Valid attacks in Argumentation Frameworks with Recursive Attacks

Claudette Cayrol,
Jorge Fandinno,*
Luis Fariñas del Cerro,
M-Christine Lagasquie-Schiex

Université de Toulouse, IRIT,
118 route de Narbonne, 31062 Toulouse, France
{ccayrol, jorge.fandinno, luis, lagasq}@irit.fr

Tech. Report
IRIT/RR- -2017- -16- -FR

*The second author is funded by the Centre International de Mathématiques et d'Informatique de Toulouse (CIMI) through contract ANR-11-LABEX-0040-CIMI within the program ANR-11-IDEX-0002-02.

Abstract

The purpose of this work is to study a generalisation of Dung's abstract argumentation frameworks that allows representing *recursive attacks*, that is, a class of attacks whose targets are other attacks. We do this by developing a theory of argumentation where the classic role of *attacks* in defeating arguments is replaced by a subset of them, which is extension dependent and which, intuitively, represents a set of "valid attacks" with respect to the extension. The studied theory displays a conservative generalisation of Dung's semantics (complete, preferred and stable) and also of its principles (conflict-freeness, acceptability and admissibility). Furthermore, despite its conceptual differences, we are also able to show that our theory agrees with the AFRA interpretation of recursive attacks for the complete, preferred and stable semantics.

Contents

1 Introduction

Argumentation has become an essential paradigm for Knowledge Representation and, especially, for reasoning from contradictory information [1, 10] and for formalizing the exchange of arguments between agents in, *e.g.*, negotiation [2]. Formal abstract frameworks have greatly eased the modelling and study of argumentation. For instance, a Dung's argumentation framework (AF) [10] consists of a collection of arguments interacting with each other through an attack relation, enabling to determine "acceptable" sets of arguments called *extensions*.

A natural generalisation of Dung's argumentation frameworks consists in allowing high-order attacks that target other attacks. Here is an example in the legal field, borrowed from [3].

Example 1. *The lawyer says that the defendant did not have intention to kill the victim (Argument b). The prosecutor says that the defendant threw a sharp knife towards the victim (Argument a). So, there is an attack from a to b. And the intention to kill should be inferred. Then the lawyer says that the defendant was in a habit of throwing the knife at his wife's foot once drunk. This latter argument (Argument c) is better considered attacking the attack from a to b, than argument a itself. Now the prosecutor's argumentation seems no longer sufficient for proving the intention to kill. This example is represented as a recursive framework in Fig. 1.*

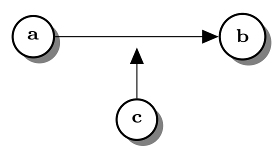

Figure 1: An acyclic recursive framework

Another example, borrowed from [4, 8], will be taken as a running example.

Example 2. *Suppose Bob is making decisions about his Christmas holidays, and is willing to buy cheap last minute offers. He knows there are deals for travelling to Gstaad (g) or Cuba (c). Suppose that Bob has a preference for skiing (p) and knows that Gstaad is a renowned ski resort. However, the weather service reports that it has not snowed in Gstaad (n). So it might not be possible to ski there. Suppose finally that Bob is informed that the ski resort in Gstaad has a good amount of artificial snow, that makes it anyway possible to ski there (a). The different attacks are represented on Fig. 2.* □

A semantics for these classes of *recursive frameworks* has been first introduced in [13] in the context of preferences in argumentation frameworks, and latter studied in [4] under the name of AFRA (Argumentation Framework with Recursive Attacks). This version describes abstract argumentation frameworks in which the interactions can be either attacks between arguments or attacks

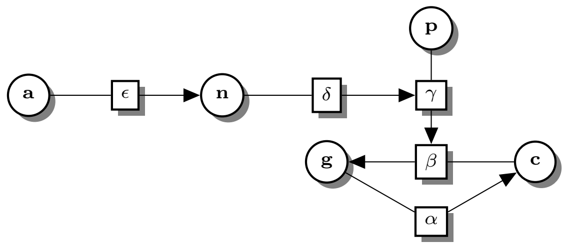

Figure 2: Bob's dilemma: arguments are in circle and attacks in square.

from an argument to another attack. A translation of an AFRA into an AF is defined by the addition of some new arguments and the attacks they produce or they receive. Note that AFRA have been extended in order to handle recursive support interactions together with recursive attacks [8, 9]. However, when supports are removed, these approaches go back to AFRA.

A similar work is described in [7] using the addition of meta-arguments that enable to encode the notions of "grounded attack" and "valid attack". The notion of grounded attack is about the source of the attack and the notion of valid attack is about the link between the source and the target of the attack (*i.e.* the role of the interaction itself). Despite the intuitive results obtained by these approaches regarding complete, stable or grounded extensions, it somehow changes the role that attacks play in Dung's frameworks.

Example 3. *Consider the argumentation framework corresponding to Fig. 3. According to Dung's theory, this framework has three conflict-free sets:* $\varnothing$, $\{a\}$

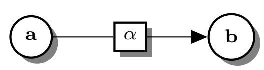

Figure 3: A simple Dung's framework

and $\{b\}$. *On the other hand,* $\{a, b\}$ *is a conflict-free set according to AFRA because the attack* α *is not in the set. In fact, in AFRA, such an argumentation framework can be turned into an AF by converting* α *into a new argument as in Fig. 4. In this new framework, it is easy to observe that* $\{a, b\}$ *is considered*

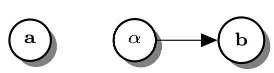

Figure 4: AF framework for AFRA of Fig. 3

conflict-free in AFRA because there is no attack between a *and* b. *In some sense, the connection between an attack and its source has been lost. As another example of this behaviour, the set* $\{\alpha, b\}$ *is not AFRA-conflict-free despite the fact that the source of* α*, the argument* a*, is not in the set.* □

In this paper, we study an alternative semantics for argumentation frameworks with recursive attacks based on the following intuitive principles:

P1 The role played in Dung's argumentation frameworks by attacks in defeating arguments is now played by a subset of these attacks, which is extension dependent and represents the "valid attacks" with respect to that extension.

P2 It is a conservative generalisation of Dung's framework for the definitions of conflict-free, admissible, complete, preferred, and stable extensions.

For instance, in the proposed semantics, the conflict-free extensions of the framework of Fig. 3 are precisely Dung's conflict-free extensions: $\varnothing$, $\{a\}$ and $\{b\}$. Besides, as we will see later, the attack α is valid with respect to all three extensions because it is not the target of any attack. It is worth noting that, despite its conceptual difference with respect to AFRA, we are able to prove an one-to-one correspondence between our complete, preferred and stable extensions and the corresponding AFRA extensions, in which the set of "acceptable" arguments are the same. This offers an alternative view for the semantics of recursive attacks that we believe to be closer to Dung's intuitive understanding.

2 Background

Definition 1. *A* Dung's abstract argumentation framework (D-framework *for short) is a pair* $\mathbf{AF} = \langle \mathbf{A}, \mathbf{R} \rangle$ *where* $\mathbf{A}$ *is a set of arguments and* $\mathbf{R} \subseteq \mathbf{A} \times \mathbf{A}$ *is a relation representing attacks over arguments.* □

For instance, the graph depicted in Fig. 3 corresponds to the D-framework $\mathbf{AF}_3 = \langle \mathbf{A}_3, \mathbf{R}_3 \rangle$ with the set of arguments $\mathbf{A}_3 = \{a, b\}$ and the attack relation $\mathbf{R}_3 = \{(a, b)\}$.

