A constrained argumentation system for practical reasoning

Leila Amgoud[a], Caroline Devred[b], Marie-Christine Lagasquie-Schiex[a]
[a] IRIT-Université Paul Sabatier, 31062 Toulouse Cedex 9, France, {amgoud, lagasq}@irit.fr
[b] CRIL-Université d'Artois, 62307 Lens, Cedex, devred@cril.univ-artois.fr

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Abstract

Practical reasoning (PR), which is concerned with the generic question of what to do, is generally seen as a two steps process: (1) *deliberation*, in which an agent decides what state of affairs it wants to reach –that is, its *desires*; and (2) *means-ends reasoning*, in which the agent looks for plans for achieving these desires. A desire is justified if it holds in the current state of the world, and feasible if there is a plan for achieving it. The agent's *intentions* are thus a consistent subset of desires that are both justified and feasible. This paper proposes the first argumentation system for PR that computes in one step the intentions of an agent. We show that the system satisfies the rationality postulates identified in argumentation literature.

Contents

1 Introduction

Practical reasoning (PR) [17], is concerned with the generic question "what is the right thing to do for an agent in a given situation". In [22], it has been argued that PR is a two steps process. The first step, often called *deliberation*, consists of identifying the desires of an agent. In the second step, called *means-end reasoning*, one looks for ways for achieving those desires, *i.e.* for actions or plans. A desire is *justified* if it holds in the current state of the world, and is *feasible* if it has a plan for achieving it. The agent's intentions, *i.e.* what an agent decides to do, is a consistent subset of desires that are both justified and feasible.

What is worth noticing in most works on practical reasoning is the use of argument for providing reasons for choosing or discarding a desire as an intention. Indeed, several argumentation-based systems for PR have been proposed in the literature [3, 14, 16]. However, in most of these works, the problem of PR is modeled in terms of at least two separate systems, each of them capturing one step of the process. Such an approach may suffer from a serious drawback. In fact, some desires that are not feasible may be accepted at the deliberation step to the detriment of other justified and feasible desires. Another limitation of those systems is that their properties are not investigated.

This paper proposes the first argumentation system that computes the intentions of an agent in one step. The system is grounded on a recent work on *constrained* argumentation systems [10]. These last extend the well-known general system of Dung [11] by adding constraints on arguments that need to be satisfied by the extensions returned by the system. Our system takes then as input i) three categories of arguments: *epistemic* arguments that support beliefs, *explanatory* arguments that show that a desire holds in the current state of the world, and *instrumental* arguments that show that a desire is feasible, ii) different conflicts among those arguments, and iii) a particular constraint on arguments that captures the idea that for a desire to be pursued it should be both feasible and justified. This is translated by the fact that in a given extension each instrumental argument for a desire should be accompanied by at least an explanatory argument in favor of that desire. The output of our system is different sets of intentions. We show that the results of such a system are safe, and satisfy the rationality postulates identified in [6].

The paper is organized as follows: Section 2 recalls the basics of an argumentation system. Section 3 presents the logical language. Section 4 studies the different types of arguments involved in a practical reasoning problem, and Section 5 investigates the conflicts that may exist between them. Section 6 presents the constrained argumentation system for PR, and its properties are given in Section 7. The system is then illustrated through some examples in Section 8. All the proofs are given in an appendix.

2 Basics of argumentation

Argumentation is an established approach for reasoning with inconsistent knowledge, based on the construction and the comparison of arguments. Many argumentation formalisms are built around an underlying logical language and an associated notion of logical consequence, defining the notion of argument. The argument construction is a monotonic process: new knowledge cannot rule out an argument but only gives rise to new arguments which may interact with the first argument. Since knowledge bases may give rise to inconsistent conclusions, the arguments may be conflicting too. Consequently, it is important to determine among all the available arguments, the ones that are ultimately "acceptable". In [11], an abstract argumentation system has been proposed, and different acceptability semantics have been defined.

Definition 1 (Basic argumentation system) *An* argumentation system *is a pair* $\mathsf{AF} = \langle \mathcal{A}, \mathcal{R} \rangle$ *with* $\mathcal{A}$ *is a set of arguments, and* $\mathcal{R}$ *is an attack relation (*$\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$*).*

Before recalling the acceptability semantics of Dung [11], let us first introduce some useful concepts.

Definition 2 (Conflict-free, Defence) *Let* $\mathcal{E} \subseteq \mathcal{A}$*.*

- $\mathcal{E}$ *is* conflict-free *iff* $\nexists\, \alpha, \beta \in \mathcal{E}$ *such that* $\alpha \;\mathcal{R}\; \beta$*.*
- $\mathcal{E}$ defends *an argument* α *iff* $\forall\, \beta \in \mathcal{A}$*, if* $\beta \;\mathcal{R}\; \alpha$*, then* $\exists\, \delta \in \mathcal{E}$ *such that* $\delta \;\mathcal{R}\; \beta$*.*

Dung's semantics are all based on a notion of admissibility.

Definition 3 (Acceptability semantics) *Let* $\mathcal{E}$ *be a conflict-free set of arguments.*

- $\mathcal{E}$ *is an* admissible *set iff it is conflict-free and defends every element in* $\mathcal{E}$*.*
- $\mathcal{E}$ *is a* preferred extension *iff it is a maximal (w.r.t. set-inclusion) admissible set.*
- $\mathcal{E}$ *is a* stable extension *iff it is a preferred extension that attacks all arguments in* $\mathcal{A} \backslash \mathcal{E}$*.*

Note that every stable extension is also a preferred one, but the reverse is not always true.

The above argumentation system has been generalized in [10]. The basic idea is to explicit *constraints* on arguments that should be satisfied by the above Dung's extensions. For instance, one may want that the two arguments α and β belong to the same stable extension. These constraints are generally expressed in terms of a propositional formula built from a language using $\mathcal{A}$ as an alphabet.

Definition 4 ([10] – Constraints on arguments, Completion of a set of arguments) *Let* $\mathcal{A}$ *be a set of arguments and* $\mathcal{L}_{\mathcal{A}}$ *be the propositional language defined using* $\mathcal{A}$ *as the set of propositional variables.*

- C *is a* constraint *on arguments of* $\mathcal{A}$ *iff* C *is a formula of* $\mathcal{L}_{\mathcal{A}}$*.*
- *The* completion *of a set* $\mathcal{E} \subseteq \mathcal{A}$ *is:* $\widehat{\mathcal{E}} = \{\alpha \mid \alpha \in \mathcal{E}\} \cup \{\neg\alpha \mid \alpha \in \mathcal{A} \setminus \mathcal{E}\}$.
- *A set* $\mathcal{E} \subseteq \mathcal{A}$ satisfies C *iff* $\widehat{\mathcal{E}}$ *is a model of* C *(*$\widehat{\mathcal{E}} \vdash C$*).*

A constrained argumentation system is defined as follows:

Definition 5 (Constrained argumentation system) *A* constrained argumentation system *is a triple* $\mathsf{CAF} = \langle \mathcal{A}, \mathcal{R}, C \rangle$ *with* C *is a constraint on arguments of* $\mathcal{A}$.

Let us now recall how Dung's extensions are extended to the case of a constrained argumentation system. As said before, the basic idea is to compute Dung's extensions, and then to keep among those extensions the ones that satisfy the constraint C.

Definition 6 (C-admissible set) *Let* $\mathcal{E} \subseteq \mathcal{A}$. $\mathcal{E}$ *is* C-admissible *iff*

1. $\mathcal{E}$ *is admissible,*
2. $\mathcal{E}$ *satisfies the constraint* C.

Note that the empty set is admissible, however, it is not always C-admissible since $\widehat{\varnothing}$ does not always imply C.

Definition 7 (C-preferred extension, C-stable extension) *Let* $\mathcal{E} \subseteq \mathcal{A}$.

- $\mathcal{E}$ *is a* C-preferred extension *iff* $\mathcal{E}$ *is maximal for set-inclusion among the* C*-admissible sets.*
- $\mathcal{E}$ *is a* C-stable extension *iff* $\mathcal{E}$ *is a* C*-preferred extension that attacks all arguments in* $\mathcal{A} \backslash \mathcal{E}$.

Now that the acceptability semantics are defined, we are ready to define the status of any argument.

Definition 8 (Argument status) *Let* CAF *be a constrained argumentation system, and* $\mathcal{E}_1, \ldots, \mathcal{E}_x$ *its extensions under a given semantics. Let* $\alpha \in \mathcal{A}$.

1. α *is* accepted *iff* $\alpha \in \mathcal{E}_i$, $\forall \mathcal{E}_i$ *with* $i = 1, \ldots, x$.
2. α *is* rejected *iff* $\nexists \mathcal{E}_i$ *such that* $\alpha \in \mathcal{E}_i$.
3. α *is* undecided *iff* α *is neither accepted nor rejected. This means that* α *is in some extensions and not in others.*

One can easily check that if an argument is rejected in the basic argumentation system AF, then it will also be rejected in CAF.

Property 1 *Let* $\alpha \in \mathcal{A}$. *If* α *is rejected in* AF, *then* α *is also rejected in* CAF.

3 Logical language

In this section we start by presenting the logical language that will be used throughout the paper. Let $\mathcal{L}$ be a *propositional language*, and $\equiv$ be the classical equivalence relation.

From $\mathcal{L}$, a subset $\mathcal{D}$ is distinguished and is used for encoding *desires*. By desire we mean a state of affairs that an agent wants to reach. Elements of $\mathcal{D}$ are *literals*. We will write $d_1, \ldots, d_n$ to denote desires and the lowercase letters will denote formulas of $\mathcal{L}$.

From the above sets, *desire-generation* rules can be defined. A desire-generation rule expresses under which conditions an agent may adopt a given desire. A desire may come from beliefs. For instance, "if the weather is sunny, then I desire to go to the park". In this case, the desire of going to the park depends on my belief about the weather. A desire may also come from other desires. For example, if there is a conference in India, and I have the desire to attend, then I desire also to attend the tutorials. In this example, the desire of attending the tutorials depends on my beliefs about the existence of a conference in India, and on my desire to attend that conference. Finally, a desire may be unconditional, this means that the desire depends on neither beliefs nor desires. These three sources of desires are captured by the following desire-generation rules.

Definition 9 (Desire-Generation Rules) *A* desire-generation rule *(or a* desire rule*) is an expression of the form*

$$b \wedge d_1 \wedge \ldots \wedge d_{m-1} \hookrightarrow^1 d_m$$

where b is a propositional formula of $\mathcal{L}$, and each d_i is an element of the set $\mathcal{D}$. Moreover, $\nexists d_i, d_j$ with $i, j \leq m$ such that $d_i \equiv d_j$. $b \wedge d_1 \wedge \ldots \wedge d_{m-1}$ is called the body *of the rule (this body may be empty; this is the case of an unconditional desire), and d_m its* consequent.

The meaning of the rule is "if the agent *believes* b and *desires* $d_1, \ldots, d_{m-1}$, then the agent will *desire* d_m as well". Note that the same desire d_i may appear in the consequent of several rules. This means that the same desire may depend on different beliefs or desires. In what follows, a desire rule is consistent if it depends on consistent beliefs and on non contradictory desires.

Definition 10 (Consistent Desire Rule) *A desire rule $b \wedge d_1 \wedge \ldots \wedge d_{m-1} \hookrightarrow d_m$ is* consistent *iff*

- $b \nvdash \bot$,
- $\forall i = 1 \ldots m, b \nvdash \neg d_i$ *and*
- $\nexists d_i, d_j$ *with* $i, j \leq m$ *such that* $d_i \equiv \neg d_j$.

Otherwise, the rule is said inconsistent.

We assume also that an agent is equipped with different *plans* provided by a given planning system. The generation of such plans is beyond the scope of this paper. A plan is a way of achieving a desire. It is defined as a triple: i) a set of pre-conditions that should be satisfied before executing the plan, ii) a set of post-conditions that hold after executing the plan, and iii) the desire that is reached by the plan. Formally:

Definition 11 (Plan) *A plan is a triple $\langle S, T, x \rangle$ such that*

- S *and* T *are* consistent *sets of propositional formulas of $\mathcal{L}$,*

[1] The symbol $\hookrightarrow$ is not the material implication.

- $x \in \mathcal{D}$,
- $T \vdash x$ *and* $S \not\vdash x$.

Of course, there exists a link between S and T. But this link is not explicitly defined here because we are not interested by this aspect of the process. We just consider that the plan is given by a correct and sound planning system (for instance [12, 18]).

In the remaining of the paper, we suppose that an agent is equipped with three *finite bases*:

1. a base $\mathcal{K}$ containing its *basic beliefs* about the environment (elements of $\mathcal{K}$ are propositional formulas of the language $\mathcal{L}$). The base $\mathcal{K}$ is supposed to be non-empty, and does not contain $\bot$, *i.e.* $\mathcal{K} \neq \varnothing$ and $\mathcal{K} \neq \{\bot\}$,
2. a base $\mathcal{B}_d$ containing its "consistent" desire rules,
3. a base $\mathcal{P}$ containing its plans.

Using $\mathcal{B}_d$, we can characterize the *potential desires* of an agent as follows:

Definition 12 (Potential Desires) *The set of* potential desires *of an agent is* $\mathcal{PD} = \{d_m | \exists b \wedge d_1 \wedge \ldots \wedge d_{m-1} \hookrightarrow d_m \in \mathcal{B}_d\}$.

These are "potential" desires because, when the body of the rule is not empty, the agent does not know yet whether the antecedents (*i.e.* bodies) of the corresponding rules are true or not.

4 Typology of arguments

The aim of this section is to present the different kinds of arguments involved in practical reasoning. As we will show, there are mainly three categories of arguments: one category for supporting/attacking beliefs, and two categories for justifying the adoption of desires. Note that the arguments will be denoted with lowercase greek letters.

4.1 Justifying beliefs

The first category of arguments is that studied in argumentation literature, especially for handling inconsistency in knowledge bases. Indeed, arguments are built from a knowledge base in order to support or to attack potential conclusions or inferences. These arguments are called *epistemic* in [13]. In our application, such arguments may be built from the base $\mathcal{K}$. In what follows, we will use the definition proposed in [19].

