Acceptability semantics in recursive argumentation frameworks: Logical encoding and computation

Claudette Cayrol,
M-Christine Lagasquie-Schiex

Université de Toulouse, IRIT,
118 route de Narbonne, 31062 Toulouse, France
{ccayrol,lagasq}@irit.fr

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Abstract

This work follows [10] and gives a logical translation of semantics in the case of abstract argumentation frameworks using recursive attacks. These semantics, proposed in [7, 8] for recursive argumentation frameworks, are defined using a notion of “structure” in place of the notion of “extension” classically used in Dung’s abstract argumentation framework.
So, in this report, as it has been done in [10] for classical argumentation frameworks, we show that the logical translation proposed in [10] also allows the computation of structures in the case of recursive argumentation frameworks.

Contents

Chapter 1

Introduction

The main feature of an argumentation framework is the ability to deal with incomplete and / or contradictory information, especially for reasoning [16, 2]. Moreover, argumentation can be used to formalize dialogues between several agents by modeling the exchange of arguments in, *e.g.*, negotiation between agents [3]. An argumentation framework (AF) consists of a collection of arguments interacting with each other through a relation reflecting conflicts between them, called *attack*. The issue of argumentation is then to determine *acceptable* sets of arguments (*i.e.*, sets able to defend themselves collectively while avoiding internal attacks), called *extensions*, and thus to reach a coherent conclusion. Another form of analysis of an AF is the study of the particular status of each argument, this status is based on membership (or non-membership) in the extensions. Formal frameworks have greatly eased the modeling and study of AF. In particular, the framework of [16] allows completely abstracting the *concrete* meaning of the arguments and relies only on binary interactions that may exist between them.

AF have been extended along different lines. For instance, bipolar AF (BAF) correspond to AF with a second kind of interaction, the support relation. This relation represents a positive interaction between arguments and has been first introduced by [17, 25]. Several variants of the support relation have been introduced according to different interpretations of the support (deductive support [5], necessary support [19, 20], evidential support [21, 22]). Recent work have emphasized the central role of the necessary support [11, 23, 24].

AF have been also extended so as to take into account interactions between arguments and other interactions. A first version has been introduced by [18], then studied in [4] under the name of AFRA (Argumentation Framework with Recursive Attacks). This version describes abstract argumentation frameworks in which the interactions can be either attacks between arguments or attacks from an argument to another attack. In this case, as for the bipolar case, a translation of an AFRA into an equivalent AF can be defined by the addition of some new arguments and the attacks they produce or they receive. A generalization of AFRA has been proposed in [13] in order to take into account supports on arguments or on interactions. These frameworks are called ASAF (Attack-Support Argumentation Frameworks). And, once again, a translation of an ASAF into an equivalent AF is proposed by the addition of arguments and attacks. More recently, alternative acceptability semantics have been defined in a direct way for argumentation frameworks with recursive attacks [7, 8].

The subject of the current report is to propose a logical description of argumentation frameworks with recursive interactions. We think that a logical formalization will be helpful for justifiying the introduction of all the new attacks described above. Moreover, a logical formalization enables to take advantage of logical tools for computing semantics. This logical point of view was inspired by works in bioinformatics (see [15, 1]). In this domain, we can find *metabolic networks* that describe the chemical reactions of cells; these reactions can be negative (inhibition of a protein) or positive (production of a new protein) and they can depend on other proteins or other reactions. A translation from metabolic networks to classical logic has been proposed in [1]. This translation allows for the use of automated deduction methods for reasoning on these networks. A very preliminary work has been done for adapting this approach to argumentation frameworks (see [9]).

In this new work, we restrict our study to argumentation frameworks with only attacks (recursive or not) and propose a new logical vision of argumentation frameworks. So, the interactions corresponding to the notion of support will not be considered (this case is left for future work).

Context. Two different cases are considered in this work:

- The case called "classic": we study Dung's argumentation frameworks (D-frameworks), denoted by AF, where only attacks between arguments (*simple attacks*, see Definition 1 on page 5) can be found;
- The case called "recursive": we study argumentation frameworks, where there exist arguments, attacks between arguments and attacks between an argument and another attack, (called *recursive attacks*, see Definition 4 on page 6). These frameworks will be called recursive argumentation frameworks and denoted by RAF.

According to the considered case (classic or recursive), there exist two ways for weakening an attack:

- either by weakening the source of the attack (this is possible in both cases),
- or because the attack is the target of another attack (this is possible only in the recursive case).

This leads to propose the notions of "grounded attack" and "valid attack" ([6]). The notion of grounded attack is about the source of the attack and the notion of valid attack is about the link between the source and the target of the attack (*i.e.* the role of the interaction itself).

Moreover, in [6], the recursive argumentation framework is translated into a D-framework, using the addition of meta-arguments. This translation allows taking into account the notion of grounded (resp. valid) attack in the computation of the extensions of the resulting framework.

Contents:

1. The main notions about abstract argumentation are given in Chapter 2 on page 5, including the definitions of semantics for recursive frameworks.
2. In Chapter 3 on page 19, we propose a formal description of the argumentation framework in terms of first-order logic formulae: our aim is to give a formal explicit description of the meaning attached to an attack, and so of the notions of accepted argument, grounded or valid attack, independently of any argumentation semantics. The proposed formal language is able to into account recursive attacks.

3. Then, using the language described in the previous chapter, a formalisation of argumentation semantics is given in Chapter 4 on page 21. Our aim is first to give a logical description of the principles that govern the semantics for recursive frameworks, then to characterize these semantics in logical terms.

4. In Chapter 5 on page 25, semantics for recursive frameworks are illustrated on various examples.

5. Finally, Chapter 6 on page 49 outlines comparisons between the different methods proposed in this report for defining semantics of recursive argumentation frameworks.

Chapter 2

Background on abstract argumentation

This chapter gives the definitions of different kinds of abstract argumentation framework (D-framework, bipolar framework, recursive framework) and the associated semantics.

2.1 Different abstract argumentation frameworks

The classic case concerns argumentation frameworks with only one kind of interaction: attacks between arguments.

Def. 1 (AF) *A* Dung's argumentation framework, or D-framework for short, (AF) *is a tuple* $\langle \mathbf{A}, \mathbf{R} \rangle$*, where*

- $\mathbf{A}$ *is a finite and non-empty set of arguments,*
- $\mathbf{R} \subseteq \mathbf{A} \times \mathbf{A}$ *is a relation representing attacks over arguments, called* attack relation.

A first generalization takes into account an additional kind of interaction: supports between arguments.

Def. 2 (BAF) *A* bipolar argumentation framework (BAF) *is a tuple* $\langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$*, where*

- $\mathbf{A}$ *is a finite and non-empty set of arguments,*
- $\mathbf{R}_{\text{att}} \subseteq \mathbf{A} \times \mathbf{A}$ *is a relation representing attacks over arguments, called* attack relation *and*
- $\mathbf{R}_{\text{sup}} \subseteq \mathbf{A} \times \mathbf{A}$ *is a relation representing supports over arguments, called* support relation.

It is assumed that $\mathbf{R}_{\text{att}} \cap \mathbf{R}_{\text{sup}} = \varnothing$.

Another possible extension concerns recursive interactions (support or attack), *i.e.* from an argument to either another argument or another interaction [14].

Def. 3 (ASAF) *An* Attack-Support Argumentation Framework (ASAF) *is a tuple* $\langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$ *where*

- $\mathbf{A}$ *is a set of arguments,*
- $\mathbf{R}_{\text{att}}$ *is a subset of* $\mathbf{A} \times (\mathbf{A} \cup \mathbf{R}_{\text{att}} \cup \mathbf{R}_{\text{sup}})$ *corresponding to a set of attacks, and*
- $\mathbf{R}_{\text{sup}}$ *is a subset of* $\mathbf{A} \times (\mathbf{A} \cup \mathbf{R}_{\text{att}} \cup \mathbf{R}_{\text{sup}})$ *corresponding to a set of supports. Note that* $\mathbf{R}_{\text{sup}}$ *is assumed to be irreflexive and transitive.*

It is assumed that $\mathbf{R}_{\text{att}} \cap \mathbf{R}_{\text{sup}} = \varnothing$.

We propose an alternative formalisation in which each interaction is labelled.[1]

Def. 4 (RAF) *A* recursive argumentation framework (RAF) *is a tuple* $\langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}}, s, t \rangle$ *where*

- $\mathbf{A}$ *is a set of arguments,*
- $\mathbf{R}_{\text{att}}$ *(resp.* $\mathbf{R}_{\text{sup}}$*) is a set disjunct from* $\mathbf{A}$*, representing attack (resp. support) names,*
- s *is a function from* $(\mathbf{R}_{\text{att}} \cup \mathbf{R}_{\text{sup}})$ *to* $\mathbf{A}$*, mapping each interaction to its source,*
- t *is a function from* $(\mathbf{R}_{\text{att}} \cup \mathbf{R}_{\text{sup}})$ *to* $(\mathbf{A} \cup \mathbf{R}_{\text{att}} \cup \mathbf{R}_{\text{sup}})$ *mapping each interaction to its target.*

It is assumed that $\mathbf{R}_{\text{att}} \cap \mathbf{R}_{\text{sup}} = \varnothing$.

Note that a D-framework can be recovered as a particular case of a RAF as follows. Given any D-framework AF = $\langle \mathbf{A}, \mathbf{R} \rangle$, we may obtain its corresponding recursive argumentation framework RAF = $\langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}}, s, t \rangle$ by defining

- a set of attack names $\mathbf{R}_{\text{att}} = \{\alpha_{(a,b)} | (a, b) \in \mathbf{R}\}$
- a set of support names $\mathbf{R}_{\text{sup}} = \varnothing$
- the functions s (resp. t) as $s(\alpha_{(a,b)}) = a$ (resp. $t(\alpha_{(a,b)}) = b$).

Similarly, a BAF can be recovered as a particular case of a RAF, where t is a mapping from $(\mathbf{R}_{\text{att}} \cup \mathbf{R}_{\text{sup}})$ to $\mathbf{A}$.

In the remainder of this paper, we do not consider the support relation. So we consider either an AF, or a RAF with $\mathbf{R}_{\text{sup}} = \varnothing$ (in that case, the set representing attacks will be denoted by $\mathbf{R}$).

That means, we consider a 4-tuple $\langle \mathbf{A}, \mathbf{R}, s, t \rangle$ where

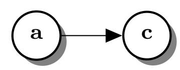
- $\mathbf{A}$ is a set of arguments (denoted by $a, b, \ldots$) and $\mathbf{R}$ is a set disjunct from $\mathbf{A}$, representing attack names (denoted by greek letters),
- s is a function from $\mathbf{R}$ to $\mathbf{A}$, mapping each interaction to its source,
- t is a function from $\mathbf{R}$ to $(\mathbf{A} \cup \mathbf{R})$ mapping each interaction to its target.

[1] Another formalisation is also given in [6]: A *labelled ASAF (LASAF)* is a 5-uple $\langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}}, \mathcal{V}, \mathcal{L} \rangle$ where $\mathbf{A}$ is a set of arguments, $\mathbf{R}_{\text{att}} \subseteq \mathbf{A} \times (\mathbf{A} \cup \mathbf{R}_{\text{att}} \cup \mathbf{R}_{\text{sup}})$ is an attack relation, $\mathbf{R}_{\text{sup}} \subseteq \mathbf{A} \times (\mathbf{A} \cup \mathbf{R}_{\text{att}} \cup \mathbf{R}_{\text{sup}})$ is a support relation, $\mathcal{V}$ is a set of labels (denoted by greek letters) and $\mathcal{L}$ is a bijection from $\mathbf{R} \subseteq (\mathbf{R}_{\text{att}} \cup \mathbf{R}_{\text{sup}})$ to $\mathcal{V}$.

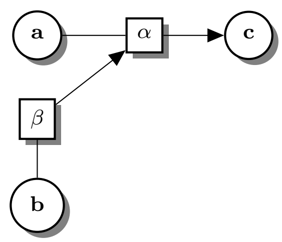
In the particular case of an AF, t is a mapping from $\mathbf{R}$ to $\mathbf{A}$.

For each of these argumentation frameworks, we will use a graphical representation defined as follows.

In the case of an AF, an attack $(a, c) \in \mathbf{R}$ will be represented by two vertices a, c and an edge from a to c.



In the case of a RAF, an attack named α (with $s(\alpha) = a$ and $t(\alpha) = c \in \mathbf{A}$) being the target of an attack β with $s(\beta) = b$ will be represented as:



In the following, the graphical representation of an argumentation framework will be denoted by G. For convenience, given a RAF = $\langle \mathbf{A}, \mathbf{R}, s, t \rangle$, we will often also use G to denote the RAF.

2.2 Argumentation semantics for AF

In the following, we recall the definitions of the standard semantics. Note that these definitions only concern Dung's abstract argumentation frameworks (with simple attacks). It is important to note that no semantics has been clearly defined in the other cases, *i.e.* when supports and/or recursive interactions exist (some suggestions have been done but there exists no consensus about them).

In the extension-based approach, a semantics specifies requirements that a set of arguments should satisfy. These requirements have been extensively analysed in [BG07]. There are three basic requirements, corresponding to three principles for semantics:

- An extension is a set of arguments that "can stand together". This corresponds to the *conflict-free principle*.
- An extension is a set of arguments that "can stand on its own", namely is able to counter all the attacks it receives. This corresponds to the *concept of defence* and leads to the *admissibility principle*.
- *Reinstatement* is a kind of dual principle. If an extension defends an argument, this argument is reinstated by the extension and should belong to the extension.

We give below the formal definitions.

Def. 5 (Basic concepts used in extension-based semantics) *Let AF* = $\langle \mathbf{A}, \mathbf{R} \rangle$ *and* $S \subseteq \mathbf{A}$.

- S *is* conflict-free *if and only if there are no arguments* $a, b \in S$*, such that* $a\mathbf{R}b$.

- $a \in \mathbf{A}$ *is* acceptable *with respect to* S *(or equivalently* S *defends* a*) if and only if* $\forall b \in \mathbf{A}$ *such that* $b\mathbf{R}a$*,* $\exists c \in S$ *such that* $c\mathbf{R}b$.

- S *is* admissible *if and only if* S *is conflict-free and each argument in* S *is acceptable with respect to* S.

Standard extension-based semantics are classically defined as follows:

Def. 6 (AF extensions in standard semantics) *Let AF =* $\langle \mathbf{A}, \mathbf{R} \rangle$ *and* $S \subseteq A$.

- S *is* a naive extension *of AF if and only if it is a maximal (with respect to* $\subseteq$*) conflict-free set.*

- S *is* a preferred extension *of AF if and only if it is a maximal (with respect to* $\subseteq$*) admissible set.*

- S *is* a complete extension *of AF if and only if* S *is admissible and each argument which is acceptable with respect to* S *belongs to* S.

- S *is* a stable extension *of AF if and only if it is conflict-free and* $\forall a \notin S$*,* $\exists b \in S$ *such that* $b\mathbf{R}a$.

Most of the standard semantics can be alternatively defined using the characteristic function $\mathcal{F}$.

Def. 7 (Extensions defined by $\mathcal{F}$) *Let AF =* $\langle \mathbf{A}, \mathbf{R} \rangle$ *and* $S \subseteq A$.

- *The* characteristic function *of AF is defined by:* $\mathcal{F}(S) = \{a \in \mathbf{A}$ *such that* a *is acceptable with respect to* $S\}$.

- S *is* admissible *if and only if* S *is conflict-free and* $S \subseteq \mathcal{F}(S)$.

- S *is* a complete extension *of AF if and only if it is conflict-free and a fixed point of* $\mathcal{F}$.

- S *is* the grounded extension *of AF if and only if it is the minimal (with respect to* $\subseteq$*) fixed point*[2] *of* $\mathcal{F}$.

- S *is* a preferred extension *of AF if and only if it is a maximal (with respect to* $\subseteq$*) complete extension if and only if it is a maximal conflict-free fixed point of* $\mathcal{F}$.

Note that due to Definition 6 the complete semantics is based on both principles of admissibility and resinstatement. Moreover, as the grounded extension, the preferred extensions and the stable extensions are also complete extensions, the grounded (resp. preferred, stable) semantics satisfies the admissibility and reinstatement principles.

[2] It can be proved that the minimal fixed point of $\mathcal{F}$ is conflict-free.

2.3 Argumentation semantics for BAF

Handling support and attack at an abstract level has the advantage to keep genericity and to give an analytic tool for studying complex attacks and new semantics considering both attack and support relations, among others. However, the drawback is the lack of guidelines for choosing the appropriate definitions and semantics depending on the application. Several variants of the support relation have been proposed according to different interpretations of the support. In the following we consider the necessary support, which can also be used for handling recursive attacks, as shown later. This kind of support was initially proposed in [20] with the following interpretation: If $c\mathbf{R}_{\text{sup}}b$ then the acceptance of c is necessary to get the acceptance of b, or equivalently the acceptance of b implies the acceptance of c. Suppose now that $a\mathbf{R}_{\text{att}}c$. The acceptance of a implies the non-acceptance of c and so the non-acceptance of b. Also, if $c\mathbf{R}_{\text{sup}}a$ and $c\mathbf{R}_{\text{att}}b$, the acceptance of a implies the acceptance of c and the acceptance of c implies the non-acceptance of b. So, the acceptance of a implies the non-acceptance of b. These constraints relating a and b are enforced by adding new complex attacks from a to b:

Def. 8 ([20] Extended attack) *Let $\langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}}\rangle$ and $a, b \in \mathbf{A}$. There is an* extended attack *from a to b iff*

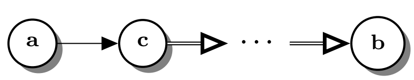
- *either (1) $a\mathbf{R}_{\text{att}}b$;*
- *or (2) $a_1\mathbf{R}_{\text{att}}a_2\mathbf{R}_{\text{sup}} \ldots \mathbf{R}_{\text{sup}}a_n$, $n \geq 3$, with $a_1 = a$, $a_n = b$;*
- *or (3) $a_1\mathbf{R}_{\text{sup}} \ldots \mathbf{R}_{\text{sup}}a_n$, and $a_1\mathbf{R}_{\text{att}}a_p$, $n \geq 2$, with $a_n = a$, $a_p = b$.*

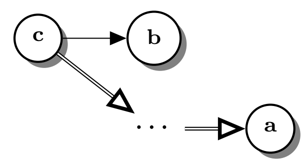
Consider the following graphical notations:

the simple edges $\longrightarrow$ represent attacks,

the double edges $\Longrightarrow$ represent supports.

The following figures illustrate the cases 2 and 3 of Definition 8:

Case 2: 

Case 3: 

Among the frameworks proposed in [12] for handling necessary supports, we focus on the one encoding the following interpretation: If $c\mathbf{R}_{\text{sup}}b$, "the acceptance of c is necessary to get the acceptance of b" because "c is the *only* attacker of a particular attacker of b":

Def. 9 ([12] MAF associated with a BAF) *Let $\langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}}\rangle$ be a BAF with $\mathbf{R}_{\text{sup}}$ being a set of necessary supports. Let $\mathbf{A}_n = \{N_{cb} | (c, b) \in \mathbf{R}_{\text{sup}}\}$ and $\mathbf{R}_n = \{(c, N_{cb}) | (c, b) \in \mathbf{R}_{\text{sup}}\} \cup \{(N_{cb}, b) | (c, b) \in \mathbf{R}_{\text{sup}}\}$. The tuple $\langle \mathbf{A} \cup \mathbf{A}_n, \mathbf{R}_{\text{att}} \cup \mathbf{R}_n\rangle$ is the meta-argumentation framework (MAF[3]) associated with the BAF.*

Note that a MAF is a D-framework, so any Dung's semantics can be applied.

[3] In [12], it is called a MAS (meta-argumentation system).

2.4 Argumentation semantics for recursive frameworks

Let us recall that we restrict the presentation to recursive frameworks without any support relation.

As in the bipolar case, there is no consensus about semantics for recursive frameworks. Concerning an ASAF, at least four distinct methods exist. The first third ones ([4, 13, 6]) consist in a translation of the original ASAF into an AF (in which all Dung's semantics can be reused) whereas the last one ([14]) gives direct definitions for ASAF semantics without using a translation into an AS.

More recently ([7, 8]), acceptability semantics have been defined for a RAF, where an extension is composed of a set of arguments and a set of attacks.

Note that if supports are removed, an ASAF exactly corresponds to an AFRA. So in this section, the expressions "ASAF without support" or "AFRA" may be used indifferently.

2.4.1 Method of [4]

The proposed translation uses the notion of *defeat* defined as follows:[4]

Def. 10 ([4] Defeat in ASAF without support) *Let* $\langle \mathbf{A}, \mathbf{R} \rangle$ *be an ASAF without support. Let* $\alpha, \beta \in \mathbf{R}$. *Let* $X \in \mathbf{A} \cup \mathbf{R}$.

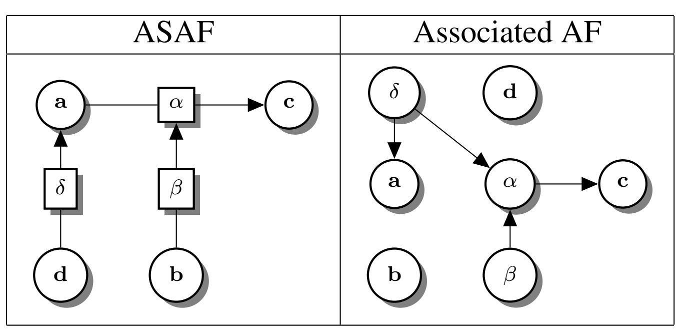
- α directly defeats X *iff* X *is the target of* α.
- α indirectly defeats β *iff the target of* α *is an argument that is the source of* β.

Then, a translation of an ASAF into an AF is provided:

Def. 11 ([4] AF associated with an ASAF) *Let* $\langle \mathbf{A}, \mathbf{R} \rangle$ *be an ASAF without support. The AF associated with this ASAF is* $\langle \mathbf{A}', \mathbf{R}' \rangle$ *defined by:*

- $\mathbf{A}' = \mathbf{A} \cup \mathbf{R}$,
- $\mathbf{R}' = \{(X, Y)$ *s.t.* $X \in \mathbf{R}, Y \in \mathbf{A} \cup \mathbf{R}$ *and* X *directly or indirectly defeats* $Y\}$.

The previous notions are illustrated on the following example:



[4] In [4], the argumentation framework with recursive attacks is called AFRA.

For instance, α directly defeats c, β directly defeats α and δ indirectly defeats α.

The following points seem counterintuitive:

- there is no attack between a and c (more generally, no argument from $\mathbf{A}$ can be an attacker in the associated AF of the ASAF),
- there is no link between a and α (more generally, there is no link between an attack and its source); that is surprizing since, without a, the attack α does not exist.

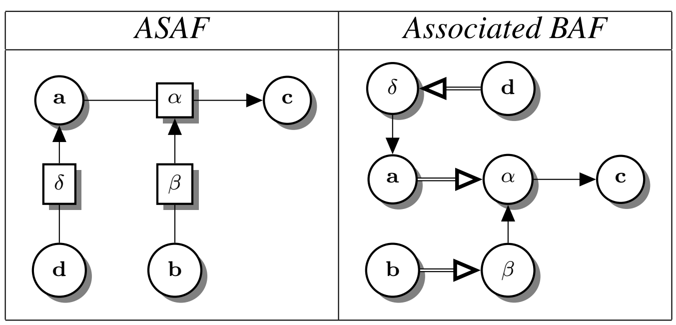
2.4.2 Method of [13]

The translation from an ASAF into an AF follows two steps:[5]

1. First, the ASAF is turned into a BAF with necessary support.
2. Then, this BAF is turned into an AF by adding extended attacks.

Here, we restrict the definitions of [13] to ASAF without support.

Def. 12 ([13] BAF associated with ASAF) *The following schemas describe the encoding of attacks (attacked or not):*

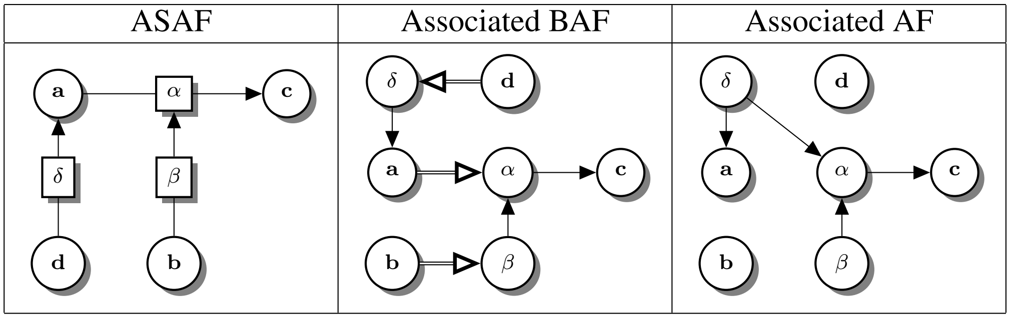


Given the BAF associated with the ASAF, the second step followed in [13] is to create an AF by adding complex attacks, namely Case 2 - extended attacks (see Def. 8):

Def. 13 ([13] AF associated with BAF and ASAF) *Let* $\langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$ *be the BAF associated with a given ASAF. The pair* $\langle \mathbf{A}', \mathbf{R}' \rangle$*, where* $\mathbf{A}' = \mathbf{A}$ *and* $\mathbf{R}' = \mathbf{R}_{\text{att}} \cup \{(a,b)|$ *there is a sequence* $a_1\ \mathbf{R}_{\text{att}}\ a_2\ \mathbf{R}_{\text{sup}} \ldots \mathbf{R}_{\text{sup}}\ a_n, n \geq 3,$ *with* $a_1 = a, a_n = b\}$ *is the AF associated with the BAF and the ASAF.*

For instance (in this case the attack from δ to α is added following Definition 13):

[5] In [13], the argumentation framework contains recursive attacks and supports, so the definitions are more complex in order to take into account supports.