Definition 2. *Given some D-framework* $\mathbf{AF} = \langle \mathbf{A}, \mathbf{R} \rangle$ *and some set of arguments* $S \subseteq \mathbf{A}$*, an argument* $a \in \mathbf{A}$ *is said to be*

i) defeated w.r.t. S *iff* $\exists b \in S$ *such that* $(b, a) \in \mathbf{R}$*, and*

ii) acceptable w.r.t. S *iff for every argument* $b \in \mathbf{A}$ *with* $(b, a) \in \mathbf{R}$*, there is* $c \in S$ *such that* $(c, b) \in \mathbf{R}$*.* □

To obtain shorter definitions we will also use the following notations:

$$Def(S) \stackrel{\text{def}}{=} \{\, a \in \mathbf{A} \mid \exists b \in S \text{ s.t. } (b, a) \in \mathbf{R} \,\}$$
$$Acc(S) \stackrel{\text{def}}{=} \{\, a \in \mathbf{A} \mid \forall b \in \mathbf{A},\ (b, a) \in \mathbf{R} \text{ implies } b \in Def(S) \,\}$$

respectively denote the set of all defeated and acceptable arguments w.r.t. S.

Definition 3. *Given a D-framework* $\mathbf{AF} = \langle \mathbf{A}, \mathbf{R} \rangle$*, a set of arguments* $S \subseteq \mathbf{A}$ *is said to be*

i) conflict-free *iff* $S \cap Def(S) = \varnothing$*,*

ii) admissible *iff it is conflict-free and* $S \subseteq Acc(S)$*,*

iii) complete *iff it is conflict-free and* $S = Acc(S)$*,* *iv)* preferred *iff it is* $\subseteq$*-maximal*[1] *admissible,*

v) stable *iff it is conflict-free and* $S \cup Def(S) = \mathbf{A}$*.* □

Theorem 1 ([10])**.** *Given a D-framework* $\mathbf{AF} = \langle \mathbf{A}, \mathbf{R} \rangle$*, the following assertions hold:*

i) every complete set is also admissible,

ii) every preferred set is also complete, and

iii) every stable set is also preferred. □

For instance, in Example 3, the argument a is accepted w.r.t. any set S because there is no argument $x \in \mathbf{A}$ such that $(x, a) \in \mathbf{R}$. Furthermore, b is defeated and non-acceptable w.r.t. the set $\{a\}$. Then, it is easy to check that $\{a\}$ is stable (and, thus, conflict-free, admissible, complete and preferred). The empty set $\varnothing$ is admissible, but not complete; and the set $\{b\}$ is conflict-free, but not admissible.

3 Semantics for recursive attacks

Definition 4. *A* recursive argumentation framework $\mathbf{RAF} = \langle \mathbf{A},\mathbf{K},\mathbf{s},\mathbf{t} \rangle$ *is a quadruple where* $\mathbf{A}$ *and* $\mathbf{K}$ *are (possibly infinite) disjunct sets respectively representing arguments and attack names, and where* $\mathbf{s} : \mathbf{K} \longrightarrow \mathbf{A}$ *and* $\mathbf{t} : \mathbf{K} \longrightarrow \mathbf{A} \cup \mathbf{K}$ *are functions respectively mapping each attack to its source and its target.* □

For instance, the argumentation framework of Example 3 corresponds to $\mathbf{RAF}_3 = \langle \mathbf{A}_3, \mathbf{K}_3, \mathbf{s}_3, \mathbf{t}_3 \rangle$ where $\mathbf{A}_3 = \{a, b\}$, $\mathbf{K}_3 = \{\alpha\}$, $\mathbf{s}_3(\alpha) = a$ and $\mathbf{t}_3(\alpha) = b$. In general, given any D-framework $\mathbf{AF} = \langle \mathbf{A},\mathbf{R} \rangle$, we may obtain its corresponding argumentation framework $\mathbf{RAF} = \langle \mathbf{A},\mathbf{K},\mathbf{s},\mathbf{t} \rangle$ by defining a set of attack names $\mathbf{K} = \{\ \alpha_{(a,b)} \mid (a, b) \in \mathbf{R}\ \}$. Functions $\mathbf{s}$ and $\mathbf{t}$ are straightforwardly defined by mapping each attack $(a, b) \in \mathbf{R}$ as follows: $\mathbf{s}(\alpha_{(a,b)}) = a$ and $\mathbf{t}(\alpha_{(a,b)}) = b$.

It is worth noting that our definition allows the existence of several attacks between the same arguments. Though this does not make any difference for frameworks without recursive attacks, for recursive ones, it allows representing attacks between the same arguments that are valid in different contexts. For instance, in the example of Figure 5, there are two attacks between a and b,

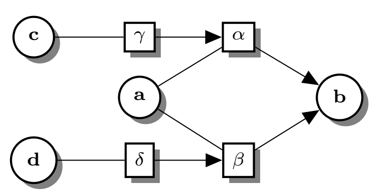

Figure 5: A recursive framework representing attacks in different contexts

[1]With $\subseteq$ denoting the standard set inclusion relation.

namely α and β, which represent different contexts as they are attacked by different arguments.

Definition 5 (Structure). *A pair* $\mathfrak{A} = \langle S, \Gamma\rangle$ *is said to be a* structure *of some* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t}\rangle$ *iff it satisfies:* $S \subseteq \mathbf{A}$ *and* $\Gamma \subseteq \mathbf{K}$. □

Intuitively, the set S represents the set of "acceptable" arguments w.r.t. the structure $\mathfrak{A}$, while Γ represents the set of "valid attacks" w.r.t. $\mathfrak{A}$. Any attack[2] $\alpha \in \overline{\Gamma}$ is understood as non-valid and, in this sense, it cannot defeat the argument or attack that it is targeting.

For the rest of this section we assume that all definitions and results are relative to some given framework $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t}\rangle$. We extend now the definition of defeated arguments (Definition 2) using the set Γ instead of the attack relation $\mathbf{R}$: given a structure of the form $\mathfrak{A} = \langle S, \Gamma\rangle$, we define:

$$Def(\mathfrak{A}) \stackrel{\text{def}}{=} \{\, a \in \mathbf{A} \mid \exists \alpha \in \Gamma,\ \mathbf{s}(\alpha) \in S \text{ and } \mathbf{t}(\alpha) = a \,\} \tag{1}$$

In other words, an argument $a \in \mathbf{A}$ is defeated w.r.t. $\mathfrak{A}$ iff there is a "valid attack" w.r.t. $\mathfrak{A}$ that targets a and whose source is "acceptable" w.r.t. $\mathfrak{A}$. It is interesting to observe that we may define the *attack relation* associated with some structure $\mathfrak{A} = \langle S, \Gamma\rangle$ as follows:

$$\mathbf{R}_{\mathfrak{A}} \stackrel{\text{def}}{=} \{\ (\mathbf{s}(\alpha), \mathbf{t}(\alpha)) \mid \alpha \in \Gamma\ \} \tag{2}$$

and that, using this relation, we can rewrite (1) as:

$$Def(\mathfrak{A}) \stackrel{\text{def}}{=} \{\ a \in \mathbf{A} \mid \exists b \in S \text{ s.t. } (b, a) \in \mathbf{R}_{\mathfrak{A}}\ \} \tag{3}$$

Now, it is easy to see that our definition can be obtained from Dung's definition of defeat (Definition 2) just by replacing the attack relation $\mathbf{R}$ by the attack relation $\mathbf{R}_{\mathfrak{A}}$ associated with the structure $\mathfrak{A}$, or in other words, by replacing the set of all attacks in the argumentation framework by the set of the "valid attacks" w.r.t. the structure $\mathfrak{A}$, as stated in **P1**. Analogously, by

$$Inh(\mathfrak{A}) \stackrel{\text{def}}{=} \{\ \alpha \in \mathbf{K} \mid \exists b \in S \text{ s.t. } (b, \alpha) \in \mathbf{R}_{\mathfrak{A}}\ \} \tag{4}$$

we denote a set of attacks that, intuitively, represents the "inhibited attacks[3]" w.r.t. $\mathfrak{A}$.

We are now ready to extend the definition of acceptable argument w.r.t. a set (Definition 2):

Definition 6 (Acceptability). *An element* $x \in (\mathbf{A} \cup \mathbf{K})$ *is said to be* acceptable *w.r.t. some structure* $\mathfrak{A}$ *iff every attack* $\alpha \in \mathbf{K}$ *with* $\mathbf{t}(\alpha) = x$ *satisfies one of the following conditions:* (*i*) $\mathbf{s}(\alpha) \in Def(\mathfrak{A})$ *or* (*ii*) $\alpha \in Inh(\mathfrak{A})$. □

[2]By $\overline{\Gamma} \stackrel{\text{def}}{=} \mathbf{K} \backslash \Gamma$ we denote the set complement of Γ.
[3]We will deepen about the intuition of inhibited attacks in Section 6.

