{A, C, F\}$ is not strong-admissible since $\{A, D, F\} \gg \{A, C, F\}$. So, the max-strong-admissible sets are $\{A, C\}$ and $\{A, D, F\}$.

An alternative proposal of semantics consists in comparing the global defence offered by preferred vs-extensions (Definition 5). In other words, the quality of the defence is considered only for maximal vs-admissible sets. That leads to select strong-preferred sets defined as follows:

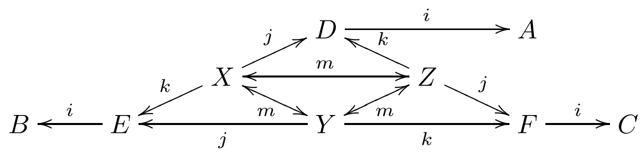
Definition 14 (Strong-preferred set) *Let $S \subseteq \mathbf{A}$ be a preferred vs-extension. S is* strong-preferred *iff S is maximal for the relation $\gg$ (see Definition 11) among the preferred vs-extensions.*

So, a maximal vs-admissible set is strong-preferred iff $\nexists S'$ maximal vs-admissible such that $S' \gg S$.

Example 8 (cont'd) *The preferred vs-extensions are* $\{A, D, F\}$ *and* $\{A, C, F\}$. *As* $\{A, D, F\} \gg \{A, C, F\}$, *the only strong-preferred set is* $\{A, D, F\}$.

Note that in some cases, there exists no strong-preferred extension, as illustrated by the following example.

Example 9 *Consider the* $\mathbf{AS_{vs}}$ *depicted by the following graph:*



with the assumptions $\xrightarrow{i} \not\succ \xrightarrow{j}$, $\xrightarrow{i} \not\succ \xrightarrow{k}$ *and* $\xrightarrow{j} \succ \xrightarrow{k}$, $\xrightarrow{j} \succ \xrightarrow{m}$ $\xrightarrow{k} \succ \xrightarrow{m}$

In this case, there are three preferred vs-extensions: $E_1 = \{A, B, X\}$, $E_2 = \{B, C, Y\}$ *and* $E_3 = \{A, C, Z\}$. *Nevertheless there is no strong-preferred set because:* $E_1 \gg E_3 \gg E_2 \gg E_1$.

The max-strong-admissible sets are: $\{A, X\}$, $\{B, Y\}$ *and* $\{C, Z\}$.

Both semantics aim at selecting maximal vs-admissible sets defending at strongest their elements. However, they do not consider the quality of the defence at the same level. Indeed, S is a strong-admissible set means that S contains a strongest defence for each of its elements. So S is max-strong-admissible means that S gathers all arguments for which S offers a strongest defence. It is as each argument of S would be treated separately. For instance, in Example 8, $S = \{A, X\}$ is a max-strong-admissible set, but does not contain B which is defended by S. It means that there exists another strong-admissible set S' containing a better vs-defender of B than the one contained in S: $S' = \{B, Y\}$.

In contrast, a strong-preferred set offers a globally strongest defense for a maximal set of arguments. So, there may exist a max-strong-admissible set which is not a preferred vs-extension (see Example 8), and there may exist a strong-preferred set which is not strong-admissible (see Example 7).

Example 7 (cont'd) *The preferred vs-extensions are* $S_1 = \{A, C, F\}$ *and* $S_2 = \{A, D, F\}$. *We have* $S_1 \gg_A S_2$ *and* $S_2 \gg_F S_1$, *so* S_1 *and* S_2 *are strong-preferred. However,* $\{D, F\} \gg \{A, C, F\}$ *and* $\{A, C\} \gg \{A, D, F\}$, *so* $\{D, F\}$ *and* $\{A, C\}$ *are the only max-strong-admissible sets.*

However, some interesting particular cases can be encountered:

Proposition 6 *If a preferred vs-extension is strong-admissible, then it is both strong-preferred and max-strong-admissible.*

Proof: Let S be a preferred vs-extension. If S is strong-admissible, S is maximal for $\gg$ among the vs-admissible sets. As preferred vs-extensions are vs-admissible sets, S is also maximal for $\gg$ among the preferred vs-extensions. Now, if S is not max-strong-admissible, there exists S' strong-admissible such that $S \subset S'$. S' is vs-admissible, so it is in contradiction with S preferred vs-extension. $\square$

Note the the converse of Proposition 6 trivially holds, giving a characterization of the sets selected by both semantics.