Note that it has been proved in [13] that in case of ASAF without support, the approaches developed in [4] and [13] were equivalent.

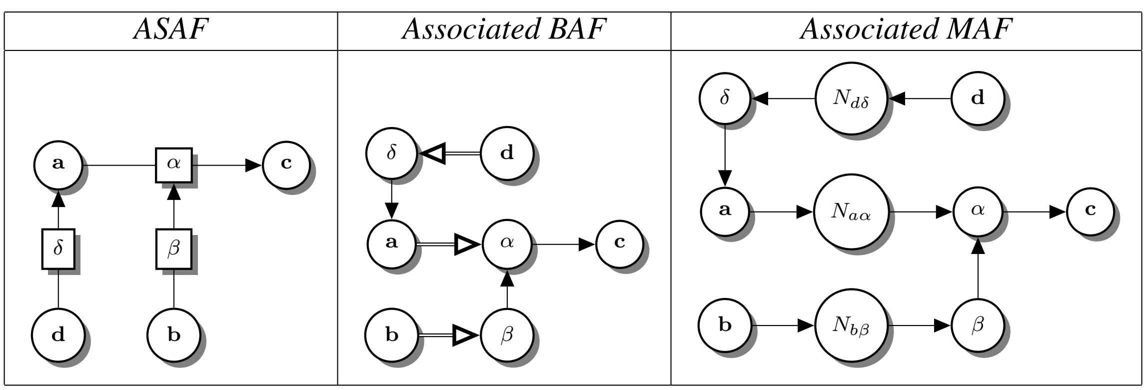
2.4.3 Method of [6]

The proposed translation consists in encoding recursive interactions into a MAF in two steps:[6]

1. First, the ASAF is turned into a BAF with necessary support (exactly in the same way as done in [13]).
2. Then, this BAF is turned into an MAF following Definition 9 on page 9.

Here, we restrict the definitions of [6] to ASAF without support.

Def. 14 ([6] Encoding of labelled attacks) *The following schemas describe the encoding of a labelled attack (attacked or not):*



Note that 3 new notions are also defined in [6] for characterizing interactions:

- the validity of an interaction α encodes the fact that α can or cannot be affected by interactions on it;
- the groundness of an interaction α encodes the link between α and its source;

[6] In [6], the argumentation framework is called LASAF and contains recursive attacks and supports, so the definitions are more complex in order to take into account supports.

- the activation of an interaction α = its validity + its groundness; in this case, α can be taken into account for determining the status of its target.

Note that the N_{ij} code the ground-links, *i.e.* the links between the source of an interaction and the interaction.

2.4.4 Method of [14]

The first idea is similar to the one developed in [4]: introducing a notion of *defeat*. As done above, we only give the definitions corresponding to ASAF without support, *i.e.* the definitions relied to the notion of *unconditional defeat*.[7]

There are two cases of unconditional defeats: the first case corresponds to conflicts already captured by the attack relation of the ASAF (called direct defeats), the second case (called indirect defeat) captures the intuition that attacks are strictly related to their source, as in [4].

Def. 15 ([14] Defeats) *Let* $\langle \mathbf{A}, \mathbf{R} \rangle$ *be an ASAF without support,* $\alpha, \beta \in \mathbf{R}$ *and* $X \in \mathbf{A} \cup \mathbf{R}$*.*

Direct defeat α *directly defeats* X*, noted* α `d-def` X*, iff the target of* α *is* X*.*

Indirect defeat α *indirectly defeats* β*, noted* α `i-def` β*, iff* α `d-def` *the source of* β*.*

Unconditional defeat α *unconditionally defeats* X*, noted* α `u-def` X*, iff* α `d-def` X *or* α `i-def` X*.*

Then a redefinition of the main concepts used in argumentation semantics is given:

Once again we restrict the definitions of [14] to ASAF without support.

Def. 16 ([14] Main semantics concepts for semantics) *Let* $\langle \mathbf{A}, \mathbf{R} \rangle$ *be an ASAF without support,* $S \in \mathbf{A} \cup \mathbf{R}$*.*

Conflict-Freeness S *is conflict-free iff:* $\nexists \alpha, X \in S$ *s.t.* α `u-def` X*.*

Acceptability *Let* $X \in \mathbf{A} \cup \mathbf{R}$*,* X *is acceptable wrt* S *iff:* $\forall \alpha \in \mathbf{R}$ *s.t.* α `u-def` X*,* $\exists \beta \in S$ *s.t.* β `u-def` α*.*

Then, from these new notions of conflict-freeness and acceptability as in Dung's definitions, it is possible to redefine all standard semantics.

Even if this approach is methodologically distinct from the one of [4], in the case of an ASAF without support, the same results are obtained.

[7] In [14], the argumentation framework contains recursive attacks and supports so the definitions are more complex in order to take into account supports.

2.4.5 Method of [7, 8]

In [7, 8], alternative acceptability semantics are defined, where the notion of extension (set of arguments) is replaced by a pair of a set of arguments and a set of attacks, called a "structure". The intuition is the fact that two arguments may be conflicting depends on the validity of the attack between them. So it would not be sound to give a definition of a set of arguments being conflict-free, independently of a set of attacks. More generally, the classic role of attacks in defeating arguments will be played by a subset of attacks, which is extension dependent, and represents the valid attacks with respect to the extension.

For the rest of this section, we consider a given recursive argumentation framework without support RAF = $\langle \mathbf{A}, \mathbf{R}, s, t\rangle$.

Def. 17 ([7]) *A structure on RAF is a pair $U = (S, \Gamma)$ such that $S \subseteq \mathbf{A}$ and $\Gamma \subseteq \mathbf{R}$.*

Intuitively, we are interested in structures $U = (S, \Gamma)$ such that S contains arguments that are accepted "owing to" U and Γ contains attacks which are valid "owing to" U. The precise meaning of "owing to" will depend on the considered semantics.

In the following, we recall the main notions used in semantics about structures that are defined in [7].

For that purpose, we define the set of arguments (resp. attacks) that are "defeated" (resp. "inhibited") wrt a given structure.

Def. 18 ([7]) *Given $U = (S, \Gamma)$ a structure on RAF = $\langle \mathbf{A}, \mathbf{R}, s, t\rangle$. Let $a \in \mathbf{A}$ and $\alpha \in \mathbf{R}$.*

- *a is defeated wrt (S, Γ) iff $\exists \beta \in \Gamma$ such that $s(\beta) \in S$ and $t(\beta) = a$,*
- *α is inhibited wrt (S, Γ) iff $\exists \beta \in \Gamma$ such that $s(\beta) \in S$ and $t(\beta) = \alpha$.*

$Def(U)$ (resp. $Inh(U)$) will denote the set of arguments (resp. attacks) that are defeated (resp. inhibited) wrt the structure U.

Conflict-free structures The minimal requirement for a structure (S, Γ) is that two arguments of S cannot be related by an attack of the structure, and similarly there cannot be an attack grounded in S and whose target is an element of Γ. In other words, a structure is conflict-free if no argument (resp. attack) of the structure is defeated (resp. inhibited) wrt the structure:

Def. 19 ([7]) *A structure $U = (S, \Gamma)$ on RAF is conflict-free iff $S \cap Def(U) = \varnothing$ and $\Gamma \cap Inh(U) = \varnothing$.*

Note that for any $\Gamma \subseteq \mathbf{R}$, the structure $(\varnothing, \Gamma)$ is conflict-free. The same holds for AF, where an empty set of arguments is always conflict-free. Moreover for any $S \subseteq \mathbf{A}$ the structure $(S, \varnothing)$ is conflict-free.

Admissible structures As done for conflict-freeness, the definition of an argument a being acceptable wrt a set of arguments S should be relative to a set of attacks:

Def. 20 ([7]) *Given a structure $U = (S, \Gamma)$ on RAF. Let $a \in \mathbf{A}$ and $\alpha \in \mathbf{R}$.*

- *a is acceptable wrt U iff $\forall \beta \in \mathbf{R}$ such that $t(\beta) = a$, either $\beta \in Inh(U)$ or $s(\beta) \in Def(U)$.*
- *α is acceptable wrt U iff $\forall \beta \in \mathbf{R}$ such that $t(\beta) = \alpha$, either $\beta \in Inh(U)$ or $s(\beta) \in Def(U)$.*

$Acc(U)$ will denote the set containing all acceptable arguments and attacks wrt U.

In other words, $x \in \mathbf{A} \cup \mathbf{R}$ is acceptable wrt a structure $U = (S, \Gamma)$ iff for each attack $\beta \in \mathbf{R}$ such that $t(\beta) = x$, there exists $\gamma \in \Gamma$ with $s(\gamma) \in S$ and $t(\gamma) \in \{\beta, s(\beta)\}$.

Then admissible structures can be defined as follows:

Def. 21 ([7]) *A structure $U = (S, \Gamma)$ on RAF is admissible iff it is conflict-free and $\forall x \in (S \cup \Gamma)$, x is acceptable wrt U.*

Remark Let $\alpha \in \mathbf{R}$ with $t(\alpha) = b \in \mathbf{A}$. If α and $s(\alpha)$ are unattacked, there is no admissible structure $U = (S, \Gamma)$ such that $b \in S$.

Similarly, let $\gamma \in \mathbf{R}$ with $t(\gamma) = \beta \in \mathbf{R}$. If γ and $s(\gamma)$ are unattacked, there is no admissible structure $U = (S, \Gamma)$ such that $\beta \in \Gamma$.

Complete structures Following the definitions of standard semantics in Dung's frameworks, complete structures are defined as admissible structures which contain all the arguments (resp. attacks) that are acceptable wrt the structure:

Def. 22 ([7]) *A structure $U = (S, \Gamma)$ on RAF is complete iff it is conflict-free and $Acc(U) = S \cup \Gamma$.*

Note that each complete structure must contain all the unattacked arguments and all the unattacked attacks.

Stable structures In Dung's framework, stable extensions are defined as conflict-free extensions that attack external arguments. It can be proved that stable extensions are also admissible and even complete extensions. In [7], a definition of stable structures is provided that preserves the same relation between semantics:

Def. 23 ([7]) *A structure $U = (S, \Gamma)$ on RAF is stable iff it is conflict-free and satisfies :*

1. *$\forall a \in \mathbf{A} \setminus S$, $a \in Def(U)$ and*
2. *$\forall \alpha \in \mathbf{R} \setminus \Gamma$, $\alpha \in Inh(U)$.*

Preferred structures The definition of preferred structures requires the definition of the following order relation: Let $U = (S, \Gamma)$ and $U' = (S', \Gamma')$ be structures on RAF. We write $U \subseteq U'$ iff $(S \cup \Gamma) \subseteq (S' \cup \Gamma')$. We say that the structure U is $\subseteq$-maximal iff every structure U' that satisfies $U \subseteq U'$ also satisfies $U' \subseteq U$. Similarly, we say that U is $\subseteq$-minimal iff every structure U' that satisfies $U' \subseteq U$ also satisfies $U \subseteq U'$.

Def. 24 *A structure $U = (S, \Gamma)$ on RAF is preferred iff it is a $\subseteq$-maximal admissible structure.*

It has been proved in [7] that every complete structure is admissible, every preferred structure is also complete and every stable structure is also preferred.

Grounded structure In Dung's framework, the grounded structure is defined as the $\subseteq$-minimal complete extension. It is unique.

In a similar way, we could study the $\subseteq$-minimal complete structures. The following results hold:

Prop. 1 *Let $U = (S, \Gamma)$ be a structure such that $Acc(U) \subseteq S \cup \Gamma$. The following assertions are equivalent:*

1. *U is a $\subseteq$-minimal conflict-free structure satisfying $Acc(U) \subseteq S \cup \Gamma$*
2. *U is a $\subseteq$-minimal complete structure*

Prop. 2 *There is only one $\subseteq$-minimal structure which is conflict-free and satisfies $Acc(U) \subseteq S \cup \Gamma$.*

So we may define:

Def. 25 *The grounded structure on RAF is the $\subseteq$-minimal conflict-free structure $U = (S, \Gamma)$ satisfying $Acc(U) \subseteq S \cup \Gamma$.*

D-structures The notion of structure can be strengthened so that we obtain a conservative generalization of Dung's frameworks for the conflict-free, admissible, complete, stable and preferred definitions. It is worth to note that in Dung's frameworks, every attack is considered as valid, in the sense that it may affect its target. The following definition strengthens the notion of structure by adding a condition on attacks that will force every acceptable attack to be valid.

Def. 26

1. *A d-structure on RAF is a structure $U = (S, \Gamma)$ such that $(Acc(U) \cap \mathbf{R}) \subseteq \Gamma$.*
2. *A conflict-free (resp. admissible, complete, preferred, stable) d-structure is a conflict-free (resp. admissible, complete, preferred, stable) structure which is also a d-structure.*

It follows from Definition 22 on the previous page that every complete (resp. stable, preferred) structure is a d-structure. However it is not the case for admissible and conflict-free structures. Indeed, as a direct consequence of Definition 26, we have that each conflict-free d-structure must contain all the unattacked attacks.

In order to establish the conservative generalization of Dung's semantics, we need to:

- define what it means for a set of arguments to be an extension of a RAF,
- establish a correspondence between a RAF in which no attack targets another attack, and a D-framework.

First, a set of arguments $S \subseteq \mathbf{A}$ is said to be a conflict-free (resp. admissible, complete, stable, preferred) extension of RAF iff there is some $\Gamma \subseteq \mathbf{R}$ such that (S, Γ) is a d-structure of RAF.

Then, let us consider a particular RAF in which no attack targets another attack, called a non-recursive RAF. In other words, a non-recursive RAF is such that t is a mapping from $\mathbf{R}$ to $\mathbf{A}$. It is easy to build a D-framework associated with some non-recursive RAF.

Let RAF = $\langle \mathbf{A}, \mathbf{R}, s, t \rangle$ be non recursive. The D-framework associated with RAF is denoted by RAF^D and defined as $\langle \mathbf{A}, \{(s(\alpha), t(\alpha))/\alpha \in \mathbf{R}\} \rangle$.

In the case of a non-recursive RAF, as no attack is attacked, every d-structure $U = (S, \Gamma)$ satisfies $\Gamma = \mathbf{R}$.

The conservative generalization of Dung's semantics has been proved in [7, 8] according to the following result: A set of arguments S is a conflict-free (resp. admissible, complete, stable, preferred) extension of some non-recursive RAF iff S is conflict-free (resp. admissible, complete, stable, preferred) wrt the D-framework RAF^D.

Chapter 3

Logical description of a recursive framework

[10] proposes a logical description of an argumentation framework, that allows an explicit representation of arguments, attacks (being themselves possibly attacked) and their properties (accepted argument, attacked argument, valid attack, ...). Let G denote a recursive argumentation framework, $\Sigma(G)$ will denote the set of first-order logic formulae describing G.

3.1 Vocabulary

The following unary predicate symbols are used: Acc, $NAcc$, Val, $Attack$, $Argument$[1] and the following unary functions symbols : T, S, with the following meaning:

- $Acc(x)$ (resp. $NAcc(x)$) means "x is accepted" (resp. "x cannot be accepted"), when x denotes an argument
- $Val(\alpha)$ means "α is valid when α denotes an attack
- $Attack(x)$ means "x is an attack"
- $Argument(x)$ means "x is an argument"
- $T(x)$ (resp. $S(x)$) denotes the target (resp. source) of x, when x denotes an attack

The binary equality predicate is also used. Note that the quantifiers $\exists$ and $\forall$ range over some domain D. To restrict them to subsets of D, bounded quantifiers will be also used:

$(\forall x \in E)(P(x))$ means $(\forall x)(x \in E \to P(x))$ or equivalently $(\forall x)(E(x) \to P(x))$.

So we will use:

- $(\forall x \in Attack)(\Phi(x))$ (resp. $(\exists x \in Attack)(\Phi(x))$)
- and $(\forall x \in Argument)(\Phi(x))$ (resp. $(\exists x \in Argument)(\Phi(x))$).

[1]Here, we only give a simplified version that does not explicitly represent the notions of *grounded* and *active* attacks, since in [6, 10] an attack is grounded iff its source is accepted and an attack is active iff it is grounded and valid.

3.2 Properties

The properties introduced in [10] can be partitioned into two sets:

- the properties describing the general behaviour of an attack, possibly recursive, in an argumentation framework, *i.e.* how an attack interacts with arguments and other attacks related to it.
- and the properties encoding the specificities of the current argumentation framework.

Using the vocabulary defined in Section 3.1 on the previous page, the general properties can be expressed by the following set of first-order formulae:

(1) $\forall x \in \mathit{Attack}(\forall y \in \mathit{Attack}((Val(y) \wedge (T(y) = x) \wedge \mathit{Acc}(S(y)) \rightarrow \neg Val(x)))$

(2) $\forall x \in Argument(\forall y \in \mathit{Attack}((Val(y) \wedge (T(y) = x) \wedge \mathit{Acc}(S(y)) \rightarrow N\mathit{Acc}(x)))$

(3 $\forall x \in Argument(N\mathit{Acc}(x) \rightarrow \neg \mathit{Acc}(x))$

(4) $\forall x(\mathit{Attack}(x) \rightarrow \neg Argument(x))$

(5) $\forall x(Argument(x) \vee \mathit{Attack}(x))$

Concerning the logical encoding of specificities of the RAF, we have the following set of formulas:

(6) $(S(\alpha) = a) \wedge (T(\alpha) = b)$ for all $\alpha \in \mathbf{R}$ with $s(\alpha) = a$ and $t(\alpha) = b$

(7) $\forall x(Argument(x) \leftrightarrow (x = a_1) \vee \ldots \vee (x = a_n))$

(8 $\forall x(\mathit{Attack}(x) \leftrightarrow (x = \alpha_1) \vee \ldots \vee (x = \alpha_m))$

(9) $a_i \neq a_j$ for all $a_i, a_j \in \mathbf{A}$ with $i \neq j$

(10) $\alpha_i \neq \alpha_j$ for all $\alpha_i, \alpha_j \in \mathbf{R}$ with $i \neq j$

The logical theory $\Sigma(G)$ corresponding to G consists of the above 10 formulae.

Note that we assume that the argumentation framework is finite, with $\mathbf{A} = \{a_1, \ldots a_n\}$ and $\mathbf{R} = \{\alpha_1, \ldots, \alpha_m\}$. Moreover, in the following, we will write s_α (resp. t_α) in place of $S(\alpha)$ (resp. $T(\alpha)$) for simplicity.

Chapter 4

Logical formalization of semantics in a RAF

[10] considers the basic principles used for defining semantics (conflict-freeness, defence, reinstatement, stability) and gives a logical expression for each of these principles, thus leading to add formulas to the base $\Sigma(G)$ and producing new bases.

4.1 Logical encoding of semantics

Conflict-freeness In presence of recursive attacks, the conflict-freeness principle is reformulated in [10] as follows (one for arguments and one for attacks):

- If there is a valid attack between two arguments, they cannot be jointly accepted.
- If there is an attack from an accepted argument to an attack, these attacks cannot be both valid.

Due to the formulae encoding an attack α in the base $\Sigma(G)$, these properties are already expressed in $\Sigma(G)$.

Defence [10] reformulates the defence principle as follows (one for arguments and one for attacks):

- An attacked argument may be accepted only if for each attack against it, either the source or the attack itself is in turn attacked by a valid attack from an accepted argument.
- An attack may be valid only if for each attack against it, either the source or the attack itself is in turn attacked by a valid attack from an accepted argument.

These properties are expressed by the following formulae:

(11) $\forall \alpha \in Attack$ (
$Acc(t_\alpha)$
$\rightarrow (\exists \beta \in Attack$
$(t_\beta \in \{s_\alpha, \alpha\}^1 \wedge Val(\beta) \wedge Acc(s_\beta)))$)

[1]Strictly speaking, should be written as follows : $t_\beta = s_\alpha \vee t_\beta = \alpha$.

(12) $\forall \alpha \in \mathit{Attack}\ (\forall \delta \in \mathit{Attack}\ ($
$$((\delta = t_\alpha) \wedge Val(\delta))$$
$$\rightarrow (\exists \beta \in \mathit{Attack}$$
$$(t_\beta \in \{s_\alpha, \alpha\} \wedge Val(\beta) \wedge Acc(s_\beta)))\ ))$$

These formulae are added to the base $\Sigma(G)$, thus producing the base $\Sigma_d(G)$.

Reinstatement The reinstatement principle is reformulated in [10] as follows (one for arguments and one for attacks):

- An argument must be accepted provided that, for each attack against it, the source or the attack itself is in turn attacked by a valid attack from an accepted argument.
- An attack may be valid provided that for each attack against it, either the source or the attack itself is in turn attacked by a valid attack from an accepted argument.

These properties are expressed by the following formulae:

(13) $\forall c \in \mathit{Argument}\ ($
$$(\forall \alpha \in \mathit{Attack}\ (t_\alpha = c$$
$$\rightarrow (\exists \beta \in \mathit{Attack}$$
$$(t_\beta \in \{s_\alpha, \alpha\} \wedge Val(\beta) \wedge Acc(s_\beta)))))$$
$$\rightarrow Acc(c)\ )$$

(14) $\forall \delta \in \mathit{Attack}\ ($
$$(\forall \alpha \in \mathit{Attack}\ (t_\alpha = \delta$$
$$\rightarrow (\exists \beta \in \mathit{Attack}$$
$$(t_\beta \in \{s_\alpha, \alpha\} \wedge Val(\beta) \wedge Acc(s_\beta)))))$$
$$\rightarrow Val(\delta)\ )$$

These formulae are added to the base $\Sigma(G)$, thus producing the base $\Sigma_r(G)$.

Stability The stability property is reformulated in [10] as follows (one for arguments and one for attacks):

- If an argument is not accepted, it must be attacked by a valid attack from an accepted argument.
- If an attack is not valid, it must be attacked by a valid attack from an accepted argument.

These properties are expressed by the following formulae:

(15) $\forall c \in \mathit{Argument}\ ($
$$\neg Acc(c)$$
$$\rightarrow (\exists \beta \in \mathit{Attack}$$
$$((t_\beta = c) \wedge Val(\beta) \wedge Acc(s_\beta)))\ )$$

(16) $\forall \alpha \in Attack$ (
$\neg Val(\alpha)$
$\to (\exists \beta \in Attack$
$((t_\beta = \alpha) \wedge Val(\beta) \wedge Acc(s_\beta))$)

These formulae are added to the base $\Sigma(G)$, thus producing the base $\Sigma_s(G)$.

4.2 Characterizing semantics for a RAF

Our purpose is to propose characterizations of the structures in different semantics in terms of models of the bases $\Sigma(G)$, $\Sigma_d(G)$, $\Sigma_r(G)$, $\Sigma_s(G)$ following the same method that is used in the classical case (see [10]).

Let $\mathcal{I}$ be an interpretation of a set of formulae Σ of the language introduced in Section 3.1 on page 19. Let us define:

- $S_\mathcal{I} = \{x \in \mathbf{A} | \mathcal{I}(Acc(x)) = \textit{true}\}$
- $\Gamma_\mathcal{I} = \{x \in \mathbf{R} | \mathcal{I}(Val(x)) = \textit{true}\}$

Moreover, let $\mathcal{I}$ be a model of Σ:

- I is a $\subseteq$-maximal model of Σ iff there is no model I' of Σ such that $(S_\mathcal{I} \cup \Gamma_\mathcal{I}) \subset (S_{\mathcal{I}'} \cup \Gamma_{\mathcal{I}'})$
- I is a $\subseteq$-minimal model of Σ iff there is no model I' of Σ such that $(S_{\mathcal{I}'} \cup \Gamma_{\mathcal{I}'}) \subset (S_\mathcal{I} \cup \Gamma_\mathcal{I})$

Let G denote the RAF $\langle \mathbf{A}, \mathbf{R}, s, t \rangle$. We have the following characterizations:

Prop. 3 *Let $U = (S, \Gamma)$ a structure on G.*

1. *U is conflict-free if and only if $\exists \mathcal{I}$ model of $\Sigma(G)$ such that $S_\mathcal{I} = S$ and $\Gamma_\mathcal{I} = \Gamma$.*
2. *U is admissible if and only if $\exists \mathcal{I}$ model of $\Sigma_d(G)$ such that $S = S_\mathcal{I}$ and $\Gamma_\mathcal{I} = \Gamma$.*
3. *U is complete if and only if $\exists \mathcal{I}$ model of $\Sigma_d(G) \cup \Sigma_r(G)$ such that $S = S_\mathcal{I}$ and $\Gamma_\mathcal{I} = \Gamma$.*
4. *U is a stable structure if and only if $\exists \mathcal{I}$ model of $\Sigma_s(G)$ such that $S_\mathcal{I} = S$ and $\Gamma_\mathcal{I} = \Gamma$.*
5. *U is a preferred structure if and only if $\exists \mathcal{I}$ $\subseteq$-maximal model of $\Sigma_d(G)$ such that $S_\mathcal{I} = S$ and $\Gamma_\mathcal{I} = \Gamma$.*
6. *U is the grounded structure if and only if $S = S_\mathcal{I}$ and $\Gamma_\mathcal{I} = \Gamma$ where $\mathcal{I}$ is a $\subseteq$-minimal model of $\Sigma_r(G)$.*[2]

[2] Note that it also holds that U is the grounded structure iff $S = S_\mathcal{I}$ and $\Gamma_\mathcal{I} = \Gamma$ where $\mathcal{I}$ is a $\subseteq$-minimal model of $\Sigma_d(G) \cup \Sigma_r(G)$. Considering $\Sigma_d(G) \cup \Sigma_r(G)$ instead of $\Sigma_r(G)$ might be useful from a computational point of view, when searching for minimal models.