By $Acc(\mathfrak{A})$, we denote the set containing all acceptable arguments and attacks with respect to $\mathfrak{A}$. We also define the following order relations that will help us defining preferred structures: for any pair of structures $\mathfrak{A} = \langle S, \Gamma \rangle$ and $\mathfrak{A}' = \langle S', \Gamma' \rangle$, we write $\mathfrak{A} \sqsubseteq \mathfrak{A}'$ iff $(S \cup \Gamma) \subseteq (S' \cup \Gamma')$ and we write $\mathfrak{A} \sqsubseteq_{ar} \mathfrak{A}'$ iff $S \subseteq S'$. As usual, we say that a structure $\mathfrak{A}$ is $\sqsubseteq$-maximal (resp. $\sqsubseteq_{ar}$-maximal) iff every $\mathfrak{A}'$ that satisfies $\mathfrak{A} \sqsubseteq \mathfrak{A}'$ (resp. $\mathfrak{A} \sqsubseteq_{ar} \mathfrak{A}'$) also satisfies $\mathfrak{A}' \sqsubseteq \mathfrak{A}$ (resp. $\mathfrak{A}' \sqsubseteq_{ar} \mathfrak{A}$).

Definition 7. *A structure $\mathfrak{A} = \langle S, \Gamma \rangle$ is said to be:*

i) conflict-free *iff* $S \cap Def(\mathfrak{A}) = \varnothing$ *and* $\Gamma \cap Inh(\mathfrak{A}) = \varnothing$*,*

ii) admissible *iff it is conflict-free and* $(S \cup \Gamma) \subseteq Acc(\mathfrak{A})$*,*

iii) complete *iff it is conflict-free and* $Acc(\mathfrak{A}) = (S \cup \Gamma)$*,*

iv) preferred *iff it is a* $\sqsubseteq$*-maximal admissible structure,*

v) arg-preferred *iff it is a* $\sqsubseteq_{ar}$*-maximal preferred structure,*

vi) stable[4] *iff* $S = \overline{Def(\mathfrak{A})}$ *and* $\Gamma = \overline{Inh(\mathfrak{A})}$*.* □

Example 1 (cont'd) *Let* **RAF** *be the recursive argumentation framework corresponding to Fig. 6 (Fig. 6 is Fig. 1 completed with the attack names). It is*

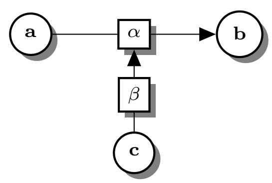

Figure 6: An acyclic recursive framework

easy to check that this framework has a unique complete, preferred and stable structure $\mathfrak{A} = \langle \{a, b, c\}, \{\beta\} \rangle$*. Furthermore, there are nine admissible structures that are not complete:* $\langle \{a, c\}, \{\beta\} \rangle$, $\langle \{b, c\}, \{\beta\} \rangle$, $\langle \{a\}, \{\beta\} \rangle$, $\langle \{c\}, \{\beta\} \rangle$, $\langle \varnothing, \{\beta\} \rangle$, $\langle \{a, c\}, \varnothing \rangle$, $\langle \{a\}, \varnothing \rangle$, $\langle \{c\}, \varnothing \rangle$ *and* $\langle \varnothing, \varnothing \rangle$*. There are also other conflict-free structures that are not admissible:* $\langle \varnothing, \{\alpha, \beta\} \rangle$, $\langle \{a\}, \{\alpha, \beta\} \rangle$, $\langle \{b\}, \{\alpha, \beta\} \rangle$, $\langle \{a, b\}, \{\beta\} \rangle$, $\langle \{b\}, \{\beta\} \rangle$, $\langle \{a, c\}, \{\alpha\} \rangle$, $\langle \{b, c\}, \{\alpha\} \rangle$, $\langle \{a\}, \{\alpha\} \rangle$, $\langle \{b\}, \{\alpha\} \rangle$, $\langle \{c\}, \{\alpha\} \rangle$, $\langle \varnothing, \{\alpha\} \rangle$, $\langle \{a, b\}, \varnothing \rangle$, $\langle \{a, b, c\}, \varnothing \rangle$, $\langle \{b, c\}, \varnothing \rangle$ *and* $\langle \{b\}, \varnothing \rangle$*.* □

It is worth to mention that preferred and arg-preferred structures do not necessarily coincide, since there exist preferred structures that do not contain a maximal set of arguments as shown by the following example:

Example 4. *Let* **RAF** *be the argumentation framework corresponding to the the graph depicted in Figure 7. Both* $\mathfrak{A} = \langle \{a, b\}, \{\beta\} \rangle$ *and* $\mathfrak{A}' = \langle \{a\}, \{\alpha, \beta\} \rangle$

[4]By $\overline{Def(\mathfrak{A})} \stackrel{\text{def}}{=} \mathbf{A} \backslash Def(\mathfrak{A})$ we denote the non-defeated arguments. Similarly, by $\overline{Inh(\mathfrak{A})} \stackrel{\text{def}}{=} \mathbf{K} \backslash Inh(\mathfrak{A})$ we denote the non-inhibited attacks. Note also that $S = \overline{Def(\mathfrak{A})}$ and $\Gamma = \overline{Inh(\mathfrak{A})}$ already implies conflict-freeness.

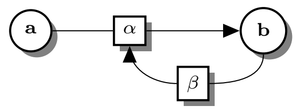

Figure 7: A RAF in which preferred and arg-preferred structures do not coincide

are preferred structures of **RAF***, but only the former contains a maximal set of arguments and thus* $\mathfrak{A}$ *is the unique arg-preferred structure.* □

We show now[5] that, as in Dung's argumentation theory, there is also a kind of Fundamental Lemma for argumentation frameworks with recursive attacks. For the sake of compactness, we will adopt the following notations: Given a structure $\mathfrak{A} = \langle S, \Gamma \rangle$ and a set $T \subseteq (\mathbf{A} \cup \mathbf{K})$ containing arguments and attacks, by $\mathfrak{A} \cup T \stackrel{\text{def}}{=} \langle S \cup (T \cap \mathbf{A}), \Gamma \cup (T \cap \mathbf{K}) \rangle$ we denote the result of extending $\mathfrak{A}$ with the elements in T.

Lemma 1 (Fundamental Lemma)**.** *Let* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be an admissible structure and* $x, y \in Acc(\mathfrak{A})$ *be any pair of acceptable elements. Then, (i)* $\mathfrak{A}' = \mathfrak{A} \cup \{x\}$ *is an admissible structure, and (ii)* $y \in Acc(\mathfrak{A}')$*.* □

Moreover, admissible structures form a complete partial order with preferred structures as maximal elements:

Proposition 1. *The set of all admissible structures forms a complete partial order with respect to* $\sqsubseteq$*. Furthermore, for every admissible structure* $\mathfrak{A}$*, there exists an (arg-)preferred one* $\mathfrak{A}'$ *such that* $\mathfrak{A} \sqsubseteq \mathfrak{A}'$*.* □

The following result shows that the usual relation between extensions also holds for structures.