Definition 13 (Epistemic Argument) *Let $\mathcal{K}$ be a knowledge base. An* epistemic argument α *is a pair* $\alpha = \langle H, h\rangle$ *s.t:*

1. $H \subseteq \mathcal{K}$,
2. H *is consistent,*

3. $H \vdash h$ *and*

4. H *is minimal (for set $\subseteq$) among the sets satisfying conditions 1, 2, 3.*

The support *of the argument is given by the function* $\texttt{SUPP}(\alpha) = H$*, whereas its* conclusion *is returned by* $\texttt{CONC}(\alpha) = h$*. $\mathcal{A}_b$ stands for the set of all epistemic arguments that can be built from the base $\mathcal{K}$.*

4.2 Justifying desires

A desire may be pursued by an agent only if it is justified and feasible. Thus, there are two kinds of reasons for adopting a desire:

- the conditions underlying the desire hold in the current state of world;
- there is a plan for reaching the desire.

The definition of the first kind of arguments involves two bases: the belief base $\mathcal{K}$ and the base of desire rules $\mathcal{B}_d$. In what follows, we will use a tree-style definition of arguments [20]. Before presenting that definition, let us first introduce some functions. In what follows, the functions $\texttt{BELIEFS}(\delta)$, $\texttt{DESIRES}(\delta)$, $\texttt{CONC}(\delta)$ and $\texttt{SUB}(\delta)$ return respectively, for a given argument δ, the beliefs used in δ, the desires supported by δ, the conclusion and the set of sub-arguments of the argument δ.

Definition 14 (Explanatory Argument) *Let $\langle \mathcal{K}, \mathcal{B}_d \rangle$ be two bases.*

- *If $\exists \hookrightarrow d \in \mathcal{B}_d$ then $\longrightarrow d$ is an* explanatory argument *(δ) with*
  - $\texttt{BELIEFS}(\delta) = \varnothing$,
  - $\texttt{DESIRES}(\delta) = \{d\}$,
  - $\texttt{CONC}(\delta) = d$,
  - $\texttt{SUB}(\delta) = \{\delta\}$.
- *If α is an epistemic argument, and $\delta_1, \ldots, \delta_m$ are explanatory arguments, and $\exists\ \texttt{CONC}(\alpha) \wedge \texttt{CONC}(\delta_1) \wedge \ldots \wedge \texttt{CONC}(\delta_m) \hookrightarrow d \in \mathcal{B}_d$ then $\alpha, \delta_1, \ldots, \delta_m \longrightarrow d$ is an* explanatory argument *(δ) with*
  - $\texttt{BELIEFS}(\delta) = \texttt{SUPP}(\alpha) \cup \texttt{BELIEFS}(\delta_1) \cup \ldots \cup \texttt{BELIEFS}(\delta_m)$,
  - $\texttt{DESIRES}(\delta) = \texttt{DESIRES}(\delta_1) \cup \ldots \cup \texttt{DESIRES}(\delta_m) \cup \{d\}$,
  - $\texttt{CONC}(\delta) = d$,
  - $\texttt{SUB}(\delta) = \{\alpha\} \cup \texttt{SUB}(\delta_1) \cup \ldots \cup \texttt{SUB}(\delta_m) \cup \{\delta\}$.

$\mathcal{A}_d$ stands for the set of all explanatory arguments that can be built from $\langle \mathcal{K}, \mathcal{B}_d \rangle$ respecting the fact that their $\texttt{DESIRES}$ *set is consistent*[2].

One can easily show that the set $\texttt{BELIEFS}$ of an explanatory argument is a subset of the knowledge base $\mathcal{K}$, and that the set $\texttt{DESIRES}$ is a subset of $\mathcal{PD}$.

[2] We do not accept cases where contradictory desires are necessary for justifying another desire.

Property 2 *Let $\delta \in \mathcal{A}_d$.*

- $\texttt{BELIEFS}(\delta) \subseteq \mathcal{K}$.
- $\texttt{DESIRES}(\delta) \subseteq \mathcal{PD}$.

Note that the same desire may be supported by several explanatory arguments since a desire may be the consequent of different desire rules.

The last category of arguments claims that "a desire may be pursued since it has a plan for achieving it". The definition of this kind of arguments involves the belief base $\mathcal{K}$, the base of plans $\mathcal{P}$, and the set $\mathcal{PD}$.

Definition 15 (Instrumental Argument) *Let $\langle \mathcal{K}, \mathcal{P}, \mathcal{PD} \rangle$ be three bases, and $d \in \mathcal{PD}$. An* instrumental *argument is a pair $\pi = \langle \langle S, T, x \rangle, d \rangle$ where*

- $\langle S, T, x \rangle \in \mathcal{P}$,
- $S \subseteq \mathcal{K}$,
- $x \equiv d$.

$\mathcal{A}_p$ stands for the set of all instrumental arguments that can be built from $\langle \mathcal{K}, \mathcal{P}, \mathcal{PD} \rangle$. The function CONC *will return for an argument π the desire d. Similarly, the functions* PLAN, Prec *and* Postc *will return respectively the plan $\langle S, T, x \rangle$ of the argument, the pre-conditions S of the plan, its post-conditions T.*

The second condition of the above definition says that the pre-conditions of the plan hold in the current state of the world. In other words, the plan can be executed. Indeed, it may be the case that the base $\mathcal{P}$ contains plans whose pre-conditions are not true. Such plans cannot be executed and their corresponding instrumental arguments do not exist.
In what follows, $\mathcal{A} = \mathcal{A}_b \cup \mathcal{A}_d \cup \mathcal{A}_p$. Note that $\mathcal{A}$ is *finite* since the three initial bases ($\mathcal{K}$, $\mathcal{B}_d$ and $\mathcal{P}$) are finite.

5 Interactions between arguments

Arguments built from a knowledge base cannot generally be considered separately in an inference problem. Indeed, an argument constitutes a reason for believing, or adopting a desire. However, it is not a proof that the belief is true, or in our case that the desire should be adopted. The reason is that an argument can be attacked by other arguments. In this section, we will investigate the different kinds of conflicts among the arguments identified in the previous section.

5.1 Conflicts among epistemic arguments

An argument can be attacked by another argument for three main reasons: i) they have contradictory conclusions (this is known as *rebuttal*), ii) the conclusion of an argument contradicts a premise of another argument (*assumption attack*), iii) the conclusion of an argument contradicts an inference rule used in order to build the other argument (*undercutting*). Since the base $\mathcal{K}$ is built around a propositional language, it has been shown in [2] that the notion of assumption attack is sufficient to capture conflicts between epistemic arguments.

Definition 16 *Let* α_1, $\alpha_2 \in \mathcal{A}_b$. α_1 $\mathcal{R}_b$ α_2 *iff* $\exists h \in \mathtt{SUPP}(\alpha_2)$ *such that* $\mathtt{CONC}(\alpha_1) \equiv \neg h$.

Note that assumption attack is a binary relation that *is not symmetric*. Moreover, one can show that there are no self-defeating arguments.

Property 3 $\nexists \alpha \in \mathcal{A}_b$ *such that* α $\mathcal{R}_b$ α.

In [7], the argumentation system $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$ has been applied for handling inconsistency in a knowledge base, say $\mathcal{K}$. In this particular case, a full correspondence has been established between the stable extensions of the system and the maximal consistent subsets of the base $\mathcal{K}$. Before presenting formally the result, let us introduce some useful notations. Let $\mathcal{E} \subseteq \mathcal{A}_b$, $\mathtt{Base}(\mathcal{E}) = \bigcup H_i$ such that $\langle H_i, h_i \rangle \in \mathcal{E}$. Let $T \subseteq \mathcal{K}$, $\mathtt{Arg}(T) = \{\langle H_i, h_i \rangle | H_i \subseteq T\}$.

Property 4 ([7]) *Let* $\mathcal{E}$ *be a stable extension of* $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$.

- $\mathtt{Base}(\mathcal{E})$ *is a maximal (for set inclusion) consistent subset of* $\mathcal{K}$.
- $\mathtt{Arg}(\mathtt{Base}(\mathcal{E})) = \mathcal{E}$.

Property 5 ([7]) *Let* T *be a maximal (for set inclusion) consistent subset of* $\mathcal{K}$.

- $\mathtt{Arg}(T)$ *is a stable extension of* $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$.
- $\mathtt{Base}(\mathtt{Arg}(T)) = T$.

A direct consequence of the above result is that if the base $\mathcal{K}$ is not reduced to $\perp$, then the system $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$ has at least one non-empty stable extension.

Property 6 *The argumentation system* $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$ *has non-empty stable extensions.*

5.2 Conflicts among explanatory arguments

Explanatory arguments may also be conflicting. Indeed, two explanatory arguments may be based on two contradictory desires. This kind of conflict is captured by the following relation:

Definition 17 *Let* δ_1, $\delta_2 \in \mathcal{A}_d$. δ_1 $\mathcal{R}_d$ δ_2 *iff* $\exists d_1 \in \mathtt{DESIRES}(\delta_1)$, $d_2 \in \mathtt{DESIRES}(\delta_2)$ *such that* $d_1 \equiv \neg d_2$.

Property 7 *The relation* $\mathcal{R}_d$ *is symmetric and irreflexive.*

Property 8 *Let* d_1, $d_2 \in \mathcal{PD}$. *If* $d_1 \equiv \neg d_2$, *then* $\forall \delta_1, \delta_2 \in \mathcal{A}_d$ *such that: (1)* $\exists \delta'_1 \in \mathtt{SUB}(\delta_1)$ *with* $\mathtt{CONC}(\delta'_1) = d_1$, *and (2)* $\exists \delta'_2 \in \mathtt{SUB}(\delta_2)$ *with* $\mathtt{CONC}(\delta'_2) = d_2$, *then* δ_1 $\mathcal{R}_d$ δ_2.

Note that, by definition, the set DESIRES of the argument cannot be inconsistent.

However, it is noticing that the set BELIEFS of an explanatory argument may be inconsistent, or even the union of the beliefs of two explanatory arguments is inconsistent. However, later in the paper, we will show that it is useless to explicit this kind of conflict, since they are captured by conflicts between the explanatory arguments and epistemic ones (see Prop. 11 and Prop. 12).

5.3 Conflicts among instrumental arguments

Two plans may be conflicting for four main reasons:

1. incompatibility of their pre-conditions (indeed, both plans cannot be executed at the same time),
2. incompatibility of their post-conditions (the execution of both plans will lead to contradictory states of the world),
3. incompatibility between the post-conditions of a plan and the preconditions of the other (this means that the execution of a plan will prevent the execution of the second plan in the future),
4. incompatibility of their supporting desires (indeed, plans for achieving contradictory desires are conflicting; their execution will in fact lead to a contradictory state of the world).

The above reasons are captured in the following definition of attack among instrumental arguments.

Definition 18 *Let* $\pi_1, \pi_2 \in \mathcal{A}_p$ *and* $\pi_1 \neq \pi_2$. $\pi_1 \ \mathcal{R}_p \ \pi_2$ *iff:*

- $\texttt{Prec}(\pi_1) \wedge \texttt{Prec}(\pi_2) \models \perp$, *or*
- $\texttt{Postc}(\pi_1) \wedge \texttt{Postc}(\pi_2) \models \perp$, *or*
- $\texttt{Postc}(\pi_1) \wedge \texttt{Prec}(\pi_2) \models \perp$ *or* $\texttt{Prec}(\pi_1) \wedge \texttt{Postc}(\pi_2) \models \perp$

From the above definition, one can show that if two plans realize conflicting desires, then their corresponding instrumental arguments are conflicting too.

Property 9 *Let* $d_1, d_2 \in \mathcal{PD}$. *If* $d_1 \equiv \neg d_2$, *then* $\forall \pi_1, \pi_2 \in \mathcal{A}_p$ *s.t.* $\texttt{CONC}(\pi_1) = d_1$ *and* $\texttt{CONC}(\pi_2) = d_2$, *then* $\pi_1 \ \mathcal{R}_p \ \pi_2$.

It is clear from the above definition that the relation $\mathcal{R}_p$ is symmetric and irreflexive[3].

Property 10 *The relation* $\mathcal{R}_p$ *is symmetric and irreflexive.*

[3]The fact that the post-conditions of a plan are inconsistent with its pre-conditions is not considering as a conflict. In this case, after the execution of the plan, we must have an update mechanism which will modify the beliefs. It is also for this reason that there is no conflict between epistemic arguments and instrumental arguments concerning the post-conditions (see Def. 19).

In this section, we have considered only binary conflicts between plans, and consequently between their corresponding instrumental arguments. However, in every-day life, one may have for instance three plans such that any pair of them is not conflicting, but the three together are incompatible. For simplicity reasons, in this paper we suppose that we do not have such conflicts (it will be the subject of a future work).

5.4 Conflicts among mixed arguments

In the previous sections we have shown how arguments of the same category can interact with each other. In this section, we will show that arguments of different categories can also interact. Indeed, epistemic arguments play a key role in ensuring the acceptability of explanatory or instrumental arguments. Namely, an epistemic argument can attack both types of arguments. The idea is to invalidate any belief used in an explanatory or an instrumental argument. An explanatory argument may also conflicts with an instrumental argument when this last achieves a desire whose negation is among the desires of the explanatory argument.

Definition 19 *Let* $\alpha \in \mathcal{A}_b$, $\delta \in \mathcal{A}_d$, $\pi \in \mathcal{A}_p$.

- $\alpha \, \mathcal{R}_{bd} \, \delta$ *iff* $\exists h \in \texttt{BELIEFS}(\delta)$ *s.t.* $h \equiv \neg\texttt{CONC}(\alpha)$.
- $\alpha \, \mathcal{R}_{bp} \, \pi$ *iff* $\exists h \in \texttt{Prec}(\pi)$, *s.t.* $h \equiv \neg\texttt{CONC}(\alpha)$.
- $\delta \, \mathcal{R}_{pdp} \, \pi$ *and* $\pi \, \mathcal{R}_{pdp} \, \delta$ *iff* $\texttt{CONC}(\pi) \equiv \neg d$ *with* $d \in \texttt{DESIRES}(\delta)$[4].