Then we consider characterizations of d-structures. We recall that d-structures are particular structures in which acceptable attacks are forced to be valid. So, we consider the base $\Sigma(G)$ augmented with the formula that expresses the reinstatement principle for attacks, that is the formula **(14)** (denoted by $\Phi_r^{att}(G)$):

$$(\forall\delta \in Attack)(((\forall\alpha \in Attack)(t_\alpha = \delta \to (\exists\beta \in Attack)(t_\beta \in \{s_\alpha, \alpha\} \wedge Val(\beta) \wedge Acc(s_\beta)))) \to Val(\delta)).$$

Thus, we obtain the following characterizations:

Prop. 4 *Let $U = (S, \Gamma)$ a structure on G.*

1. *U is a conflict-free d-structure if and only if $\exists\mathcal{I}$ model of $\Sigma(G) \cup \{\Phi_r^{att}(G)\}$ such that $S_\mathcal{I} = S$ and $\Gamma_\mathcal{I} = \Gamma$.*
2. *U is an admissible d-structure if and only if $\exists\mathcal{I}$ model of $\Sigma_d(G) \cup \{\Phi_r^{att}(G)\}$ such that $S = S_\mathcal{I}$ and $\Gamma_\mathcal{I} = \Gamma$.*

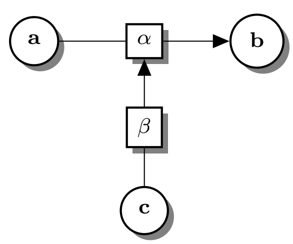
Chapter 5

Examples with recursivity (ASAF)

The following examples are extracted from [10] in which they are numbered from 14 to 25.

5.1 Example 14 issued from [10]

Ex. 14



$\Sigma(G)$	$= \{$	$(Val(\beta) \wedge Acc(c)) \to \neg Val(\alpha)$ $(Val(\alpha) \wedge Acc(a)) \to NAcc(b)$ $NAcc(b) \to \neg Acc(b)$	$\}$
$\Sigma_d(G)$	$= \Sigma(G) \cup \{$	$Acc(b) \to (Val(\beta) \wedge Acc(c))$ $\neg Val(\alpha)$	$\}$
$\Sigma_r(G)$	$= \Sigma(G) \cup \{$	$Acc(a)$ $Acc(c)$ $(Val(\beta) \wedge Acc(c)) \to Acc(b)$ $Val(\beta)$	$\}$
$\Sigma_s(G)$	$= \Sigma(G) \cup \{$	$Acc(a)$ $\neg Acc(b) \to (Val(\alpha \wedge Acc(a))$ $Acc(c)$ $\neg Val(\alpha) \to (Val(\beta) \wedge Acc(c))$ $Val(\beta)$	$\}$

Then using Proposition 3 on page 23, the computation of the models gives the following results:

Type of structure	*Type of model*	*Computed structures*
Conflict-free	*model of* $\Sigma(G)$	$(\varnothing,\varnothing)$ $(\{b\},\varnothing)$ $(\{c\},\varnothing)$ $(\{b,c\},\varnothing)$ $(\varnothing,\{\alpha\})$ $(\{b\},\{\alpha\})$ $(\{c\},\{\alpha\})$ $(\{b,c\},\{\alpha\})$ $(\varnothing,\{\alpha,\beta\})$ $(\varnothing,\{\beta\})$ $(\{b\},\{\beta\})$ $(\{c\},\{\beta\})$ $(\{b,c\},\{\beta\})$ $(\{b\},\{\alpha,\beta\})$ $(\{a\},\{\alpha,\beta\})$ $(\{a\},\varnothing)$ $(\{a,b\},\varnothing)$ $(\{a,c\},\varnothing)$ $(\{a,b,c\},\varnothing)$ $(\{a\},\{\alpha\})$ $(\{a,c\},\{\alpha\})$ $(\{a\},\{\beta\})$ $(\{a,b\},\{\beta\})$ $(\{a,c\},\{\beta\})$ $(\{a,b,c\},\{\beta\})$
Naive	$\subseteq$*-maximal model of* $\Sigma(G)$	$(\{b,c\},\{\alpha\})$ $(\{b\},\{\alpha,\beta\})$ $(\{a\},\{\alpha,\beta\})$ $(\{a,c\},\{\alpha\})$ $(\{a,b,c\},\{\beta\})$
Admissible	*model of* $\Sigma_d(G)$	$(\varnothing,\varnothing)$ $(\{c\},\varnothing)$ $(\varnothing,\{\beta\})$ $(\{c\},\{\beta\})$ $(\{b,c\},\{\beta\})$ $(\{a\},\{\beta\})$ $(\{a\},\varnothing)$ $(\{a,c\},\varnothing)$ $(\{a,c\},\{\beta\})$ $(\{a,b,c\},\{\beta\})$
Preferred	$\subseteq$*-maximal model of* $\Sigma_d(G)$	$(\{a,b,c\},\{\beta\})$
Grounded	$\subseteq$*-minimal model of* $\Sigma_r(G)$	$(\{a,b,c\},\{\beta\})$
Complete	*model of* $\Sigma_d(G)\cup\Sigma_r(G)$	$(\{a,b,c\},\{\beta\})$
Stable	*model of* $\Sigma_s(G)$	$(\{a,b,c\},\{\beta\})$

Then, from Definition 26 on page 16 and Proposition 4 on page 24, we obtain:

$\Phi_r^{att}(G) = Val(\beta)$

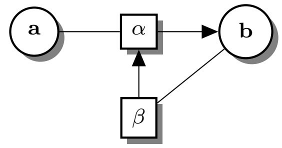
Semantics	*D-structures*
Admissible	$(\varnothing,\{\beta\})$, $(\{a\},\{\beta\})$, $(\{c\},\{\beta\})$,$(\{a,c\},\{\beta\})$, $(\{b,c\},\{\beta\})$, $(\{a,b,c\},\{\beta\})$
Stable	$(\{a,b,c\},\{\beta\})$
Complete	$(\{a,b,c\},\{\beta\})$

Moreover, the following table gives the extensions obtained with [6] or [4, 13] (concerning only arguments and interactions given in the initial framework):

Semantics	*Extensions*
Grounded	$\{a,b,c,\beta\}$
Preferred	$\{a,b,c,\beta\}$
Stable	$\{a,b,c,\beta\}$
Complete	$\{a,b,c,\beta\}$

5.2 Example 15 issued from [10]

Ex. 15



$\Sigma(G)$	$= \{$	$(Val(\beta) \wedge Acc(b)) \to \neg Val(\alpha)$ $(Val(\alpha) \wedge Acc(a)) \to NAcc(b)$ $NAcc(b) \to \neg Acc(b)$	$\}$
$\Sigma_d(G)$	$= \Sigma(G) \cup \{$	$Acc(b) \to Val(\beta)$ $Val(\alpha) \to Acc(a)$	$\}$
$\Sigma_r(G)$	$= \Sigma(G) \cup \{$	$Acc(a)$ $Val(\beta)$	$\}$
$\Sigma_s(G)$	$= \Sigma(G) \cup \{$	$Acc(a)$ $\neg Acc(b) \to (Val(\alpha) \wedge Acc(a))$ $Val(\beta)$ $\neg Val(\alpha) \to (Val(\beta) \wedge Acc(b))$	$\}$

Then using Proposition 3 on page 23, the computation of the models gives the following results:

Type of structure	*Type of model*	*Computed structures*
Conflict-free	*model of* $\Sigma(G)$	$(\varnothing, \varnothing)$ $(\{b\}, \varnothing)$ $(\varnothing, \{\alpha\})$ $(\{b\}, \{\alpha\})$ $(\varnothing, \{\alpha, \beta\})$ $(\varnothing, \{\beta\})$ $(\{b\}, \{\beta\})$ $(\{a\}, \{\alpha, \beta\})$ $(\{a\}, \varnothing)$ $(\{a, b\}, \varnothing)$ $(\{a\}, \{\alpha\})$ $(\{a\}, \{\beta\})$ $(\{a, b\}, \{\beta\})$
Naive	$\subseteq$*-maximal model of* $\Sigma(G)$	$(\{b\}, \{\alpha\})$ $(\{a\}, \{\alpha, \beta\})$ $(\{a, b\}, \{\beta\})$
Admissible	*model of* $\Sigma_d(G)$	$(\varnothing, \varnothing)$ $(\varnothing, \{\beta\})$ $(\{b\}, \{\beta\})$ $(\{a\}, \{\beta\})$ $(\{a\}, \varnothing)$ $(\{a\}, \{\alpha\})$ $(\{a\}, \{\alpha, \beta\})$ $(\{a, b\}, \{\beta\})$
Preferred	$\subseteq$*-maximal model of* $\Sigma_d(G)$	$(\{a, b\}, \{\beta\})$ $(\{a\}, \{\alpha, \beta\})$
Grounded	$\subseteq$*-minimal model of* $\Sigma_r(G)$	$(\{a\}, \{\beta\})$
Complete	*model of* $\Sigma_d(G) \cup \Sigma_r(G)$	$(\{a\}, \{\beta\})$ $(\{a, b\}, \{\beta\})$ $(\{a\}, \{\alpha, \beta\})$
Stable	*model of* $\Sigma_s(G)$	$(\{a, b\}, \{\beta\})$ $(\{a\}, \{\alpha, \beta\})$

Then, from Definition 26 on page 16 and Proposition 4 on page 24, we obtain:

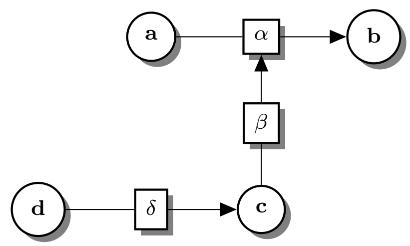
Semantics	*D-structures*
Admissible	$(\varnothing, \{\beta\})$, $(\{a\}, \{\beta\})$, $(\{b\}, \{\beta\})$, $(\{a\}, \{\alpha, \beta\})$, $(\{a, b\}, \{\beta\})$
Stable	$(\{a\}, \{\alpha, \beta\})$, $(\{a, b\}, \{\beta\})$
Complete	$(\{a\}, \{\beta\})$, $(\{a\}, \{\alpha, \beta\})$, $(\{a, b\}, \{\beta\})$

Moreover, the following table gives the extensions obtained with [6] or [4, 13] (concerning only arguments and interactions given in the initial graph):

Semantics	*Extensions*
Grounded	$\{a\}$
Preferred	$\{a, b, \beta\}$ $\{a, \alpha\}$
Stable	$\{a, b, \beta\}$ $\{a, \alpha\}$
Complete	$\{a\}$ $\{a, b, \beta\}$ $\{a, \alpha\}$

5.3 Example 16 issued from [10]

Ex. 16



$\Sigma(G)$	$= \{$	$(Val(\beta) \wedge Acc(c)) \rightarrow \neg Val(\alpha)$ $(Val(\alpha) \wedge Acc(a)) \rightarrow NAcc(b)$ $NAcc(b) \rightarrow \neg Acc(b)$ $(Val(\delta) \wedge Acc(d)) \rightarrow NAcc(c)$ $NAcc(c) \rightarrow \neg Acc(c)$	$\}$
$\Sigma_d(G)$	$= \Sigma(G) \cup \{$	$Acc(b) \rightarrow (Val(\beta) \wedge Acc(c))$ $\neg Acc(c)$ $Val(\alpha) \rightarrow (Val(\delta) \wedge Acc(d))$	$\}$
$\Sigma_r(G)$	$= \Sigma(G) \cup \{$	$Acc(a)$ $Acc(d)$ $(Val(\beta) \wedge Acc(c)) \rightarrow Acc(b)$ $Val(\beta)$ $Val(\delta)$ $(Val(\delta) \wedge Acc(d)) \rightarrow Val(\alpha)$	$\}$
$\Sigma_s(G)$	$= \Sigma(G) \cup \{$	$Acc(a)$ $Acc(d)$ $\neg Acc(c) \rightarrow (Val(\delta) \wedge Acc(d))$ $\neg Acc(b) \rightarrow (Val(\alpha) \wedge Acc(a))$ $Val(\beta)$ $Val(\delta)$ $\neg Val(\alpha) \rightarrow (Val(\beta) \wedge Acc(c))$	$\}$

Then using Proposition 3 on page 23, the computation of the models gives the following results:

Type of structure	*Type of model*	*Computed structures*
Conflict-free	*model of* $\Sigma(G)$	*89 structures*
Naive	$\subseteq$*-maximal model of* $\Sigma(G)$	$(\{b,c,d\},\{\alpha\})$ $(\{b,c\},\{\alpha,\delta\})$ $(\{b,d\},\{\alpha,\beta,\delta\})$ $(\{a,d\},\{\alpha,\beta,\delta\})$ $(\{a,c,d\},\{\alpha\})$ $(\{a,b,c,d\},\{\beta\})$ $(\{a,c\},\{\alpha,\delta\})$ $(\{a,b,c\},\{\beta,\delta\})$ $(\{a,b,d\},\{\beta,\delta\})$
Admissible	*model of* $\Sigma_d(G)$	*20 structures (only 6 contain* β *and* δ*)*
Preferred	$\subseteq$*-maximal model of* $\Sigma_d(G)$	$(\{a,d\},\{\alpha,\beta,\delta\})$
Grounded	$\subseteq$*-minimal model of* $\Sigma_r(G)$	$(\{a,d\},\{\alpha,\beta,\delta\})$
Complete	*model of* $\Sigma_d(G) \cup \Sigma_r(G)$	$(\{a,d\},\{\alpha,\beta,\delta\})$
Stable	*model of* $\Sigma_s(G)$	$(\{a,d\},\{\alpha,\beta,\delta\})$

Then, from Definition 26 on page 16 and Proposition 4 on page 24, we obtain:

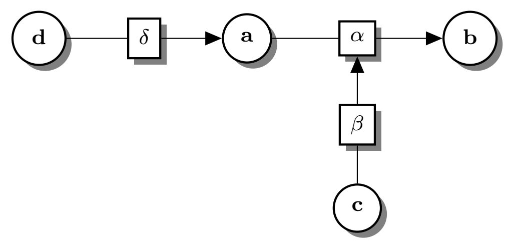
Semantics	*D-structures*
Admissible	$(\varnothing,\{\beta,\delta\})$, $(\{a\},\{\beta,\delta\})$, $(\{d\},\{\beta,\delta\})$, $(\{d\},\{\alpha,\beta,\delta\})$, $(\{a,d\},\{\beta,\delta\})$, $(\{a,d\},\{\alpha,\beta,\delta\})$
Stable	$(\{a,d\},\{\alpha,\beta,\delta\})$
Complete	$(\{a,d\},\{\alpha,\beta,\delta\})$

Moreover, the following table gives the extensions obtained with [6] or [4, 13] (concerning only arguments and interactions given in the initial graph):

Semantics	*Extensions*
Grounded	$\{a,d,\alpha,\delta\}$
Preferred	$\{a,d,\alpha,\delta\}$
Stable	$\{a,d,\alpha,\delta\}$
Complete	$\{a,d,\alpha,\delta\}$

5.4 Example 17 issued from [10]

Ex. 17



$\Sigma(G)$	$= \{$	$(Val(\beta) \wedge Acc(c)) \rightarrow \neg Val(\alpha)$ $(Val(\alpha) \wedge Acc(a)) \rightarrow NAcc(b)$ $NAcc(b) \rightarrow \neg Acc(b)$ $(Val(\delta) \wedge Acc(d)) \rightarrow NAcc(a)$ $NAcc(a) \rightarrow \neg Acc(a)$	$\}$
$\Sigma_d(G)$	$= \Sigma(G) \cup \{$	$\neg Acc(a)$ $Acc(b) \rightarrow ((Val(\beta) \wedge Acc(c)) \vee (Val(\delta) \wedge Acc(d)))$ $\neg Val(\alpha)$	$\}$
$\Sigma_r(G)$	$= \Sigma(G) \cup \{$	$Acc(d)$ $Acc(c)$ $((Val(\beta) \wedge Acc(c)) \vee (Val(\delta) \wedge Acc(d))) \rightarrow Acc(b)$ $Val(\beta)$ $Val(\delta)$	$\}$
$\Sigma_s(G)$	$= \Sigma(G) \cup \{$	$Acc(d)$ $Acc(c)$ $\neg Acc(a) \rightarrow (Val(\delta) \wedge Acc(d))$ $\neg Acc(b) \rightarrow (Val(\alpha) \wedge Acc(a))$ $Val(\beta)$ $Val(\delta)$ $\neg Val(\alpha) \rightarrow (Val(\beta) \wedge Acc(c))$	$\}$

Then using Proposition 3 on page 23, the computation of the models gives the following results:

Type of structure	*Type of model*	*Computed structures*
Conflict-free	*model of* $\Sigma(G)$	*89 structures*
Naive	$\subseteq$*-maximal model of* $\Sigma(G)$	$(\{b,c,d\},\{\alpha,\delta\})$ $(\{b,c,d\},\{\beta,\delta\})$ $(\{b,d\},\{\alpha,\beta,\delta\})$ $(\{a\},\{\alpha,\beta,\delta\})$ $(\{a,c,d\},\{\alpha\})$ $(\{a,d\},\{\alpha,\beta\})$ $(\{a,b,c,d\},\{\beta\})$ $(\{a,c\},\{\alpha,\delta\})$ $(\{a,b,c\},\{\beta,\delta\})$
Admissible	*model of* $\Sigma_d(G)$	*23 structures (only 7 contain* β *and* δ*)*
Preferred	$\subseteq$*-maximal model of* $\Sigma_d(G)$	$(\{b,c,d\},\{\beta,\delta\})$
Grounded	$\subseteq$*-minimal model of* $\Sigma_r(G)$	$(\{b,c,d\},\{\beta,\delta\})$
Complete	*model of* $\Sigma_d(G) \cup \Sigma_r(G)$	$(\{b,c,d\},\{\beta,\delta\})$
Stable	*model of* $\Sigma_s(G)$	$(\{b,c,d\},\{\beta,\delta\})$

Then, from Definition 26 on page 16 and Proposition 4 on page 24, we obtain:

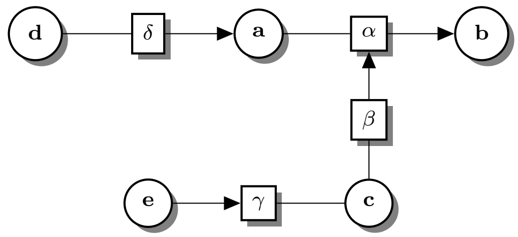
Semantics	*D-structures*
Admissible	$(\varnothing,\{\beta,\delta\})$, $(\{c\},\{\beta,\delta\})$, $(\{d\},\{\beta,\delta\})$, $(\{b,c\},\{\beta,\delta\})$, $(\{b,d\},\{\beta,\delta\})$, $(\{c,d\},\{\beta,\delta\})$, $(\{b,c,d\},\{\beta,\delta\})$
Stable	$(\{b,c,d\},\{\beta,\delta\})$
Complete	$(\{b,c,d\},\{\beta,\delta\})$

Moreover, the following table gives the extensions obtained with [6] or [4, 13] (concerning only arguments and interactions given in the initial graph):

Semantics	*Extensions*
Grounded	$\{c,d,\delta,\beta,b\}$
Preferred	$\{c,d,\delta,\beta,b\}$
Stable	$\{c,d,\delta,\beta,b\}$
Complete	$\{c,d,\delta,\beta,b\}$

5.5 Example 18 issued from [10]

Ex. 18



$\Sigma(G)$	$= \{$	$(Val(\beta) \wedge Acc(c)) \rightarrow \neg Val(\alpha)$ $(Val(\alpha) \wedge Acc(a)) \rightarrow NAcc(b)$ $NAcc(b) \rightarrow \neg Acc(b)$ $(Val(\delta) \wedge Acc(d)) \rightarrow NAcc(a)$ $NAcc(a) \rightarrow \neg Acc(a)$ $(Val(\gamma) \wedge Acc(e)) \rightarrow NAcc(c)$ $NAcc(c) \rightarrow \neg Acc(c)$	$\}$
$\Sigma_d(G)$	$= \Sigma(G) \cup \{$	$\neg Acc(a)$ $\neg Acc(c)$ $Acc(b) \rightarrow ((Val(\beta) \wedge Acc(c)) \vee (Val(\delta) \wedge Acc(d)))$ $Val(\alpha) \rightarrow (Val(\gamma) \wedge Acc(e))$	$\}$
$\Sigma_r(G)$	$= \Sigma(G) \cup \{$	$Acc(d)$ $Acc(e)$ $((Val(\beta) \wedge Acc(c)) \vee (Val(\delta) \wedge Acc(d))) \rightarrow Acc(b)$ $Val(\beta)$ $Val(\delta)$ $Val(\gamma)$ $(Val(\gamma) \wedge Acc(e)) \rightarrow Val(\alpha)$	$\}$
$\Sigma_s(G)$	$= \Sigma(G) \cup \{$	$Acc(d)$ $Acc(e)$ $\neg Acc(a) \rightarrow (Val(\delta) \wedge Acc(d))$ $\neg Acc(b) \rightarrow (Val(\alpha) \wedge Acc(a))$ $\neg Acc(c) \rightarrow (Val(\gamma) \wedge Acc(e))$ $Val(\beta)$ $Val(\delta)$ $Val(\gamma)$ $\neg Val(\alpha) \rightarrow (Val(\beta) \wedge Acc(c))$	$\}$

Then using Proposition 3 on page 23, the computation of the models gives the following results:

Type of structure	*Type of model*	*Computed structures*
Conflict-free	*model of* $\Sigma(G)$	*317 structures*
Naive	$\subseteq$*-maximal model of* $\Sigma(G)$	*17 structures*
Admissible	*model of* $\Sigma_d(G)$	*50 structures (only 9 contain* β*,* δ *and* γ*)*
Preferred	$\subseteq$*-maximal model of* $\Sigma_d(G)$	$(\{b,d,e\},\{\alpha,\delta,\beta,\gamma\})$
Grounded	$\subseteq$*-minimal model of* $\Sigma_r(G)$	$(\{b,d,e\},\{\alpha,\delta,\beta,\gamma\})$
Complete	*model of* $\Sigma_d(G)\cup\Sigma_r(G)$	$(\{b,d,e\},\{\alpha,\delta,\beta,\gamma\})$
Stable	*model of* $\Sigma_s(G)$	$(\{b,d,e\},\{\alpha,\delta,\beta,\gamma\})$

Then, from Definition 26 on page 16 and Proposition 4 on page 24, we obtain:

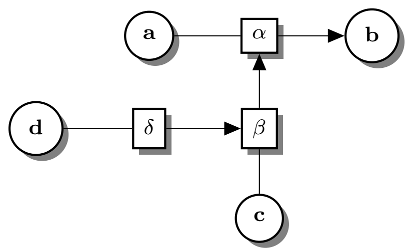
Semantics	*D-structures*
Admissible	$(\varnothing,\{\beta,\delta,\gamma\})$, $(\{d\},\{\beta,\delta,\gamma\})$, $(\{b,d\},\{\beta,\delta,\gamma\})$, $(\{e\},\{\beta,\delta,\gamma\})$, $(\{e\},\{\alpha,\beta,\delta,\gamma\})$, $(\{d,e\},\{\beta,\delta,\gamma\})$, $(\{d,e\},\{\alpha,\beta,\delta,\gamma\})$, $(\{b,d,e\},\{\beta,\delta,\gamma\})$, $(\{b,d,e\},\{\alpha,\beta,\delta,\gamma\})$
Stable	$(\{b,d,e\},\{\alpha,\beta,\delta,\gamma\})$
Complete	$(\{b,d,e\},\{\alpha,\beta,\delta,\gamma\})$

Moreover, the following table gives the extensions obtained with [6] or [4, 13] (concerning only arguments and interactions given in the initial graph):

Semantics	*Extensions*
Grounded	$\{b,d,e,\delta,\gamma\}$
Preferred	$\{b,d,e,\delta,\gamma\}$
Stable	$\{b,d,e,\delta,\gamma\}$
Complete	$\{b,d,e,\delta,\gamma\}$

5.6 Example 19 issued from [10]

Ex. 19



$\Sigma(G)$	$= \{$	$(Val(\beta) \wedge Acc(c)) \rightarrow \neg Val(\alpha)$ $(Val(\delta) \wedge Acc(d)) \rightarrow \neg Val(\beta)$ $(Val(\alpha) \wedge Acc(a)) \rightarrow NAcc(b)$ $NAcc(b) \rightarrow \neg Acc(b)$	$\}$
$\Sigma_d(G)$	$= \Sigma(G) \cup \{$	$Acc(b) \rightarrow (Val(\beta) \wedge Acc(c))$ $\neg Val(\beta)$ $Val(\alpha) \rightarrow (Val(\delta) \wedge Acc(d))$	$\}$
$\Sigma_r(G)$	$= \Sigma(G) \cup \{$	$Acc(a)$ $Acc(c)$ $Acc(d)$ $(Val(\beta) \wedge Acc(c)) \rightarrow Acc(b)$ $Val(\delta)$ $(Val(\delta) \wedge Acc(d)) \rightarrow Val(\alpha)$	$\}$
$\Sigma_s(G)$	$= \Sigma(G) \cup \{$	$Acc(a)$ $Acc(c)$ $Acc(d)$ $\neg Acc(b) \rightarrow (Val(\alpha) \wedge Acc(a))$ $Val(\delta)$ $\neg Val(\alpha) \rightarrow (Val(\beta) \wedge Acc(c))$ $\neg Val(\beta) \rightarrow (Val(\delta) \wedge Acc(d))$	$\}$

Then using Proposition 3 on page 23, the computation of the models gives the following results:

Type of structure	*Type of model*	*Computed structures*
Conflict-free	*model of* $\Sigma(G)$	*89 structures*
Naive	$\subseteq$*-maximal model of* $\Sigma(G)$	$(\{b,d\},\{\alpha,\beta\})$ $(\{b,c,d\},\{\alpha,\delta\})$ $(\{b\},\{\alpha,\beta,\delta\})$ $(\{a\},\{\alpha,\beta,\delta\})$ $(\{a,d\},\{\alpha,\beta\})$ $(\{a,b,c,d\},\{\beta\})$ $(\{a,b,c,d\},\{\delta\})$ $(\{a,c,d\},\{\alpha,\delta\})$ $(\{a,b,c\},\{\beta,\delta\})$
Admissible	*model of* $\Sigma_d(G)$	*20 structures (only 12 contain* δ*)*
Preferred	$\subseteq$*-maximal model of* $\Sigma_d(G)$	$(\{a,c,d\},\{\alpha,\delta\})$
Grounded	$\subseteq$*-minimal model of* $\Sigma_r(G)$	$(\{a,c,d\},\{\alpha,\delta\})$
Complete	*model of* $\Sigma_d(G) \cup \Sigma_r(G)$	$(\{a,c,d\},\{\alpha,\delta\})$
Stable	*model of* $\Sigma_s(G)$	$(\{a,c,d\},\{\alpha,\delta\})$

Then, from Definition 26 on page 16 and Proposition 4 on page 24, we obtain:

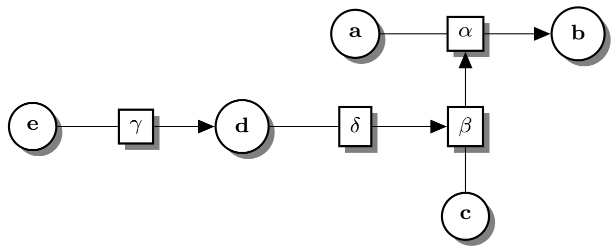
Semantics	*D-structures*
Admissible	$(\varnothing,\{\delta\})$, $(\{a\},\{\delta\})$, $(\{c\},\{\delta\})$, $(\{d\},\{\delta\})$ $(\{d\},\{\alpha,\delta\})$, $(\{a,c\},\{\delta\})$, $(\{a,d\},\{\delta\})$, $(\{a,d\},\{\alpha,\delta\})$, $(\{c,d\},\{\delta\})$, $(\{c,d\},\{\alpha,\delta\})$, $(\{a,c,d\},\{\delta\})$, $(\{a,c,d\},\{\alpha,\delta\})$
Stable	$(\{a,c,d\},\{\alpha,\delta\})$
Complete	$(\{a,c,d\},\{\alpha,\delta\})$

Moreover, the following table gives the extensions obtained with [6] or [4, 13] (concerning only arguments and interactions given in the initial graph):

Semantics	*Extensions*
Grounded	$\{a,c,d,\delta,\alpha\}$
Preferred	$\{a,c,d,\delta,\alpha\}$
Stable	$\{a,c,d,\delta,\alpha\}$
Complete	$\{a,c,d,\delta,\alpha\}$

5.7 Example 20 issued from [10]

Ex. 20



$\Sigma(G)$	$= \{$	$(Val(\beta) \wedge Acc(c)) \rightarrow \neg Val(\alpha)$ $(Val(\delta) \wedge Acc(d)) \rightarrow \neg Val(\beta)$ $(Val(\alpha) \wedge Acc(a)) \rightarrow NAcc(b)$ $NAcc(b) \rightarrow \neg Acc(b)$ $(Val(\gamma) \wedge Acc(e)) \rightarrow NAcc(d)$ $NAcc(d) \rightarrow \neg Acc(d)$	$\}$
$\Sigma_d(G)$	$= \Sigma(G) \cup \{$	$Acc(b) \rightarrow (Val(\beta) \wedge Acc(c))$ $\neg Acc(d)$ $Val(\alpha) \rightarrow (Val(\delta) \wedge Acc(d))$ $Val(\beta) \rightarrow (Val(\gamma) \wedge Acc(e))$	$\}$
$\Sigma_r(G)$	$= \Sigma(G) \cup \{$	$Acc(a)$ $Acc(c)$ $Acc(e)$ $(Val(\beta) \wedge Acc(c)) \rightarrow Acc(b)$ $Val(\delta)$ $Val(\gamma)$ $(Val(\delta) \wedge Acc(d)) \rightarrow Val(\alpha)$ $(Val(\gamma) \wedge Acc(e)) \rightarrow Val(\beta)$	$\}$
$\Sigma_s(G)$	$= \Sigma(G) \cup \{$	$Acc(a)$ $Acc(c)$ $Acc(e)$ $\neg Acc(b) \rightarrow (Val(\alpha) \wedge Acc(a))$ $\neg Acc(d) \rightarrow (Val(\gamma) \wedge Acc(e))$ $Val(\delta)$ $Val(\gamma)$ $\neg Val(\alpha) \rightarrow (Val(\beta) \wedge Acc(c))$ $\neg Val(\beta) \rightarrow (Val(\delta) \wedge Acc(d))$	$\}$

Then using Proposition 3 on page 23, the computation of the models gives the following results:

Type of structure	*Type of model*	*Computed structures*
Conflict-free	*model of* $\Sigma(G)$	*317 structures*
Naive	$\subseteq$*-maximal model of* $\Sigma(G)$	*17 structures*
Admissible	*model of* $\Sigma_d(G)$	*44 structures (only 14 contain* δ *and* γ*)*
Preferred	$\subseteq$*-maximal model of* $\Sigma_d(G)$	$(\{a,b,c,e\},\{\beta,\gamma,\delta\})$
Grounded	$\subseteq$*-minimal model of* $\Sigma_r(G)$	$(\{a,b,c,e\},\{\beta,\gamma,\delta\})$
Complete	*model of* $\Sigma_d(G) \cup \Sigma_r(G)$	$(\{a,b,c,e\},\{\beta,\gamma,\delta\})$
Stable	*model of* $\Sigma_s(G)$	$(\{a,b,c,e\},\{\beta,\gamma,\delta\})$

Then, from Definition 26 on page 16 and Proposition 4 on page 24, we obtain:

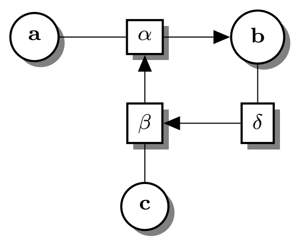
Semantics	*D-structures*
Admissible	$(\varnothing,\{\delta,\gamma\})$, $(\{a\},\{\delta,\gamma\})$, $(\{c\},\{\delta,\gamma\})$, $(\{a,c\},\{\delta,\gamma\})$ $(\{e\},\{\delta,\gamma\})$, $(\{e\},\{\beta,\delta,\gamma\})$, $(\{a,e\},\{\delta,\gamma\})$, $(\{a,e\},\{\beta,\delta,\gamma\})$, $(\{c,e\},\{\delta,\gamma\})$, $(\{c,e\},\{\beta,\delta,\gamma\})$, $(\{a,c,e\},\{\delta,\gamma\})$, $(\{a,c,e\},\{\beta,\delta,\gamma\})$, $(\{b,c,e\},\{\beta,\delta,\gamma\})$, $(\{a,b,c,e\},\{\beta,\delta,\gamma\})$
Stable	$(\{a,b,c,e\},\{\beta,\delta,\gamma\})$
Complete	$(\{a,b,c,e\},\{\beta,\delta,\gamma\})$

Moreover, the following table gives the extensions obtained with [6] or [4, 13] (concerning only arguments and interactions given in the initial graph):

Semantics	*Extensions*
Grounded	$\{a,c,e,\gamma,\beta,b\}$
Preferred	$\{a,c,e,\gamma,\beta,b\}$
Stable	$\{a,c,e,\gamma,\beta,b\}$
Complete	$\{a,c,e,\gamma,\beta,b\}$

5.8 Example 21 issued from [10]

Ex. 21



$\Sigma(G)$	$= \{$	$(Val(\beta) \wedge Acc(c)) \rightarrow \neg Val(\alpha)$ $(Val(\delta) \wedge Acc(b)) \rightarrow \neg Val(\beta)$ $(Val(\alpha) \wedge Acc(a)) \rightarrow NAcc(b)$ $NAcc(b) \rightarrow \neg Acc(b)$	$\}$
$\Sigma_d(G)$	$= \Sigma(G) \cup \{$	$Acc(b) \rightarrow (Val(\beta) \wedge Acc(c))$ $Val(\alpha) \rightarrow (Val(\delta) \wedge Acc(b))$ $Val(\beta) \rightarrow (Val(\alpha) \wedge Acc(a))$	$\}$
$\Sigma_r(G)$	$= \Sigma(G) \cup \{$	$Acc(a)$ $Acc(c)$ $(Val(\beta) \wedge Acc(c)) \rightarrow Acc(b)$ $Val(\delta)$ $(Val(\delta) \wedge Acc(b)) \rightarrow Val(\alpha)$ $(Val(\alpha) \wedge Acc(a)) \rightarrow Val(\beta)$	$\}$
$\Sigma_s(G)$	$= \Sigma(G) \cup \{$	$Acc(a)$ $Acc(c)$ $\neg Acc(b) \rightarrow (Val(\alpha) \wedge Acc(a))$ $Val(\delta)$ $\neg Val(\alpha) \rightarrow (Val(\beta) \wedge Acc(c))$ $\neg Val(\beta) \rightarrow (Val(\delta) \wedge Acc(b))$	$\}$

Then using Proposition 3 on page 23, the computation of the models gives the following results:

Type of structure	*Type of model*	*Computed structures*
Conflict-free	*model of* $\Sigma(G)$	*45 structures*
Naive	$\subseteq$*-maximal model of* $\Sigma(G)$	$(\{b\},\{\alpha,\beta\})$ $(\{b,c\},\{\alpha,\delta\})$ $(\{a\},\{\alpha,\beta,\delta\})$ $(\{a,b,c\},\{\beta\})$ $(\{a,b,c\},\{\delta\})$ $(\{a,c\},\{\alpha,\delta\})$ $(\{a,c\},\{\beta,\delta\})$
Admissible	*model of* $\Sigma_d(G)$	$(\varnothing,\varnothing)$ $(\{c\},\varnothing)$ $(\varnothing,\{\delta\})$ $(\{c\},\{\delta\})$ $(\{a\},\{\delta\})$ $(\{a\},\varnothing)$ $(\{a,c\},\varnothing)$ $(\{a,c\},\{\delta\})$
Preferred	$\subseteq$*-maximal model of* $\Sigma_d(G)$	$(\{a,c\},\{\delta\})$
Grounded	$\subseteq$*-minimal model of* $\Sigma_r(G)$	$(\{a,c\},\{\delta\})$
Complete	*model of* $\Sigma_d(G)\cup\Sigma_r(G)$	$(\{a,c\},\{\delta\})$
Stable	*model of* $\Sigma_s(G)$	*None*

Then, from Definition 26 on page 16 and Proposition 4 on page 24, we obtain:

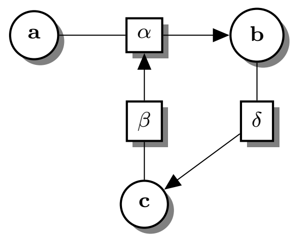
Semantics	*D-structures*
Admissible	$(\varnothing,\{\delta\})$, $(\{a\},\{\delta\})$, $(\{c\},\{\delta\})$, $(\{a,c\},\{\delta\})$
Stable	$\nexists$
Complete	$(\{a,c\},\{\delta\})$

Moreover, the following table gives the extensions obtained with [6] or [4, 13] (concerning only arguments and interactions given in the initial graph):

Semantics	*Extensions*
Grounded	$\{a,c\}$
Preferred	$\{a,c\}$
Stable	$\nexists$
Complete	$\{a,c\}$

5.9 Example 22 issued from [10]

Ex. 22



$\Sigma(G)$	$= \{$	$(Val(\beta) \wedge Acc(c)) \rightarrow \neg Val(\alpha)$ $(Val(\alpha) \wedge Acc(a)) \rightarrow NAcc(b)$ $NAcc(b) \rightarrow \neg Acc(b)$ $(Val(\delta) \wedge Acc(b)) \rightarrow NAcc(c)$ $NAcc(c) \rightarrow \neg Acc(c)$	$\}$
$\Sigma_d(G)$	$= \Sigma(G) \cup \{$	$Acc(b) \rightarrow (Val(\beta) \wedge Acc(c))$ $Acc(c) \rightarrow (Val(\alpha) \wedge Acc(a))$ $Val(\alpha) \rightarrow (Val(\delta) \wedge Acc(b))$	$\}$
$\Sigma_r(G)$	$= \Sigma(G) \cup \{$	$Acc(a)$ $(Val(\beta) \wedge Acc(c)) \rightarrow Acc(b)$ $(Val(\alpha) \wedge Acc(a)) \rightarrow Acc(c)$ $Val(\beta)$ $Val(\delta)$ $(Val(\delta) \wedge Acc(b)) \rightarrow Val(\alpha)$	$\}$
$\Sigma_s(G)$	$= \Sigma(G) \cup \{$	$Acc(a)$ $\neg Acc(b) \rightarrow (Val(\alpha) \wedge Acc(a))$ $\neg Acc(c) \rightarrow (Val(\delta) \wedge Acc(b))$ $Val(\beta)$ $Val(\delta)$ $\neg Val(\alpha) \rightarrow (Val(\beta) \wedge Acc(c))$	$\}$

Then using Proposition 3 on page 23, the computation of the models gives the following results:

Type of structure	*Type of model*	*Computed structures*
Conflict-free	*model of* $\Sigma(G)$	*45 structures*
Naive	$\subseteq$*-maximal model of* $\Sigma(G)$	$(\{b,c\},\{\alpha\})$ $(\{b\},\{\alpha,\beta,\delta\})$ $(\{a\},\{\alpha,\beta,\delta\})$ $(\{a,b,c\},\{\beta\})$ $(\{a,c\},\{\alpha,\delta\})$ $(\{a,c\},\{\beta,\delta\})$ $(\{a,b\},\{\beta,\delta\})$
Admissible	*model of* $\Sigma_d(G)$	$(\varnothing,\varnothing)$ $(\varnothing,\{\beta\})$ $(\varnothing,\{\beta,\delta\})$ $(\varnothing,\{\delta\})$ $(\{a\},\{\beta,\delta\})$ $(\{a\},\varnothing)$ $(\{a\},\{\beta\})$ $(\{a\},\{\delta\})$
Preferred	$\subseteq$*-maximal model of* $\Sigma_d(G)$	$(\{a\},\{\beta,\delta\})$
Grounded	$\subseteq$*-minimal model of* $\Sigma_r(G)$	$(\{a\},\{\beta,\delta\})$
Complete	*model of* $\Sigma_d(G)\cup\Sigma_r(G)$	$(\{a\},\{\beta,\delta\})$
Stable	*model of* $\Sigma_s(G)$	*None*

Then, from Definition 26 on page 16 and Proposition 4 on page 24, we obtain:

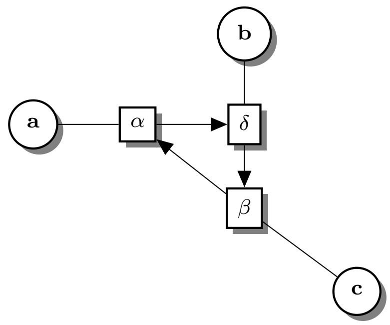
Semantics	*D-structures*
Admissible	$(\varnothing,\{\beta,\delta\})$, $(\{a\},\{\beta,\delta\})$
Stable	$\nexists$
Complete	$(\{a\},\{\beta,\delta\})$

Moreover, the following table gives the extensions obtained with [6] or [4, 13] (concerning only arguments and interactions given in the initial graph):

Semantics	*Extensions*
Grounded	$\{a\}$
Preferred	$\{a\}$
Stable	$\nexists$
Complete	$\{a\}$

5.10 Example 23 issued from [10]

Ex. 23



$\Sigma(G)$	$= \{$	$(Val(\beta) \wedge Acc(c)) \rightarrow \neg Val(\alpha)$ $(Val(\delta) \wedge Acc(b)) \rightarrow \neg Val(\beta)$ $(Val(\alpha) \wedge Acc(a)) \rightarrow \neg Val(\delta)$	$\}$
$\Sigma_d(G)$	$= \Sigma(G) \cup \{$	$Val(\alpha) \rightarrow (Val(\delta) \wedge Acc(b))$ $Val(\beta) \rightarrow (Val(\alpha) \wedge Acc(a))$ $Val(\delta) \rightarrow (Val(\beta) \wedge Acc(c))$	$\}$
$\Sigma_r(G)$	$= \Sigma(G) \cup \{$	$Acc(a)$ $Acc(b)$ $Acc(c)$ $(Val(\delta) \wedge Acc(b)) \rightarrow Val(\alpha)$ $(Val(\alpha) \wedge Acc(a)) \rightarrow Val(\beta)$ $(Val(\beta) \wedge Acc(c)) \rightarrow Val(\delta)$	$\}$
$\Sigma_s(G)$	$= \Sigma(G) \cup \{$	$Acc(a)$ $Acc(b)$ $Acc(c)$ $\neg Val(\alpha) \rightarrow (Val(\beta) \wedge Acc(c))$ $\neg Val(\beta) \rightarrow (Val(\delta) \wedge Acc(b))$ $\neg Val(\delta) \rightarrow (Val(\alpha) \wedge Acc(a))$	$\}$

Then using Proposition 3 on page 23, the computation of the models gives the following results:

Type of structure	*Type of model*	*Computed structures*
Conflict-free	*model of* $\Sigma(G)$	*33 structures*
Naive	$\subseteq$*-maximal model of* $\Sigma(G)$	$(\varnothing, \{\alpha, \beta, \delta\})$ $(\{b, c\}, \{\alpha, \delta\})$ $(\{a, b, c\}, \varnothing)$ $(\{a, b\}, \{\alpha, \beta\})$ $(\{a, c\}, \{\beta, \delta\})$
Admissible	*model of* $\Sigma_d(G)$	$(\varnothing, \varnothing)$ $(\{b\}, \varnothing)$ $(\{c\}, \varnothing)$ $(\{b, c\}, \varnothing)$ $(\{a, c\}, \varnothing)$ $(\{a\}, \varnothing)$ $(\{a, b\}, \varnothing)$ $(\{a, b, c\}, \varnothing)$
Preferred	$\subseteq$*-maximal model of* $\Sigma_d(G)$	$(\{a, b, c\}, \varnothing)$
Grounded	$\subseteq$*-minimal model of* $\Sigma_r(G)$	$(\{a, b, c\}, \varnothing)$
Complete	*model of* $\Sigma_d(G) \cup \Sigma_r(G)$	$(\{a, b, c\}, \varnothing)$
Stable	*model of* $\Sigma_s(G)$	*None*

Then, from Definition 26 on page 16 and Proposition 4 on page 24, we obtain:

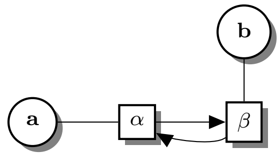
Semantics	*D-structures*
Admissible	$(\varnothing, \varnothing)$, $(\{a\}, \varnothing)$, $(\{b\}, \varnothing)$, $(\{c\}, \varnothing)$, $(\{a, b\}, \varnothing)$, $(\{a, c\}, \varnothing)$, $(\{b, c\}, \varnothing)$, $(\{a, b, c\}, \varnothing)$
Stable	$\nexists$
Complete	$(\{a, b, c\}, \varnothing)$

Moreover, the following table gives the extensions obtained with [6] or [4, 13] (concerning only arguments and interactions given in the initial graph):

Semantics	*Extensions*
Grounded	$\{a, b, c\}$
Preferred	$\{a, b, c\}$
Stable	$\nexists$
Complete	$\{a, b, c\}$

5.11 Example 24 issued from [10]

Ex. 24



$\Sigma(G)$	$= \{$	$(Val(\beta) \wedge Acc(b)) \rightarrow \neg Val(\alpha)$ $(Val(\alpha) \wedge Acc(a)) \rightarrow \neg Val(\beta)$	$\}$
$\Sigma_d(G)$	$= \Sigma(G) \cup \{$	$Val(\alpha) \rightarrow Acc(a)$ $Val(\beta) \rightarrow Acc(b)$	$\}$
$\Sigma_r(G)$	$= \Sigma(G) \cup \{$	$Acc(a)$ $Acc(b)$	$\}$
$\Sigma_s(G)$	$= \Sigma(G) \cup \{$	$Acc(a)$ $Acc(b)$ $\neg Val(\alpha) \rightarrow (Val(\beta) \wedge Acc(b))$ $\neg Val(\beta) \rightarrow (Val(\alpha) \wedge Acc(a))$	$\}$

Then using Proposition 3 on page 23, the computation of the models gives the following results:

Type of structure	*Type of model*	*Computed structures*
Conflict-free	*model of* $\Sigma(G)$	$(\varnothing, \varnothing)$ $(\{b\}, \varnothing)$ $(\varnothing, \{\alpha\})$ $(\{b\}, \{\alpha\})$ $(\varnothing, \{\alpha, \beta\})$ $(\varnothing, \{\beta\})$ $(\{b\}, \{\beta\})$ $(\{a\}, \{\alpha\})$ $(\{a\}, \varnothing)$ $(\{a, b\}, \varnothing)$ $(\{a, b\}, \{\alpha\})$ $(\{a\}, \{\beta\})$ $(\{a, b\}, \{\beta\})$
Naive	$\subseteq$*-maximal model of* $\Sigma(G)$	$(\varnothing, \{\alpha, \beta\})$ $(\{a, b\}, \{\alpha\})$ $(\{a, b\}, \{\beta\})$
Admissible	*model of* $\Sigma_d(G)$	$(\varnothing, \varnothing)$ $(\{b\}, \varnothing)$ $(\{b\}, \{\beta\})$ $(\{a, b\}, \{\beta\})$ $(\{a\}, \varnothing)$ $(\{a, b\}, \varnothing)$ $(\{a\}, \{\alpha\})$ $(\{a, b\}, \{\alpha\})$
Preferred	$\subseteq$*-maximal model of* $\Sigma_d(G)$	$(\{a, b\}, \{\beta\})$ $(\{a, b\}, \{\alpha\})$
Grounded	$\subseteq$*-minimal model of* $\Sigma_r(G)$	$(\{a, b\}, \varnothing)$
Complete	*model of* $\Sigma_d(G) \cup \Sigma_r(G)$	$(\{a, b\}, \varnothing)$ $(\{a, b\}, \{\alpha\})$ $(\{a, b\}, \{\beta\})$
Stable	*model of* $\Sigma_s(G)$	$(\{a, b\}, \{\beta\})$ $(\{a, b\}, \{\alpha\})$

Then, from Definition 26 on page 16 and Proposition 4 on page 24, we obtain:

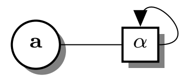
Semantics	*D-structures*
Admissible	$(\varnothing, \varnothing)$, $(\{a\}, \varnothing)$, $(\{a\}, \{\alpha\})$, $(\{b\}, \varnothing)$, $(\{b\}, \{\beta\})$, $(\{a, b\}, \varnothing)$, $(\{a, b\}, \{\alpha\})$, $(\{a, b\}, \{\beta\})$
Stable	$(\{a, b\}, \{\alpha\})$, $(\{a, b\}, \{\beta\})$
Complete	$(\{a, b\}, \varnothing)$, $(\{a, b\}, \{\alpha\})$, $(\{a, b\}, \{\beta\})$

Moreover, the following table gives the extensions obtained with [6] or [4, 13] (concerning only arguments and interactions given in the initial graph):

Semantics	*Extensions*
Grounded	$\{a, b\}$
Preferred	$\{a, b, \alpha\}$, $\{a, b, \beta\}$
Stable	$\{a, b, \alpha\}$, $\{a, b, \beta\}$
Complete	$\{a, b\}$, $\{a, b, \alpha\}$, $\{a, b, \beta\}$

5.12 Example 25 issued from [10]

Ex. 25



$\Sigma(G)$	$= \{$	$(Val(\alpha) \wedge Acc(a)) \rightarrow$	$\}$
$\Sigma_d(G)$	$= \Sigma(G) \cup \{$	$Val(\alpha) \rightarrow Acc(a)$	$\}$
$\Sigma_r(G)$	$= \Sigma(G) \cup \{$	$Acc(a)$	$\}$
$\Sigma_s(G)$	$= \Sigma(G) \cup \{$	$Acc(a)$ $Val(\alpha)$	$\}$

Then using Proposition 3 on page 23, the computation of the models gives the following results:

Type of structure	*Type of model*	*Computed structures*
Conflict-free	*model of* $\Sigma(G)$	$(\varnothing, \varnothing)$ $(\varnothing, \{\alpha\})$ $(\{a\}, \varnothing)$
Naive	$\subseteq$*-maximal model of* $\Sigma(G)$	$(\varnothing, \{\alpha\})$ $(\{a\}, \varnothing)$
Admissible	*model of* $\Sigma_d(G)$	$(\varnothing, \varnothing)$ $(\{a\}, \varnothing)$
Preferred	$\subseteq$*-maximal model of* $\Sigma_d(G)$	$(\{a\}, \varnothing)$
Grounded	$\subseteq$*-minimal model of* $\Sigma_r(G)$	$(\{a\}, \varnothing)$
Complete	*model of* $\Sigma_d(G) \cup \Sigma_r(G)$	$(\{a\}, \varnothing)$
Stable	*model of* $\Sigma_s(G)$	*None*

Then, from Definition 26 on page 16 and Proposition 4 on page 24, we obtain:

Semantics	*D-structures*
Admissible	$(\varnothing, \varnothing)$, $(\{a\}, \varnothing)$
Stable	$\nexists$
Complete	$(\{a\}, \varnothing)$

Moreover, the following table gives the extensions obtained with [6] or [4, 13] (concerning only arguments and interactions given in the initial graph):

Semantics	*Extensions*
Grounded	$\{a\}$
Preferred	$\{a\}$
Stable	$\nexists$
Complete	$\{a\}$

Chapter 6

Comparative analysis

In this chapter, we compare the different methods presented above for defining semantics of recursive argumentation frameworks. Three kinds of methods have been considered:

Translation methods: These methods, described in [6] or [4, 13], consist in a translation of the original framework into a D-framework, with the addition of meta-arguments representing the initial interactions and new interactions concerning them.