Theorem 2. *The following assertions hold:*

i) every complete structure is also admissible,

ii) every preferred structure is also complete, and

iii) every stable structure is also preferred. □

Example 5. *As a further example, consider the framework* **RAF** *corresponding to Fig. 8. This framework has a unique complete and (arg-)preferred struc-*

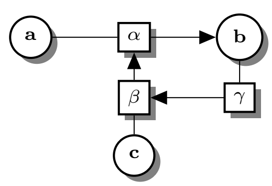

Figure 8: A cyclic recursive framework

[5]The proofs of propositions, lemmas, theorems given in this paper can be found in A.

ture $\mathfrak{A} = \langle\{a, c\}, \{\gamma\}\rangle$*, but no stable one. Note that* α *and* β *are neither acceptable nor inhibited w.r.t.* $\mathfrak{A}$*:* β *is not inhibited because* b *is not in the structure, so* α *is not acceptable.* α *is not inhibited because* β *is not in the structure. And* β *is not acceptable because* b *is not defeated (as* α *is not in the structure).* $\square$

Example 2 (cont'd) *Consider the framework RAF represented in Fig. 2. This framework has a unique complete, preferred and stable structure:* $\mathfrak{A}_0 = \langle\{a, g, p\}, \{\alpha, \epsilon, \gamma, \delta\}\rangle$*. Among the 63 admissible structures, we find* $\mathfrak{A}_1 = \langle\{a\}, \{\epsilon\}\rangle$*,* $\mathfrak{A}_2 = \langle\{a\}, \{\epsilon, \delta\}\rangle$*, and* $\mathfrak{A}_3 = \langle\{a\}, \{\alpha, \epsilon, \gamma, \delta\}\rangle$*.* $\square$

4 Relation with AFRA

In this section, we establish correspondences between our semantics for recursive attacks and the semantics for AFRA. In [4] a recursive framework is turned into a Dung's framework by adding new arguments and attacks using the following notion of defeat:

Definition 8 (Defeat)**.** *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t}\rangle$*. An attack* $\alpha \in \mathbf{K}$ *is said to* directly defeat $x \in \mathbf{A} \cup \mathbf{K}$ *iff* $\mathbf{t}(\alpha) = x$*. It is said to* indirectly defeat $\beta \in \mathbf{K}$ *iff* α *directly defeats* $\mathbf{s}(\beta)$*. Then,* α *is said to* defeat $x \in \mathbf{A} \cup \mathbf{K}$ *iff* α *directly defeats* x *or* α *indirectly defeats* x*.* $\square$

For instance, in Example 5, it is easy to see that α directly defeats b and indirectly defeats γ. Hence, α defeats both b and γ. Attacks β and γ directly defeat α and β, respectively. It has been shown in [4] that AFRA extensions can be characterized as the extensions of a Dung's framework whose new set of arguments contains both arguments and attacks and whose new attack relation is the defeat relation of Definition 8. In this sense, under AFRA, the argumen-

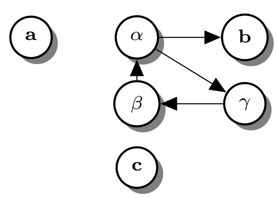

Figure 9: AF framework for AFRA framework of Ex. 5

tation framework of Example 5 is turned into the one in Fig. 9 and it can be checked that it has a unique complete and preferred extension $\{a, c\}$ and no stable one. We recall next the formal definitions of AFRA from [4]:

Definition 9. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t}\rangle$ *and* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$*. Then, an element* $x \in (\mathbf{A} \cup \mathbf{K})$ *is said to be* AFRA-acceptable *w.r.t.* $\mathcal{E}$ *iff for every* $\alpha \in \mathbf{K}$ *that defeats* x*, there is* $\beta \in \mathcal{E}$ *that defeats* α*.* $\square$

Definition 10 (AFRA-extensions)**.** *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *and a set* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$*,* $\mathcal{E}$ *is said to be:*

i) AFRA-conflict-free *iff* $\nexists \alpha, x \in \mathcal{E}$ *s.t.* α *defeats* x*,*

ii) AFRA-admissible *iff* $\mathcal{E}$ *is AFRA-conflict-free and each element of* $\mathcal{E}$ *is AFRA-acceptable w.r.t.* $\mathcal{E}$*,*

iii) AFRA-complete *iff it is AFRA-admissible and every* $x \in (\mathbf{A} \cup \mathbf{K})$ *which is AFRA-acceptable w.r.t.* $\mathcal{E}$ *belongs to* $\mathcal{E}$*,*

iv) AFRA-preferred *iff it is a* $\subseteq$*-maximal AFRA-admissible extension,*

v) AFRA-stable *iff it is AFRA-conflict-free and, for every* $x \in (\mathbf{A} \cup \mathbf{K})$*,* $x \notin \mathcal{E}$ *implies that* x *is defeated by some* $\alpha \in \mathcal{E}$*.* □

As illustrated by Example 3, AFRA does not preserve Dung's notion of conflict-freeness.

Observation 1. *AFRA is not a conservative generalisation of Dung's approach.* □

In order to characterize the relation between our approach and AFRA, we will need the following notation. Given some structure $\mathfrak{A} = \langle S, \Gamma \rangle$, by

$$\mathcal{E}_{\mathfrak{A}} \stackrel{\text{def}}{=} S \cup \{ \alpha \in \Gamma \mid \mathbf{s}(\alpha) \in S \}$$

we denote its corresponding AFRA-extension.

Note that the AFRA-extension corresponding to a given structure only contains the attacks of the structure whose source belongs to the structure. The other attacks of the structure do not appear in the AFRA-extension. Intuitively, this selection is motivated by the fact that any attack in an AFRA-extension directly carries a conflict against its target, even if its source is not accepted, something which we avoid in our framework.

Proposition 2. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *and a structure* $\mathfrak{A} = \langle S, \Gamma \rangle$*, the following assertions hold:*

i) if $\mathfrak{A}$ *is conflict-free, then* $\mathcal{E}_{\mathfrak{A}}$ *is AFRA-conflict-free,*

ii) if $\mathfrak{A}$ *is admissible, then* $\mathcal{E}_{\mathfrak{A}}$ *is AFRA-admissible,*

iii) if $\mathfrak{A}$ *is complete, then* $\mathcal{E}_{\mathfrak{A}}$ *is AFRA-complete,*

iv) if $\mathfrak{A}$ *is preferred, then* $\mathcal{E}_{\mathfrak{A}}$ *is AFRA-preferred,*

v) if $\mathfrak{A}$ *is stable, then* $\mathcal{E}_{\mathfrak{A}}$ *is AFRA-stable.* □

For the converse of Prop. 2, we need to introduce some extra notation: Given some set $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$, by $S_{\mathcal{E}} \stackrel{\text{def}}{=} (\mathcal{E} \cap \mathbf{A})$, we denote the set of arguments of $\mathcal{E}$. Then, considering the structure $\mathfrak{A}' = \langle S_{\mathcal{E}}, (\mathcal{E} \cap \mathbf{K}) \rangle$, by

$$\Gamma_{\mathcal{E}} \stackrel{\text{def}}{=} (\mathcal{E} \cap \mathbf{K}) \cup \{ \alpha \in (Acc(\mathfrak{A}') \cap \mathbf{K}) \mid \mathbf{s}(\alpha) \notin \mathcal{E} \} \quad (5)$$

we denote the set of "valid attacks" with respect to $\mathcal{E}$. Finally, by $\mathfrak{A}_{\mathcal{E}} \stackrel{\text{def}}{=} \langle S_{\mathcal{E}}, \Gamma_{\mathcal{E}} \rangle$, we denote the structure corresponding to some AFRA-extension $\mathcal{E}$. Here, we have to add attacks that do not belong to the AFRA-extension. Intuitively, this is due to the fact that, in AFRA, an attack is not acceptable whenever its source is not acceptable [4, Lemma 1]. Hence, we need to add to the structure all those attacks that are non-AFRA-acceptable only because of attacks towards their source.