As already said, the set of beliefs of an explanatory argument may be inconsistent. In such a case, the explanatory argument is attacked (in the sense of $\mathcal{R}_{bd}$) for sure by an epistemic argument. Formally:

Property 11 *Let* $\delta \in \mathcal{A}_d$. *If* $\texttt{BELIEFS}(\delta) \vdash \bot$, *then* $\exists \alpha \in \mathcal{A}_b$ *such that* $\alpha \, \mathcal{R}_{bd} \, \delta$.

Similarly, when the beliefs of two explanatory arguments are inconsistent, it can be checked that there exists an epistemic argument that attacks at least one of the two explanatory arguments. Formally:

Property 12 *Let* $\delta_1, \ \delta_2 \in \mathcal{A}_d$ *with* $\texttt{BELIEFS}(\delta_1) \nvdash \bot$ *and* $\texttt{BELIEFS}(\delta_2) \nvdash \bot$. *If* $\texttt{BELIEFS}(\delta_1) \cup \texttt{BELIEFS}(\delta_2) \vdash \bot$, *then* $\exists \alpha \in \mathcal{A}_b$ *such that* $\alpha \, \mathcal{R}_{bd} \, \delta_1$, *or* $\alpha \, \mathcal{R}_{bd} \, \delta_2$.

The beliefs of an explanatory argument may also be conflicting with the preconditions of an instrumental argument. Here again, we'll show that there exists an epistemic argument that attacks at least one of the two arguments. Formally:

Property 13 *Let* $\delta \in \mathcal{A}_d$ *and* $\pi \in \mathcal{A}_p$ *with* $\texttt{BELIEFS}(\delta) \nvdash \bot$. *If* $\texttt{BELIEFS}(\delta) \cup \texttt{Prec}(\pi) \vdash \bot$ *then* $\exists \alpha \in \mathcal{A}_b$ *such that* $\alpha \, \mathcal{R}_{bd} \, \delta$, *or* $\alpha \, \mathcal{R}_{bp} \, \pi$.

Later in the paper, it will be shown that the three above propositions are sufficient for ignoring these conflicts (between two explanatory arguments, and between an explanatory argument and an instrumental one).

Note that explanatory arguments and instrumental arguments are not allowed to attack epistemic arguments. In particular a desire cannot invalidate a belief. The main reason is to avoid any form of *wishful thinking*.

[4]Note that if $\delta_1 \mathcal{R}_{pdp} \pi_2$ and there exists δ_2 such that $\texttt{CONC}(\delta_2) = \texttt{CONC}(\pi_2)$ then $\delta_1 \mathcal{R}_d \delta_2$.

6 Argumentation system for PR

The notion of constraint which forms the backbone of constrained argumentation systems allows, in the context of practical reasoning, the representation of the link between the justification of a desire and the plan for achieving it (so between the explanatory argument in favor of a given desire and the instrumental arguments in favor of that desire). A constrained argumentation system for PR is defined as follows:

Definition 20 (Constrained argumentation system for PR) *The constrained argumentation system for practical reasoning is the triple* $\mathsf{CAF}_{\mathsf{PR}} = \langle \mathcal{A}, \mathcal{R}, C \rangle$ *with:*

- $\mathcal{A} = \mathcal{A}_b \cup \mathcal{A}_d \cup \mathcal{A}_p$,
- $\mathcal{R} = \mathcal{R}_b \cup \mathcal{R}_d \cup \mathcal{R}_p \cup \mathcal{R}_{bd} \cup \mathcal{R}_{bp} \cup \mathcal{R}_{pdp}$
- *and* C *a constraint on arguments defined on* $\mathcal{A}$ *respecting* $C = \wedge_i(\pi_i \Rightarrow (\vee_j \delta_j))$ *for each* $\pi_i \in \mathcal{A}_p$ *and* $\delta_j \in \mathcal{A}_d$ *such that* $\mathtt{CONC}(\pi_i) \equiv \mathtt{CONC}(\delta_j)$.

At some places of the paper, we will refer by $\mathsf{AF}_{\mathsf{PR}} = \langle \mathcal{A}, \mathcal{R} \rangle$ to a basic argumentation system for PR, *i.e.* an argumentation system without the constraint, and $\mathcal{A}$ and $\mathcal{R}$ are defined as in Definition 20.

Note that the satisfaction of the constraint C implies that each plan of a desire must be taken into account only if this desire is justified. Note also that we consider that there may be several plans for one desire but only one desire for each plan. Nevertheless, for each desire there may exist several explanatory arguments.

An important remark concerns the notion of defence: this notion has two different semantics in PR context. When we consider only epistemic or explanatory arguments, the defence corresponds exactly to the notion defined in Dung's argumentation systems and in its constrained extension: an argument α attacks the attacker of another argument β; so α "reinstates" β; without the defence, we cannot keep β in an admissible set. Things are different with instrumental arguments: when an instrumental argument attacks another argument, this attack is always symmetric (so, each argument defends itself against an instrumental argument). In this case, it would be sufficient to take into account the notion of conflict-free in order to identify the plans which belong to an admissible set[5]. However, in order to keep an homogeneous definition of admissibility, the notion of defence is also used for instrumental arguments knowing that it is without impact when conflicts from an instrumental argument are concerned.

Note that $\varnothing$ is always a C-admissible set for $\mathsf{CAF}_{\mathsf{PR}}$ ($\varnothing$ is admissible and all π_i variables are false in $\widehat{\varnothing}$, so $\widehat{\varnothing} \vdash C$)[6]. So, there is always a C-preferred extension of $\mathsf{CAF}_{\mathsf{PR}}$.

Let us recall that the purpose of a practical reasoning problem is to compute the intentions to be pursued by an agent, *i.e.* the desires that are both justified and feasible. These intentions are defined as follows:

[5]This property can be extended to $\langle \mathcal{A}_d \cup \mathcal{A}_p, \mathcal{R}_d \cup \mathcal{R}_p \cup \mathcal{R}_{pdp} \rangle$ because this subpart of $\mathsf{AF}_{\mathsf{PR}}$ is symmetric in the sense of [9]; in this case, the admissibility is equivalent to the conflict-free notion.

[6]This is due to the particular form of the constraint for practical reasoning. This is not true for all constraints (see Section2 and [10]).

Definition 21 (Set of intentions) *Let $\mathcal{I} \subseteq \mathcal{PD}$. $\mathcal{I}$ is a* set of intentions *of* $\mathsf{CAF_{PR}}$ *iff there exists a C-extension $\mathcal{E}$ (under a given semantics) of* $\mathsf{CAF_{PR}}$ *such that for each $d \in \mathcal{I}$, there exists $\pi \in \mathcal{A}_p \cap \mathcal{E}$ such that $d = \texttt{CONC}(\pi)$.*

Note that each extension gives a set of intentions, the state of the world which justifies these intentions and the plans which can realize them.

As noticed above, different intention sets may be returned by our $\mathsf{CAF_{PR}}$. The set that an agent decides to pursue is merely a decision problem as argued in [4]. This choice is beyond the scope of this paper. Recall that the aim of this paper is only to identify the different possibilities for an agent. In the next section we will show that our $\mathsf{CAF_{PR}}$ satisfies the rationality postulates identified in [6], this confirms that its results are safe and complete.

7 Properties of the system

The aim of this section is to study the properties of the proposed argumentation system for PR.

The first results concern the extensions of the system, and are mainly direct consequences of results got in [10].

The first property establishes a link between C-admissible sets and C-preferred extensions, and shows the impact of applying constraints on the notion of admissibility.

Property 14 *Let* $\mathsf{CAF_{PR}} = \langle \mathcal{A}, \mathcal{R}, C \rangle$. *Let Ω be the set of C-admissible sets of* $\mathsf{CAF_{PR}}$.

1. *Let $\mathcal{E} \in \Omega$. There exists a C-preferred extension $\mathcal{E}'$ of* $\mathsf{CAF_{PR}}$ *such that $\mathcal{E} \subseteq \mathcal{E}'$.*
2. *Let* $\mathsf{CAF_{PR}}' = \langle \mathcal{A}, \mathcal{R}, C' \rangle$ *such that $C' \models C$. Let Ω' be the set of C'-admissible sets of* $\mathsf{CAF_{PR}}'$. *We have $\Omega' \subseteq \Omega$.*

The two following properties show that the constrained argumentation system is more general than a classical argumentation system. However, they may coincide in some circumstances.

Property 15 *Let* $\mathsf{CAF_{PR}} = \langle \mathcal{A}, \mathcal{R}, C \rangle$. *For each C-preferred extension $\mathcal{E}$ of* $\mathsf{CAF_{PR}}$, *there exists a preferred extension $\mathcal{E}'$ of* $\mathsf{AF_{PR}}$ *such that $\mathcal{E} \subseteq \mathcal{E}'$.*

Property 16 *Let* $\mathsf{CAF_{PR}} = \langle \mathcal{A}, \mathcal{R}, C \rangle$ *such that C is a valid formula on $\mathcal{A}$. Then the preferred extensions of* $\mathsf{AF_{PR}}$ *are the C-preferred extensions of* $\mathsf{CAF_{PR}}$.

Property 17 *Let* $\mathsf{CAF_{PR}} = \langle \mathcal{A}, \mathcal{R}, C \rangle$. *Let Ω be the set of C-admissible sets of* $\mathsf{CAF_{PR}}$. *Ω defines a complete partial order for $\subseteq$.*

It is worth noticing that the above property is not true for any constrained argumentation system[7].

Another important property shows that the system $\mathsf{AF_{PR}}$ has stable extensions. Formally:

[7]Because in the general case, $\varnothing$ is not a C-admissible set, and one can find examples of a constrained argumentation system in which the set of C-admissible sets has several lower bounds.

Property 18 *The system* $\mathsf{AF}_{\mathsf{PR}}$ *has at least one non-empty stable extension.*

Note that even if the system $\mathsf{AF}_{\mathsf{PR}}$ has at least one stable extension, this does not mean that there is at least one non-empty intentions set. Indeed, the above result is mainly due to epistemic arguments. In fact, we show that the set of epistemic arguments in a given stable extension of $\mathsf{AF}_{\mathsf{PR}}$ is itself a stable extension of the system $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$. This shows clearly that stable extensions are "complete" w.r.t. epistemic arguments.

Property 19 *Let* $\mathcal{E}$ *be a stable extension of* $\mathsf{AF}_{\mathsf{PR}}$.
The set $\mathcal{E} \cap \mathcal{A}_b$ *is a stable extension of* $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$.

One can show that if an explanatory argument belongs to a stable extension of $\mathsf{AF}_{\mathsf{PR}}$, then all its sub-arguments belong to that extension.

Property 20 *Let* $\delta \in \mathcal{A}_d$. *Let* $\mathcal{E}_i$ *be a stable extension of* $\mathsf{AF}_{\mathsf{PR}}$. *If* $\delta \in \mathcal{E}_i$, *then* $\mathtt{SUB}(\delta) \subseteq \mathcal{E}_i$.

The above result is also true for epistemic arguments. In a previous section, we have shown that an explanantory argument may be based on contradictory beliefs. We have also shown that such an argument is attacked by an epistemic argument. In what follows, we will show that the situation is worse since such an argument is attacked by each stable extension of the system $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$. That's why such arguments will be discarded in PR.

Property 21 *Let* $\delta \in \mathcal{A}_d$. *If* $\mathtt{BELIEFS}(\delta) \vdash \bot$, *then* $\forall \mathcal{E}_i$ *with* $\mathcal{E}_i$ *is a stable extension of* $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$, $\exists \alpha \in \mathcal{E}_i$ *such that* $\alpha \mathcal{R}_{bd} \delta$.

A direct consequence of the above result is that such explanatory argument (with contradictory beliefs) will never belong to a stable extension of the system $\mathsf{AF}_{\mathsf{PR}}$.

Property 22 *Let* $\delta \in \mathcal{A}_d$ *with* $\mathtt{BELIEFS}(\delta) \vdash \bot$. *The argument* δ *is rejected in* $\mathsf{AF}_{\mathsf{PR}}$.

Since an explanatory argument with contradictory beliefs is rejected in $\mathsf{AF}_{\mathsf{PR}}$, then it will be also rejected in $\mathsf{CAF}_{\mathsf{PR}}$.

Property 23 *Let* $\delta \in \mathcal{A}_d$ *with* $\mathtt{BELIEFS}(\delta) \vdash \bot$. δ *is a rejected argument in* $\mathsf{CAF}_{\mathsf{PR}}$.

Besides in Property 12, we have shown when two explanatory arguments are based on contradictory beliefs, then at least one of the two arguments is attacked by an epistemic argument. Again, we will show that one of the two arguments is attacked by each stable extension of the system $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$.

Property 24 *Let* δ_1, $\delta_2 \in \mathcal{A}_d$ *with* $\mathtt{BELIEFS}(\delta_1) \not\vdash \bot$ *and* $\mathtt{BELIEFS}(\delta_2) \not\vdash \bot$.
If $\mathtt{BELIEFS}(\delta_1) \cup \mathtt{BELIEFS}(\delta_2) \vdash \bot$, *then* $\forall \mathcal{E}_i$ *with* $\mathcal{E}_i$ *is a stable extension of* $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$, $\exists \alpha \in \mathcal{E}_i$ *such that* $\alpha \mathcal{R}_{bd} \delta_1$, *or* $\alpha \mathcal{R}_{bd} \delta_2$.

We go further, and we show that the two arguments cannot be accepted at the same time, *i.e.* they cannot belong to the same stable extension at the same time. This guarantees that the system proposed here returns safe results.

Property 25 *Let δ_1, $\delta_2 \in \mathcal{A}_d$ with* $\texttt{BELIEFS}(\delta_1) \nvdash \bot$ *and* $\texttt{BELIEFS}(\delta_2) \nvdash \bot$. *If* $\texttt{BELIEFS}(\delta_1) \cup \texttt{BELIEFS}(\delta_2) \vdash \bot$, *then $\nexists \mathcal{E}$ with $\mathcal{E}$ a stable extension of* $\mathsf{CAF}_{\mathsf{PR}}$ *such that $\delta_1 \in \mathcal{E}$ and $\delta_2 \in \mathcal{E}$.*

Similarly, in section 5.4, some conflicts between explanatory and instrumental arguments were discarded. We have shown in Prop. 13 that in such a case, at least one of the two arguments will be attacked by an epistemic argument. Here we will show that the explanatory argument cannot be accepted at the same time with the instrumental one. One of them will be for sure rejected in the system.