Note that, even if the added arguments and interactions are not exactly the same in both methods, it seems that the resulting extensions are the same considering only the elements belonging to the initial graph. This equivalence remains to be formally proved.

Direct method: This method, described in [7, 8], defines structure-based semantics, in a way similar to the definition of extension-based semantics, using structures in place of extensions.

Logical method: This method describes semantics in terms of models of a logical encoding of the argumentation framework.

First, we show that the direct method and the logical method are equivalent. Then we compare the translation methods with the direct method (or equivalently the logical method).

6.1 Comparison between the logical method and the direct method

Prop. 3 on page 23 and Prop. 4 on page 24 give an exact characterization of structure-based semantics in terms of logical models. More precisely, the table 6.1 on the next page synthetizes the links between particular interpretations[1] $\mathcal{I}$ and properties of the associated sets $S_{\mathcal{I}} = \{x \in \mathbf{A} | \mathcal{I}(Acc(x)) = \textit{true}\}$ and $\Gamma_{\mathcal{I}} = \{x \in \mathbf{R} | \mathcal{I}(Val(x)) = \textit{true}\}$.

[1] $\mathcal{I}$ is an interpretation of a set of formulae Σ of the language introduced in Section 3.1 on page 19.

$\mathcal{I}$	$(S_{\mathcal{I}}, \Gamma_{\mathcal{I}})$	Number of Prop.
model of $\Sigma(G)$	conflict-free structure	3.1
model of $\Sigma_d(G)$	admissible structure	3.2
$\subseteq$-maximal model of $\Sigma_d(G)$	preferred structure	3.5
$\subseteq$-minimal model of $\Sigma_r(G)$	grounded structure	3.6
model of $\Sigma_d(G) \cup \Sigma_r(G)$	complete structure	3.3
model of $\Sigma_s(G)$	stable structure	3.4
model of $\Sigma(G) \cup \{\Phi_r^{att}(G)\}$	conflict-free d-structure	4.1
model of $\Sigma_d(G) \cup \{\Phi_r^{att}(G)\}$	admissible d-structure	4.2

with $\Phi_r^{att}(G) \stackrel{def}{\equiv} (\forall \delta \in Attack)(((\forall \alpha \in Attack)(t_\alpha = \delta \to (\exists \beta \in Attack)(t_\beta \in \{s_\alpha, \alpha\} \wedge Val(\beta) \wedge Acc(s_\beta)))) \to Val(\delta))$.

Table 6.1: Models vs structures

6.2 Comparison between the direct/logical method and the translation methods

The examples presented in Chapter 5 show the grounded (resp. preferred, complete, stable) structures, and the grounded (resp. preferred, complete, stable) extensions obtained by a translation method. Note that in the case of the method using MAF ([6]), these extensions are restricted to the arguments and interactions appearing in the original framework (the meta-arguments of the form N_{ij} were hidden).

It can be noticed that, for each semantics (grounded, preferred, complete, stable):

- There is a one-to-one correspondence between the structures and the extensions.
- For each structure, the structure and the corresponding extension contain exactly the same arguments of A.
- For each structure, the structure contains (at least) the interactions of $\mathbf{R}$ that belong to the corresponding extension. However, the structure may contain other interactions (see Ex. 15 on page 27).

  Indeed, following definitions given in [6] or [4, 13], an interaction belonging to an extension is valid *and* grounded. Whereas, in a structure $U = (S, \Gamma)$, Γ contains *all* the attacks that are valid wrt U, whether grounded or not.

  This difference is illustrated on Ex. 15 on page 27: $(\{a\}, \{\alpha, \beta\})$ is a preferred structure, as β is valid since it is unattacked. However, the corresponding extension $\{a, \alpha\}$ does not contain β since its source b does not belong to the extension.

In the following, we pursue these comparisons with formal results.

6.2.1 Translation method of [4, 13]

Let us recall that in case of ASAF without support, the approaches proposed in [4] and [13] are equivalent.

$\mathcal{I}$		$\mathcal{E}_{\mathcal{I}}$
model of $\Sigma(G)$	$\Rightarrow$	AFRA-conflict-free
model of $\Sigma_d(G)$	$\Rightarrow$	AFRA-admissible
$\subseteq$-maximal model of $\Sigma_d(G)$	$\Rightarrow$	AFRA-preferred
model of $\Sigma_d(G) \cup \Sigma_r(G)$	$\Rightarrow$	AFRA-complete
model of $\Sigma_s(G)$	$\Rightarrow$	AFRA-stable

Table 6.2: From models to AFRA-extensions

$\mathcal{E}$		$\mathcal{I}_{\mathcal{E}}$
AFRA-conflict-free	$\Rightarrow$	model of $\Sigma(G)$
AFRA-admissible	$\Rightarrow$	model of $\Sigma(G)$
closed AFRA-admissible	$\Rightarrow$	model of $\Sigma_d(G)$
AFRA-preferred	$\Rightarrow$	$\subseteq$-maximal model of $\Sigma_d(G)$
AFRA-complete	$\Rightarrow$	model of $\Sigma_d(G) \cup \Sigma_r(G)$
AFRA-stable	$\Rightarrow$	model of $\Sigma_s(G)$

Table 6.3: From AFRA-extensions to models

In [7, 8], a formal correspondence has been established between semantics proposed in [4] and the structure-based semantics. Then, using propositions 3 on page 23 and 4 on page 24, comparisons can be drawn between the logical method and the addition method given in [4].

Let $\mathcal{I}$ be an interpretation, $S_{\mathcal{I}} = \{x \in \mathbf{A} | \mathcal{I}(Acc(x)) = \textit{true}\}$, $\Gamma_{\mathcal{I}} = \{x \in \mathbf{R} | \mathcal{I}(Val(x)) = \textit{true}\}$ and $\mathcal{E}_{\mathcal{I}} = S_{\mathcal{I}} \cup \{x \in \Gamma_{\mathcal{I}} | s(x) \in S_{\mathcal{I}}\}$, the table 6.2 synthetizes the correspondences from particular interpretations $\mathcal{I}$ to AFRA-extensions of [4].

Moreover, correspondences can be established in the other direction. Let $\mathcal{E}$ be an AFRA-extension, consider the structure $(S_{\mathcal{E}}, \Gamma_{\mathcal{E}})$ with $S_{\mathcal{E}} = \mathcal{E} \cap \mathbf{A}$ and $\Gamma_{\mathcal{E}} = (\mathcal{E} \cap \mathbf{R}) \cup \mathcal{X}$, $\mathcal{X}$ being the set of attacks that are not AFRA-acceptable wrt $\mathcal{E}$ only because of attacks towards their source. Consider an interpretation $\mathcal{I}_{\mathcal{E}}$ such that $S_{\mathcal{I}_{\mathcal{E}}} = S_{\mathcal{E}}$ and $\Gamma_{\mathcal{I}_{\mathcal{E}}} = \Gamma_{\mathcal{E}}$. The table 6.3 synthetizes the correspondences from AFRA-extensions of [4] to particular interpretations $\mathcal{I}$.

For more details about these correspondences, the reader is invited to refer to [7, 8].

6.2.2 Translation method of [6]

In this section, the comparison between the logical method and the method of [6] is conducted in two steps:[2]

1. first, structure-based semantics are compared with MAF semantics and vice-versa;

2. then we can use the above results (the comparison between the logical method and the direct method) for concluding about the comparison between the logical method and the MAF semantics.

Moreover, let us recall that we restrict the approches of [6] to frameworks without support.

[2] Note that another way could be to study the links between AFRA and MAF semantics.

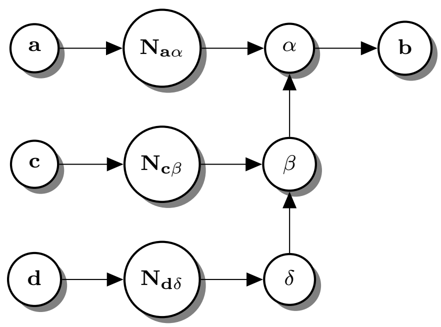
6.2.2.1 Preliminary results

Following Def. 14 on page 12, we define the MAF associated with a given RAF.

Def. 27 *Let $\langle \mathbf{A}, \mathbf{R}, s, t\rangle$ be a RAF without support. The associated MAF is $\langle \mathbf{A}', \mathbf{R}'\rangle$ with:*

- $\mathbf{A}' = \mathbf{A} \cup \mathbf{R} \cup \{N_{s(\alpha)\alpha} | \alpha \in \mathbf{R}\}$ *(this last subset will be denoted by $\mathbf{N}$),*
- $\mathbf{R}' = \{(s(\alpha), N_{s(\alpha)\alpha}) | \alpha \in \mathbf{R}\} \cup \{(N_{s(\alpha)\alpha}, \alpha) | \alpha \in \mathbf{R}\} \cup \{(\alpha, t(\alpha)) | \alpha \in \mathbf{R}\}$.

Ex. 19 (cont'd) *In the case when recursive attacks are encoded with meta-arguments, as in [6], the graph is turned into:*



The following properties of $\langle \mathbf{A}', \mathbf{R}'\rangle$ directly follow from the above definition:

Observation 1

1. *There are only three types of attacks in $\mathbf{R}'$: either from $\mathbf{A}$ to $\mathbf{N}$, or from $\mathbf{N}$ to $\mathbf{R}$, or from $\mathbf{R}$ to $\mathbf{A} \cup \mathbf{R}$.*
2. *$\forall N_{a\alpha} \in \mathbf{N}$, $N_{a\alpha}$ is involved in only two attacks belonging to $\mathbf{R}'$: $(a, N_{a\alpha})$ and $(N_{a\alpha}, \alpha)$.*
3. *$\forall \alpha \in \mathbf{R}$, $s(\alpha)$ is the only attacker of $N_{s(\alpha)\alpha}$ and so the only defender of α against $N_{s(\alpha)\alpha}$.*
4. *$\forall a \in \mathbf{A}$, a is unttacked in the RAF iff a is unattacked in the MAF.*
5. *$\forall \alpha \in \mathbf{R}$, α is always attacked in the MAF.*

Then it can be easily proved that:

Prop. 5 *Let $\langle \mathbf{A}, \mathbf{R}, s, t\rangle$ be a RAF without support and $\langle \mathbf{A}', \mathbf{R}'\rangle$ its associated MAF. Let $S \subseteq \mathbf{A}'$ be a conflict-free set of $\langle \mathbf{A}', \mathbf{R}'\rangle$.*

1. *Let $a \in \mathbf{A}$. $S \cup \{a\}$ is also conflict-free iff $\nexists \alpha \in S$ s.t. $t(\alpha) = a$ and $\nexists N_{a\alpha} \in S$.*
2. *Let $\beta \in \mathbf{R}$. $S \cup \{\beta\}$ is also conflict-free iff $t(\beta) \notin S$ and $N_{s(\beta)\beta} \notin S$ and $\nexists \alpha \in S$ s.t. $t(\alpha) = \beta$.*

In the following, given a RAF without support $\langle \mathbf{A}, \mathbf{R}, s, t\rangle$, and its associated MAF $\langle \mathbf{A}', \mathbf{R}'\rangle$, we establish correspondences between the structure-based semantics of the RAF and the semantics of the MAF $\langle \mathbf{A}', \mathbf{R}'\rangle$.
So, for each semantics, starting with a given structure of the RAF, we build a corresponding extension of the MAF. Then we consider the opposite case, starting from a MAF extension and building a corresponding structure. For that purpose, we need the following notations:

Notation 1 (From RAF structures to MAF extensions) *Given $U = (S, \Gamma)$ a structure of the RAF:*

- $\mathcal{E}'_U$ *denotes the set* $S \cup \{\alpha \in \Gamma \text{ s.t. } s(\alpha) \in S\}$.
- $\mathcal{E}_U$ *denotes the set* $\mathcal{E}'_U \cup \{N_{s(\alpha)\alpha} \text{ s.t. } s(\alpha) \notin S \text{ and } s(\alpha) \in Def(U)\}$.

In other words, $\mathcal{E}'_U$ is made of the arguments of S and the attacks of Γ whose source belongs to S. $\mathcal{E}_U$ is obtained from $\mathcal{E}'_U$ by adding the elements $N_{s(\alpha)\alpha}$ of $\mathbf{N}$ such that $\alpha \notin \mathcal{E}'_U$ and $N_{s(\alpha)\alpha}$ is defended by $\mathcal{E}'_U$.[3] The following properties directly follow from the above notation and from Observation 1 on the facing page:

Observation 2 *Let $a \in \mathbf{A}$ and $\alpha \in \mathbf{R}$,*

1. *If α is acceptable wrt $\mathcal{E}_U$ in the MAF, then $s(\alpha) \in (\mathcal{E}_U \cap \mathbf{A}) = S$.*
2. *$a \in Def(U)$ if and only if $\mathcal{E}_U$ attacks a in the MAF.*
3. *$\alpha \in Inh(U)$ if and only if $\mathcal{E}_U \cap \mathbf{R}$ attacks α in the MAF.*

For the opposite case, we also need the following notations:

Notation 2 (From MAF extensions to RAF structures) *Given some set $\mathcal{E} \subseteq \mathbf{A}'$:*

- *$\mathcal{E}_a$ (resp. $\mathcal{E}_r$, $\mathcal{E}_n$) denotes the set $\mathcal{E} \cap \mathbf{A}$ (resp. $\mathcal{E} \cap \mathbf{R}$, $\mathcal{E} \cap \mathbf{N}$).*
- *Considering the structure $U'_{\mathcal{E}} = (\mathcal{E}_a, \mathcal{E}_r)$, $\Gamma_{\mathcal{E}}$ denotes the set $\mathcal{E}_r \cup \{\alpha \notin \mathcal{E}_r \text{ s.t. } \alpha \in Acc(U'_{\mathcal{E}})$ and $s(\alpha) \notin \mathcal{E}_a\}$.*
- *$U_{\mathcal{E}}$ denotes the structure $(S_{\mathcal{E}}, \Gamma_{\mathcal{E}})$ where $S_{\mathcal{E}} = \mathcal{E}_a$.*

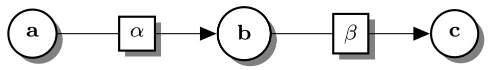
Indeed, $\Gamma_{\mathcal{E}}$ contains the attacks that belong to $\mathcal{E}$ and also the attacks that do not belong to $\mathcal{E}$ but are acceptable wrt $U'_{\mathcal{E}}$, even if they are not acceptable wrt $\mathcal{E}$, because of their source. Intuitively, this is due to the fact that an attack α cannot be acceptable wrt $\mathcal{E}$ if $s(\alpha) \notin \mathcal{E}$, whereas this is not a problem for structure-based semantics.

Note that $\Gamma_{\mathcal{E}} \setminus \mathcal{E}_r$ could be defined in terms of $\mathcal{E}$ as follows: $\alpha \in \Gamma_{\mathcal{E}} \setminus \mathcal{E}_r$ iff $\forall \beta \in \mathbf{R}$ s.t. $(\beta, \alpha) \in \mathbf{R}'$, $\exists \gamma \in \mathcal{E}_r$ s.t. $s(\gamma) \in \mathcal{E}_a$ and either $(\gamma, \beta) \in \mathbf{R}'$ or $(\gamma, s(\beta)) \in \mathbf{R}'$.

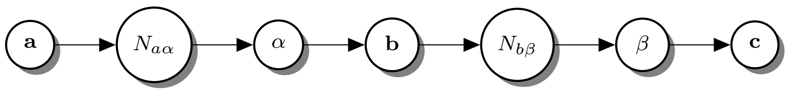
The following example illustrates the previous notations:

Ex. 26 *Consider the RAF depicted by the following figure:*

[3] This last condition will be mandatory for proving acceptability and admissibility.



The associated MAF is represented by the following graph:



Consider the structure $U = (\{a,c\},\{\alpha,\beta\})$ in the RAF. β belongs to Γ, but its source (b) does not belong to S. So, $\mathcal{E}'_U = \{a,c,\alpha\}$ and $\mathcal{E}_U = \{a,c,\alpha,N_{b\beta}\}$.

Consider the set $\mathcal{E} = \{a,\alpha,N_{b\beta}\}$ in the MAF. $U'_{\mathcal{E}} = (\{a\},\{\alpha\})$. β is acceptable wrt $U'_{\mathcal{E}}$; so $\Gamma_{\mathcal{E}} = \{\alpha,\beta\}$ and $U_{\mathcal{E}} = (\{a\},\{\alpha,\beta\})$.

6.2.2.2 Conflict-freeness

Concerning the mapping from conflict-free RAF structures to conflict-free MAF sets, we have:

Prop. 6 *Let $U = (S,\Gamma)$ be a conflict-free structure of the RAF.*

1. *$\mathcal{E}'_U$ is conflict-free in the MAF $\langle \mathbf{A}', \mathbf{R}' \rangle$.*
2. *$\mathcal{E}_U$ is conflict-free in the MAF $\langle \mathbf{A}', \mathbf{R}' \rangle$.*

And conversely, we have:

Prop. 7 *Let $\mathcal{E}$ be a conflict-free subset in the MAF $\langle \mathbf{A}', \mathbf{R}' \rangle$.*

1. *$U'_{\mathcal{E}}$ is a conflict-free structure of the RAF.*
2. *$U_{\mathcal{E}}$ is a conflict-free structure of the RAF.*

6.2.2.3 Admissibility

The following proposition gives a correspondence from RAF structures to MAF sets concerning the notion of acceptability:

Prop. 8 *Let $U = (S,\Gamma)$ be a conflict-free structure of the RAF. Let $a \in \mathbf{A}$ and $\alpha \in \mathbf{R}$.*

1. *If a is acceptable wrt U in the RAF, then a is acceptable wrt $\mathcal{E}_U$ in the MAF $\langle \mathbf{A}', \mathbf{R}' \rangle$.*
2. *If α is acceptable wrt U in the RAF, and $s(\alpha) \in S$, then α is acceptable wrt $\mathcal{E}_U$ in the MAF $\langle \mathbf{A}', \mathbf{R}' \rangle$.*

Note that the second result of the above proposition does not hold if we drop the condition $s(\alpha) \in S$, as shown on the following example:

Ex. 26 (cont'd) *Consider the conflict-free structure $U = (\{a, c\}, \{\alpha\})$. We have $\mathcal{E}_U = \{a, c, \alpha, N_{b\beta}\}$. β is acceptable wrt U since it is not attacked in the RAF. However, β is not acceptable wrt $\mathcal{E}_U$ in the MAF, since b does not belong to $\mathcal{E}_U$.*

Note also that Proposition 8 does not hold if we replace $\mathcal{E}_U$ by $\mathcal{E}'_U$.

Ex. 26 (cont'd) *Let $U = (\{a, c\}, \{\alpha\})$. We have $\mathcal{E}'_U = \{a, c, \alpha\}$. c is acceptable wrt U. However, c is not acceptable wrt $\mathcal{E}'_U$ in the MAF.*

And finally, the following proposition shows the link between admissible structures of the RAF and admissible extensions of its associated MAF:

Prop. 9 *Let $U = (S, \Gamma)$ be an admissible structure of the RAF. Then $\mathcal{E}_U$ is an admissible extension of the MAF $\langle \mathbf{A}', \mathbf{R}' \rangle$.*

Conversely, Ex. 26 on page 53 illustrates how acceptability differs when going from MAF to RAF:

Ex. 26 (cont'd) *Consider the set $\mathcal{E} = \{a, \alpha, N_{b\beta}\}$ in the MAF. $U'_{\mathcal{E}} = (\{a\}, \{\alpha\})$. β is acceptable wrt $U'_{\mathcal{E}}$, whereas it is not acceptable wrt $\mathcal{E}$ since $b \notin \mathcal{E}$.*

Indeed, from a given admissible extension $\mathcal{E}$ of the MAF, several admissible structures can be obtained that contained the arguments of $\mathcal{E} \cap \mathbf{A}$ and the attacks of $\mathcal{E} \cap \mathbf{R}$. The following proposition shows two such admissible structures.

Prop. 10 *Let $\mathcal{E}$ be an admissible extension of the MAF $\langle \mathbf{A}', \mathbf{R}' \rangle$.*

1. *$U'_{\mathcal{E}}$ is an admissible structure of the RAF.*
2. *$U_{\mathcal{E}}$ is an admissible structure of the RAF.*

6.2.2.4 Complete semantics

Concerning the correspondence from complete RAF structures to complete MAF extensions, the following proposition holds:

Prop. 11 *Let $U = (S, \Gamma)$ be a complete structure of the RAF. Then $\mathcal{E}_U$ is a complete extension of the MAF $\langle \mathbf{A}', \mathbf{R}' \rangle$.*

Conversely, we have:

Prop. 12 *Let $\mathcal{E}$ be a complete extension of the MAF $\langle \mathbf{A}', \mathbf{R}' \rangle$. Then $U_{\mathcal{E}}$ is a complete structure of the RAF.*

Moreover, we also have a one to one correspondence for complete semantics:

Prop. 13 *Let $\langle \mathbf{A}, \mathbf{R}, s, t \rangle$ be a RAF without support and $\langle \mathbf{A}', \mathbf{R}' \rangle$ its associated MAF. The following assertions hold:*

1. *If U is a complete structure of the RAF, then $U_{\mathcal{E}_U} = U$.*
2. *If $\mathcal{E}$ is a complete extension of the MAF, then $\mathcal{E}_{U_{\mathcal{E}}} = \mathcal{E}$.*

6.2.2.5 Stable semantics

Concerning the correspondence from stable RAF structures to stable MAF extensions, the following proposition holds:

Prop. 14 *Let $U = (S, \Gamma)$ be a stable structure of the RAF. Then $\mathcal{E}_U$ is a stable extension of the MAF $\langle \mathbf{A}', \mathbf{R}' \rangle$.*

Conversely, we have:

Prop. 15 *Let $\mathcal{E}$ be a stable extension of the MAF $\langle \mathbf{A}', \mathbf{R}' \rangle$. Then $U_{\mathcal{E}}$ is a stable structure of the RAF.*

6.2.2.6 Preferred semantics

Concerning the correspondence from preferred RAF structures to preferred MAF extensions, the following proposition holds:

Prop. 16 *Let $U = (S, \Gamma)$ be a preferred structure of the RAF. Then $\mathcal{E}_U$ is a preferred extension of the MAF $\langle \mathbf{A}', \mathbf{R}' \rangle$.*

Conversely, we have:

Prop. 17 *Let $\mathcal{E}$ be a preferred extension of the MAF $\langle \mathbf{A}', \mathbf{R}' \rangle$. Then $U_{\mathcal{E}}$ is a preferred structure of the RAF.*

6.2.2.7 Grounded semantics

Concerning the correspondence from the grounded RAF structure to the grounded MAF extension, the following proposition holds:

Prop. 18 *Let $U = (S, \Gamma)$ be the grounded structure of the RAF. Then $\mathcal{E}_U$ is the grounded extension of the MAF $\langle \mathbf{A}', \mathbf{R}' \rangle$.*

Conversely, we have:

Prop. 19 *Let $\mathcal{E}$ be the grounded extension of the MAF $\langle \mathbf{A}', \mathbf{R}' \rangle$. Then $U_{\mathcal{E}}$ is the grounded structure of the RAF.*

6.2.2.8 Synthesis

The correspondences between RAF and MAF are synthetized in Tables 6.4 on the facing page and 6.5 on the next page.

Then using the correspondences between RAF and MAF, we can establish a comparison between the logical method and the method of [6]. This comparison is synthetized in Tables 6.6 on the facing page and 6.7 on page 58.