Proposition 7 *If $\mathbf{AS_{vs}}$ admits only one preferred vs-extension S, then*

- *S is the only strong-preferred set.*
- *S is also strong-admissible and it is the only max-strong-admissible set.*

Proof: Let S_0 be the only preferred vs-extension. The first result trivially holds, due to Definition 14. Let us prove the second point.

- If S_0 is not strong-admissible, there exists S vs-admissible s.t. $S \gg S_0$. As S is vs-admissible and S_0 is the only vs-admissible set maximal for set-inclusion, we have $S \subseteq S_0$. Then, following Proposition 5, $S {\gg}{=} S_0$ does not hold. That is in contradiction with the initial assumption. So, S_0 is strong-admissible.
- If S_0 is not max-strong-admissible, there exists S_1 strong-admissible s.t. $S_0 \subset S_1$. By definition, S_1 is vs-admissible. That is in contradiction with S_0 vs-admissible maximal for set-inclusion. So, S_0 is max-strong-admissible.
- Assume that there exists another max-strong-admissible denoted by S_2. S_2 is vs-admissible, so $S_2 \subseteq S_0$. As S_0 and S_2 are both max-strong-admissible, $S_2 = S_0$. So, S_0 is the only max-strong-admissible set.

$\square$

The following examples illustrate the proposed semantics for cyclic $\mathbf{AS_{vs}}$.

Example 10

$$\begin{tikzcd}
B \arrow[r, "i"] & A \arrow[d, "j"] \\
C \arrow[u, "j"] & D \arrow[l, "i"]
\end{tikzcd}$$

The max-strong-admissible sets are $\{A, C\}$ if $\xrightarrow{j}\succ\xrightarrow{i}$, $\{B, D\}$ if $\xrightarrow{i}\succ\xrightarrow{j}$, $\{A, C\}$ and $\{B, D\}$ otherwise. In each case, the strong-preferred sets are exactly the max-strong-admissible sets. Preference between attacks may reduce the number of extensions, in case of even-length cycle.

Example 11

$$\begin{tikzcd}
 & A \arrow[dd, "k"] \\
B \arrow[ur, "i"] & \\
 & C \arrow[ul, "j"]
\end{tikzcd}$$

The empty set is the only max-strong-admissible set and the only strong-preferred set whatever preference is assumed between the attacks. As vs-admissibility requires the classical strict notion of conflict-free, our new semantics have the same behaviour as classical preferred semantics on that odd-length cycle.

As said before, strong-admissibility refines classical admissibility by the restriction to vs-defence, and by requiring that the collective defence is at strongest. Analogous ideas have been developed in [MGS08a], leading to the notion of top-admissibility. However, our proposal comes to strong-admissibility through several defence comparisons. In contrast, [MGS08a] gives a direct definition of top-admissible sets. Moreover, nothing is proposed to select some top-admissible sets.

In order to compare both notions more precisely, let us recall that vs-acceptability is a particular case of "constrained acceptability" (see Section 2). In that particular case, the definition of top-admissibility can be restated as follows:

Definition 15 (top-admissibility) *Let $S \subseteq \mathbf{A}$ be a vs-admissible set. S is* top-admissible *iff for each argument A belonging to S, there exists no vs-admissible set S' such that $\mathbf{Def}(A, S') \sqsupset \mathbf{Def}(A, S)$).*

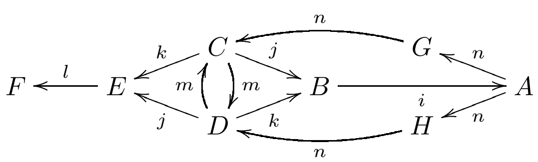
Then, it can be proved that:

Proposition 8 *Let $S \subseteq \mathbf{A}$ be a vs-admissible set. If S is top-admissible, then S is a strong-admissible set.*

> **Proof:** Let S be top-admissible. If S is not strong-admissible, there exists S' vs-admissible such that $S' \gg S$. So, there exists $A \in S \cap S'$ such that $S' \gg_A S$. That is in contradiction with S being top-admissible. $\square$

However, the converse does not generally hold as shown by:

Example 12



Assume that C and D are vs-defenders of A and F, A is a vs-defender of C and D, and $\xrightarrow{j} \succ \xrightarrow{k}$. $\{A, C, F\}$ is a preferred vs-extension and is strong-admissible (indeed, the defence of F by C could be improved by D, but the only vs-admissible set containing F and D is $\{A, D, F\}$ – due to H – which is equivalent to $\{A, C, F\}$ for $\gg$=). However, $\{A, C, F\}$ is not top-admissible, as $\{A, D, F\} \gg_F \{A, C, F\}$.