Property 26 *Let $\delta \in \mathcal{A}_d$ and $\pi \in \mathcal{A}_p$ with* $\texttt{BELIEFS}(\delta) \nvdash \bot$. *If* $\texttt{BELIEFS}(\delta) \cup \texttt{Prec}(\pi) \vdash \bot$ *then $\forall \mathcal{E}_i$ with $\mathcal{E}_i$ is a stable extension of $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$, $\exists \alpha \in \mathcal{E}_i$ such that $\alpha \mathcal{R}_{bd} \delta$, or $\alpha \mathcal{R}_{bp} \pi$.*

Property 27 *Let $\delta \in \mathcal{A}_d$ and $\pi \in \mathcal{A}_p$ with* $\texttt{BELIEFS}(\delta) \nvdash \bot$. *If* $\texttt{BELIEFS}(\delta) \cup \texttt{Prec}(\pi) \vdash \bot$ *then $\nexists \mathcal{E}$ with $\mathcal{E}$ a stable extension of* $\mathsf{CAF}_{\mathsf{PR}}$ *such that $\delta \in \mathcal{E}$ and $\pi \in \mathcal{E}$.*

The next results are of great importance. They show that the proposed argumentation system for PR satisfies the "consistency" rationality postulate identified in [6]. Indeed, we show that each stable extension of our system supports a consistent set of desires and a consistent set of beliefs.

Let $\mathcal{E} \subseteq \mathcal{A}$. We use the following notations:

$$\begin{aligned}
\texttt{Bel}(\mathcal{E}) &= \Big(\bigcup_{\alpha_i \in \mathcal{E} \cap \mathcal{A}_b} \texttt{SUPP}(\alpha_i)\Big) \\
&\cup \Big(\bigcup_{\delta_j \in \mathcal{E} \cap \mathcal{A}_d} \texttt{BELIEFS}(\delta_j)\Big) \\
&\cup \Big(\bigcup_{\pi_k \in \mathcal{E} \cap \mathcal{A}_p} \texttt{Prec}(\pi_k)\Big) \\
\texttt{Des}(\mathcal{E}) &= \Big(\bigcup_{\delta_j \in \mathcal{E} \cap \mathcal{A}_d} \texttt{DESIRES}(\delta_j)\Big) \\
&\cup \Big(\bigcup_{\pi_k \in \mathcal{E} \cap \mathcal{A}_p} \texttt{CONC}(\pi_k)\Big)
\end{aligned}$$

Theorem 1 *(Consistency) Let* $\mathsf{AF}_{\mathsf{PR}}$ *be an argumentation system for PR, and $\mathcal{E}_1, \ldots, \mathcal{E}_n$ its stable extensions. $\forall \mathcal{E}_i$, $i = 1, \ldots, n$, it holds that:*

1. *The set* $\texttt{Bel}(\mathcal{E}_i)$ = $\texttt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)$.
2. *The set* $\texttt{Bel}(\mathcal{E}_i)$ *is a maximal (for set inclusion) consistent subset of $\mathcal{K}$.*
3. *The set* $\texttt{Des}(\mathcal{E}_i)$ *is consistent.*

As direct consequence of the above result, an intention set is consistent. Formally:

Theorem 2 *Let $\mathcal{I}_1, \ldots, \mathcal{I}_n$ be the intention sets of* $\mathsf{CAF}_{\mathsf{PR}}$. *Each intentions set $\mathcal{I}_j$ is consistent.*

We have also shown that our system satisfies the rationality postulate concerning the closedness of the extensions [6]. Namely, we have shown that the set of arguments that can be built from the beliefs, desires, and plans involved in a given stable extension, is that extension itself.

Let $\mathcal{E}_i$ be a stable extension of $\mathsf{AF}_{\mathsf{PR}}$. $\mathtt{Bel}(\mathcal{E}_i)$ denotes the set of beliefs used in arguments of $\mathcal{E}_i$, $\mathtt{Des}(\mathcal{E}_i)$ is the set of desires involved in arguments of $\mathcal{E}_i$. Finally, $\mathtt{Plans}(\mathcal{E}_i) = \{\mathtt{PLAN}(\pi)$ s.t. $\pi \in \mathcal{E}_i \cap \mathcal{A}_p\}$. Let $\mathcal{A}s$ be the set of all (epistemic, explanatory and instrumental) arguments that can be built from the four bases $\langle \mathtt{Bel}(\mathcal{E}_i), \mathtt{Des}(\mathcal{E}_i), \mathtt{Plans}(\mathcal{E}_i), \mathcal{B}_d \rangle$.

Theorem 3 *(Closedness) Let $\mathsf{AF}_{\mathsf{PR}}$ be an argumentation system for PR, and $\mathcal{E}_1, \ldots, \mathcal{E}_n$ its stable extensions.*
$\forall \mathcal{E}_i$, $i = 1, \ldots, n$, it holds that:

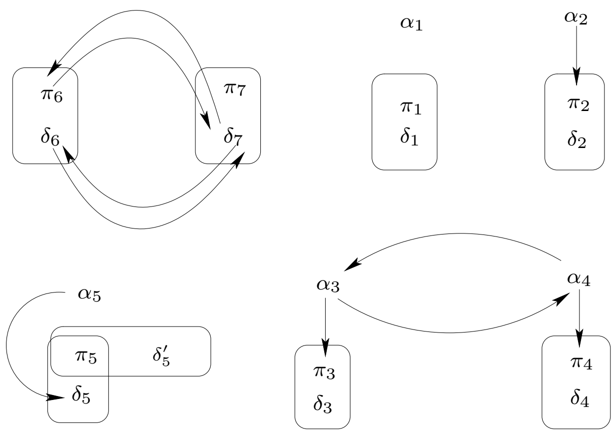
- $\mathtt{Arg}(\mathtt{Bel}(\mathcal{E}_i))$ = $\mathcal{E}_i \cap \mathcal{A}_b$.
- $\mathcal{A}s$ = $\mathcal{E}_i$.

In fact, this shows that every 'good' argument is included in a stable extension.

8 Examples

The aim of this section is to illustrate the above system through some examples in which each type of conflict is illustrated. In what follows, argumentation systems are given in terms of directed graphs. The arrows represent the attack relation. The arguments put in a same box support the same desire.

Example 1



Let us suppose the constrained argumentation system depicted in the above figure. The corresponding constraint is:

$$\begin{aligned} C \quad = \quad & (\pi_1 \Rightarrow \delta_1) \wedge (\pi_2 \Rightarrow \delta_2) \wedge (\pi_3 \Rightarrow \delta_3) \wedge (\pi_4 \Rightarrow \delta_4) \\ & \wedge (\pi_5 \Rightarrow (\delta_5 \vee \delta'_5)) \wedge (\pi_6 \Rightarrow \delta_6) \wedge (\pi_7 \Rightarrow \delta_7) \end{aligned}$$

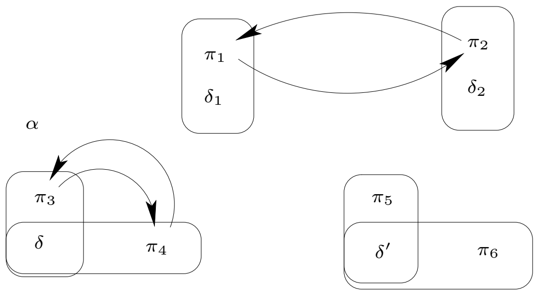
The stable extensions of this $\mathsf{AF_{PR}}$ *are:*

- $\mathcal{E}_1 = \{\alpha_1, \pi_1, \delta_1, \alpha_2, \delta_2, \alpha_3, \delta_3, \pi_4, \delta_4, \alpha_5, \pi_5, \delta'_5, \pi_6, \delta_6\}$,
- $\mathcal{E}_2 = \{\alpha_1, \pi_1, \delta_1, \alpha_2, \delta_2, \alpha_4, \delta_4, \pi_3, \delta_3, \alpha_5, \pi_5, \delta'_5, \pi_6, \delta_6\}$,
- $\mathcal{E}_3 = \{\alpha_1, \pi_1, \delta_1, \alpha_2, \delta_2, \alpha_3, \delta_3, \pi_4, \delta_4, \alpha_5, \pi_5, \delta'_5, \pi_7, \delta_7\}$,
- $\mathcal{E}_4 = \{\alpha_1, \pi_1, \delta_1, \alpha_2, \delta_2, \alpha_4, \delta_4, \pi_3, \delta_3, \alpha_5, \pi_5, \delta'_5, \pi_7, \delta_7\}$.

The intentions sets are:

- $\mathcal{I}_1 = \{d_1, d_4, d'_5(= d_5), d_6\}$,
- $\mathcal{I}_2 = \{d_1, d_3, d'_5(= d_5), d_6\}$,
- $\mathcal{I}_3 = \{d_1, d_4, d'_5(= d_5), d_7\}$ *and*
- $\mathcal{I}_4 = \{d_1, d_3, d'_5(= d_5), d_7\}$.

Example 2



- *The constraint:*

$$
\begin{aligned}
C &= (\pi_1 \Rightarrow \delta_1) \wedge (\pi_2 \Rightarrow \delta_2) \wedge (\pi_3 \Rightarrow \delta) \\
&\quad \wedge (\pi_4 \Rightarrow \delta) \wedge (\pi_5 \Rightarrow \delta') \wedge (\pi_6 \Rightarrow \delta')
\end{aligned}
$$

- *The stable extensions of this* $\mathsf{AF_{PR}}$ *are:*
  - $\mathcal{E}_1 = \{\alpha, \pi_1, \delta_1, \delta_2, \pi_3, \delta, \pi_5, \pi_6, \delta'\}$,
  - $\mathcal{E}_2 = \{\alpha, \pi_1, \delta_1, \delta_2, \pi_4, \delta, \pi_5, \pi_6, \delta'\}$,
  - $\mathcal{E}_3 = \{\alpha, \pi_2, \delta_1, \delta_2, \pi_3, \delta, \pi_5, \pi_6, \delta'\}$,
  - $\mathcal{E}_4 = \{\alpha, \pi_2, \delta_1, \delta_2, \pi_4, \delta, \pi_5, \pi_6, \delta'\}$;
- *the sets of intentions are* $\{d_1, d, d'\}$ *and* $\{d_2, d, d'\}$.

9 Related Works

A number of attempts have been made to use formal models of argumentation as a basis for practical reasoning. In fact the use of arguments for justifying an action has already been advocated by philosophers like Walton [21] who proposed the famous *practical*

syllogism:
- G is a goal for agent X
- Doing action A is sufficient for agent X to carry out G
- Then, agent X ought to do action A

The above syllogism, which would apply to the means-end reasoning step, is in essence already an argument in favor of doing action A. However, this does not mean that the action is warranted, since other arguments (called counter-arguments) may be built or provided against the action.

In [1], Amgoud presented an argumentation system for generating consistent plans from a given set of desires and planning rules. This was later extended with argumentation systems that generate the desires themselves [3]. This system suffers from three main drawbacks: i) exhibiting a form of wishful thinking, ii) desires may depend only on beliefs, and iii) some undesirable results may be returned due to the separation of the two steps of PR. This system has been later extended in [16]. In the new system the problem of wishful thinking has been solved. However, the separation of the two steps was kept.

Other researchers in AI like Atkinson and Bench Capon [5] are more interested in studying the different argument schemes that one may encounter in practical reasoning. Their starting point was the above practical syllogism of Walton. The authors have defined different variants of this syllogism as well as different ways of attacking it. However, it is not clear how all these arguments can be put together in order to answer the critical question of PR "what is the right thing to do in a given situation?"

10 Conclusion

The paper has tackled the problem of practical reasoning, which is concerned with the question "what is the best thing to do at a given situation"? The approach followed here for answering this question is based on argumentation theory, in which choices are explained and justified by arguments. The contribution of this paper is two-fold. To the best of our knowledge, this paper proposes the first argumentation system that computes the intentions in one step, *i.e.* by combining desire generation and planning. This avoids undesirable results encountered by previous proposals in the literature. The second contribution of the paper consists of studying deeply the properties of argumentation-based PR.

This work can be extended in different ways. First, we are currently working on relaxing the assumption that the attack relation among instrumental arguments is binary. Indeed, it may be the case that more than two plans may be conflicting while each pair of them is compatible. Another urgent extension would be to introduce preferences to the system. The idea is that beliefs may be pervaded with uncertainty, desires may not have equal priorities, and plans may have different costs. Thus, taking into account these preferences will help to reduce the intention sets into more relevant ones.

In [8, 15], it has been shown that an argument may not only be attacked by other arguments, but may also be supported by arguments. It would be interesting to study deeply the impact of such a relation between arguments in the context of PR.

Finally, an interesting area of future work is investigating the proof theories of this system. The idea is to answer the question “is a given potential desire a possible intention of the agent ?” without computing the whole preferred extensions.

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A Proofs of properties

Property 1. Let $\alpha \in \mathcal{A}$. If α is rejected in AF, then α is also rejected in CAF.
Proof: Let us suppose that $\alpha \in \mathcal{A}$, and α is rejected in AF. Let us also suppose that α is not rejected in CAF.

Case of stable semantics: Since α is not rejected in CAF, then there exists $\mathcal{E}_i$ that is a C-stable extension of CAF, and $\alpha \in \mathcal{E}_i$. In [10], it has been shown (Proposition 6) that every C-stable extension is also a stable extension. Consequently, $\mathcal{E}_i$ is also a stable extension. Since α is rejected in AF, then $\alpha \notin \mathcal{E}_i$, contradiction.

Case of preferred semantics: Since α is not rejected in CAF, then there exists $\mathcal{E}_i$ that is a C-preferred extension of CAF, and $\alpha \in \mathcal{E}_i$. In [10], it has been shown (Proposition 4) that each C-preferred extension is a subset of a preferred extension. This means that $\exists E$ such $\mathcal{E}$ is a preferred extension of AF and $\mathcal{E}_i \subseteq \mathcal{E}$. However, since α is rejected in AF, then $\alpha \notin \mathcal{E}$, contradiction with the fact that $\alpha \in \mathcal{E}_i$.