U		$\mathcal{E}_U$	Number of Prop.
conflict-free structure	$\Rightarrow$	conflict-free set	6
admissible structure	$\Rightarrow$	admissible set	9
preferred structure	$\Rightarrow$	preferred extension	16
grounded structure	$\Rightarrow$	grounded extension	18
complete structure	$\Rightarrow$	complete extension	11
stable structure	$\Rightarrow$	stable extension	14

with $\mathcal{E}_U = S \cup \{\alpha \in \Gamma \text{ s.t. } s(\alpha) \in S\} \cup \{N_{s(\alpha)\alpha} \text{ s.t. } s(\alpha) \notin S \text{ and } s(\alpha) \in Def(U)\}$

Table 6.4: From RAF structures to MAF extensions

$\mathcal{E}$		$U_{\mathcal{E}}$	Number of Prop.
conflict-free set	$\Rightarrow$	conflict-free structure	7
admissible set	$\Rightarrow$	admissible structure	10
preferred extension	$\Rightarrow$	preferred structure	17
grounded extension	$\Rightarrow$	grounded structure	19
complete extension	$\Rightarrow$	complete structure	12
stable extension	$\Rightarrow$	stable structure	15

with $U_{\mathcal{E}} = (S_{\mathcal{E}}, \Gamma_{\mathcal{E}})$ with $S_{\mathcal{E}} = \mathcal{E} \cap \mathbf{A}$ and $\Gamma_{\mathcal{E}} = (\mathcal{E} \cap \mathbf{R}) \cup \{\alpha \notin (\mathcal{E} \cap \mathbf{R})$ s.t. $\alpha \in Acc(\mathcal{E} \cap \mathbf{A}, \mathcal{E} \cap \mathbf{R})$ and $s(\alpha) \notin \mathcal{E} \cap \mathbf{A}\})$

Table 6.5: From MAF extensions to RAF structures

$\mathcal{I}$		$\mathcal{E}_{\mathcal{I}}$
model of $\Sigma(G)$	$\Rightarrow$	conflict-free set
model of $\Sigma_d(G)$	$\Rightarrow$	admissible set
$\subseteq$-maximal model of $\Sigma_d(G)$	$\Rightarrow$	preferred extension
$\subseteq$-minimal model of $\Sigma_r(G)$	$\Rightarrow$	grounded extension
model of $\Sigma_d(G) \cup \Sigma_r(G)$	$\Rightarrow$	complete extension
model of $\Sigma_s(G)$	$\Rightarrow$	stable extension

with $S_{\mathcal{I}} = \{x \in \mathbf{A} | \mathcal{I}(Acc(x)) = \mathit{true}\}$, $\Gamma_{\mathcal{I}} = \{x \in \mathbf{R} | \mathcal{I}(Val(x)) = \mathit{true}\}$ and $\mathcal{E}_{\mathcal{I}} = S_{\mathcal{I}} \cup \{\alpha \in \Gamma_{\mathcal{I}} \text{ s.t. } s(\alpha) \in S_{\mathcal{I}}\} \cup \{N_{s(\alpha)\alpha} \text{ s.t. } s(\alpha) \notin S_{\mathcal{I}} \text{ and } s(\alpha) \in Def(S_{\mathcal{I}}, \Gamma_{\mathcal{I}})\}$

Table 6.6: From models to MAF extensions

$\mathcal{E}$		$\mathcal{I}_{\mathcal{E}}$
conflict-free set	$\Rightarrow$	model of $\Sigma(G)$
admissible set	$\Rightarrow$	model of $\Sigma_d(G)$
preferred extension	$\Rightarrow$	$\subseteq$-maximal model $\Sigma_d(G)$
grounded extension	$\Rightarrow$	$\subseteq$-minimal model of $\Sigma_r(G)$
complete extension	$\Rightarrow$	model of $\Sigma_d(G) \cup \Sigma_r(G)$
stable extension	$\Rightarrow$	model of $\Sigma_s(G)$

with $\mathcal{I}_{\mathcal{E}}$ is such that $S_{\mathcal{I}_{\mathcal{E}}} = S_{\mathcal{E}}$ and $\Gamma_{\mathcal{I}_{\mathcal{E}}} = \Gamma_{\mathcal{E}}$, that is:

$\forall x \in \mathbf{A}$, $\mathcal{I}_{\mathcal{E}}(Acc(x)) =$ *true* iff $x \in (\mathcal{E} \cap \mathbf{A})$ and

$\forall x \in \mathbf{R}$, $\mathcal{I}_{\mathcal{E}}(Val(x)) =$ *true* iff $x \in (\mathcal{E} \cap \mathbf{R}) \cup$
$\{\alpha \notin (\mathcal{E} \cap \mathbf{R})$ s.t. $\alpha \in Acc(\mathcal{E} \cap \mathbf{A}, \mathcal{E} \cap \mathbf{R})$ and $s(\alpha) \notin \mathcal{E} \cap \mathbf{A}\}$

Table 6.7: From MAF extensions to models

Chapter 7

Proofs

7.1 Proofs of Section 2.4.5

Proof of Prop. 1 on page 16:

$\Rightarrow$ Assume that U is conflict-free and $\subseteq$-minimal among the conflict-free structures satisfying $Acc(U) \subseteq S \cup \Gamma$.

First, we prove that U is a complete structure. Indeed we just have to prove that $S \cup \Gamma \subseteq Acc(U)$. If it is not the case, there is $x \in S \cup \Gamma$ such that $x \notin Acc(U)$. Consider the structure $U' = (S', \Gamma')$ obtained by removing x from U. As $U' \subset U$, it is easy to see that $Acc(U') \subseteq Acc(U)$, so $x \notin Acc(U')$. As $Acc(U) \subseteq S \cup \Gamma$, we have $Acc(U') \subseteq S \cup \Gamma$. From $x \notin Acc(U')$, we deduce that $Acc(U') \subseteq S \cup \Gamma \setminus \{x\} = S' \cup \Gamma'$. That is in contradiction with the fact that U is a $\subseteq$-minimal conflict-free structure satisfying $Acc(U) \subseteq S \cup \Gamma$. So, we have proved that $S \cup \Gamma \subseteq Acc(U)$, and so U is complete.

It remains to prove that U is $\subseteq$-minimal complete. Assume that there is a complete structure $U' = (S', \Gamma')$ strictly included in U. As U' is conflict-free and satisfies $Acc(U') \subseteq S' \cup \Gamma'$, there is a contradiction with the assumption on U.

$\Leftarrow$ Let U be a $\subseteq$-minimal complete structure. Assume that there exists $U' = (S', \Gamma')$ a conflict-free structure strictly included in U and satisfying $Acc(U') \subseteq S' \cup \Gamma'$. Without loss of generality we may assume that U' is minimal. From the first part of the proof, we know that U' is complete. So there is a contradiction with the fact that U is a $\subseteq$-minimal complete structure.

$\square$

Proof of Prop. 2 on page 16: Assume that there are two different $\subseteq$-minimal conflict-free structures satisfying $Acc(U) \subseteq S \cup \Gamma$, let $U_1 = (S_1, \Gamma_1)$ and $U_2 = (S_2, \Gamma_2)$. Let us denote by $U_1 \cap U_2$ the structure $(S_1 \cap S_2, \Gamma_1 \cap \Gamma_2)$. As U_1 and U_2 are different and minimal, the structure $U_1 \cap U_2$ is strictly included in U_1 and also in U_2. Then, it is easy to see that $Acc(U_1 \cap U_2) \subseteq Acc(U_1) \cap Acc(U_2)$. So, by hypothesis on U_1 and U_2, $Acc(U_1 \cap U_2) \subseteq (S_1 \cup \Gamma_1) \cap (S_2 \cup \Gamma_2)$. As a set of arguments and a set of attacks are disjoint, we have $(S_1 \cup \Gamma_1) \cap (S_2 \cup \Gamma_2) = (S_1 \cap S_2) \cup (\Gamma_1 \cap \Gamma_2)$. So, $Acc(U_1 \cap U_2) \subseteq (S_1 \cap S_2) \cup (\Gamma_1 \cap \Gamma_2)$. Moreover, the structure $U_1 \cap U_2$ is conflict-free. So we obtain a contradiction with the fact that U_1 (and U_2) is a $\subseteq$-minimal conflict-free structure satisfying $Acc(U) \subseteq S \cup \Gamma$. $\square$

7.2 Proofs of Section 4.2

Proof of Prop. 3 on page 23:

1. Let us recall that $\Sigma(G)$ includes formulae **(1, 2, 3)**.

   $\Rightarrow$ Assume that the structure $U = (S, \Gamma)$ is conflict-free. Let us define an interpretation $\mathcal{I}$ of $\Sigma(G)$ as follows :
   - For all $x \in \mathbf{A} \cup \mathbf{R}$, $\mathcal{I}(Argument(x)) =$ *true* if and only if $x \in \mathbf{A}$ and $\mathcal{I}(Attack(x)) =$ *true* if and only if $x \in \mathbf{R}$
   - For all $x \in \mathbf{A}$, $\mathcal{I}(Acc(x)) =$ *true* if and only if $x \in S$ and $\mathcal{I}(NAcc(x)) =$ *true* if and only if $\mathcal{I}(Acc(x)) =$ *false*.
   - For all $x \in \mathbf{R}$, $\mathcal{I}(Val(x)) =$ *true* if and only if $x \in \Gamma$.
   We have $S_{\mathcal{I}} = S$ and $\Gamma_{\mathcal{I}} = \Gamma$. It remains to prove that $\mathcal{I}$ is a model of $\Sigma(G)$.
   Obviously $\mathcal{I}$ satisfies Formula **(3)**.
   If $\mathcal{I}$ does not satisfy Formula **(2)**, there exist $x \in \mathbf{A}$ and $\alpha \in \mathbf{R}$ such that $t(\alpha) = x$, $\mathcal{I}(Val(\alpha)) =$ *true*, $\mathcal{I}(Acc(s(\alpha))) =$ *true* and $\mathcal{I}(NAcc(x)) =$ *false*.
   In other words, $\alpha \in \Gamma$, $s(\alpha) \in S$ and $x \in S$. That is in contradiction with (S, Γ) being conflict-free.
   If $\mathcal{I}$ does not satisfy Formula **(1)**, there exist $\alpha, \beta \in \mathbf{R}$ such that $t(\alpha) = \beta$, $\mathcal{I}(Val(\alpha)) =$ *true*, $\mathcal{I}(Acc(s(\alpha))) =$ *true* and $\mathcal{I}(Val(\beta)) =$ *true*.
   In other words, $\alpha, \beta \in \Gamma$ and $s(\alpha) \in S$. That is in contradiction with (S, Γ) being conflict-free.
   It follows easily that $\mathcal{I}$ is a model of $\Sigma(G)$.

   $\Leftarrow$ Let $\mathcal{I}$ be a model of $\Sigma(G)$. We prove that the structure $(S_{\mathcal{I}}, \Gamma_{\mathcal{I}})$ is conflict-free.
   If it is not the case, either there exist $a, b \in S_{\mathcal{I}}$ and $\alpha \in \Gamma_{\mathcal{I}}$ with $s(\alpha) = a$ and $t(\alpha) = b$, or there exist $\alpha, \beta \in \Gamma_{\mathcal{I}}$ with $s(\alpha) \in S_{\mathcal{I}}$ and $t(\alpha) = \beta$.
   In the first case, Formula **(2)** is falsified. In the second case, Formula **(1)** is falsified. That is in contradiction with $\mathcal{I}$ being a model of $\Sigma(G)$.

2. Let us recall that $\Sigma_d(G)$ includes formulae **(11, 12)**.

   $\Rightarrow$ Assume that the structure $U = (S, \Gamma)$ is admissible. Due to the proof of the first item, $\exists \mathcal{I}$ model of $\Sigma(G)$ with $S_{\mathcal{I}} = S$ and $\Gamma_{\mathcal{I}} = \Gamma$ and such that:
   - $\forall x \in S, \mathcal{I}(Acc(x)) =$ *true* and $\mathcal{I}(NAcc(x)) =$ *false*;
   - $\forall x \in \mathbf{A} \setminus S, \mathcal{I}(Acc(x)) =$ *false* and $\mathcal{I}(NAcc(x)) =$ *true*.
   - For all $x \in \mathbf{R}$, $\mathcal{I}(Val(x)) =$ *true* if and only if $x \in \Gamma$

   We have to prove that $\mathcal{I}$ satisfies formulae **(11, 12)**. Let us first consider formula **(11)**. Let $\alpha \in \mathbf{R}$ and $x \in \mathbf{A}$ such that $x = t_\alpha$ and $\mathcal{I}(Acc(x)) =$ *true*. That means that $x \in S$. As U is an admissible structure, we know that x is acceptable wrt U. So there exists $\beta \in \Gamma$ with $s_\beta \in S$ and $t_\beta = s_\alpha$ or $t_\beta = \alpha$. So we have $\mathcal{I}(Acc(s_\beta)) =$ *true* and $\mathcal{I}(Val(\beta)) =$ *true*. We have proved that $\mathcal{I}$ satisfies formula **(11)**. Similarly, it is easy to prove that $\mathcal{I}$ satisfies formulae **(12)**.

   $\Leftarrow$ Let $\mathcal{I}$ be a model of $\Sigma_d(G)$. We have to prove that the structure $U_{\mathcal{I}} = (S_{\mathcal{I}}, \Gamma_{\mathcal{I}})$ is admissible.
   As $\Sigma_d(G)$ contains $\Sigma(G)$, from the first item, we know that the structure $(S_{\mathcal{I}}, \Gamma_{\mathcal{I}})$ is conflict-free.
   Assume that $x \in S_{\mathcal{I}}$ is the target of an attack α. From formula **(11)** and $\mathcal{I}(Acc(x)) =$ *true*, it follows that there is $\beta \in \mathbf{R}$ with $t_\beta = s_\alpha$ or $t_\beta = \alpha$, and $\mathcal{I}(Acc(s_\beta)) =$ *true* and $\mathcal{I}(Val(\beta)) =$ *true*. That means that $s_\beta \in S_{\mathcal{I}}$ and $\beta \in \Gamma_{\mathcal{I}}$. So either $\alpha \in Inh(U_{\mathcal{I}})$ or $s_\alpha \in Def(U_{\mathcal{I}})$. We have proved that x is acceptable wrt $U_{\mathcal{I}}$. Similarly it is easy to prove that each attack of $\Gamma_{\mathcal{I}}$ is acceptable wrt $U_{\mathcal{I}}$.

3. Let us recall that $\Sigma_r(G)$ includes formulae **(13, 14)**.

   ⇒ Assume that the structure $U = (S, \Gamma)$ is complete. Due to the proof of the second item, $\exists \mathcal{I}$ model of $\Sigma_d(G)$ with $S_\mathcal{I} = S$ and $\Gamma_\mathcal{I} = \Gamma$ and such that:

   - $\forall x \in S, \mathcal{I}(Acc(x)) =$ *true* and $\mathcal{I}(NAcc(x)) =$ *false*;
   - $\forall x \in \mathbf{A} \setminus S, \mathcal{I}(Acc(x)) =$ *false* and $\mathcal{I}(NAcc(x)) =$ *true*.
   - For all $x \in \mathbf{R}$, $\mathcal{I}(Val(x)) =$ *true* if and only if $x \in \Gamma$

   We have to prove that $\mathcal{I}$ satisfies formulae **(13, 14)**. Let us first consider formula **(13)**. Let $c \in \mathbf{A}$ such that $(\forall\alpha \in Attack)(t_\alpha = c \to (\exists\beta \in Attack)(t_\beta \in \{s_\alpha, \alpha\} \wedge Val(\beta) \wedge Acc(s_\beta)))$ is satisfied by $\mathcal{I}$. We have to prove that $\mathcal{I}(Acc(c)) =$ *true*. If $\mathcal{I}$ satisfies $(\forall\alpha \in Attack)(t_\alpha = c \to (\exists\beta \in Attack)(t_\beta \in \{s_\alpha, \alpha\} \wedge Val(\beta) \wedge Acc(s_\beta)))$, $\forall\alpha \in \mathbf{R}$ s.t. $t_\alpha = c$, $\exists\beta \in \mathbf{R}$ s.t. $\mathcal{I}(Val(\beta)) =$ *true* and $\mathcal{I}(Acc(s_\beta)) =$ *true* and $t_\beta = s_\alpha$ or $t_\beta = \alpha$. In other words, due to the definition of $\mathcal{I}$, $\forall\alpha \in \mathbf{R}$ s.t. $t_\alpha = c$, $\exists\beta \in \Gamma$ s.t. $s_\beta \in S$ and $t_\beta = s_\alpha$ or $t_\beta = \alpha$. That exactly means that $\forall\alpha \in \mathbf{R}$ s.t. $t_\alpha = c$, $s_\alpha \in Def(U)$ or $\alpha \in Inh(U)$, or in other words that $c \in Acc(U)$. As U is complete, it follows that $c \in S$, so we obtain $\mathcal{I}(Acc(c)) =$ *true*. Similarly, it is easy to prove that $\mathcal{I}$ satisfies formula **(14)**.

   ⇐ Let $\mathcal{I}$ be a model of $\Sigma_d(G) \cup \Sigma_r(G)$. We know that the structure $(S_\mathcal{I}, \Gamma_\mathcal{I})$ is admissible.
   It remains to prove that each $x \in \mathbf{A}$ (resp. $x \in \mathbf{R}$) which is acceptable wrt $(S_\mathcal{I}, \Gamma_\mathcal{I})$ belongs to $S_\mathcal{I}$ (resp. $\Gamma_\mathcal{I}$). In other words, we have to prove that each $x \in \mathbf{A}$ (resp. $x \in \mathbf{R}$) which is acceptable wrt $(S_\mathcal{I}, \Gamma_\mathcal{I})$ satisfies $\mathcal{I}(Acc(x)) =$ *true* (resp. $\mathcal{I}(Val(x)) =$ *true*). That follows easily from the fact that $\mathcal{I}$ satisfies formula **(13)** (resp. **(14)**) instantiated with $c = x$ (resp. $\delta = x$).

4. Let us recall that $\Sigma_s(G)$ includes formulae **(15, 16)**.

   ⇒ Assume that the structure (S, Γ) is stable. Due to the proof of the first item, $\exists\mathcal{I}$ model of $\Sigma(G)$ with $S_\mathcal{I} = S$ and $\Gamma_\mathcal{I} = \Gamma$ and such that:

   - $\forall x \in S, \mathcal{I}(Acc(x)) =$ *true* and $\mathcal{I}(NAcc(x)) =$ *false*;
   - $\forall x \in \mathbf{A} \setminus S, \mathcal{I}(Acc(x)) =$ *false* and $\mathcal{I}(NAcc(x)) =$ *true*.
   - For all $x \in \mathbf{R}$, $\mathcal{I}(Val(x)) =$ *true* if and only if $x \in \Gamma$

   We have to prove that $\mathcal{I}$ satisfies formulae **(15, 16)**. Let us first consider formula **(15)**. Let $c \in \mathbf{A}$ s.t. $\mathcal{I}(\neg Acc(c)) =$ *true*. It means that $c \in \mathbf{A} \setminus S$. As U is stable, we have that $c \in Def(U)$. So there exists $\beta \in \Gamma$ s.t. $t_\beta = c$ and $s_\beta \in S$. Then we have that $\mathcal{I}(Val(\beta)) =$ *true* and $\mathcal{I}(Acc(s_\beta)) =$ *true* which proves that $\mathcal{I}$ satisfies formula **(15)**. Similarly, it is easy to prove that $\mathcal{I}$ satisfies formula **(16)**.

   ⇐ Let $\mathcal{I}$ be a model of $\Sigma_s(G)$. As $\Sigma_s(G)$ contains $\Sigma(G)$, from the first item, we know that the structure $(S_\mathcal{I}, \Gamma_\mathcal{I})$ is conflict-free. Then the fact that $\mathcal{I}$ satisfies formulae **(15, 16)** enables to prove that the structure $(S_\mathcal{I}, \Gamma_\mathcal{I})$ satisfies the two following conditions: $\forall a \in \mathbf{A} \setminus S_\mathcal{I}$, a is defeated wrt $(S_\mathcal{I}, \Gamma_\mathcal{I})$, and $\forall\alpha \in \mathbf{R} \setminus \Gamma_\mathcal{I}$, α is inhibited wrt $(S_\mathcal{I}, \Gamma_\mathcal{I})$. So the structure $(S_\mathcal{I}, \Gamma_\mathcal{I})$ is stable.

5. Let $\mathcal{I}$ be an interpretation of a set of formulae Σ. Let $U_\mathcal{I}$ denote the structure $(S_\mathcal{I}, \Gamma_\mathcal{I})$. It is easy to see that $\mathcal{I}$ is a $\subseteq$-maximal model of Σ iff the structure $U_\mathcal{I}$ is $\subseteq$-maximal among all the structures of the form $U_\mathcal{J} = (S_\mathcal{J}, \Gamma_\mathcal{J})$, where $\mathcal{J}$ denotes an interpretation of Σ. Then taking $\Sigma = \Sigma_d(G)$, it follows that the preferred structures correspond to the structures $U_\mathcal{I}$ where $\mathcal{I}$ is a $\subseteq$-maximal model of $\Sigma_d(G)$.

6. Let $\mathcal{I}$ be an interpretation of a set of formulae Σ. Let $U_\mathcal{I}$ denote the structure $(S_\mathcal{I}, \Gamma_\mathcal{I})$. It is easy to see that $\mathcal{I}$ is a $\subseteq$-minimal model of Σ iff the structure $U_\mathcal{I}$ is $\subseteq$-minimal among all the structures of the form $U_\mathcal{J}$, where $\mathcal{J}$ denotes an interpretation of Σ. In particular, it holds for $\Sigma = \Sigma_r(G)$.

From the third item of the current proof, it holds that $\mathcal{I}$ satisfies formulae **(13, 14)** if and only if $Acc(U_{\mathcal{I}}) \subseteq (S_{\mathcal{I}} \cup \Gamma_{\mathcal{I}})$. By definition, (see Definition 25) the grounded structure is the $\subseteq$-minimal conflict-free structure $U = (S, \Gamma)$ satisfying $Acc(U) \subseteq S \cup \Gamma$. It follows that the grounded structure corresponds to the structure $U_{\mathcal{I}}$ where $\mathcal{I}$ is a $\subseteq$-minimal model of $\Sigma_r(G)$.

□

Proof of Prop. 4 on page 24: Let $U = (S, \Gamma)$ be a conflict-free structure. Let us consider the model $\mathcal{I}$ of $\Sigma(G)$ defined in the first item of the proof of Proposition 3. We have $S = S_{\mathcal{I}}$ and $\Gamma = \Gamma_{\mathcal{I}}$. Let us prove that $\mathcal{I}$ satisfies formula **(14)** iff $Acc(U_{\mathcal{I}}) \cap \mathbf{R} \subseteq \Gamma_{\mathcal{I}}$. Then, from this result and Proposition 3, it will follow easily that U is a conflict-free (resp. admissible) d-structure iff $\exists\mathcal{I}$ model of $\Sigma(G) \cup \{\Phi_r^{att}(G)\}$ such that $S_{\mathcal{I}} = S$ and $\Gamma_{\mathcal{I}} = \Gamma$.

Formula **(14)** (or $\Phi_r^{att}(G)$) writes:

$$(\forall\delta \in \mathit{Attack})(((\forall\alpha \in \mathit{Attack})(t_\alpha = \delta \to (\exists\beta \in \mathit{Attack})(t_\beta \in \{s_\alpha, \alpha\} \wedge Val(\beta) \wedge Acc(s_\beta)))) \to Val(\delta))$$

1. Assume that $\mathcal{I}$ satisfies formula $\Phi_r^{att}(G)$. Let $\delta \in Acc(U_{\mathcal{I}}) \cap \mathbf{R}$. If δ is not attacked, the formula $(\forall\alpha \in \mathit{Attack})(t_\alpha = \delta \to (\exists\beta \in \mathit{Attack})(t_\beta \in \{s_\alpha, \alpha\} \wedge Val(\beta) \wedge Acc(s_\beta)))$ is trivially true. So, $Val(\delta)$ must also be satisfied by $\mathcal{I}$. That means $\delta \in \Gamma_{\mathcal{I}}$. If δ is attacked, as $\delta \in Acc(U_{\mathcal{I}})$, for each α attacking δ, either $\alpha \in Inh(U_{\mathcal{I}})$ or $s(\alpha) \in Def(U_{\mathcal{I}})$. So, for each α attacking δ, there is $\beta \in \Gamma_{\mathcal{I}}$ such that $s(\beta) \in S_{\mathcal{I}}$ and $t(\beta) \in \{\alpha, s(\alpha)\}$. In other words, for each α attacking δ, there is $\beta \in \mathbf{R}$ such that $\mathcal{I}$ satisfies the formulae $Val(\beta)$, $Acc(s_\beta)$ and $t_\beta \in \{\alpha, s_\alpha\}$. So, $Val(\delta)$ must also be satisfied by $\mathcal{I}$. That means $\delta \in \Gamma_{\mathcal{I}}$.

2. Assume that $Acc(U_{\mathcal{I}}) \cap \mathbf{R} \subseteq \Gamma_{\mathcal{I}}$. Let us prove that $\mathcal{I}$ satisfies formula $\Phi_r^{att}(G)$. Let δ be an attack such that $\mathcal{I}$ satisfies the formula $(\forall\alpha \in \mathit{Attack})(t_\alpha = \delta \to (\exists\beta \in \mathit{Attack})(t_\beta \in \{s_\alpha, \alpha\} \wedge Val(\beta) \wedge Acc(s_\beta)))$. Then for each attack α attacking δ, there is an attack β such that $\mathcal{I}$ satisfies the formula $t_\beta \in \{s_\alpha, \alpha\} \wedge Val(\beta) \wedge Acc(s_\beta)$. As $\mathcal{I}$ satisfies $Val(\beta) \wedge Acc(s_\beta)$ means that $\beta \in \Gamma_{\mathcal{I}}$ and $s(\beta) \in S_{\mathcal{I}}$, we have that $\delta \in Acc(U_{\mathcal{I}}) \cap \mathbf{R}$. By hypothesis $Acc(U_{\mathcal{I}}) \cap \mathbf{R} \subseteq \Gamma_{\mathcal{I}}$, so $\delta \in \Gamma_{\mathcal{I}}$ and then $\mathcal{I}$ satisfies $Val(\delta)$. We have proved that $\mathcal{I}$ satisfies formula $\Phi_r^{att}(G)$.

□

7.3 Proofs of Section 6

Proof of Prop. 5 on page 52: It follows directly from Definition 27 and Observation 1. □

The following lemmas are used in the proofs of the other propositions of Section 6 on page 49.

Lemma 1 *Let $\mathcal{E}$ be an admissible extension of the MAF $\langle \mathbf{A}', \mathbf{R}' \rangle$.*

1. *If $\mathcal{E} \cap \mathbf{R}$ contains α, then $\mathcal{E}$ contains $s(\alpha)$.*

2. *If $\mathcal{E}$ contains $N_{s(\alpha)\alpha}$ then $\mathcal{E} \cap \mathbf{R}$ attacks $s(\alpha)$.*

3. *Moreover, if $\mathcal{E}$ is complete the equivalence holds: $\mathcal{E}$ contains $N_{s(\alpha)\alpha}$ if and only if $\mathcal{E} \cap \mathbf{R}$ attacks $s(\alpha)$.*

Proof of Lem. 1 on the facing page: Let $N_{s(\alpha)\alpha} \in \mathbf{N}$. Due to the definition of $\mathbf{R}'$ (see Def. 27), the only attack to $N_{s(\alpha)\alpha}$ is the attack $(s(\alpha), N_{s(\alpha)\alpha})$.