Proposition 3. *Given a* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *and a set* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$*, the following assertions hold:*

i) if $\mathcal{E}$ *is AFRA-conflict-free, then* $\mathfrak{A}_\mathcal{E}$ *is conflict-free,*

ii) if $\mathcal{E}$ *is AFRA-admissible, then* $\mathfrak{A}_\mathcal{E}$ *is conflict-free,*

iii) if $\mathcal{E}$ *is AFRA-complete,* $\mathfrak{A}_\mathcal{E}$ *is a complete structure,*

iv) if $\mathcal{E}$ *is AFRA-preferred,* $\mathfrak{A}_\mathcal{E}$ *is a preferred structure,*

v) if $\mathcal{E}$ *is AFRA-stable,* $\mathfrak{A}_\mathcal{E}$ *is a stable structure.* □

It is worth to emphasise that for an AFRA-admissible extension, Proposition 3 only ensures that the corresponding structure $\mathfrak{A}_\mathcal{E}$ is a conflict-free structure. In fact, there exist AFRA-admissible extensions, whose corresponding structures are not admissible. For instance, considering the argumentation framework of

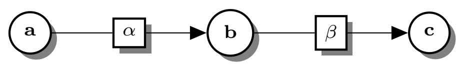

Figure 10: A Dung's framework with two attacks

Fig. 10, the set $\{\alpha, c\}$ is AFRA-admissible, but the corresponding structure $\langle \{c\}, \{\alpha, \beta\} \rangle$ is not an admissible structure (since a is not in the structure). This discrepancy follows from the fact that, in AFRA, α defeats β despite of the absence of a while in our approach attacks whose source is not accepted cannot defeat other arguments or attacks. This difference disappears if we consider what we call *closed* sets. We say that $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ is *closed* iff every attack $\alpha \in (\mathcal{E} \cap \mathbf{K})$ satisfies $\mathbf{s}(\alpha) \in \mathcal{E}$.

Proposition 4. *Let* $\mathcal{E}$ *be a closed AFRA-admissible extension. Then,* $\mathfrak{A}_\mathcal{E}$ *is an admissible structure.* □

Note that for conflict-freeness and admissibility, the correspondence is not necessarily one-to-one. For instance, $\mathfrak{A} = \langle \{a, c\}, \{\alpha\} \rangle$ and $\mathfrak{A}' = \langle \{a, c\}, \{\alpha, \beta\} \rangle$ are both admissible structures of the framework of Fig. 10 and both of them correspond to the same AFRA-admissible extension $\mathcal{E}_\mathfrak{A} = \mathcal{E}_{\mathfrak{A}'} = \{a, c, \alpha\}$. Recall that β is acceptable w.r.t. $\mathfrak{A}'$ because it is not attacked. However, it is not AFRA-acceptable w.r.t. $\{a, c, \alpha, \beta\}$ because, in AFRA, α defeats β and α is not itself defeated (in fact, $\{a, c, \alpha, \beta\}$ is not even AFRA-conflict-free). On the other hand, note that only $\mathfrak{A}'$ is a complete structure. In fact, for complete structures the correspondence is one-to-one.

Let us denote by $\texttt{Afra}(\cdot)$ the function mapping each structure $\mathfrak{A}$ to its corresponding AFRA-extension $\mathcal{E}_\mathfrak{A}$.

Proposition 5. *The following assertions hold:*

i) if $\mathcal{E}$ is AFRA-complete (or just a closed AFRA-conflict-free extension), then $\texttt{Afra}(\mathfrak{A}_{\mathcal{E}}) = \mathcal{E}$*, and*

ii) if $\mathfrak{A}$ is a complete structure, then $\mathfrak{A}_{\texttt{Afra}(\mathfrak{A})} = \mathfrak{A}$*.* □

Theorem 3. *The function* $\texttt{Afra}(\cdot)$ *is a one-to-one correspondence between the sets of all complete (resp. preferred and stable) structures and the set of all AFRA-complete (resp. preferred and stable) extensions.* □

Note that given the one-to-one correspondence between preferred structures and AFRA-preferred extensions, there are AFRA-preferred extensions that do not correspond to arg-preferred ones and thus, they do not contain a maximal set of arguments. For instance, $\{a, b, \beta\}$ and $\{a, \alpha\}$ are both AFRA-preferred extensions in Example 4, but only the former contains a maximal set of arguments.

An interesting consequence of Theorem 3 and Proposition 12 in [4] is that complexity for RAFs' semantics does not increase w.r.t. Dung's frameworks. That is, credulous acceptance w.r.t. the complete, preferred and the stable semantics is NP-complete. Sceptical acceptance w.r.t. the preferred (resp. stable) semantics is Π_2^{P}-complete (resp. coNP-complete) [11].

Example 2 (cont'd) *For the framework represented in Fig. 2, there is a unique AFRA-complete, AFRA-preferred and AFRA-stable extension:* $\mathcal{E} = \{a, g, p, \alpha, \epsilon, \gamma\}$*. Note that* $\delta \notin \mathcal{E}$ *whereas* $\mathcal{E} = \mathcal{E}_{\mathfrak{A}_0}$*. Indeed, no AFRA-admissible extension contains* δ*. Analogously, we have* $\mathcal{E}_{\mathfrak{A}_1} = \mathcal{E}_{\mathfrak{A}_2} = \mathcal{E}_{\mathfrak{A}_3} = \{a, \epsilon\}$*. Moreover, among the AFRA-admissible extensions, we find* $\{a, g, \epsilon, \gamma\}$ *which is not closed. The associated structure* $\mathfrak{A}_4 = \langle\{a, g\}, \{\epsilon, \gamma\}\rangle$ *is not an admissible structure.*

5 Conservative generalisation

As mentioned in the introduction, our theory aims to be a conservative generalisation of Dung's theory (**P2**). Indeed, given the one-to-one correspondence between complete, preferred and stable structures and their corresponding AFRA-extensions and between the latter and Dung's extensions [4] in the case of non-recursive frameworks, it immediately follows that there exists a one-to-one correspondence between complete, preferred and stable structures and their corresponding Dung's extensions.

On the other hand, this is not the case when we consider only conflict-freeness or admissibility. As mentioned in the introduction, $\{a, b\}$ is an AFRA-conflict-free extension of the non-recursive argumentation framework of Example 3. From Proposition 3, this implies that the corresponding structure $\langle\{a, b\}, \varnothing\rangle$, is a conflict-free structure.

It is worth to note that, in Dung's argumentation frameworks, every attack is considered as "valid" in the sense that it may affect its target. The following definition strengthens the notion of structure by adding a kind of reinstatement principle on attacks, that forces every attack that cannot be defeated to be "valid".

Definition 11 (D-structure)**.** *A* d-structure $\mathfrak{A} = \langle S, \Gamma \rangle$ *is a structure that satisfies* $(Acc(\mathfrak{A}) \cap \mathbf{K}) \subseteq \Gamma$. □

Definition 12. *A* conflict-free *(resp.* admissible, complete, preferred, stable*)* d-structure *is a conflict-free (resp. admissible, complete, preferred, stable) structure which is also a d-structure.* □

As a direct consequence of Definition 7, we have:

Observation 2. *Every complete structure is also a d-structure.* □

Observation 2 plus Theorem 3 immediately imply the existence of a one-to-one correspondence between complete (resp. preferred or stable) d-structures and their corresponding AFRA and Dung's extensions. In order to establish a correspondence between conflict-free (resp. admissible) d-structures and their corresponding Dung's extensions, we need to define what it means for a set of arguments to be an extension of some recursive framework.

Definition 13 (Argument extensions)**.** *A set of arguments* $S \subseteq \mathbf{A}$ *is* conflict-free *(resp.* admissible, complete, preferred, stable*) iff there is some* $\Gamma \subseteq \mathbf{K}$ *such that* $\mathfrak{A} = \langle S, \Gamma \rangle$ *is a conflict-free (resp. admissible, complete, preferred, stable) d-structure.* □

Definition 13 allows us to talk about sets of arguments instead of structures. Before formalising the fact that Definition 13 characterizes a conservative generalisation of Dung's argumentation framework, we define the attack relation associated with some framework in a similar way to the attack relation associated with some structure: $\mathbf{R_{RAF}} \stackrel{\text{def}}{=} \{\, (\mathbf{s}(\alpha), \mathbf{t}(\alpha)) \mid \alpha \in \mathbf{K} \,\}$. Note that, since every structure $\mathfrak{A} = \langle S, \Gamma \rangle$ satisfies $\Gamma \subseteq \mathbf{K}$, it clearly follows that $\mathbf{R}_{\mathfrak{A}} \subseteq \mathbf{R_{RAF}}$. We also precise what we mean by non-recursive framework:

Definition 14 (Non-recursive framework)**.** *A framework* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *is said to be* non-recursive *iff* $\mathbf{R_{RAF}} \subseteq \mathbf{A} \times \mathbf{A}$. □

That is, non-recursive frameworks are those in which no attack targets another attack. Given a non-recursive framework **RAF**, it is easy to observe that $\mathbf{AF} = \langle \mathbf{A}, \mathbf{R_{RAF}} \rangle$ is a D-framework (Definition 1). In this sense, by $\mathbf{RAF}^D \stackrel{\text{def}}{=} \langle \mathbf{A}, \mathbf{R_{RAF}} \rangle$, we denote the D-framework associated with some **RAF**.