■

Property 2. Let $\delta \in \mathcal{A}_d$.

- $\texttt{BELIEFS}(\delta) \subseteq \mathcal{K}$.
- $\texttt{DESIRES}(\delta) \subseteq \mathcal{PD}$.

Proof: Let $\delta \in \mathcal{A}_d$.

- Let us show that $\texttt{BELIEFS}(\delta) \subseteq \mathcal{K}$.
  $\texttt{BELIEFS}(\delta) = \bigcup \texttt{SUPP}(\alpha_i)$ with $\alpha_i \in \mathcal{A}_b \cap \texttt{SUB}(\delta)$. According to the definition of an epistemic argument α_i, $\texttt{SUPP}(\alpha_i) \subseteq \mathcal{K}$, thus $\texttt{BELIEFS}(\delta) \subseteq \mathcal{K}$.
- Let us show that $\texttt{DESIRES}(\delta) \subseteq \mathcal{PD}$.
  This is a direct consequence from the definition of an explanatory argument and the definition of the set $\mathcal{PD}$.

■

Property 3. $\nexists \alpha \in \mathcal{A}_b$ such that $\alpha \ \mathcal{R}_b \ \alpha$.
Proof: Let $\alpha \in \mathcal{A}_b$. Let us suppose that $\alpha \ \mathcal{R}_b \ \alpha$. According to Definition 16, $\exists h \in \texttt{SUPP}(\alpha)$ such that $\texttt{CONC}(\alpha) \equiv \neg h$. Moreover, according to the definition of an epistemic argument, it holds that $\texttt{SUPP}(\alpha) \vdash \texttt{CONC}(\alpha)$, thus, $\texttt{SUPP}(\alpha) \vdash \neg h$. Since $h \in \texttt{SUPP}(\alpha)$, this means that $\texttt{SUPP}(\alpha) \vdash h, \neg h$, thus $\texttt{SUPP}(\alpha) \vdash \bot$. This contradicts the fact that the support of an epistemic argument (α in our case) should be consistent.
■

Property6. The argumentation system $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$ has non-empty stable extensions.

Proof: Since $\mathcal{K} \neq \{\perp\}$ and $\mathcal{K} \neq \varnothing$ then the base $\mathcal{K}$ has at least one maximal (for set inclusion) consistent subset, say T. According to Prop. 5, $\mathtt{Arg}(T)$ is a stable extension of $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$. ■

Property 7. The relation $\mathcal{R}_d$ is symmetric and irreflexive.
Proof: This is a direct consequence of Definition 17. ■

Property 8. Let d_1, $d_2 \in \mathcal{PD}$. If $d_1 \equiv \neg d_2$, then $\forall \delta_1, \delta_2 \in \mathcal{A}_d$ such that: (1) $\exists \delta'_1 \in \mathtt{SUB}(\delta_1)$ with $\mathtt{CONC}(\delta'_1) = d_1$, and (2) $\exists \delta'_2 \in \mathtt{SUB}(\delta_2)$ with $\mathtt{CONC}(\delta'_2) = d_2$, then $\delta_1 \; \mathcal{R}_d \; \delta_2$.
Proof: Let d_1, $d_2 \in \mathcal{PD}$. Suppose that $d_1 \equiv \neg d_2$. Let $\delta_1, \delta_2 \in \mathcal{A}_d$ such that: (1) $\exists \delta'_1 \in \mathtt{SUB}(\delta_1)$ with $\mathtt{CONC}(\delta'_1) = d_1$, and (2) $\exists \delta'_2 \in \mathtt{SUB}(\delta_2)$ with $\mathtt{CONC}(\delta'_2) = d_2$. According to the definition of an explanatory argument, it is clear that $d_1 \in \mathtt{DESIRES}(\delta_1)$ and $d_2 \in \mathtt{DESIRES}(\delta_2)$. Since $d_1 \equiv \neg d_2$ then $\delta_1 \; \mathcal{R}_d \; \delta_2$. ■

Property 9. Let d_1, $d_2 \in \mathcal{PD}$. If $d_1 \equiv \neg d_2$, then $\forall \pi_1, \pi_2 \in \mathcal{A}_p$ s.t. $\mathtt{CONC}(\pi_1) = d_1$ and $\mathtt{CONC}(\pi_2) = d_2$, then $\pi_1 \; \mathcal{R}_p \; \pi_2$.
Proof: Let d_1, $d_2 \in \mathcal{PD}$. Suppose that $d_1 \equiv \neg d_2$. Let us also suppose that $\exists$ $\pi_1, \pi_2 \in \mathcal{A}_p$ with $\mathtt{CONC}(\pi_1) = d_1$, and $\mathtt{CONC}(\pi_2) = d_2$. According to Definition 15, it holds that $\mathtt{Postc}(\pi_1) \vdash d_1$ and $\mathtt{Postc}(\pi_2) \vdash d_2$. Since $d_1 \equiv \neg d_2$, then $\mathtt{Postc}(\pi_2) \vdash \neg d_1$. However, the two sets $\mathtt{Postc}(\pi_1)$ and $\mathtt{Postc}(\pi_2)$ are both consistent (according to Definition 11), thus $\mathtt{Postc}(\pi_1) \cup \mathtt{Postc}(\pi_2) \vdash \perp$. Consequently, $\pi_1 \; \mathcal{R}_p \; \pi_2$. ■

Property 10. The relation $\mathcal{R}_p$ is symmetric and irreflexive.
Proof: This is a direct consequence of Definition 18. ■

Property 11. Let $\delta \in \mathcal{A}_d$. If $\mathtt{BELIEFS}(\delta) \vdash \perp$, then $\exists \alpha \in \mathcal{A}_b$ such that $\alpha \; \mathcal{R}_{bd} \; \delta$.
Proof: Let $\delta \in \mathcal{A}_d$. Suppose that $\mathtt{BELIEFS}(\delta) \vdash \perp$. This means that $\exists T$ that is minimal for set inclusion among subsets of $\mathtt{BELIEFS}(\delta)$ with $T \vdash \perp$. Thus[8], $\exists h \in T$ such that $T \backslash \{h\} \vdash \neg h$ with $T \backslash \{h\}$ is consistent. Since $\mathtt{BELIEFS}(\delta) \subseteq \mathcal{K}$ (according to Prop. 2), then $T \backslash \{h\} \subseteq \mathcal{K}$. Consequently, $\exists \; \langle T \backslash \{h\}, \neg h \rangle \in \mathcal{A}_b$ with $h \in \mathtt{BELIEFS}(\delta)$. Thus, $\langle T \backslash \{h\}, \neg h \rangle \; \mathcal{R}_{bd} \; \delta$. ■

Property 12. Let $\delta_1, \delta_2 \in \mathcal{A}_d$ with $\mathtt{BELIEFS}(\delta_1) \nvdash \perp$ and $\mathtt{BELIEFS}(\delta_2) \nvdash \perp$.
If $\mathtt{BELIEFS}(\delta_1) \cup \mathtt{BELIEFS}(\delta_2) \vdash \perp$, then $\exists \alpha \in \mathcal{A}_b$ such that $\alpha \; \mathcal{R}_{bd} \; \delta_1$, or $\alpha \; \mathcal{R}_{bd} \; \delta_2$.
Proof: Let $\delta_1, \delta_2 \in \mathcal{A}_d$ with $\mathtt{BELIEFS}(\delta_1) \nvdash \perp$ and $\mathtt{BELIEFS}(\delta_2) \nvdash \perp$.

Suppose that $\mathtt{BELIEFS}(\delta_1) \cup \mathtt{BELIEFS}(\delta_2) \vdash \perp$. So, $\exists T_1 \subseteq \mathtt{BELIEFS}(\delta_1)$ and $\exists T_2 \subseteq \mathtt{BELIEFS}(\delta_2)$ with $T_1 \cup T_2 \vdash \perp$ and $T_1 \cup T_2$ is minimal for set inclusion, *i.e.* $T_1 \cup T_2$ is a minimal conflict. Since $\mathtt{BELIEFS}(\delta_1) \nvdash \perp$ and $\mathtt{BELIEFS}(\delta_2) \nvdash \perp$, then $T_1 \neq \varnothing$ and $T_2 \neq \varnothing$. Thus, $\exists h \in T_1 \cup T_2$ such that $(T_1 \cup T_2) \setminus \{h\} \vdash \neg h$. Since $T_1 \cup T_2$ is a minimal conflict, then each subset of $T_1 \cup T_2$ is consistent, thus the set $(T_1 \cup T_2) \setminus \{h\}$ is consistent. Moreover, according to Prop. 2, $\mathtt{BELIEFS}(\delta_1) \subseteq \mathcal{K}$ and

[8]Since T is $\subseteq$-minimal among inconsistent subsets of $\mathtt{BELIEFS}(\delta)$, each subset of T is consistent; so, $\exists T' = T \setminus \{h\}$ strictly included in T s.t. $T' \nvdash \perp$; so $T' \vdash \neg h$ (otherwise, $T' \cup \{h\} = T$ would be consistent).

$\texttt{BELIEFS}(\delta_2) \subseteq \mathcal{K}$. Thus, $T_1 \subseteq \mathcal{K}$ and $T_2 \subseteq \mathcal{K}$. It is then clear that $(T_1 \cup T_2) \setminus \{h\} \subseteq \mathcal{K}$. Consequently $\langle (T_1 \cup T_2) \setminus \{h\} \neg h \rangle$ is an argument of $\mathcal{A}_b$.

If $h \in T_1$, then $\langle (T_1 \cup T_2) \setminus \{h\} \neg h \rangle \; \mathcal{R}_{bd} \; \delta_1$, and if $h \in T_2$, then $\langle (T_1 \cup T_2) \setminus \{h\} \neg h \rangle \; \mathcal{R}_{bd} \; \delta_2$. ■

Property 13. Let $\delta \in \mathcal{A}_d$ and $\pi \in \mathcal{A}_p$ with $\texttt{BELIEFS}(\delta) \nvdash \bot$. If $\texttt{BELIEFS}(\delta) \cup \texttt{Prec}(\pi) \vdash \bot$ then $\exists \alpha \in \mathcal{A}_b$ such that $\alpha \; \mathcal{R}_{bd} \; \delta$, or $\alpha \; \mathcal{R}_{bp} \; \pi$.
Proof: Let $\delta \in \mathcal{A}_d$ and $\pi \in \mathcal{A}_p$. Suppose that $\texttt{BELIEFS}(\delta) \nvdash \bot$. Since $\texttt{BELIEFS}(\delta) \nvdash \bot$ and $\texttt{Prec}(\pi) \nvdash \bot$, then $\exists T \subseteq \texttt{BELIEFS}(\delta) \cup \texttt{Prec}(\pi)$ with $\texttt{BELIEFS}(\delta) \cap T \neq \varnothing$, $\texttt{Prec}(\pi) \cap T \neq \varnothing$ and T is the smallest inconsistent subset of $\texttt{BELIEFS}(\delta) \cup \texttt{Prec}(\pi)$.

Since $T \vdash \bot$, then $\exists h \in T$ such that $T \backslash \{h\} \vdash \neg h$ with $T \backslash \{h\}$ is consistent. Since $\texttt{BELIEFS}(\delta) \subseteq \mathcal{K}$ and Since $\texttt{Prec}(\pi) \subseteq \mathcal{K}$, then $T \subseteq \mathcal{K}$. Consequently, $T \backslash \{h\} \subseteq \mathcal{K}$. Thus, $\langle T \backslash \{h\}, \neg h \rangle \in \mathcal{A}_b$.

If $h \in \texttt{BELIEFS}(\delta)$, then $\langle T \backslash \{h\}, \neg h \rangle \; \mathcal{R}_{bd} \; \delta$. If $h \in \texttt{Prec}(\pi)$, then $\langle T \backslash \{h\}, \neg h \rangle \; \mathcal{R}_{bp} \; \pi$. ■

Property 14. Let $\mathsf{CAF}_{\mathsf{PR}} = \langle \mathcal{A}, \mathcal{R}, C \rangle$. Let Ω be the set of C-admissible sets of $\mathsf{CAF}_{\mathsf{PR}}$.

1. Let $\mathcal{E} \in \Omega$. There exists a C-preferred extension $\mathcal{E}'$ of $\mathsf{CAF}_{\mathsf{PR}}$ such that $\mathcal{E} \subseteq \mathcal{E}'$.
2. Let ${\mathsf{CAF}_{\mathsf{PR}}}' = \langle \mathcal{A}, \mathcal{R}, C' \rangle$ such that $C' \models C$. Let Ω' be the set of C'-admissible sets of ${\mathsf{CAF}_{\mathsf{PR}}}'$. We have $\Omega' \subseteq \Omega$.

Proof: This is a direct consequence of Proposition in [10]. ■

Property 15. Let $\mathsf{CAF}_{\mathsf{PR}} = \langle \mathcal{A}, \mathcal{R}, C \rangle$. For each C-preferred extension $\mathcal{E}$ of $\mathsf{CAF}_{\mathsf{PR}}$, there exists a preferred extension $\mathcal{E}'$ of $\mathsf{AF}_{\mathsf{PR}}$ such that $\mathcal{E} \subseteq \mathcal{E}'$.
Proof: This is a direct consequence of Proposition in [10]. ■

Property 16. Let $\mathsf{CAF}_{\mathsf{PR}} = \langle \mathcal{A}, \mathcal{R}, C \rangle$ such that C is a valid formula on $\mathcal{A}$. Then the preferred extensions of $\mathsf{AF}_{\mathsf{PR}}$ are the C-preferred extensions of $\mathsf{CAF}_{\mathsf{PR}}$.
Proof: This is a direct consequence of Proposition in [10]. ■

Property 17. Let $\mathsf{CAF}_{\mathsf{PR}} = \langle \mathcal{A}, \mathcal{R}, C \rangle$. Let Ω be the set of C-admissible sets of $\mathsf{CAF}_{\mathsf{PR}}$. Ω defines a complete partial order for $\subseteq$.
Proof: This is due to Proposition 14 and the facts that $\mathcal{A}$ is finite and $\varnothing$ is a C-admissible set. ■

Property 18. The system $\mathsf{AF}_{\mathsf{PR}}$ has at least one non-empty stable extension.
Proof: $\mathsf{AF}_{\mathsf{PR}}$ can be viewed as the union of 2 argumentation systems:

- The first one is $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$ and it has at least one non-empty stable extension (see Prop. 6).
- The second one is $\langle \mathcal{A}_d \cup \mathcal{A}_p, \mathcal{R}_d \cup \mathcal{R}_p \cup \mathcal{R}_{pdp} \rangle$ and it is symmetric in the sense of [9].