5 Conclusion and related works

Our proposal in this paper is a further contribution to the development of argumentation with various attacks of different strength, based on the abstract framework introduced by [MGS08a]. The basic idea is to use the relative strength of the attacks for refining the concept of reinstatement: we define a new notion of defence, the vs-defence, requiring that the counter-attack is not weaker than the attack. This enables us to revisit Dung's classical semantics and to define vs-admissible sets, preferred vs-extensions and stable vs-extensions. A further step is to compare the defence offered by sets of arguments. We propose comparisons at different levels: between vs-defenders, between sets of vs-defenders and between vs-admissible sets. This enables us to define two semantics accounting for two requirements: selecting maximal vs-admissible sets and selecting sets offering a strongest defence for their elements. Depending on the way these requirements are combined, we obtain max-strong-admissible sets and strong-preferred sets. These semantics are investigated and related to the notion of top-admissibility proposed by [MGS08a].

Our work has been clearly inspired from previous proposals ([MGS07, MGS08b, MGS08a]). The common basic idea is to use the relative strength of the attacks for disregarding some of the defences. In the most abstract and general proposal [MGS08a], this idea is formalised by the notion of defense condition, a set of requirements in the relative strength of attacks and counter-attacks. [MGS08a] handles expansive sets of defence conditions, and proposes several interesting semantic notions. In constrast, we focus on only one defence condition, the vs-defence, and we come to extensional semantics, by revisiting classical ones, and by investigating defence comparisons.

Another recent work dealing explicitely with attacks of relative strength is described in [DHM$^+$09], where the strength of an attack is indicated by a numerical weight. However, these weigths are not used for disregarding or comparing defences, but for disregarding attacks. Given a threshold, the idea is to combine the weights additively, and to disregard subsets of attacks which sum to no more than the threshold. Then extensions are computed relatively to a given threshold. For instance, given a threshold of b, S is a preferred extension if S can be obtained as a classical preferred extension by disregarding a subset of attacks whose total amount does not exceed b.

We have identified several directions for future work. One is to investigate the relationship between the strength of arguments and the strength of attacks. The distinction is not always clear in realistic situations, and it would be interesting to see up to which point some of the works dealing with preference between arguments could be restated in the abstract framework with attacks of various strength.

We are also interested in the definition of other semantics, related to the decision problem of credulous acceptability: namely, focussing on one particular argument, a classical issue is to compute a proof, under the form of a minimal admissible set containing this argument. Taking into account attacks of various strength suggests to search for the strongest proofs (see [CDLS10a, CDLS10b] for a preliminary work on this subject).

Another perspective is to go beyond the framework of ordered attacks. In this paper, the strength of the attacks is only used to compare them, and is not handled as a weight. It would be interesting to propose a more general framework allowing to subsume both aspects (ordered vs weighted attacks), so that, for instance, a collection of weaker attacks could be aggregated in order to overrule a stronger attack.

Acknowledgements

We would like to thank the reviewers for their help and their very interesting suggestions.

References

[AC02a] Leila Amgoud and Claudette Cayrol. Inferring from inconsistency in preference-based argumentation frameworks. *Journal of Automated Reasoning*, 29:125–169, 2002.

[AC02b] Leila Amgoud and Claudette Cayrol. A reasoning model based on the production of acceptable arguments. *Annals of Mathematics and Artificial Intelligence*, 34:197–216, 2002.

[ADM07] Leila Amgoud, Yannis Dimopoulos, and Pavlos Moraïtis. A unified and general framework dor argumentation-based negotiation. In *Proc. of International Joint Conference on Autonomous Agents and Multiagent Systems (AAMAS)*, pages 963–970, 2007.

[BC03] Trevor Bench-Capon. Persuasion in practical argument using value-based argumentation frameworks. *Journal of Logic and Computation*, 13(3):429–448, 2003.