Observation 3. *Every d-structure* $\mathfrak{A} = \langle S, \Gamma \rangle$ *of any non-recursive framework satisfies* $\Gamma = \mathbf{K}$. □

Theorem 4. *A set of arguments* $S \subseteq \mathbf{A}$ *is conflict-free (resp. admissible, complete, preferred or stable) w.r.t. some non-recursive* **RAF** *(Definition 13) iff it is conflict-free (resp. admissible, complete, preferred or stable) w.r.t.* $\mathbf{RAF}^D$ *(Definition 3).* □

Due to Observation 2, it follows directly that:

Corollary 1. *A structure* $\mathfrak{A} = \langle S, \mathbf{K} \rangle$ *is complete (resp. preferred, stable) w.r.t. a non-recursive* **RAF** *(Definition 7) iff* S *is complete (resp. preferred or stable) w.r.t.* $\mathbf{RAF}^D$ *(Definition 3).* □

It is worth to note that the notion of d-structure provides alternative semantics for the principles of conflict-freeness and admissibility.

Example 1 (cont'd) *Among the conflict-free structures that are not admissible, only five are conflict-free d-structures:* $\langle \varnothing, \{\alpha, \beta\} \rangle$, $\langle \{a\}, \{\alpha, \beta\} \rangle$, $\langle \{b\}, \{\alpha, \beta\} \rangle$, $\langle \{a, b\}, \{\beta\} \rangle$, $\langle \{b\}, \{\beta\} \rangle$. *Similarly, among the admissible structures that are not complete, only five are admissible d-structures:* $\langle \{a, c\}, \{\beta\} \rangle$, $\langle \{b, c\}, \{\beta\} \rangle$, $\langle \{a\}, \{\beta\} \rangle$, $\langle \{c\}, \{\beta\} \rangle$ *and* $\langle \varnothing, \{\beta\} \rangle$. □

Example 2 (cont'd) *There are admissible structures w.r.t. the framework represented in Fig. 2 that are not d-structures: for instance* $\mathfrak{A}_1$ *and* $\mathfrak{A}_2$. *Indeed, each d-structure must contain the attacks thas are not targeted by any other attack, that is,* $\{\epsilon, \alpha, \delta\}$. *Moreover each d-structure containing a must also contain* γ. □

6 Inhibited attacks

In this section, the intuition behind the concept of inhibited attacks is deepened and precisely defined. Indeed, we may expect that attacks that are inhibited do not have any effect on their targets, that is, we may remove them without modifying the condition of the structure.

Example 6. *Let* **RAF** *be the recursive argumentation framework of Fig. 6 and* $\mathfrak{A} = \langle \{a, b, c\}, \{\beta\} \rangle$ *its unique complete structure. It is easy to check that* α *is inhibited w.r.t.* $\mathfrak{A}$ *because c and* β *belong to the structure and* α *is the target of* β. *According to the above intuition, we may expect that this would imply that there is a "somehow" corresponding structure* $\mathfrak{A}'$ *which is complete w.r.t. some* $\mathbf{RAF}'$ *obtained by removing* α. *Note that, in this case, removing* α *also implies removing* β *because there cannot be attacks without target. In fact, the resulting* $\mathbf{RAF}'$ *is a recursive framework with arguments* $\{a, b, c\}$ *and no attack. It is easy to check that* $\mathfrak{A}' = \langle \{a, b, c\}, \varnothing \rangle$ *is complete (also preferred and stable) w.r.t* $\mathbf{RAF}'$ *and that it shares with* $\mathfrak{A}$ *the set of "acceptable" arguments.* □

Let us now formalise this intuition:

Definition 15. *Given some framework* **RAF** *and two different attacks* β, α, *we define:* $\beta \prec \alpha$ *iff there is some chain of attacks* $\delta_1, \delta_2, \ldots \delta_n$ *such that* $\delta_1 = \beta$, $\delta_n = \alpha$ *and* $\mathbf{t}(\delta_i) = \delta_{i+1}$ *for* $1 \leq i < n$. □

For instance, in the argumentation framework of Fig. 6, we have that $\beta \prec \alpha$. On the other hand, neither $\alpha \prec \beta$, nor $\beta \prec \alpha$ hold for the argumentation framework of Fig. 10. Note that $\prec$ is the empty relation for any non-recursive framework. As usual, by $\preceq$ we denote the reflexive closure of $\prec$.

Given an attack α, and a set of attacks Γ, by $\Gamma^{-\alpha} \stackrel{\text{def}}{=} \Gamma \backslash \{ \beta \in \mathbf{K} \mid \beta \preceq \alpha \}$ we denote the set of attacks obtained by removing the attack α from Γ. Furthermore, by $\mathbf{RAF}^{-\alpha} = \langle \mathbf{A}, \mathbf{K}^{-\alpha}, \mathbf{s}^{-\alpha}, \mathbf{t}^{-\alpha} \rangle$, with $\mathbf{s}^{-\alpha}$ and $\mathbf{t}^{-\alpha}$ the restrictions of $\mathbf{s}$ and $\mathbf{t}$ to $\mathbf{K}^{-\alpha}$, we denote the framework obtained by removing the attack α

from $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$. Similarly, by $\mathfrak{A}^{-\alpha} = \langle S, \Gamma^{-\alpha} \rangle$ we denote the structure obtained by removing the attack α from the structure $\mathfrak{A} = \langle S, \Gamma \rangle$.

Example 1 (cont'd) *Let* $\mathbf{RAF}$ *be the recursive argumentation framework of Fig. 6. Then* $\mathbf{RAF}^{-\alpha} = \langle \mathbf{A}, \varnothing, \mathbf{s}^{-\alpha}, \mathbf{t}^{-\alpha} \rangle$ *with* $\mathbf{A} = \{a, b, c\}$ *because* $\beta \prec \alpha$ *implies that* $\beta \notin \mathbf{K}^{-\alpha}$*. Furthermore, if* $\mathfrak{A} = \langle \{a, b, c\}, \{\beta\} \rangle$*, then* $\mathfrak{A}^{-\alpha} = \langle \{a, b, c\}, \varnothing \rangle$ *which is a stable structure of* $\mathbf{RAF}^{-\alpha}$*.* □

Proposition 6 below formalises the intuitions presented in the previous example.

Proposition 6. *Let* $\mathbf{RAF}$ *be some framework,* $\mathfrak{A}$ *be some conflict-free (resp. admissible, complete, preferred, stable) structure and* $\alpha \in Inh(\mathfrak{A})$ *be some inhibited attack w.r.t.* $\mathfrak{A}$*. Then,* $\mathfrak{A}^{-\alpha}$ *is a conflict-free (resp. admissible, complete, preferred, stable) structure of* $\mathbf{RAF}^{-\alpha}$*.* □

7 Conclusion and future works

In this work we have extended Dung's abstract argumentation framework with recursive attacks. One of the essential characteristics of this extension is its conservative nature with respect to Dung's approach (when d-structures are considered). The other one is that semantics are given with respect to the notion of "valid attacks" which play a role analogous to attacks in Dung's frameworks. The notions of "grounded attack" and "valid attack" have been introduced in [7]. However, these notions have been encoded through a two-step translation into a meta-argumentation framework. In the first step, a meta-argument is associated to an attack, and a support relation is added from the source of the attack to the meta-argument. In the second step, a support relation is encoded by the addition of a new meta-argument and new attacks. So [7] uses a method for flattening a recursive framework. As a consequence, extensions contain different kinds of argument. In contrast, we propose a theory where valid attacks remain explicit, and distinct from arguments, within the notion of structure. Despite these differences with respect to other generalisations, we proved a one-to-one correspondence with AFRA-extensions in the case of the complete, preferred and stable semantics, while retaining a one-to-one correspondence with Dung's frameworks in the case of conflict-free and admissible extensions.