And these two argumentation systems are linked with the $\mathcal{R}_{bd} \cup \mathcal{R}_{bp}$ relation.

In order to prove that $\mathsf{AF}_{\mathsf{PR}}$ has at least one stable extension, we use an inductive process:

Basic case: $\mathsf{AF}_{\mathsf{PR}}$ is reduced to $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$; so, by Prop 6, $\mathsf{AF}_{\mathsf{PR}}$ has at least one non-empty stable extension.

Inductive assumption: $\mathsf{AF}_{\mathsf{PR}}$ has at least one non-empty stable extension denoted by $\mathcal{E}$. Let study $\mathsf{AF}'_{\mathsf{PR}} = \langle \mathcal{A} \cup \{x\}, \mathcal{R} \cup \{u\} \rangle$ with x being an argument which is not epistemic and u being an interaction from an argument x_1 to an argument x_2 knowing that x_2 cannot be epistemic.

There are 3 cases:

1. $\{u\} = \varnothing$ and $\{x\} \neq \varnothing$; so x is added to $\mathcal{A}$ without any interaction with the arguments of $\mathcal{A}$; so, x can be added to $\mathcal{E}$ and $\mathcal{E}$ is a stable extension of $\mathsf{AF}'_{\mathsf{PR}}$.
2. $\{x\} = \varnothing$ and $\{u\} \neq \varnothing$; for u, there are 2 cases: either u is an interaction between two non-epistemic arguments and the addition of u corresponds in fact with the addition of 2 symmetric interactions (x_1, x_2) and (x_2, x_1) with $x_1 \neq x_2$, or u is an interaction from an epismetic argument x_1 to a non-epistemic argument x_2 (in this case u cannot be symmetric). For all cases, there exist 2 possibilities:
   (a) x_1 or $x_2 \notin \mathcal{E}$; so, $\mathcal{E}$ remains a stable extension of $\mathsf{AF}'_{\mathsf{PR}}$.
   (b) x_1 and $x_2 \in \mathcal{E}$; suppose that $\nexists \mathcal{E}'$ stable extension of $\mathsf{AF}'_{\mathsf{PR}}$ then there exists at least one new odd-length cycle[9] in $\mathsf{AF}'_{\mathsf{PR}}$ which is not in $\mathsf{AF}_{\mathsf{PR}}$; this is impossible because the introduction of (x_1, x_2) and eventually (x_2, x_1) cannot create such a cycle (in particular, let recall that there is no possible attack of an epismetic argument by a non-epistemic argument); so there exists a stable extension of $\mathsf{AF}'_{\mathsf{PR}}$.
3. $\{x\} \neq \varnothing$, $\{u\} \neq \varnothing$ and u corresponds to (x, y) and (y, x) with x and y non-epistemic arguments and $y \in \mathcal{A}$; suppose that $\nexists \mathcal{E}'$ stable extension of $\mathsf{AF}'_{\mathsf{PR}}$ then there exists at least one new odd-length cycle in $\mathsf{AF}'_{\mathsf{PR}}$ which is not in $\mathsf{AF}_{\mathsf{PR}}$; this is impossible because the introduction of x and of of $(x_, y)$ and (y, x) cannot create such a cycle; so there exists a stable extension of $\mathsf{AF}'_{\mathsf{PR}}$.

■

Property 19. Let $\mathcal{E}$ be a stable extension of $\mathsf{AF}_{\mathsf{PR}}$. The set $\mathcal{E} \cap \mathcal{A}_b$ is a stable extension of $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$.

[9]An odd-length cycle in $\mathsf{AF} = \langle \mathcal{A}, \mathcal{R} \rangle$ is a sequence of arguments $x_1, \dots, x_{2n+1}$ belonging to $\mathcal{A}$ defined by:

i. $\forall i = 1 \dots 2n$, $x_i \mathcal{R} x_{i+1}$, and $x_{2n+1} \mathcal{R} x_1$.

ii. $\nexists T \subseteq \{x_1, \dots, x_{2n+1}\}$ such that T satisfies the previous condition.

In a symmetric argumentation system (in the sense of [9]), there never exist odd-length cycles.

Proof: Let $\mathcal{E}$ be a stable extension of $\mathsf{AF}_{\mathsf{PR}}$. Let us suppose that $\mathcal{E}' = \mathcal{E} \cap \mathcal{A}_b$ is not a stable extension of $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$. Two cases can be distinguished:
Case 1: $\mathcal{E}'$ is not conflict-free. This means that there exist $\alpha, \alpha' \in \mathcal{E}'$ such that $\alpha \mathcal{R}_b \alpha'$. Since $\mathcal{E}' = \mathcal{E} \cap \mathcal{A}_b$, then $\alpha, \alpha' \in \mathcal{E}$. This means that $\mathcal{E}$ is not conflict-free. This contradicts the fact that $\mathcal{E}$ is a stable extension.
Case 2: $\mathcal{E}'$ does not attack every argument that is not in $\mathcal{E}'$. This means that $\exists \alpha \in \mathcal{A}_b$ and $\notin \mathcal{E}'$ and $\mathcal{E}'$ does not attack (w.r.t. $\mathcal{R}_b$) α. This means that $\mathcal{E}' \cup \{\alpha\}$ is conflict-free, thus $\mathcal{E} \cup \{\alpha\}$ is also conflict-free, and does not attack an argument that is not in it (because only an epistemic argument can attack another epistemic argument and all epistemic arguments of $\mathcal{E}$ belong to $\mathcal{E}'$). This contradicts the fact that $\mathcal{E}$ is a stable extension. ■

Property 20 Let $\delta \in \mathcal{A}_d$. Let $\mathcal{E}$ be a stable extension of $\mathsf{AF}_{\mathsf{PR}}$. If $\delta \in \mathcal{E}$, then $\mathtt{SUB}(\delta) \subseteq \mathcal{E}$.
Proof: Let $\mathcal{E}$ be a stable extension of $\mathsf{AF}_{\mathsf{PR}}$. Let $\delta \in \mathcal{A}_d$. Let us suppose that $\delta \in \mathcal{E}$ and $\exists \delta' \in \mathtt{SUB}(\delta)$ such that $\delta' \notin \mathcal{E}$. Since $\delta' \notin \mathcal{E}$, $\exists x \in \mathcal{E}$ such that $x \mathcal{R} \delta'$. There are three possible cases:

1. $x \in \mathcal{A}_b$, thus $x \mathcal{R}_{bd} \delta'$. This means that $\exists h \in \mathtt{BELIEFS}(\delta')$ such that $\mathtt{CONC}(x) \equiv \neg h$. However, $\delta' \in \mathtt{SUB}(\delta)$, thus $\mathtt{BELIEFS}(\delta') \subseteq \mathtt{BELIEFS}(\delta)$. Thus, $x \mathcal{R}_{bd} \delta$ and consequently, $x \mathcal{R} \delta$. This contradicts the fact that $\mathcal{E}$ is conflict-free.

2. $x \in \mathcal{A}_d$, thus $x \mathcal{R}_d \delta'$. Thus, $\exists d_1 \in \mathtt{DESIRES}(x)$ and $\exists d_2 \in \mathtt{DESIRES}(\delta')$ such that $d_1 \equiv \neg d_2$. However, $\mathtt{DESIRES}(\delta') \subseteq \mathtt{DESIRES}(\delta)$, thus $x \mathcal{R}_d \delta$ and consequently, $x \mathcal{R} \delta$. This contradicts the fact that $\mathcal{E}$ is conflict-free.

3. $x \in \mathcal{A}_p$, thus $x \mathcal{R}_{pdp} \delta'$. This means that $\mathtt{CONC}(x) \equiv \neg d$ with $d \in \mathtt{DESIRES}(\delta')$. However, $\mathtt{DESIRES}(\delta') \subseteq \mathtt{DESIRES}(\delta)$, thus $x \mathcal{R}_{pdp} \delta$ and consequently, $x \mathcal{R} \delta$. This contradicts the fact that $\mathcal{E}$ is conflict-free.

■

Property 21. Let $\delta \in \mathcal{A}_d$. If $\mathtt{BELIEFS}(\delta) \vdash \bot$, then $\forall \mathcal{E}_i$ with $\mathcal{E}_i$ is a stable extension of $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$, $\exists \alpha \in \mathcal{E}_i$ such that $\alpha \mathcal{R}_{bd} \delta$.
Proof: Let $\delta \in \mathcal{A}_d$ with $\mathtt{BELIEFS}(\delta) \vdash \bot$. Let $\mathcal{E}_1, \ldots \mathcal{E}_n$ be the stable extensions of the system $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$. Suppose that $\exists \mathcal{E}_i$ such that $\mathcal{E}_i$ does not attack δ, *i.e.* $\nexists \alpha \in \mathcal{E}_i$ such that $\alpha \mathcal{R}_{bd} \delta$.

According to Prop. 4, $\mathtt{Base}(\mathcal{E}_i)$ is a maximal (for set inclusion) consistent subset of $\mathcal{K}$. Since $\mathtt{BELIEFS}(\delta) \vdash \bot$, then $\exists T \subseteq \mathtt{BELIEFS}(\delta)$ with T is the smallest inconsistent subset of $\mathtt{BELIEFS}(\delta)$ (*i.e.* $T \vdash \bot$). Moreover, according to Prop. 2, $\mathtt{BELIEFS}(\delta) \subseteq \mathcal{K}$, thus $T \subseteq \mathcal{K}$.

Since $\mathtt{Base}(\mathcal{E}_i)$ is a maximal (for set inclusion) consistent subset of $\mathcal{K}$, and T a minimal conflict of $\mathcal{K}$, then we have two cases:

- Case 1: $\mathtt{Base}(\mathcal{E}_i) \cap T = \varnothing$. This means that $\forall h \in T$, $\mathtt{Base}(\mathcal{E}_i) \cup \{h\} \vdash \bot$. Thus, $\mathtt{Base}(\mathcal{E}_i) \vdash \neg h$. Consequently, $\exists H \subseteq \mathtt{Base}(\mathcal{E}_i)$ with H is minimal for set-inclusion among subsets of $\mathtt{Base}(\mathcal{E}_i)$ that satisfy $H \vdash \neg h$. The pair $\langle H, \neg h \rangle$

is then an argument of $\mathcal{A}_b$. However, according to Prop. 4, $\texttt{Arg}(\texttt{Base}(\mathcal{E}_i))$ = $\mathcal{E}_i$, this means that $\langle H, \neg h\rangle \in \mathcal{E}_i$ and $\langle H, \neg h\rangle\ \mathcal{R}_{bd}\ \delta$.

- Case 2: $\texttt{Base}(\mathcal{E}_i) \cap T \neq \varnothing$. Since $\texttt{Base}(\mathcal{E}_i) \nvdash \perp$ and $T \vdash \perp$, then $\exists h \in T$ and $h \notin \texttt{Base}(\mathcal{E}_i)$ such that $\texttt{Base}(\mathcal{E}_i) \vdash \neg h$ (this is due to the fact that $\texttt{Base}(\mathcal{E}_i)$ is a maximal consistent subset of $\mathcal{K}$). Consequently, $\exists H \subseteq \texttt{Base}(\mathcal{E}_i)$ with H is minimal for set-inclusion among subsets of $\texttt{Base}(\mathcal{E}_i)$ that satisfy $H \vdash \neg h$. The pair $\langle H, \neg h\rangle$ is then an argument of $\mathcal{A}_b$. According to Prop. 4, $\texttt{Arg}(\texttt{Base}(\mathcal{E}_i))$ = $\mathcal{E}_i$, this means that $\langle H, \neg h\rangle \in \mathcal{E}_i$ and $\langle H, \neg h\rangle\ \mathcal{R}_{bd}\ \delta$.

■

Property 22. Let $\delta \in \mathcal{A}_d$ with $\texttt{BELIEFS}(\delta) \vdash \perp$. The argument δ is rejected in $\mathsf{AF}_{\mathsf{PR}}$.
Proof: Let $\delta \in \mathcal{A}_d$ with $\texttt{BELIEFS}(\delta) \vdash \perp$.

According to Property 18, the system $\mathsf{AF}_{\mathsf{PR}}$ has at least one stable extension. Let $\mathcal{E}$ be one of these stable extensions. Suppose that $\delta \in \mathcal{E}$.