1. As $\mathcal{E}$ is admissible, α is acceptable wrt $\mathcal{E}$. As $s(\alpha)$ is the only defender of α against $N_{s(\alpha)\alpha}$, $s(\alpha)$ must belong to $\mathcal{E}$.
2. If $\mathcal{E}$ is admissible and contains $N_{s(\alpha)\alpha}$, $N_{s(\alpha)\alpha}$ is acceptable wrt $\mathcal{E}$ so $\mathcal{E}$ must attack $s(\alpha)$. Due to the definition of $\mathbf{R}'$ again (see Def. 27), this attack comes from $\mathcal{E} \cap \mathbf{R}$.
3. If $\mathcal{E}$ is complete and $\mathcal{E} \cap \mathbf{R}$ attacks $s(\alpha)$ then $N_{s(\alpha)\alpha}$ is acceptable wrt $\mathcal{E}$. So $N_{s(\alpha)\alpha}$ must belong to $\mathcal{E}$.

□

Lemma 2 *Let $U = (S, \Gamma)$ be a structure. Let U' be the structure$(S, \mathcal{E}_U \cap \mathbf{R})$. It holds that $Acc(U)$ = $Acc(U')$.*

Proof of Lem. 2: By definition $\mathcal{E}_U \cap \mathbf{R} = \{\alpha \in \Gamma \text{ s.t. } s(\alpha) \in S\})$. Let $x \in \mathbf{A} \cup \mathbf{R}$. By definition, $x \in Acc(U)$ if and only if for each attack $\beta \in \mathbf{R}$ such that $t(\beta) = x$, there exists $\gamma \in \Gamma$ with $s(\gamma) \in S$ and $t(\gamma) \in \{\beta, s(\beta)\}$. Obviously, $\gamma \in \Gamma$ with $s(\gamma) \in S$ is equivalent to $\gamma \in \mathcal{E}_U \cap \mathbf{R}$ with $s(\gamma) \in S$. So $x \in Acc(U)$ if and only if $x \in Acc(U')$. □

Lemma 3 *Let $U = (S, \Gamma)$ be a structure, $\mathcal{E}_U$ be the associated extension of the MAF, and $U_{\mathcal{E}_U} = (S_{\mathcal{E}_U}, \Gamma_{\mathcal{E}_U})$ be the structure associated with the extension $\mathcal{E}_U$. The following assertions hold:*

1. $S_{\mathcal{E}_U} = S$
2. *If U is an admissible structure, then $\Gamma \subseteq \Gamma_{\mathcal{E}_U}$*
3. *If $(Acc(U) \cap \mathbf{R}) \subseteq \Gamma$, then $\Gamma_{\mathcal{E}_U} \subseteq \Gamma$*
4. *If U is a complete structure, then $\Gamma = \Gamma_{\mathcal{E}_U}$*

Proof of Lem. 3:

1. By definition of $U_{\mathcal{E}_U}$ we have $S_{\mathcal{E}_U} = \mathcal{E}_U \cap \mathbf{A}$. By definition of $\mathcal{E}_U$, we have $\mathcal{E}_U \cap \mathbf{A} = S$.
2. By definition, $\Gamma_{\mathcal{E}_U} = (\mathcal{E}_U \cap \mathbf{R}) \cup \{\alpha \notin (\mathcal{E}_U \cap \mathbf{R}) \text{ s.t. } \alpha \in Acc((\mathcal{E}_U \cap \mathbf{A}), (\mathcal{E}_U \cap \mathbf{R}))$ and $s(\alpha) \notin (\mathcal{E}_U \cap \mathbf{A})\}$. Note that by definition, $(\mathcal{E}_U \cap \mathbf{A}) = S$ and $(\mathcal{E}_U \cap \mathbf{R}) = \{\gamma \in \Gamma$ with $s(\gamma) \in S\}$. And from Lemma 2, we have $Acc((\mathcal{E}_U \cap \mathbf{A}), (\mathcal{E}_U \cap \mathbf{R})) = Acc(U)$.
   Let $\alpha \in \Gamma$. If $s(\alpha) \in S$, $\alpha \in (\mathcal{E}_U \cap \mathbf{R})$ so $\alpha \in \Gamma_{\mathcal{E}_U}$. If $s(\alpha) \notin S$, as U is admissible, we have $\alpha \in Acc(U)$, so α belongs to the second part of $\Gamma_{\mathcal{E}_U}$.
3. Assume that $(Acc(U) \cap \mathbf{R}) \subseteq \Gamma$. Let $\alpha \in \Gamma_{\mathcal{E}_U}$. If $\alpha \in (\mathcal{E}_U \cap \mathbf{R}) \subseteq \Gamma$, we have $\alpha \in \Gamma$. If $\alpha \notin (\mathcal{E}_U \cap \mathbf{R})$, by definition of $\Gamma_{\mathcal{E}_U}$, we have $\alpha \in Acc(U)$. Due to the assumption, it follows that $\alpha \in \Gamma$.
4. The proof follows directly from the above items of this lemma.

□

Lemma 4 *Let $\mathcal{E}$ be an extension of the MAF. Let $U_{\mathcal{E}}$ be its associated structure and $\mathcal{E}_{U_{\mathcal{E}}}$ be the MAF extension associated with $U_{\mathcal{E}}$. The following assertions hold:*

1. $\mathcal{E}_{U_\mathcal{E}} \cap \mathbf{A} = \mathcal{E} \cap \mathbf{A}$

2. $\mathcal{E}_{U_\mathcal{E}} \cap \mathbf{R} \subseteq \mathcal{E} \cap \mathbf{R}$

3. If $\mathcal{E}$ is admissible, then $\mathcal{E}_{U_\mathcal{E}} \cap \mathbf{R} = \mathcal{E} \cap \mathbf{R}$

4. If $\mathcal{E}$ is admissible, then $\mathcal{E} \cap \mathbf{N} \subseteq \mathcal{E}_{U_\mathcal{E}} \cap \mathbf{N}$

5. If $\mathcal{E}$ is complete then $\mathcal{E}_{U_\mathcal{E}} \cap \mathbf{N} = \mathcal{E} \cap \mathbf{N}$

Proof of Lem. 4 on the previous page: By definition, $U_\mathcal{E} = (S_\mathcal{E}, \Gamma_\mathcal{E})$ with $S_\mathcal{E} = \mathcal{E}_a = \mathcal{E} \cap \mathbf{A}$, $\mathcal{E}_r = \mathcal{E} \cap \mathbf{R}$, and $\Gamma_\mathcal{E} = \mathcal{E}_r \cup \{\alpha \notin \mathcal{E}_r$ s.t. $\alpha \in Acc(\mathcal{E}_a, \mathcal{E}_r)$ and $s(\alpha) \notin \mathcal{E}_a\}$.
Then, $\mathcal{E}_{U_\mathcal{E}}$ is such that $\mathcal{E}_{U_\mathcal{E}} \cap \mathbf{A} = S_\mathcal{E}$, $\mathcal{E}_{U_\mathcal{E}} \cap \mathbf{R} = \{\alpha \in \Gamma_\mathcal{E}$ such that $s(\alpha) \in S_\mathcal{E}\}$, and $\mathcal{E}_{U_\mathcal{E}} \cap \mathbf{N} = \{N_{s(\alpha)\alpha}$ s.t. $s(\alpha) \notin S_\mathcal{E}$ and $s(\alpha) \in Def(U_\mathcal{E})\}$.

1. Obviously, $\mathcal{E}_{U_\mathcal{E}} \cap \mathbf{A} = \mathcal{E} \cap \mathbf{A}$.
2. By definition of $\Gamma_\mathcal{E}$, if $\alpha \in \Gamma_\mathcal{E}$ with $s(\alpha) \in S_\mathcal{E} = \mathcal{E}_a$ then $\alpha \in \mathcal{E}_r$. So, $\mathcal{E}_{U_\mathcal{E}} \cap \mathbf{R} \subseteq \mathcal{E}_r$.
3. Assume that $\mathcal{E}$ is admissible. Let $\alpha \in \mathcal{E}_r$. From Lemma 1, we have $s(\alpha) \in \mathcal{E}_a$. As $\mathcal{E}_r \subseteq \Gamma_\mathcal{E}$ it follows that $\alpha \in \Gamma_\mathcal{E}$ and $s(\alpha) \in \mathcal{E}_a$. So $\alpha \in \mathcal{E}_{U_\mathcal{E}} \cap \mathbf{R}$. From the above item we conclude that $\mathcal{E}_{U_\mathcal{E}} \cap \mathbf{R} = \mathcal{E}_r$.
4. Assume that $\mathcal{E}$ is admissible. Let $N_{s(\alpha)\alpha} \in \mathcal{E} \cap \mathbf{N}$. From Lemma 1, we know that $\mathcal{E}_r$ attacks $s(\alpha)$. So there exists $\beta \in \mathcal{E}_r$ that attacks $s(\alpha)$. From Lemma 1 again, we have $s(\beta) \in \mathcal{E}$. So, $s(\alpha)$ is attacked by $\beta \in \mathcal{E}_r \subseteq \Gamma_\mathcal{E}$ with $s(\beta) \in \mathcal{E}_a$. That means that $s(\alpha) \in Def(U_\mathcal{E})$. Moreover, as $\mathcal{E}$ is conflict-free, $s(\alpha) \notin \mathcal{E}_a$. It follows that $N_{s(\alpha)\alpha} \in \mathcal{E}_{U_\mathcal{E}} \cap \mathbf{N}$.
5. Assume that $\mathcal{E}$ is complete. Let $N_{s(\alpha)\alpha} \in \mathcal{E}_{U_\mathcal{E}} \cap \mathbf{N}$. By definition, $s(\alpha) \notin S_\mathcal{E} = \mathcal{E}_a$ and $s(\alpha) \in Def(S_\mathcal{E}, \Gamma_\mathcal{E})$. So there exists $\beta \in \Gamma_\mathcal{E}$ with $s(\beta) \in \mathcal{E}_a$ such that β attacks $s(\alpha)$. By definition of $\Gamma_\mathcal{E}$ it follows that $\beta \in \mathcal{E}_r$. Then from Lemma 1, we conclude that $\mathcal{E}$ contains $N_{s(\alpha)\alpha}$. Then from the above item of this lemma, as E is also admissible, we conclude that $\mathcal{E}_{U_\mathcal{E}} \cap \mathbf{N} = \mathcal{E} \cap \mathbf{N}$.

□

Lemma 5 *Let $\mathcal{E}$ and $\mathcal{E}'$ be two complete extensions of the MAF such that $\mathcal{E} \subseteq \mathcal{E}'$ then $U_\mathcal{E} \subseteq U_{\mathcal{E}'}$.*

Proof of Lem. 5: By definition, $U_\mathcal{E} = (S_\mathcal{E}, \Gamma_\mathcal{E})$ with $S_\mathcal{E} = \mathcal{E}_a = \mathcal{E} \cap \mathbf{A}$, $\mathcal{E}_r = \mathcal{E} \cap \mathbf{R}$, and $\Gamma_\mathcal{E} = \mathcal{E}_r \cup \{\alpha \notin \mathcal{E}_r$ s.t. $\alpha \in Acc(\mathcal{E}_a, \mathcal{E}_r)$ and $s(\alpha) \notin \mathcal{E}_a\}$.
First, we have $\mathcal{E}_a \subseteq \mathcal{E}'_a$ and $\mathcal{E}_r \subseteq \mathcal{E}'_r$. Then, due to Lemma 1, as $\mathcal{E}$ (resp. $\mathcal{E}'$) is admissible, $s(\alpha) \notin \mathcal{E}_a$ (resp. $s(\alpha) \notin \mathcal{E}'_a$) implies that $\alpha \notin \mathcal{E}_r$ (resp. $\alpha \notin \mathcal{E}'_r$). So it is enough to prove that: $\{\alpha \in Acc(\mathcal{E}_a, \mathcal{E}_r)$ such that $s(\alpha) \notin \mathcal{E}_a\} \subseteq \mathcal{E}'_r \cup \{\alpha \in Acc(\mathcal{E}'_a, \mathcal{E}'_r)$ such that $s(\alpha) \notin \mathcal{E}'_a\}$.
Furthermore, we have $Acc(\mathcal{E}_a, \mathcal{E}_r) \subseteq Acc(\mathcal{E}'_a, \mathcal{E}'_r)$ so it is enough to show that: For each $\alpha \in Acc(\mathcal{E}_a, \mathcal{E}_r)$ such that $s(\alpha) \notin \mathcal{E}_a$, either $\alpha \in \mathcal{E}'_r$ or $s(\alpha) \notin \mathcal{E}'_a$.
Assume that the contrary holds. So there is $\alpha \in Acc(\mathcal{E}_a, \mathcal{E}_r)$ such that $\alpha \notin \mathcal{E}'_r$, $s(\alpha) \notin \mathcal{E}_a$ and $s(\alpha) \in \mathcal{E}'_a$. As $\mathcal{E}'$ is a complete extension of the MAF, α is not acceptable wrt $\mathcal{E}'$. Moreover, as $s(\alpha) \in \mathcal{E}'_a$, α is defended by $\mathcal{E}'$ against its attacker $N_{s(\alpha)\alpha}$. So there must exist another attacker of α, say β, such that $\mathcal{E}'$ does not attack β. That implies that $N_{s(\beta)\beta} \notin \mathcal{E}'$, and from Lemma 1 that $\mathcal{E}'$ does not attack $s(\beta)$. So $\mathcal{E}'$ attacks neither β, nor $s(\beta)$; this fact will be denoted by (*).
Moreover, as $\alpha \in Acc(\mathcal{E}_a, \mathcal{E}_r)$, either $\beta \in Inh(\mathcal{E}_a, \mathcal{E}_r)$ (Case 1), or $s(\beta) \in Def(\mathcal{E}_a, \mathcal{E}_r)$ (Case 2). It follows that there is $\gamma \in \mathcal{E}_r \subseteq \mathcal{E}'_r$ with $s(\gamma) \in \mathcal{E}_a$ such that $t(\gamma) = \beta$ in Case 1 (resp.

$t(\gamma) = s(\beta)$ in Case 2). So $\mathcal{E}'_r$ attacks β in Case 1 (resp. $s(\beta)$ in Case 2), which is in contradiction with the fact (*).

□

Lemma 6 *Let U and U' be two conflict-free structures such that $U \subseteq U'$ then $\mathcal{E}_U \subseteq \mathcal{E}_{U'}$.*

Proof of Lem. 6: By definition, U being the structure (S, Γ), $\mathcal{E}_U = S \cup \{\alpha \in \Gamma \text{ s.t. } s(\alpha) \in S\} \cup \{N_{s(\alpha)\alpha} \text{ s.t. } s(\alpha) \notin S \text{ and } s(\alpha) \in Def(U)\}$.
First, we have $S \subseteq S'$ and $\{\alpha \in \Gamma \text{ s.t. } s(\alpha) \in S\} \subseteq \{\alpha \in \Gamma' \text{ s.t. } s(\alpha) \in S'\}$. So it is enough to prove that $(\mathcal{E}_U \cap \mathbf{N}) \subseteq (\mathcal{E}_{U'} \cap \mathbf{N})$.
Let $x = N_{s(\alpha)\alpha} \in \mathcal{E}_U \cap \mathbf{N}$. As $U \subseteq U'$, we have $Def(U) \subseteq Def(U')$. So $s(\alpha) \in Def(U')$. Hence, as U' is a conflict-free structure, it is impossible to have $s(\alpha) \in S'$. So $x = N_{s(\alpha)\alpha} \in \mathcal{E}_{U'} \cap \mathbf{N}$.

□

Proof of Prop. 6 on page 54: Let $U = (S, \Gamma)$ be a conflict-free structure of the RAF.

1. Assume that $\mathcal{E}'_U = S \cup \{\alpha \in \Gamma \text{ s.t. } s(\alpha) \in S\}$ is not conflict-free in the MAF $\langle \mathbf{A}', \mathbf{R}' \rangle$. Due to Observation 1, the only possible conflict comes from an attack of the form $(\alpha, t(\alpha))$ with $\alpha \in (\mathcal{E}'_U \cap \Gamma)$ and $t(\alpha) \in \mathcal{E}'_U$ (*i.e.* $t(\alpha) \in S \subseteq \mathbf{A}$ or $t(\alpha) \in (\mathcal{E}'_U \cap \Gamma) \subseteq \mathbf{R}$). Moreover, for $\alpha \in (\mathcal{E}'_U \cap \Gamma)$ we have $s(\alpha) \in S$; so, $t(\alpha)$ is either defeated (if $t(\alpha) \in \mathbf{A}$) or inhibited (if $t(\alpha) \in \mathbf{R}$) wrt U. And so there is a contradiction with U being conflict-free.

2. Assume that $\mathcal{E}_U = \mathcal{E}'_U \cup \{N_{s(\alpha)\alpha} \text{ s.t. } s(\alpha) \notin S \text{ and } s(\alpha) \in Def(U)\}$ is not conflict-free. From the first part of the proof, the only possible conflict comes from an attack from S to $(\mathcal{E}_U \cap \mathbf{N})$ or from $(\mathcal{E}_U \cap \mathbf{N})$ to $(\mathcal{E}_U \cap \Gamma)$.
In the first case, there is an attack of the form $(s(\alpha), N_{s(\alpha)\alpha})$ with $s(\alpha) \in S$ and $N_{s(\alpha)\alpha} \in (\mathcal{E}_U \cap \mathbf{N})$. However, for $N_{s(\alpha)\alpha} \in (\mathcal{E}_U \cap \mathbf{N})$, we have $s(\alpha) \notin S$. So there is a contradiction about $s(\alpha)$.
In the second case, there is an attack of the form $(N_{s(\alpha)\alpha}, \alpha)$ with $N_{s(\alpha)\alpha} \in (\mathcal{E}_U \cap \mathbf{N})$ and $\alpha \in (\mathcal{E}_U \cap \Gamma)$. However, for $\alpha \in (\mathcal{E}_U \cap \Gamma)$, we have $s(\alpha) \in S$ so there is a contradiction with $N_{s(\alpha)\alpha} \in (\mathcal{E}_U \cap \mathbf{N})$.

□

Proof of Prop. 7 on page 54:

1. Assume that $U'_{\mathcal{E}} = (\mathcal{E}_a, \mathcal{E}_r)$ is not a conflict-free structure of the RAF. So either $\mathcal{E}_a \cap Def(U'_{\mathcal{E}}) \neq \varnothing$ (Case 1) or $\mathcal{E}_r \cap Inh(U'_{\mathcal{E}}) \neq \varnothing$ (Case 2).
In Case 1, $\exists a \in \mathcal{E}_a, \beta \in \mathcal{E}_r$ s.t. $s(\beta) \in \mathcal{E}_a$ and $t(\beta) = a$. So, due to Def. 27, we have $(\beta, a) \in \mathbf{R}'$. That is in contradiction with $\mathcal{E}$ being conflict-free in the MAF.
In Case 2, $\exists \alpha \in \mathcal{E}_r, \beta \in \mathcal{E}_r$ s.t. $s(\beta) \in \mathcal{E}_a$ and $t(\beta) = \alpha$. So, due to Def. 27, we have $(\beta, \alpha) \in \mathbf{R}'$. That is in contradiction with $\mathcal{E}$ being conflict-free in the MAF.

2. Assume that $U_{\mathcal{E}} = (\mathcal{E}_a, \Gamma_{\mathcal{E}})$ is not a conflict-free structure of the RAF. So either $\mathcal{E}_a \cap Def(U_{\mathcal{E}}) \neq \varnothing$ (Case 1) or $\Gamma_{\mathcal{E}} \cap Inh(U_{\mathcal{E}}) \neq \varnothing$ (Case 2).
Let us recall that $\Gamma_{\mathcal{E}} = \mathcal{E}_r \cup \{\alpha \notin \mathcal{E}_r \text{ s.t. } s(\alpha) \notin \mathcal{E}_a \text{ and } \alpha \in Acc(U'_{\mathcal{E}})\}$.
In Case 1, $\exists a \in \mathcal{E}_a, \beta \in \Gamma_{\mathcal{E}}$ s.t. $s(\beta) \in \mathcal{E}_a$ and $t(\beta) = a$. Due to the definition of $\Gamma_{\mathcal{E}}$, as $s(\beta) \in \mathcal{E}_a$, we have $\beta \in \mathcal{E}_r$. So we are back to the first part of the proof (Case 1) and we get a contradiction with $\mathcal{E}$ being conflict-free in the MAF.
In Case 2, $\exists \alpha \in \Gamma_{\mathcal{E}}, \beta \in \Gamma_{\mathcal{E}}$ s.t. $s(\beta) \in \mathcal{E}_a$ and $t(\beta) = \alpha$. Due to the definition of $\Gamma_{\mathcal{E}}$, as $s(\beta) \in \mathcal{E}_a$, we

have $\beta \in \mathcal{E}_r$. Due to the first part of the proof (Case 2), we cannot have $\alpha \in \mathcal{E}_r$. So we have $s(\alpha) \notin \mathcal{E}_a$ and $\alpha \in Acc(U'_{\mathcal{E}})$. From the second condition, it follows that $\beta \in Inh(U'_{\mathcal{E}})$ or $s(\beta) \in Def(U'_{\mathcal{E}})$. However, $s(\beta) \in \mathcal{E}_a$, and $\beta \in \mathcal{E}_r$. Moreover $U'_{\mathcal{E}} = (\mathcal{E}_a, \mathcal{E}_r)$ is a conflict-free structure, due to the first part of the proof. So we obtain a contradiction.

□

Proof of Prop. 8 on page 54:

1. Let a be acceptable wrt U in the RAF. We have to prove that a is acceptable wrt $\mathcal{E}_U$ in the MAF $\langle \mathbf{A}', \mathbf{R}' \rangle$. If a is not attacked in the MAF, it is trivially acceptable wrt $\mathcal{E}_U$. So, let us assume that $a \in \mathbf{A}$ is attacked in the MAF. Due to Observation 1, there is $\alpha \in \mathbf{R}$ with $(\alpha, a) \in \mathbf{R}'$, or in other words $\alpha \in \mathbf{R}$ with $t(\alpha) = a$. So there are also in $\mathbf{R}'$ the attacks $(s(\alpha), N_{s(\alpha)\alpha})$ and $(N_{s(\alpha)\alpha}, \alpha)$.
   As a is acceptable wrt U in the RAF, either $\alpha \in Inh(U)$, or $s(\alpha) \in Def(U)$. It means that $\exists \beta \in \Gamma$ s.t. $s(\beta) \in S$ and $t(\beta) \in \{\alpha, s(\alpha)\}$. So $\beta \in (\mathcal{E}_U \cap \Gamma)$.
   If $t(\beta) = \alpha$ then $(\beta, \alpha) \in \mathbf{R}'$ and we have that $\mathcal{E}_U$ attacks α.
   If $t(\beta) = s(\alpha)$ then $s(\alpha) \in Def(U)$ and, since U is assumed to be conflict-free, we have $s(\alpha) \notin S$; so we can prove that $N_{s(\alpha)\alpha} \in (\mathcal{E}_U \cap \mathbf{N})$. And then $\mathcal{E}_U$ attacks α.

2. Let α be acceptable wrt U in the RAF, with $s(\alpha \in S$. We have to prove that α is acceptable wrt $\mathcal{E}_U$ in the MAF $\langle \mathbf{A}', \mathbf{R}' \rangle$.
   $\alpha \in \mathbf{R}$ is always attacked in the MAF $\langle \mathbf{A}', \mathbf{R}' \rangle$. Due to Observation 1, we have to consider two kinds of attack, namely an attack of the form $(N_{s(\alpha)\alpha}, \alpha)$ and an attack of the form (γ, α) with $\gamma \in \mathbf{R}$.
   In the first case, as it is assumed that $s(\alpha) \in S$, we have $s(\alpha) \in \mathcal{E}_U$. As we also have the attack $(s(\alpha), N_{s(\alpha)\alpha})$ in $\mathbf{R}'$, we conclude that $\mathcal{E}_U$ attacks $N_{s(\alpha)\alpha}$.
   In the second case, as α is acceptable wrt U in the RAF, $\gamma \in Inh(U)$, or $s(\gamma) \in Def(U)$. It means that $\exists \beta \in \Gamma$ s.t. $s(\beta) \in S$ and $t(\beta) \in \{\gamma, s(\gamma)\}$. So $\beta \in (\mathcal{E}_U \cap \Gamma)$. As done in the first part of the proof, we prove that $\mathcal{E}_U$ attacks γ.

□

Proof of Prop. 9 on page 55: Let $U = (S, \Gamma)$ be an admissible structure of the RAF.
From Prop. 6, $\mathcal{E}_U$ is conflict-free in the MAF $\langle \mathbf{A}', \mathbf{R}' \rangle$.
It remains to prove that $\forall x \in \mathcal{E}_U$, x is acceptable wrt $\mathcal{E}_U$. Three cases must be considered for x:

1. Let $x \in \mathcal{E}_U \cap \mathbf{A}$. So $x \in S$. As U is admissible, x is acceptable wrt U. From Prop. 8, it follows that x is acceptable wrt $\mathcal{E}_U$ in the MAF.

2. Let $x \in \mathcal{E}_U \cap \mathbf{R}$. As U is admissible, and $x \in \Gamma$, x is acceptable wrt U. Moreover from the definition of $\mathcal{E}_U$ we have $s(x) \in S$. So Prop. 8 applies and we conclude that x is acceptable wrt $\mathcal{E}_U$ in the MAF.

3. Let $x \in \mathcal{E}_U \cap \mathbf{N}$. So x has the form $N_{s(\alpha)\alpha}$ with $s(\alpha) \notin S$ and $s(\alpha) \in Def(U)$.
   The only possible attack to x is from $s(\alpha)$. As $s(\alpha) \in Def(U)$, $\exists \beta \in \Gamma$ s.t. $s(\beta) \in S$ and $t(\beta) = s(\alpha)$. From the definition of $\mathcal{E}_U$, it follows that $\beta \in \mathcal{E}_U$ and then that $\mathcal{E}_U$ attacks $s(\alpha)$. So $N_{s(\alpha)\alpha}$ is acceptable wrt $\mathcal{E}_U$ in the MAF.