For a better understanding of the RAF framework, future work should include the study of other semantics (stage, semi-stable, grounded and ideal), extending our approach by taking into account bipolar interactions [8, 15] (case when arguments and attacks may be attacked or supported), and enriching the translation proposed by [5, 6, 12, 14] from Dung's framework into propositional logic and ASP, in order to capture RAF.

References

[1] L. Amgoud and C. Cayrol. A reasoning model based on the production of acceptable arguments. *Annals of Mathematics and Artificial Intelligence*, 34:197–216, 2002.

[2] L. Amgoud, N. Maudet, and S. Parsons. Modelling dialogues using argumentation. In *Proc. of ICMAS*, pages 31–38, 2000.

[3] R. Arisaka and K. Satoh. Voluntary manslaughter? a case study with meta-argumentation with supports. In S. Kurahashi, Y. Ohta, S. Arai, K. Satoh, and D. Bekki, editors, *New Frontiers in Artificial Intelligence. JSAI-isAI 2016. Lecture Notes in Computer Science, vol 10247*, pages 241–252. Springer, Cham, 2017.

[4] P. Baroni, F. Cerutti, M. Giacomin, and G. Guida. AFRA: argumentation framework with recursive attacks. *Int. J. Approx. Reasoning*, 52(1):19–37, 2011.

[5] P. Besnard and S. Doutre. Checking the acceptability of a set of arguments. In J. P. Delgrande and T. Schaub, editors, *10th International Workshop on Non-Monotonic Reasoning (NMR 2004), Whistler, Canada, June 6-8, 2004, Proceedings*, pages 59–64, 2004.

[6] J. L. Carballido, J. C. Nieves, and M. Osorio. Inferring preferred extensions by pstable semantics. *Inteligencia artificial: Revista Iberoamericana de Inteligencia Artificial*, 13(41):38–53, 2009.

[7] C. Cayrol, A. Cohen, and M-C. Lagasquie-Schiex. Towards a new framework for recursive interactions in abstract bipolar argumentation. In *Proc. of COMMA*, pages 191–198, 2016.

[8] A. Cohen, S. Gottifredi, A. J. García, and G. R. Simari. An approach to abstract argumentation with recursive attack and support. *J. Applied Logic*, 13(4):509–533, 2015.

[9] A. Cohen, S. Gottifredi, A. J. García, and G. R. Simari. On the acceptability semantics of argumentation frameworks with recursive attack and support. In *Proc. of COMMA*, pages 231–242, 2016.

[10] P. M. Dung. On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. *Artif. Intell.*, 77(2):321–358, 1995.

[11] P. E. Dunne. Computational properties of argument systems satisfying graph-theoretic constraints. *Artif. Intell.*, 171(10-15):701–729, 2007.

[12] U. Egly, S. A. Gaggl, and S. Woltran. Answer-set programming encodings for argumentation frameworks. *Argument & Computation*, 1(2):147–177, 2010.

[13] S. Modgil. Reasoning about preferences in argumentation frameworks. *Artif. Intell.*, 173(9-10):901–934, 2009.

[14] M. Osorio, J. C. Nieves, and A. Santoyo. Complete extensions as clark's completion semantics. In *Proc. of the Mexican International Conference on Computer Science*, pages 81–88, 2013.

[15] S. Villata, G. Boella, D. M. Gabbay, and L. van der Torre. Modelling defeasible and prioritized support in bipolar argumentation. *Annals of Maths and AI*, 66(1-4):163–197, 2012.

A Proofs

Lemma A.1. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathfrak{A}$ *be a conflict-free structure. Then, it follows that* $Acc(\mathfrak{A}) \cap Def(\mathfrak{A}) = \varnothing$ *and* $Acc(\mathfrak{A}) \cap Inh(\mathfrak{A}) = \varnothing$

Proof. Assume that $a \in (Acc(\mathfrak{A}) \cap Def(\mathfrak{A}))$. Then, there is $\alpha \in \Gamma$ with $s(\alpha) \in S$ and $t(\alpha) = a$. Since $a \in Acc(\mathfrak{A})$, it follows that either $s(\alpha) \in Def(\mathfrak{A})$ or $\alpha \in Inh(\mathfrak{A})$ holds. Both situations are impossible since $\mathfrak{A}$ is conflict-free, meaning that $S \cap Def(\mathfrak{A}) = \varnothing$ and $\Gamma \cap Inh(\mathfrak{A}) = \varnothing$.

The same reasoning holds for $\beta \in (Acc(\mathfrak{A}) \cap Inh(\mathfrak{A}))$ replacing a by β. □

Lemma A.2. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathfrak{A}, \mathfrak{A}'$ *be two structures such that* $\mathfrak{A} \sqsubseteq \mathfrak{A}'$. *Then, it follows that* $Def(\mathfrak{A}) \subseteq Def(\mathfrak{A}')$ *and* $Inh(\mathfrak{A}) \subseteq Inh(\mathfrak{A}')$. □

Proof. $\mathfrak{A} \sqsubseteq \mathfrak{A}'$ implies that $S \subseteq S'$ and $\Gamma \subseteq \Gamma'$. So due to definition of $Def(\mathfrak{A})$ and $Inh(\mathfrak{A})$, it is obvious that $Def(\mathfrak{A}) \subseteq Def(\mathfrak{A}')$ and $Inh(\mathfrak{A}) \subseteq Inh(\mathfrak{A}')$. □

Lemma A.3. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathfrak{A}, \mathfrak{A}'$ *be two structures such that* $\mathfrak{A} \sqsubseteq \mathfrak{A}'$. *Then, it follows that* $Acc(\mathfrak{A}) \subseteq Acc(\mathfrak{A}')$. □

Proof. Let $x \in Acc(\mathfrak{A})$. Pick any $\alpha \in \mathbf{K}$ with $\mathbf{t}(\alpha) = x$. Since $x \in Acc(\mathfrak{A})$, it follows that either $\mathbf{s}(\alpha) \in Def(\mathfrak{A})$ or $\alpha \in Inh(\mathfrak{A})$ hold. Furthermore, from Lem. A.2, we have $Def(\mathfrak{A}) \subseteq Def(\mathfrak{A}')$ and $Inh(\mathfrak{A}) \subseteq Inh(\mathfrak{A}')$. In its turn, this implies that $x \in Acc(\mathfrak{A}')$ follows. □

Lemma A.4. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be some an admissible structure. Then, any acceptable argument* $a \in (Acc(\mathfrak{A}) \cap \mathbf{A})$ *satisfies that* $\mathfrak{A}' = \langle S \cup \{a\}, \Gamma \rangle$ *is conflict-free.*

Proof. Let $S' = (S \cup \{a\})$ and suppose, for the sake of contradiction, that $\mathfrak{A}'$ is not conflict-free, that is, that either $(S' \cap Def(\mathfrak{A}')) \neq \varnothing$ or $(\Gamma \cap Inh(\mathfrak{A}')) \neq \varnothing$ holds.

a) In the first case, there is $a' \in (S' \cap Def(\mathfrak{A}'))$. So there is $\alpha \in \Gamma$ such that $\mathbf{t}(\alpha) = a'$ and $\mathbf{s}(\alpha) \in S'$. Either $a' = a$ or $a' \in S$.