According to Property 19, the set $\mathcal{E} \cap \mathcal{A}_b$ is a stable extension of $\langle \mathcal{A}_b, \mathcal{R}_b\rangle$. Moreover, according to Property 21, $\exists \alpha \in \mathcal{E} \cap \mathcal{A}_b$ such that $\alpha \mathcal{R}_{bd} \delta$. This contradicts the fact that a stable extension is conflict-free. ■

Property 23 Let $\delta \in \mathcal{A}_d$ with $\texttt{BELIEFS}(\delta) \vdash \perp$. δ is a rejected argument in $\mathsf{CAF}_{\mathsf{PR}}$.
Proof: Let $\delta \in \mathcal{A}_d$ with $\texttt{BELIEFS}(\delta) \vdash \perp$. According to Prop. 22, δ is rejected in $\mathsf{AF}_{\mathsf{PR}}$. Moreover, according to Prop. 1; we know that each argument that is rejected in $\mathsf{AF}_{\mathsf{PR}}$ is also rejected in $\mathsf{CAF}_{\mathsf{PR}}$. ■

Property 24. Let δ_1, $\delta_2 \in \mathcal{A}_d$ with $\texttt{BELIEFS}(\delta_1) \nvdash \perp$ and $\texttt{BELIEFS}(\delta_2) \nvdash \perp$.
If $\texttt{BELIEFS}(\delta_1) \cup \texttt{BELIEFS}(\delta_2) \vdash \perp$, then $\forall \mathcal{E}_i$ with $\mathcal{E}_i$ is a stable extension of $\langle \mathcal{A}_b, \mathcal{R}_b\rangle$, $\exists \alpha \in \mathcal{E}_i$ such that $\alpha \mathcal{R}_{bd} \delta_1$, or $\alpha \mathcal{R}_{bd} \delta_2$.
Proof: Let $\delta_1, \delta_2 \in \mathcal{A}_d$ with $\texttt{BELIEFS}(\delta_1) \nvdash \perp$, $\texttt{BELIEFS}(\delta_2) \nvdash \perp$, and $\texttt{BELIEFS}(\delta_1) \cup \texttt{BELIEFS}(\delta_2) \vdash \perp$. Let $\mathcal{E}_1, \ldots \mathcal{E}_n$ be the stable extensions of the system $\langle \mathcal{A}_b, \mathcal{R}_b\rangle$. Suppose that $\exists \mathcal{E}_i$ such that $\mathcal{E}_i$ does not attack δ_1 and $\mathcal{E}_i$ does not attack δ_2, *i.e.* $\nexists \alpha \in \mathcal{E}_i$ such that $\alpha \mathcal{R}_{bd} \delta_1$, or $\alpha \mathcal{R}_{bd} \delta_2$.

Since $\texttt{BELIEFS}(\delta_1) \cup \texttt{BELIEFS}(\delta_2) \vdash \perp$, then $\exists T \subseteq \texttt{BELIEFS}(\delta_1) \cup \texttt{BELIEFS}(\delta_2)$ with T is the smallest inconsistent subset of $\texttt{BELIEFS}(\delta_1) \cup \texttt{BELIEFS}(\delta_2)$ (*i.e.* $T \vdash \perp$).

Moreover, according to Property 2, we have $\texttt{BELIEFS}(\delta_1) \subseteq \mathcal{K}$ and $\texttt{BELIEFS}(\delta_2) \subseteq \mathcal{K}$, thus $T \subseteq \mathcal{K}$.

According to Property 4, $\texttt{Base}(\mathcal{E}_i)$ is a maximal (for set inclusion) consistent subset of $\mathcal{K}$. Since $\texttt{Base}(\mathcal{E}_i)$ is a maximal (for set inclusion) consistent subset of $\mathcal{K}$, and T a minimal conflict of $\mathcal{K}$, then we have two cases:

- Case 1: $\texttt{Base}(\mathcal{E}_i) \cap T = \varnothing$. This means that $\forall h \in T$, $\texttt{Base}(\mathcal{E}_i) \cup \{h\} \vdash \perp$. Thus, $\texttt{Base}(\mathcal{E}_i) \vdash \neg h$. Consequently, $\exists H \subseteq \texttt{Base}(\mathcal{E}_i)$ with H is minimal for set-inclusion among subsets of $\texttt{Base}(\mathcal{E}_i)$ that satisfy $H \vdash \neg h$. The pair $\langle H, \neg h\rangle$ is then an argument of $\mathcal{A}_b$. However, according to Property 4, $\texttt{Arg}(\texttt{Base}(\mathcal{E}_i))$ = $\mathcal{E}_i$, this means that $\langle H, \neg h\rangle \in \mathcal{E}_i$. If $h \in T \cap \texttt{BELIEFS}(\delta_1)$, then $\langle H, \neg h\rangle\ \mathcal{R}_{bd}\ \delta_1$. If $h \in T \cap \texttt{BELIEFS}(\delta_2)$, then $\langle H, \neg h\rangle\ \mathcal{R}_{bd}\ \delta_2$.

- Case 2: $\texttt{Base}(\mathcal{E}_i) \cap T \neq \varnothing$. Since $\texttt{Base}(\mathcal{E}_i) \nvdash \perp$ and $T \vdash \perp$, then $\exists h \in T$ and $h \notin \texttt{Base}(\mathcal{E}_i)$ such that $\texttt{Base}(\mathcal{E}_i) \vdash \neg h$ (this is due to the fact that $\texttt{Base}(\mathcal{E}_i)$ is a maximal consistent subset of $\mathcal{K}$). Consequently, $\exists H \subseteq \texttt{Base}(\mathcal{E}_i)$ with H is minimal for set-inclusion among subsets of $\texttt{Base}(\mathcal{E}_i)$ that satisfy $H \vdash \neg h$. The pair $\langle H, \neg h\rangle$ is then an argument of $\mathcal{A}_b$. According to Prop. 4, $\texttt{Arg}(\texttt{Base}(\mathcal{E}_i))$ = $\mathcal{E}_i$, this means that $\langle H, \neg h\rangle \in \mathcal{E}_i$. If $h \in T \cap \texttt{BELIEFS}(\delta_1)$, then and $\langle H, \neg h\rangle$ $\mathcal{R}_{bd}$ δ_1. If $h \in T \cap \texttt{BELIEFS}(\delta_2)$, then and $\langle H, \neg h\rangle$ $\mathcal{R}_{bd}$ δ_2.

■

Property 25. Let δ_1, $\delta_2 \in \mathcal{A}_d$ with $\texttt{BELIEFS}(\delta_1) \nvdash \perp$ and $\texttt{BELIEFS}(\delta_2) \nvdash \perp$.
If $\texttt{BELIEFS}(\delta_1) \cup \texttt{BELIEFS}(\delta_2) \vdash \perp$, then $\nexists \mathcal{E}$ with $\mathcal{E}$ a stable extension of $\mathsf{AF}_{\mathsf{PR}}$ such that $\delta_1 \in \mathcal{E}$ and $\delta_2 \in \mathcal{E}$.
Proof: Let $\delta_1, \delta_2 \in \mathcal{A}_d$ with $\texttt{BELIEFS}(\delta_1) \nvdash \perp$, $\texttt{BELIEFS}(\delta_2) \nvdash \perp$, and $\texttt{BELIEFS}(\delta_1) \cup \texttt{BELIEFS}(\delta_2) \vdash \perp$. According to Property 18, the system $\mathsf{AF}_{\mathsf{PR}}$ has at least one stable extension. Let $\mathcal{E}$ be one of these stable extensions. Suppose that $\delta_1 \in \mathcal{E}$ and $\delta_2 \in \mathcal{E}$.

According to Property 19, the set $\mathcal{E} \cap \mathcal{A}_b$ is a stable extension of $\langle \mathcal{A}_b, \mathcal{R}_b\rangle$. Moreover, according to Property 24, $\exists \alpha \in \mathcal{E} \cap \mathcal{A}_b$ such that $\alpha \mathcal{R}_{bd} \delta_1$, or $\alpha \mathcal{R}_{bd} \delta_2$. ■

Property 26. Let $\delta \in \mathcal{A}_d$ and $\pi \in \mathcal{A}_p$ with $\texttt{BELIEFS}(\delta) \nvdash \perp$. If $\texttt{BELIEFS}(\delta) \cup \texttt{Prec}(\pi) \vdash \perp$ then $\forall \mathcal{E}_i$ with $\mathcal{E}_i$ is a stable extension of $\langle \mathcal{A}_b, \mathcal{R}_b\rangle$, $\exists \alpha \in \mathcal{E}_i$ such that $\alpha \mathcal{R}_{bd} \delta$, or $\alpha \mathcal{R}_{bp} \pi$.
Proof: Let $\delta \in \mathcal{A}_d$, $\pi \in \mathcal{A}_p$ with $\texttt{BELIEFS}(\delta) \nvdash \perp$ and $\texttt{BELIEFS}(\delta) \cup \texttt{Prec}(\pi) \vdash \perp$. Let us suppose that $\mathcal{E}$ is a stable extension of $\langle \mathcal{A}_b, \mathcal{R}_b\rangle$, and that $\delta \in \mathcal{E}$ and $\pi \in \mathcal{E}$.

Since $\texttt{BELIEFS}(\delta) \cup \texttt{Prec}(\pi) \vdash \perp$, $\texttt{BELIEFS}(\delta) \nvdash \perp$, and $\texttt{Prec}(\pi) \nvdash \perp$, then $\exists T_1 \subseteq \texttt{BELIEFS}(\delta)$ and $\exists T_2 \subseteq \texttt{Prec}(\pi)$ such that $T_1 \cup T_2 \vdash \perp$ and $T_1 \cup T_2$ is the minimal inconsistent subset of $\texttt{BELIEFS}(\delta) \cup \texttt{Prec}(\pi)$. We know also that $T_1 \subseteq \mathcal{K}$ (since according to Property 2, $\texttt{BELIEFS}(\delta) \subseteq \mathcal{K}$) and $T_2 \subseteq \mathcal{K}$ (since $\texttt{Prec}(\pi) \subseteq \mathcal{K}$). Let $T = T_1 \cup T_2$.

According to Property 4, $\texttt{Base}(\mathcal{E})$ is a maximal (for set inclusion) consistent subset of $\mathcal{K}$. Then, two cases are distinguished:

- Case 1: $\texttt{Base}(\mathcal{E}) \cap T = \varnothing$. This means that $\forall h \in T$, $\texttt{Base}(\mathcal{E}) \cup \{h\} \vdash \perp$. Thus, $\texttt{Base}(\mathcal{E}) \vdash \neg h$. Consequently, $\exists H \subseteq \texttt{Base}(\mathcal{E})$ with H is minimal for set-inclusion among subsets of $\texttt{Base}(\mathcal{E})$ that satisfy $H \vdash \neg h$. The pair $\langle H, \neg h\rangle$ is then an argument of $\mathcal{A}_b$. However, according to Property 4, $\texttt{Arg}(\texttt{Base}(\mathcal{E}))$ = $\mathcal{E}$, this means that $\langle H, \neg h\rangle \in \mathcal{E}$. If $h \in T_1$, then $\langle H, \neg h\rangle$ $\mathcal{R}_{bd}$ δ. If $h \in T_2$, then $\langle H, \neg h\rangle$ $\mathcal{R}_{bp}$ π.

- Case 2: $\texttt{Base}(\mathcal{E}) \cap T \neq \varnothing$. Since $\texttt{Base}(\mathcal{E}) \nvdash \perp$ and $T \vdash \perp$, then $\exists h \in T$ and $h \notin \texttt{Base}(\mathcal{E})$ such that $\texttt{Base}(\mathcal{E}) \vdash \neg h$ (this is due to the fact that $\texttt{Base}(\mathcal{E})$ is a maximal consistent subset of $\mathcal{K}$). Consequently, $\exists H \subseteq \texttt{Base}(\mathcal{E})$ with H is minimal for set-inclusion among subsets of $\texttt{Base}(\mathcal{E})$ that satisfy $H \vdash \neg h$. The pair $\langle H, \neg h\rangle$ is then an argument of $\mathcal{A}_b$. According to Prop. 4, $\texttt{Arg}(\texttt{Base}(\mathcal{E}))$ = $\mathcal{E}$, this means that $\langle H, \neg h\rangle \in \mathcal{E}$. If $h \in T \cap \texttt{BELIEFS}(\delta_1)$, then $\langle H, \neg h\rangle$ $\mathcal{R}_{bd}$ δ_1. If $h \in T \cap \texttt{BELIEFS}(\delta_2)$, then $\langle H, \neg h\rangle$ $\mathcal{R}_{bd}$ δ_2.

$\blacksquare$

Property 27. Let $\delta \in \mathcal{A}_d$ and $\pi \in \mathcal{A}_p$ with $\texttt{BELIEFS}(\delta) \nvdash \perp$. If $\texttt{BELIEFS}(\delta) \cup \texttt{Prec}(\pi) \vdash \perp$ then $\nexists \mathcal{E}$ with $\mathcal{E}$ a stable extension of $\mathsf{AF}_{\mathsf{PR}}$ such that $\delta \in \mathcal{E}$ and $\pi \in \mathcal{E}$.
Proof: Let $\delta \in \mathcal{A}_d$ and $\pi \in \mathcal{A}_p$ with $\texttt{BELIEFS}(\delta) \nvdash \perp$ and $\texttt{BELIEFS}(\delta) \cup \texttt{Prec}(\pi) \vdash \perp$. Let $\mathcal{E}$ be a stable extension of $\mathsf{AF}_{\mathsf{PR}}$. Let us suppose that $\delta \in \mathcal{E}$ and $\pi \in \mathcal{E}$. Since $\mathcal{E}$ is a stable extension of $\mathsf{AF}_{\mathsf{PR}}$, then $\mathcal{E}' = \mathcal{E} \cap \mathcal{A}_b$ is a stable extension of $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$ (according to Property 19). Moreover, according to Property 26, since $\texttt{BELIEFS}(\delta) \cup \texttt{Prec}(\pi) \vdash \perp$ then $\exists \alpha \in \mathcal{E}'$ such that $\alpha \mathcal{R}_{bd} \delta$ or $\alpha \mathcal{R}_{bp} \pi$. This means that $\mathcal{E}$ attacks δ or $\mathcal{E}$ attacks π. However, $\delta \in \mathcal{E}$ and $\pi \in \mathcal{E}$. This contradicts the fact that $\mathcal{E}$ is conflict free. $\blacksquare$

Theorem 1. Let $\mathsf{CAF}_{\mathsf{PR}}$ be an argumentation system for PR, and $\mathcal{E}_1, \ldots, \mathcal{E}_n$ its C-stable extensions. $\forall \mathcal{E}_i$, $i = 1, \ldots, n$, it holds that:

1. The set $\texttt{Bel}(\mathcal{E}_i)$ = $\texttt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)$.
2. The set $\texttt{Bel}(\mathcal{E}_i)$ is a maximal (for set inclusion) consistent subset of $\mathcal{K}$.
3. The set $\texttt{Des}(\mathcal{E}_i)$ is consistent.

Proof: Let $\mathcal{E}_i$ be a stable extension of the system $\mathsf{CAF}_{\mathsf{PR}}$.