[Cam06] Martin Caminada. Semi-stable semantics. In Paul E. Dunne and Trevor J. M. Bench-Capon, editors, *Proc. of the 1^{st} COMMA (Computational models of arguments)*, pages 121–128. IOS Press, 2006.

[CDLS10a] Claudette Cayrol, Caroline Devred, and Marie-Christine Lagasquie-Schiex. Dialectical proofs accounting for strength of attacks. Technical report IRIT/RR–2010-7–FR, IRIT and LERIA, France, March 2010. ftp://ftp.irit.fr/pub/IRIT/RPDMP/rap-2010-7.pdf.

[CDLS10b] Claudette Cayrol, Caroline Devred, and Marie-Christine Lagasquie-Schiex. Dialectical proofs accounting for strength of attacks. In *submitted to International Conference on Computational Models of Argument (COMMA)*, 2010.

[CRS93] Claudette Cayrol, Véronique Royer, and Claire Saurel. Management of preferences in assumption-based reasoning. In R. Yager and B. Bouchon, editors, *Advanced methods in AI. Lecture notes in computer science 682*, pages 13–22. Springer Verlag, 1993. Extended version in Technical Report IRIT-CERT, 92-13R (University Paul Sabatier Toulouse).

[DHM$^+$09] Paul E. Dunne, Anthony Hunter, Peter McBurney, Simon Parsons, and Michael Wooldridge. Inconsistency tolerance in weighted argument systems. In *Proc of International Joint Conference on Autonomous Agents and Multi-Agent Systems (AAMAS)*, 2009.

[Dun95] Phan Minh Dung. On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. *Artificial Intelligence*, 77:321–357, 1995.

[EGH95] Morten Elvang-Gøransson and Anthony Hunter. Argumentative logics: Reasoning with classically inconsistent information. *Data and Knowledge Engineering*, 16:125–145, 1995.

[Hal97] Joseph Y. Halpern. Defining relative likelihood in partially-ordered preferential structures. *Journal of Artificial intelligence Research*, 7:1–24, 1997.

[KAEF95] Paul Krause, Simon Ambler, Morten Elvang, and John Fox. A logic of argumentation for reasoning under uncertainty. *Computational Intelligence*, 11 (1):113–131, 1995.

[KM03] Antonis C. Kakas and Pavlos Moraïtis. Argumentation based decision making for autonomous agents. In *Proc. of International Joint Conference on Autonomous Agents and Multi-Agent Systems (AAMAS)*, pages 883–890. ACM Press, 2003.

[MGS07] Diego C. Martinez, Alejandro J. Garcia, and Guillermo R. Simari. On defense strength of blocking defeaters in admissible sets. In *Proc. of International Conference on Knowledge Science, Engineering and Management (KSEM) – LNAI 4798*, pages 140–152, 2007.

[MGS08a] Diego C. Martinez, Alejandro J. Garcia, and Guillermo R. Simari. An abstract argumentation framework with varied-strength attacks. In *Proc of International Conference on the Principles of Knowledge Representation and Reasoning (KR)*, pages 135–143, 2008.

[MGS08b] Diego C. Martinez, Alejandro J. Garcia, and Guillermo R. Simari. Strong and weak forms of abstract argument defense. In *Proc of International Conference on Computational models of arguments (COMMA)*, pages 216–227, 2008.

[Mod09] Sanjay Modgil. Reasoning about preferences in argumentation frameworks. *Artificial Intelligence*, 173:901–934, 2009.

[PV02] Henry Prakken and Gerard Vreeswijk. Logics for defeasible argumentation. In D. Gabbay and F. Guenthner, editors, *Handbook of Philosophical Logic*, volume 4, pages 218–319. Kluwer Academic, 2002.

[TBS08] Fernando A. Tohm, Gustavo A. Bodanza, and Guillermo R. Simari. Aggregation of attack relations: a social-choice theoretical analysis of defeasibility criteria. In *Proc of Symposium on Foundations of Information and Knowledge Systems (FoIKS) – LNCS 4932*, pages 8–23, 2008.

[WBC07] Adam Z. Wyner and Trevor J. Bench-Capon. Towards an extensible argumentation system. In *Proc of European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty (ECSQARU) – LNAI 4724*, pages 283–294, 2007.