Hence we have proved that $\mathcal{E}_U$ is an admissible extension of the MAF $\langle \mathbf{A}', \mathbf{R}' \rangle$. □

Proof of Prop. 10 on page 55: Let $\mathcal{E}$ be an admissible extension of the MAF $\langle \mathbf{A}', \mathbf{R}' \rangle$.

1. From Prop. 7, $U'_{\mathcal{E}}$ is a conflict-free structure of the RAF.
   It remains to prove that $\forall x \in (\mathcal{E}_a \cup \mathcal{E}_r)$, x is acceptable wrt $U'_{\mathcal{E}}$. If x is unattacked in the RAF, then it is obviously acceptable wrt $U'_{\mathcal{E}}$. Otherwise two cases must be considered for x:

(a) Let $x \in \mathcal{E}_a$. Assume that x is attacked by $\alpha \in \mathbf{R}$. We have to prove that either $\alpha \in Inh(U'_\mathcal{E})$ or $s(\alpha) \in Def(U'_\mathcal{E})$.
The attack α is encoded in the MAF with the three following attacks in $\mathbf{R}'$: $(s(\alpha), N_{s(\alpha)\alpha})$, $(N_{s(\alpha)\alpha}, \alpha)$ and (α, x).
As $\mathcal{E}$ is assumed to be admissible, $\mathcal{E}$ attacks α. So either $N_{s(\alpha)\alpha} \in \mathcal{E}$ (Case 1), or $\mathcal{E}_r$ attacks α (Case 2). Moreover, due to Lemma 1, Case 1 also implies that $\mathcal{E}_r$ attacks $s(\alpha)$.
So there exists $\beta \in \mathcal{E}_r$ s.t. β attacks $s(\alpha)$ in Case 1 (resp. β attacks α in Case 2). As $\mathcal{E}$ is admissible, $\mathcal{E}$ defends β against $N_{s(\beta)\beta}$ (Observation 1). So $s(\beta) \in \mathcal{E}_a$. That fact with $\beta \in \mathcal{E}_r$ prove that $s(\alpha) \in Def(U'_\mathcal{E})$ (resp. $\alpha \in Inh(U'_\mathcal{E})$).

(b) Let $x \in \mathcal{E}_r$. Assume that x is attacked by $\alpha \in \mathbf{R}$. We have to prove that either $\alpha \in Inh(U'_\mathcal{E})$ or $s(\alpha) \in Def(U'_\mathcal{E})$. We can do exactly the same reasoning as for the first case ($x \in \mathcal{E}_a$).
So we have proved that the structure $U'_\mathcal{E}$ is admissible.

2. From Prop. 7, $U_\mathcal{E}$ is a conflict-free structure of the RAF.
It remains to prove that $\forall x \in (\mathcal{E}_a \cup \Gamma_\mathcal{E})$, x is acceptable wrt $U_\mathcal{E}$. We recall that $\Gamma_\mathcal{E} = \mathcal{E}_r \cup \{\alpha \notin \mathcal{E}_r$ s.t. $\alpha \in Acc(U'_\mathcal{E})\}$.
From the first part of the proof, $U'_\mathcal{E}$ is admissible. So $\forall x \in (\mathcal{E}_a \cup \mathcal{E}_r)$, x is acceptable wrt $U'_\mathcal{E}$ and then wrt $U_\mathcal{E}$. It remains to consider $x \in \Gamma_\mathcal{E} \setminus \mathcal{E}_r$. In that case, due to the definition of $\Gamma_\mathcal{E}$, $x \in Acc(U'_\mathcal{E})$ so x is acceptable wrt $U_\mathcal{E}$.

So we have proved that the structure $U_\mathcal{E}$ is admissible.

□

Proof of Prop. 11 on page 55: Let $U = (S, \Gamma)$ be a complete structure of the RAF. Let us recall that $\mathcal{E}_U = S \cup \{\alpha \in \Gamma$ s.t. $s(\alpha) \in S\} \cup \{N_{s(\alpha)\alpha}$ s.t. $s(\alpha) \notin S$ and $s(\alpha) \in Def(U)\}$.
By definition, U is an admissible structure and satisfies $Acc(U) \subseteq S \cup \Gamma$. From Proposition 9, we have that $\mathcal{E}_U$ is an admissible extension of the MAF $\langle \mathbf{A}', \mathbf{R}' \rangle$. So it remains to prove that $\forall x \in \mathbf{A}'$, if x is acceptable wrt $\mathcal{E}_U$, then $x \in \mathcal{E}_U$.
Three cases must be considered for x:

1. Let $x \in \mathbf{A}$ being acceptable wrt $\mathcal{E}_U$. Assume that $x \notin \mathcal{E}_U$. Then $x \notin S$ and so $x \notin Acc(U)$, due to the assumption on U. $x \notin Acc(U)$ means that there is $\beta \in \mathbf{R}$ with $t(\beta) = x$ and such that $\beta \notin Inh(U)$ and $s(\beta) \notin Def(U)$; this fact will be denoted by (*). So we have the attack (β, x) in the MAF.
As x is acceptable wrt $\mathcal{E}_U$, we know that $\mathcal{E}_U$ attacks β in the MAF. So, either $N_{s(\beta)\beta} \in \mathcal{E}_U$ (Case 1), or there exists $\gamma \in (\mathcal{E}_U \cap \mathbf{R})$ that attacks β (Case 2).
In Case 1, by definition of $\mathcal{E}_U$, $s(\beta) \in Def(U)$, which is in contradiction with the fact (*). In Case 2, by definition of $\mathcal{E}_U$, we have $\gamma \in \Gamma$ and $s(\gamma) \in S$. So $\beta \in Inh(U)$, which is in contradiction with the fact (*).
So we have proved that x must belong to $\mathcal{E}_U$.

2. Let $\alpha \in \mathbf{R}$ being acceptable wrt $\mathcal{E}_U$. From Observation 2, it follows that $s(\alpha) \in S$. Now, assume that $\alpha \notin \mathcal{E}_U$. By definition of $\mathcal{E}_U$, it follows that $\alpha \notin \Gamma$ and so $\alpha \notin Acc(U)$, due to the assumption on U.
The rest of the proof is analogous to the proof of the first item. $\alpha \notin Acc(U)$ means that there is $\beta \in \mathbf{R}$ with $t(\beta) = \alpha$ and such that $\beta \notin Inh(U)$ and $s(\beta) \notin Def(U)$; this fact will be denoted by (*). So we have the attack (β, α) in the MAF.
As α is acceptable wrt $\mathcal{E}_U$, we know that $\mathcal{E}_U$ attacks β in the MAF. So, either $N_{s(\beta)\beta} \in \mathcal{E}_U$ (Case 1), or there exists $\gamma \in (\mathcal{E}_U \cap \mathbf{R})$ that attacks β (Case 2).
In Case 1, by definition of $\mathcal{E}_U$, $s(\beta) \in Def(U)$, which is in contradiction with the fact (*). In Case 2, by definition of $\mathcal{E}_U$, we have $\gamma \in \Gamma$ and $s(\gamma) \in S$. So $\beta \in Inh(U)$, which is in contradiction with the

fact (*).
So we have proved that α must belong to $\mathcal{E}_U$.

3. Let $x \in \mathbf{N}$ being acceptable wrt $\mathcal{E}_U$. x has the form $N_{s(\alpha)\alpha}$. As $s(\alpha)$ is the only attacker of x, we know that $\mathcal{E}_U$ attacks $s(\alpha)$ in the MAF. So there exists $\beta \in \mathcal{E}_U \cap \mathbf{R}$ with $(\beta, s(\alpha)) \in \mathbf{R}'$. By definition of $\mathcal{E}_U$, we have $\beta \in \Gamma$ and $s(\beta) \in S$. Hence $s(\alpha) \in Def(U)$. As U is conflict-free, it implies that $s(\alpha) \notin S$. So we have $N_{s(\alpha)\alpha} \in \mathcal{E}_U \cap \mathbf{N}$.

□

Proof of Prop. 12 on page 55: Let $\mathcal{E}$ be a complete extension of the MAF $\langle \mathbf{A}', \mathbf{R}' \rangle$. Let us recall that $U_\mathcal{E} = (\mathcal{E}_a, \Gamma_\mathcal{E})$, where $\Gamma_\mathcal{E} = \mathcal{E}_r \cup \{\alpha \notin \mathcal{E}_r \text{ s.t. } \alpha \in Acc(U'_\mathcal{E}) \text{ and } s(\alpha) \notin \mathcal{E}_a\}$, and $U'_\mathcal{E} = (\mathcal{E}_a, \mathcal{E}_r)$
By definition, $\mathcal{E}$ is an admissible extension of the MAF and $\forall x \in \mathbf{A}'$, if x is acceptable wrt $\mathcal{E}$, then $x \in \mathcal{E}$.
From Proposition 10, we have that $U_\mathcal{E}$ is an admissible structure. So it remains to prove that $Acc(U_\mathcal{E}) \subseteq \mathcal{E}_a \cup \Gamma_\mathcal{E}$. Two cases must be considered:

1. Let $a \in \mathbf{A} \cap Acc(U_\mathcal{E})$. Assume that $a \notin \mathcal{E}_a$. As $\mathcal{E}$ is a complete extension of the MAF, a is not acceptable wrt $\mathcal{E}$. So there exists an attack (β, a) in $\mathbf{R}'$ such that $\mathcal{E}$ does not attack β. That implies that $N_{s(\beta)\beta} \notin \mathcal{E}$, and from Lemma 1 that $\mathcal{E}$ does not attack $s(\beta)$. So $\mathcal{E}$ attacks neither β, nor $s(\beta)$; this fact will be denoted by (*).
Moreover, as $a \in Acc(U_\mathcal{E})$, either $\beta \in Inh(U_\mathcal{E})$ (Case 1), or $s(\beta) \in Def(U_\mathcal{E})$ (Case 2). It follows that there is $\gamma \in \Gamma_\mathcal{E}$ with $s(\gamma) \in \mathcal{E}_a$ such that $t(\gamma) = \beta$ in Case 1 (resp. $t(\gamma) = s(\beta)$ in Case 2). These conditions on γ and the definition of $\Gamma_\mathcal{E}$ imply that γ must belong to $\mathcal{E}_r$. So we have that $\mathcal{E}$ attacks β in Case 1 (resp. $s(\beta)$ in Case 2). Hence we obtain a contradiction with the fact (*) and consequently we have proved that $a \in \mathcal{E}_a$.

2. Let $\alpha \in \mathbf{R} \cap Acc(U_\mathcal{E})$. Assume that $\alpha \notin \Gamma_\mathcal{E}$. It follows that $\alpha \notin \mathcal{E}_r$ and either $s(\alpha) \in \mathcal{E}_a$ or $\alpha \notin Acc(U'_\mathcal{E})$. Let us successively consider the two cases.

   (a) Assume that $\alpha \notin \mathcal{E}_r$ and $s(\alpha) \in \mathcal{E}_a$. As $\mathcal{E}$ is a complete extension of the MAF, α is not acceptable wrt $\mathcal{E}$. As $s(\alpha) \in \mathcal{E}_a$, α is defended by $\mathcal{E}$ against its attacker $N_{s(\alpha)\alpha}$. So there must exist another attacker of α, say β, such that $\mathcal{E}$ does not attack β. That implies that $N_{s(\beta)\beta} \notin \mathcal{E}$, and from Lemma 1 that $\mathcal{E}$ does not attack $s(\beta)$. So $\mathcal{E}$ attacks neither β, nor $s(\beta)$; this fact will be denoted by (*).
   Moreover, as $\alpha \in Acc(U_\mathcal{E})$, either $\beta \in Inh(U_\mathcal{E})$ (Case 1), or $s(\beta) \in Def(U_\mathcal{E})$ (Case 2). It follows that there is $\gamma \in \Gamma_\mathcal{E}$ with $s(\gamma) \in \mathcal{E}_a$ such that $t(\gamma) = \beta$ in Case 1 (resp. $t(\gamma) = s(\beta)$ in Case 2). These conditions on γ and the definition of $\Gamma_\mathcal{E}$ imply that γ must belong to $\mathcal{E}_r$. So we have that $\mathcal{E}$ attacks β in Case 1 (resp. $s(\beta)$ in Case 2). Hence we obtain a contradiction with the fact (*).

   (b) It remains to consider the case when $\alpha \notin \mathcal{E}_r$, $s(\alpha) \notin \mathcal{E}_a$ and $\alpha \notin Acc(U'_\mathcal{E})$. Let us recall that $U'_\mathcal{E}$ is the structure $(\mathcal{E}_a, \mathcal{E}_r)$. $\alpha \notin Acc(U'_\mathcal{E})$ implies that α is attacked in the RAF by $\beta \in \mathbf{R}$ such that $\beta \notin Inh(U'_\mathcal{E})$ and $s(\beta) \notin Def(U'_\mathcal{E})$; this fact will be denoted by (**).
   Moreover, as $\alpha \in Acc(U_\mathcal{E})$, either $\beta \in Inh(U_\mathcal{E})$ (Case 1), or $s(\beta) \in Def(U_\mathcal{E})$ (Case 2). It follows that there is $\gamma \in \Gamma_\mathcal{E}$ with $s(\gamma) \in \mathcal{E}_a$ such that $t(\gamma) = \beta$ in Case 1 (resp. $t(\gamma) = s(\beta)$ in Case 2). These conditions on γ and the definition of $\Gamma_\mathcal{E}$ imply that γ must belong to $\mathcal{E}_r$. So we have that $\beta \in Inh(U'_\mathcal{E})$ in Case 1 (resp. $s(\beta) \in Def(U'_\mathcal{E})$ in Case 2). Hence we obtain a contradiction with with the fact (**).

   So we have proved that α must belong to $\Gamma_\mathcal{E}$.

□

Proof of Prop. 13 on page 55:

1. The proof follows directly from Lemma 3.

2. The proof follows directly from Lemma 4.

□

Proof of Prop. 14 on page 56: Let $U = (S, \Gamma)$ be a stable structure of the RAF. By definition, U is a conflict-free structure that satisfies: $\mathbf{A} \setminus S \subseteq Def(U)$ and $\mathbf{R} \setminus \Gamma \subseteq Inh(U)$. First, from Proposition 6, we have that $\mathcal{E}_U$ is conflict-free in the MAF $\langle \mathbf{A}', \mathbf{R}' \rangle$. Then, we have to prove that $\forall x \in \mathbf{A}' \setminus \mathcal{E}_U$, x is attacked by $\mathcal{E}_U$. Three cases must be considered for x:

1. Let $x \in \mathbf{A} \setminus \mathcal{E}_U$. Then $x \in \mathbf{A} \setminus S$. By assumption on U, it follows that $x \in Def(U)$ and from Observation 2 it follows that $\mathcal{E}_U$ attacks x.

2. Let $\alpha \in \mathbf{R} \setminus \mathcal{E}_U$. Then either $\alpha \notin \Gamma$ (Case 1), or $\alpha \in \Gamma$ and $s(\alpha) \notin S$ (Case 2).
In Case 1, as U is a stable structure, $\alpha \in Inh(U)$ and from Observation 2 it follows that $\mathcal{E}_U$ attacks α.
In Case 2, as U is a stable structure, $s(\alpha) \in Def(U)$. Moreover, $s(\alpha) \notin S$, so $N_{s(\alpha)\alpha} \in \mathcal{E}_U \cap \mathbf{N}$. As $(N_{s(\alpha)\alpha}, \alpha) \in \mathbf{R}'$, we conclude that $\mathcal{E}_U$ attacks α.

3. Let $x \in \mathbf{N} \setminus \mathcal{E}_U$. x has the form $N_{s(\alpha)\alpha}$ with $s(\alpha) \in S$ or $s(\alpha) \notin Def(U)$. Note that, as U is stable, if $s(\alpha) \notin Def(U)$ then $s(\alpha) \in S$. Then, if $s(\alpha) \in S$, as $(s(\alpha), N_{s(\alpha)\alpha}) \in \mathbf{R}'$ and $S \subseteq \mathcal{E}_U$, we conclude that $\mathcal{E}_U$ attacks x.

So we have proved that $\mathcal{E}_U$ is a stable extension of the MAF.

□

Proof of Prop. 15 on page 56: Let $\mathcal{E}$ be a stable extension of the MAF $\langle \mathbf{A}', \mathbf{R}' \rangle$. First, from Prop. 7, we have that $U_\mathcal{E}$ is a conflict-free structure of the RAF. Then, we have to prove that $\mathbf{A} \setminus \mathcal{E}_a \subseteq Def(U_\mathcal{E})$ and $\mathbf{R} \setminus \Gamma_\mathcal{E} \subseteq Inh(U_\mathcal{E})$.

1. Let $x \in \mathbf{A} \setminus \mathcal{E}_a$. As $\mathcal{E}$ is stable, $\mathcal{E}$ attacks x. As $x \in \mathbf{A}$ all the attackers of x belong to $\mathbf{R}$. So there is $\alpha \in \mathcal{E}_r$ that attacks x. Note that $\alpha \in \Gamma_\mathcal{E}$ by definition of $\Gamma_\mathcal{E}$.
Moreover, as $\mathcal{E}$ is a stable extension of the MAF, $\mathcal{E}$ is admissible. As α is attacked by $N_{s(\alpha)\alpha}$, $\mathcal{E}$ contains the only attacker of $N_{s(\alpha)\alpha}$, that is $s(\alpha)$. So $s(\alpha) \in \mathcal{E}_a$. By definition, $\alpha \in \mathcal{E}_r$ and $s(\alpha) \in \mathcal{E}_a$ imply that $x \in Def(U'_\mathcal{E})$. As $\Gamma_\mathcal{E}$ contains $\mathcal{E}_r$ it follows that we also have $x \in Def(U_\mathcal{E})$.

2. Let $\alpha \in \mathbf{R} \setminus \Gamma_\mathcal{E}$. It follows that $\alpha \notin \mathcal{E}_r$ and either $s(\alpha) \in \mathcal{E}_a$ or $\alpha \notin Acc(U'_\mathcal{E})$. Let us successively consider the two cases.

   (a) Assume that $\alpha \notin \mathcal{E}_r$ and $s(\alpha) \in \mathcal{E}_a$. As $\mathcal{E}$ is stable, $\mathcal{E}$ attacks α. As $\mathcal{E}$ is conflict-free and contains $s(\alpha)$, it follows that $N_{s(\alpha)\alpha} \notin \mathcal{E}$. So there exists $\beta \in \mathcal{E}_r$ that attacks α.
   Moreover, as $\mathcal{E}$ is a stable extension of the MAF, $\mathcal{E}$ is admissible. As β is attacked by $N_{s(\beta)\beta}$, $\mathcal{E}$ contains the only attacker of $N_{s(\beta)\beta}$, that is $s(\beta)$. So $s(\beta) \in \mathcal{E}_a$. By definition, $\beta \in \mathcal{E}_r$ and $s(\beta) \in \mathcal{E}_a$ imply that $\alpha \in Inh(U'_\mathcal{E})$. As $\Gamma_\mathcal{E}$ contains $\mathcal{E}_r$ it follows that we also have $\alpha \in Inh(U_\mathcal{E})$.

   (b) It remains to consider the case when $\alpha \notin \mathcal{E}_r$, $s(\alpha) \notin \mathcal{E}_a$ and $\alpha \notin Acc(U'_\mathcal{E})$. Let us recall that $U'_\mathcal{E}$ is the structure $(\mathcal{E}_a, \mathcal{E}_r)$. $\alpha \notin Acc(U'_\mathcal{E})$ implies that α is attacked in the RAF by $\beta \in \mathbf{R}$ such that $\beta \notin Inh(U'_\mathcal{E})$ and $s(\beta) \notin Def(U'_\mathcal{E})$; this fact will be denoted by (*).
   If $s(\beta) \notin \mathcal{E}_a$, from the first part of this proof, it follows that $s(\beta) \in Def(U'_\mathcal{E})$. That is in

contradiction with the fact (*). So we have $s(\beta) \in \mathcal{E}_a$.
If $\beta \notin \Gamma_{\mathcal{E}}$, as $s(\beta) \in \mathcal{E}_a$, from the first item of the second part of this proof, it follows that $\beta \in Inh(U'_{\mathcal{E}})$. That is in contradiction with the fact (*). So we have $\beta \in \Gamma_{\mathcal{E}}$.
By definition, $s(\beta) \in \mathcal{E}_a$, $\beta \in \Gamma_{\mathcal{E}}$ and β attacks α imply that $\alpha \in Inh(U_{\mathcal{E}})$.

In both cases, we have proved that $\alpha \in Inh(U_{\mathcal{E}})$.

□

Proof of Prop. 16 on page 56: Let U be a preferred structure. By definition, U is a $\subseteq$-maximal admissible structure. Moreover U is a complete structure. So, from Proposition 11, $\mathcal{E}_U$ is a complete extension of the MAF.
Assume that $\mathcal{E}_U$ is not a preferred extension of the MAF. Then there exists $\mathcal{E}'$ an admissible extension of the MAF that strictly contains $\mathcal{E}_U$. It can be assumed that $\mathcal{E}'$ is a $\subseteq$-maximal admissible extension of the MAF. So $\mathcal{E}'$ is preferred and thus complete.
From Lemma 5, it follows that $U_{\mathcal{E}_U} \subseteq U_{\mathcal{E}'}$. From Proposition 13, we have $U_{\mathcal{E}_U} = U$. So, $U \subseteq U_{\mathcal{E}'}$. As U is preferred, it follows that $U = U_{\mathcal{E}'}$.
From Proposition13 again, $\mathcal{E}_{U_{\mathcal{E}'}} = \mathcal{E}'$ so $\mathcal{E}_U = \mathcal{E}'$. That is in contradiction with the assumption that $\mathcal{E}'$ strictly contains $\mathcal{E}_U$. Hence, we have proved that $\mathcal{E}_U$ is a preferred extension of the MAF.

□

Proof of Prop. 17 on page 56: Let $\mathcal{E}$ be a preferred extension of the MAF. By definition, $\mathcal{E}$ is a $\subseteq$-maximal admissible extension. Moreover $\mathcal{E}$ is a complete extension. So, from Proposition 12, $U_{\mathcal{E}}$ is a complete structure of the RAF.
Assume that $U_{\mathcal{E}}$ is not a preferred structure of the RAF. Then there exists U' an admissible structure that strictly contains $U_{\mathcal{E}}$. It can be assumed that U' is a $\subseteq$-maximal admissible structure of the RAF. So U' is preferred and thus complete (as proved in [7]).
From Lemma 6, it follows that $\mathcal{E}_{U_{\mathcal{E}}} \subseteq \mathcal{E}_{U'}$. From Proposition 13, we have $\mathcal{E}_{U_{\mathcal{E}}} = \mathcal{E}$. So, $\mathcal{E} \subseteq \mathcal{E}_{U'}$. As $\mathcal{E}$ is preferred, it follows that $\mathcal{E} = \mathcal{E}_{U'}$.
From Proposition 13 again, $U_{\mathcal{E}_{U'}} = U'$ so $U_{\mathcal{E}} = U'$.
That is in contradiction with the assumption that U' strictly contains $U_{\mathcal{E}}$. Hence, we have proved that $U_{\mathcal{E}}$ is a preferred structure.

□

Proof of Prop. 18 on page 56: Let $U = (S, \Gamma)$ be the grounded structure of the RAF. By definition, U is the $\subseteq$-minimal complete structure. From Proposition 11, $\mathcal{E}_U$ is a complete extension of the MAF. Assume that there is $\mathcal{E}'$ a complete extension that is strictly included in $\mathcal{E}_U$. From Lemma 5, we have $U_{\mathcal{E}'} \subseteq U_{\mathcal{E}_U}$. As $U_{\mathcal{E}_U} = U$, due to Proposition 13, we have $U_{\mathcal{E}'} \subseteq U$. From Proposition 12, $U_{\mathcal{E}'}$ is a complete structure, so by assumption on U it follows that $U_{\mathcal{E}'} = U$. Hence, $\mathcal{E}_{U_{\mathcal{E}'}} = \mathcal{E}_U$, and from Proposition 13 again, $\mathcal{E}' = \mathcal{E}_U$. That is in contradiction with the fact that $\mathcal{E}'$ is strictly included in $\mathcal{E}_U$. So we have proved that $\mathcal{E}_U$ is a $\subseteq$-minimal complete extension of the MAF, or in other words the grounded extension of the MAF. □

Proof of Prop. 19 on page 56: Let $\mathcal{E}$ be the grounded extension of the MAF. By definition, $\mathcal{E}$ is the $\subseteq$-minimal complete extension. From Proposition 12, $U_{\mathcal{E}}$ is a complete structure of the RAF. Assume that there is U' a complete structure that is strictly included in $U_{\mathcal{E}}$. From Lemma 6, we have $\mathcal{E}_{U'} \subseteq \mathcal{E}_{U_{\mathcal{E}}}$ As $\mathcal{E}_{U_{\mathcal{E}}} = \mathcal{E}$, due to Proposition 13, we have $\mathcal{E}_{U'} \subseteq \mathcal{E}$. From Proposition 11, $\mathcal{E}_{U'}$ is a complete extension, so by assumption on $\mathcal{E}$ it follows that $\mathcal{E}_{U'} = \mathcal{E}$. Hence, $U_{\mathcal{E}_{U'}} = U_{\mathcal{E}}$, and from Proposition 13 again, $U' = U_{\mathcal{E}}$. That is in contradiction with the fact that U' is strictly included in $U_{\mathcal{E}}$. So we have proved that $U_{\mathcal{E}}$ is a $\subseteq$-minimal complete structure of the RAF, or in other words is the grounded extension of the RAF.

□

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