1. Let us show that the set $\texttt{Bel}(\mathcal{E}_i)$ = $\texttt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)$.
   In order to prove this, one should handle two cases:
   - $\texttt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b) \subseteq \texttt{Bel}(\mathcal{E}_i)$. This is a direct consequence from the fact that $\texttt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)$ = $\bigcup \texttt{SUPP}(\alpha_i)$ with $\alpha_i \in \mathcal{E}_i \cap \mathcal{A}_b$ (cf. definition of $\texttt{Bel}(\mathcal{E})$).
   - $\texttt{Bel}(\mathcal{E}_i) \subseteq \texttt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)$. Let us suppose that $\exists h \in \texttt{Bel}(\mathcal{E}_i)$ and $h \notin \texttt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)$. According to Property 19, $\mathcal{E}_i \cap \mathcal{A}_b$ is a stable extension of $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$. Moreover, according to Property 4, $\texttt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)$ is a maximal (for set-$\subseteq$) consistent subset of $\mathcal{K}$[10]. However, $\texttt{Bel}(\mathcal{E}_i) \subseteq \mathcal{K}$, then $h \in \mathcal{K}$. Since $h \notin \texttt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)$, then $\texttt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b) \cup \{h\} \vdash \perp$ (this is due to the fact that $\texttt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)$ is a maximal (for set-$\subseteq$) consistent subset of $\mathcal{K}$). Thus, $\texttt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b) \vdash \neg h$. This means that $\exists H \subseteq \texttt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)$ such that H is the minimal consistent subset of $\texttt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)$, thus $H \vdash \neg h$. Since $H \subseteq \mathcal{K}$ (since $\texttt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b) \subseteq \mathcal{K}$), then $\langle H, \neg h \rangle \in \mathcal{A}_b$. However, according to Property 4, $\texttt{Arg}(\texttt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b))$ = $\mathcal{E}_i \cap \mathcal{A}_b$. Besides, $h \in \texttt{Bel}(\mathcal{E}_i)$, there are three possibilities:
     (a) $h \in \texttt{BELIEFS}(\delta)$ with $\delta \in \mathcal{E}_i$. In this case, $\langle H, \neg h \rangle \ \mathcal{R}_{bd} \ \delta$. This contradicts the fact that $\mathcal{E}_i$ is a stable extension that is conflict-free.
     (b) $h \in \texttt{Prec}(\pi)$ with $\pi \in \mathcal{E}_i$. In this case, $\langle H, \neg h \rangle \ \mathcal{R}_{bp} \ \pi$. This contradicts the fact that $\mathcal{E}_i$ is a stable extension that is conflict-free.
     (c) $h \in \texttt{SUPP}(\alpha)$ with $\alpha \in \mathcal{E}_i$. This is impossible since the set $\mathcal{E}_i \cap \mathcal{A}_b$ is a stable extension, thus it is conflict free.

[10] Because $\texttt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b) = \bigcup \texttt{SUPP}(\alpha_i)$ with $\alpha_i \in \mathcal{E}_i \cap \mathcal{A}_b$; so, $\texttt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b) = Base(\mathcal{E}_i \cap \mathcal{A}_b)$.

2. Let us show that the set $\texttt{Bel}(\mathcal{E}_i)$ is a maximal (for set inclusion) consistent subset of $\mathcal{K}$.
   Since $\mathcal{E}_i$ is a stable extension of $\mathsf{CAF}_{\mathsf{PR}}$, then $\mathcal{E}_i$ is also a stable extension of $\mathsf{AF}_{\mathsf{PR}}$ (according to [10]). Moreover, according to the first item of Theorem 1, $\texttt{Bel}(\mathcal{E}_i) = \texttt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)$. However, according to Property 19, $\mathcal{E}_i \cap \mathcal{A}_b$ is a stable extension of $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$, and according to Property 4, $\texttt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)$ is a maximal (for set-$\subseteq$) consistent subset of $\mathcal{K}$. Thus, $\texttt{Bel}(\mathcal{E}_i)$ is a maximal (for set inclusion) consistent subset of $\mathcal{K}$.

3. Let us show that the set $\texttt{Des}(\mathcal{E}_i)$ is consistent.
   Since $\mathcal{E}_i$ is a stable extension of $\mathsf{CAF}_{\mathsf{PR}}$, then $\mathcal{E}_i$ is also a stable extension of $\mathsf{AF}_{\mathsf{PR}}$ (according to [10]). Let us suppose that $\texttt{Des}(\mathcal{E}_i)$ is inconsistent, this means that $\bigcup \texttt{DESIRES}(\delta_k) \cup \bigcup \texttt{CONC}(\pi_j) \vdash \perp$ with $\delta_k \in \mathcal{E}_i$ and $\pi_j \in \mathcal{E}_i$. Since $\texttt{Des}(\mathcal{E}_i) \subseteq \mathcal{PD}$ (according to Property 2), then $\exists d_1, d_2 \in \texttt{Des}(\mathcal{E}_i)$ such that $d_1 \equiv \neg d_2$. Three possible situations may occur:

   (a) $\exists \pi_1, \pi_2 \in \mathcal{E}_i \cap \mathcal{A}_p$ such that $\texttt{CONC}(\pi_1) = d_1$, and $\texttt{CONC}(\pi_2) = d_2$. This means that $\pi_1 \mathcal{R}_p \pi_2$, thus $\pi_1 \mathcal{R} \pi_2$. This is impossible since $\mathcal{E}_i$ is a stable extension, thus it is supposed to be conflict-free.

   (b) $\exists \delta_1, \delta_2 \in \mathcal{E}_i \cap \mathcal{A}_d$ such that $d_1 \in \texttt{DESIRES}(\delta_1)$ and $d_2 \in \texttt{DESIRES}(\delta_2)$. This means that $\delta_1 \mathcal{R}_d \delta_2$, thus $\delta_1 \mathcal{R} \delta_2$. This is impossible since $\mathcal{E}_i$ is a stable extension, thus it is supposed to be conflict-free.

   (c) $\exists \delta \in \mathcal{E}_i \cap \mathcal{A}_d$, $\exists \pi \in \mathcal{E}_i \cap \mathcal{A}_p$ such that $d_1 \in \texttt{DESIRES}(\delta)$ and $d_2 = \texttt{CONC}(\pi)$. Since $d_1 \in \texttt{DESIRES}(\delta)$, thus $\exists \delta' \in \texttt{SUB}(\delta)$ such that $\texttt{CONC}(\delta') = d_1$. This means that $\delta' \mathcal{R}_{pdp} \pi$, thus $\delta' \mathcal{R} \pi$. However, since $\delta \in \mathcal{E}_i$, thus according to Property 20 $\delta' \in \mathcal{E}_i$. This is impossible since $\mathcal{E}_i$ is a stable extension, thus it is supposed to be conflict-free.

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Theorem 2. Each set of intentions of $\mathsf{CAF}_{\mathsf{PR}}$ is consistent.
Proof: Let $\mathcal{I}$ be a set of intentions of $\mathsf{CAF}_{\mathsf{PR}}$. Let us suppose that $\mathcal{I}$ is inconsistent. From the definition of an intention set, it is clear that $\mathcal{I} \subseteq \texttt{Des}(\mathcal{E}_i)$ with $\mathcal{E}_i$ is an extension of $\mathsf{CAF}_{\mathsf{PR}}$. However, according to Theorem 1 the set $\texttt{Des}(\mathcal{E}_i)$ is consistent. ■

Theorem 3. Let $\mathsf{CAF}_{\mathsf{PR}}$ be an argumentation system for PR, and $\mathcal{E}_1, \ldots, \mathcal{E}_n$ its C-stable extensions. $\forall \mathcal{E}_i$, $i = 1, \ldots, n$, it holds that:

1. The set $\texttt{Arg}(\texttt{Bel}(\mathcal{E}_i)) = \mathcal{E}_i \cap \mathcal{A}_b$.

2. $\mathcal{A}$s = $\mathcal{E}_i$.

Proof: Let $\mathcal{E}_i$ be a stable extension of the system $\mathsf{CAF}_{\mathsf{PR}}$. $\mathcal{E}_i$ is also a stable extension of $\mathsf{AF}_{\mathsf{PR}}$ (according to [10]).

1. Let us show that $\texttt{Arg}(\texttt{Bel}(\mathcal{E}_i)) = \mathcal{E}_i \cap \mathcal{A}_b$.
   According to Theorem 1, it is clear that $\texttt{Bel}(\mathcal{E}_i) = \texttt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)$. Moreover, according to Property 19, $\mathcal{E}_i \cap \mathcal{A}_b$ is a stable extension of $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$. Besides,

according to Property 4 $\texttt{Arg}(\texttt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b))$ = $\mathcal{E}_i \cap \mathcal{A}_b$, thus $\texttt{Arg}(\texttt{Bel}(\mathcal{E}_i))$ = $\mathcal{E}_i \cap \mathcal{A}_b$.

2. Let us show that $\mathcal{A}$s = $\mathcal{E}_i$.

   - $\mathcal{E}_i \subseteq \mathcal{A}$s: This is trivial.
   - $\mathcal{A}$s $\subseteq \mathcal{E}_i$: Let us suppose that $\exists y \in \mathcal{A}$s and $y \notin \mathcal{E}_i$. There are three possible situations:

     (a) $y \in \mathcal{A}$s $\cap \mathcal{A}_b$: Since $y \notin \mathcal{E}_i$, this means that $\exists \alpha \in \mathcal{E}_i \cap \mathcal{A}_b$ such that $\alpha \mathcal{R}_b y$. Thus, $\texttt{SUPP}(\alpha) \cup \texttt{SUPP}(y) \vdash \perp$. However, $\texttt{SUPP}(\alpha) \subseteq \texttt{Bel}(\mathcal{E}_i)$ and $\texttt{SUPP}(y) \subseteq \texttt{Bel}(\mathcal{E}_i)$, thus $\texttt{SUPP}(\alpha) \cup \texttt{SUPP}(y) \subseteq \texttt{Bel}(\mathcal{E}_i)$. This means that $\texttt{Bel}(\mathcal{E}_i)$ is inconsistent. According to Theorem 1 this is impossible.

     (b) $y \in \mathcal{A}$s $\cap \mathcal{A}_d$: Since $y \notin \mathcal{E}_i$, this means that $\exists x \in \mathcal{E}_i$ such that $x \mathcal{R} y$. There are three situations:

       Case 1: $x \in \mathcal{A}_b$ This means that $\texttt{BELIEFS}(y) \cup \texttt{SUPP}(x) \vdash \perp$. However, $\texttt{BELIEFS}(y) \cup \texttt{SUPP}(x) \subseteq \texttt{Bel}(\mathcal{E}_i)$. Thus, $\texttt{Bel}(\mathcal{E}_i)$ is inconsistent. This contradicts Theorem 1.

       Case 2: $x \in \mathcal{A}_d$ This means that $\texttt{DESIRES}(y) \cup \texttt{DESIRES}(x) \vdash \perp$. However, $\texttt{DESIRES}(y) \cup \texttt{DESIRES}(x) \subseteq \texttt{Des}(\mathcal{E}_i)$. Thus, $\texttt{Des}(\mathcal{E}_i)$ is inconsistent. This contradicts Theorem 1.

       Case 3: $x \in \mathcal{A}_p$ This means that $\texttt{DESIRES}(y) \cup \texttt{CONC}(x) \vdash \perp$. However, $\texttt{DESIRES}(y) \cup \texttt{CONC}(x) \subseteq \texttt{Des}(\mathcal{E}_i)$. Thus, $\texttt{Des}(\mathcal{E}_i)$ is inconsistent. This contradicts Theorem 1.

     (c) $y \in \mathcal{A}$s $\cap \mathcal{A}_p$: Since $y \notin \mathcal{E}_i$, this means that $\exists x \in \mathcal{E}_i$ such that $x \mathcal{R} y$. There are three situations:

       Case 1: $x \in \mathcal{A}_b$ This means that $x \mathcal{R}_{bp} y$, thus $\texttt{SUPP}(x) \cup \texttt{Prec}(y) \vdash \perp$. However, $\texttt{SUPP}(x) \cup \texttt{Prec}(y) \subseteq \texttt{Bel}(\mathcal{E}_i)$. Thus, $\texttt{Bel}(\mathcal{E}_i)$ is inconsistent. This contradicts Theorem 1.

       Case 2: $x \in \mathcal{A}_d$ This means that $x \mathcal{R}_{pdp} y$, thus $\texttt{DESIRES}(x) \cup \texttt{CONC}(y) \vdash \perp$. However, $\texttt{DESIRES}(x) \cup \texttt{CONC}(y) \subseteq \texttt{Des}(\mathcal{E}_i)$. Thus, $\texttt{Des}(\mathcal{E}_i)$ is inconsistent. This contradicts Theorem 1.

       Case 3: $x \in \mathcal{A}_p$ This means that $x \mathcal{R}_p y$. There are three different cases:

       – $\texttt{Prec}(x) \cup \texttt{Prec}(y) \vdash \perp$.
         However, $\texttt{Prec}(x) \cup \texttt{Prec}(y) \subseteq \texttt{Bel}(\mathcal{E}_i)$. Thus, $\texttt{Bel}(\mathcal{E}_i)$ is inconsistent. This contradicts Theorem 1.

       – $\texttt{Postc}(x) \cup \texttt{Prec}(y) \vdash \perp$. We know that y is built using one of the plans of $\mathcal{E}_i$, say $p = \langle S, T, d \rangle$. Thus, $\exists \pi \in \mathcal{E}_i$ such that π = $\langle p, d' \rangle$. Thus, $\texttt{Postc}(x) \cup \texttt{Prec}(\pi) \vdash \perp$, consequently, $x \mathcal{R} \pi$. This is impossible since $\mathcal{E}_i$ is a stable extension, thus it is supposed to be conflict-free.

       – $\texttt{Postc}(x) \cup \texttt{Postc}(y) \vdash \perp$. Since $y \in \mathcal{A}$s, thus y is built using one of the plans of $\mathcal{E}_i$, say $p = \langle S, T, d \rangle$. Thus, $\exists \pi \in \mathcal{E}_i$

such that $\pi = \langle p, d' \rangle$. Thus, $\texttt{Postc}(x) \cup \texttt{Postc}(\pi) \vdash \perp$, consequently, $x\mathcal{R}\pi$. This is impossible since $\mathcal{E}_i$ is a stable extension, thus it is supposed to be conflict-free.

$\blacksquare$