   Assume first that $a' = a$. Since $a \in Acc(\mathfrak{A})$, it follows that either $\alpha \in Inh(\mathfrak{A})$ or $s(\alpha) \in Def(\mathfrak{A})$ holds. However, we know that $\mathfrak{A}$ is conflict-free, so it is impossible that $\alpha \in (\Gamma \cap Inh(\mathfrak{A}))$. So, it must be the case that $s(\alpha) \in Def(\mathfrak{A})$ holds and, thus, that $s(\alpha) \notin S$ (also because $\mathfrak{A}$ is conflict-free). Hence, $s(\alpha) = a$. Due to Lem. A.1, it is impossible to have $a \in (Acc(\mathfrak{A}) \cap Def(\mathfrak{A}))$ and, thus, $s(\alpha) \in Def(\mathfrak{A})$ plus $s(\alpha) = a$ imply that $a \notin Acc(\mathfrak{A})$. This is a contradiction with the fact $a \in Acc(\mathfrak{A})$.

   Assume now that $a' \neq a$ and, thus, that $a' \in S$. Since $(S \cap Def(\mathfrak{A})) = \varnothing$, we have that $a' \notin Def(\mathfrak{A})$. Hence, $s(\alpha) \notin S$ and $s(\alpha) = a$ hold. Since $\mathfrak{A}$ is admissible and $a' \in S$, it follows that $a' \in Acc(\mathfrak{A})$. Furthermore, since $\mathbf{t}(\alpha) = a'$, it also follows that either $\alpha \in Inh(\mathfrak{A})$ or $s(\alpha) \in Def(\mathfrak{A})$. The former is in contradiction with the fact that $\mathfrak{A}$ is admissible (and thus conflict-free). Furthermore, from Lem. A.1 and the fact that $\mathbf{s}(\alpha) = a$, the latter implies that $a \notin Acc(\mathfrak{A})$ which is a contradiction, too.

b) If $(\Gamma \cap Inh(\mathfrak{A}')) \neq \varnothing$, then there is some attack $\beta \in \Gamma$ with $\beta \in Inh(\mathfrak{A}')$ and thus, there is also some $\alpha \in \Gamma$ such that $\mathbf{t}(\alpha) = \beta$ and $\mathbf{s}(\alpha) \in S'$. Since $\mathfrak{A}$ is conflict-free, $\beta \notin Inh(\mathfrak{A})$ which implies that $\mathbf{s}(\alpha) \notin S$ and thus, $\mathbf{s}(\alpha) = a$ holds. Since $\mathfrak{A}$ is admissible and $\beta \in \Gamma$, it follows that $\beta \in Acc(\mathfrak{A})$. Furthermore, since $\mathbf{t}(\alpha) = \beta$, it must be that either $\alpha \in Inh(\mathfrak{A})$ or $s(\alpha) \in Def(\mathfrak{A})$ holds. The former is in contradiction with the fact that $\alpha \in \Gamma$ and the latter implies that $a \notin Acc(\mathfrak{A})$ which is in contradiction with the hypothesis.

Consequently, $\mathfrak{A}'$ is conflict-free. □

Lemma A.5. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be some admissible structure. Then, any attack* $\alpha \in (Acc(\mathfrak{A}) \cap \mathbf{K})$ *satisfies that* $\mathfrak{A}' = \langle S, \Gamma \cup \{\alpha\} \rangle$ *is conflict-free.*

Proof. Let $\Gamma' = (\Gamma \cup \{\alpha\})$ and suppose, for the sake of contradiction, that $\mathfrak{A}'$ is not conflict free, that is, either $(S \cap Def(\mathfrak{A}')) \neq \varnothing$ or $(\Gamma' \cap Inh(\mathfrak{A}')) \neq \varnothing$.

1. In the first case, there is $a \in S$ with $a \in Def(\mathfrak{A}')$. So there is $\beta \in \Gamma'$ such that $\mathbf{t}(\beta) = a$ and $\mathbf{s}(\beta) \in S$. Since $\mathfrak{A}$ is conflict-free, $a \notin Def(\mathfrak{A})$ and thus, $\beta \notin \Gamma$ and $\beta = \alpha$ follow. Then, $\beta \in Acc(\mathfrak{A})$. Furthermore, since $\mathfrak{A}$ is admissible and $a \in S$, we have that $a \in Acc(\mathfrak{A})$ and thus, that either $\beta \in Inh(\mathfrak{A})$ or $s(\beta) \in Def(\mathfrak{A})$ holds. From Lem. A.1, the former is in contradiction with the fact that $\beta \in Acc(\mathfrak{A})$ and, since $\mathfrak{A}$ is conflict-free, the latter is in contradiction with the fact that $\mathbf{s}(\beta) \in S$.

2. If $(\Gamma' \cap Inh(\mathfrak{A}')) \neq \varnothing$, there is some attack $\alpha' \in \Gamma'$ with $\alpha' \in Inh(\mathfrak{A}')$. So there is $\beta \in \Gamma'$ such that $\mathbf{t}(\beta) = \alpha'$ and $\mathbf{s}(\beta) \in S$. Furthermore, either $\alpha' = \alpha$ or $\alpha' \in \Gamma$.

   Assume first that $\alpha' = \alpha$. Since $\alpha \in Acc(\mathfrak{A})$, it follows that either $\beta \in Inh(\mathfrak{A})$ or $s(\beta) \in Def(\mathfrak{A})$. However, we know that $\mathfrak{A}$ is conflict-free, so it is impossible that $s(\beta) \in (S \cap Def(\mathfrak{A}))$. So we must have $\beta \in Inh(\mathfrak{A})$ and thus, that $\beta \notin \Gamma$ (also because $\mathfrak{A}$ is conflict-free). Hence $\beta = \alpha$. Due to Lem. A.1, it is impossible to have $\beta \in (Acc(\mathfrak{A}) \cap Inh(\mathfrak{A}))$ and thus, $\alpha \notin Acc(\mathfrak{A})$. This is in contradiction with the hypothesis on α.

   Assume now that $\alpha' \neq \alpha$ and thus that $\alpha' \in \Gamma$. Since $\mathfrak{A}$ is conflict-free, it follows that $(\Gamma \cap Inh(\mathfrak{A})) = \varnothing$ and thus, that $\alpha' \notin Inh(\mathfrak{A})$. So $\beta \notin \Gamma$, that is, $\beta = \alpha$ and, thus, that $\beta \in Acc(\mathfrak{A})$. Furthermore, since $\mathfrak{A}$ is admissible and $\alpha' \in \Gamma$, we have that $\alpha' \in Acc(\mathfrak{A})$. As $\mathbf{t}(\beta) = \alpha'$, either $\beta \in Inh(\mathfrak{A})$ or $s(\beta) \in Def(\mathfrak{A})$. From Lem. A.1, the former is in contradiction with the fact that $\beta \in Acc(\mathfrak{A})$ and, since $\mathfrak{A}$ is conflict-free, the latter is in contradiction with the fact that $\mathbf{s}(\beta) \in S$.

Consequently, $\mathfrak{A}'$ is conflict-free. □

Lemma A.6. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be some an admissible structure. Then, any element* $x \in Acc(\mathfrak{A})$ *satisfies that* $\mathfrak{A}' = \mathfrak{A} \cup \{x\}$ *is conflict-free.*

Proof. If $x \in \mathbf{A}$, the result follows directly from Lem. A.4. Otherwise, $x \in \mathbf{K}$, and the result follows from Lem. A.5. □

Lemma A.7. *Any conflict-free structure* $\mathfrak{A} = \langle S, \Gamma \rangle$ *satisfies:* $Acc(\mathfrak{A}) \subseteq (\overline{Def(\mathfrak{A})} \cup \overline{Inh(\mathfrak{A})})$. □

Proof. It follows directly from Lem. A.1 and the definitions of $\overline{Def(\mathfrak{A})}$ and $\overline{Inh(\mathfrak{A})}$.

□

A.1 Proofs of Section 3

Proof of Lemma 1. From Lem. A.6, we know that $\mathfrak{A}' = \langle S', \Gamma' \rangle$ is conflict-free. Furthermore, since $\mathfrak{A}$ is admissible and $x \in Acc(\mathfrak{A})$, $(S \cup \Gamma \cup \{x\}) \subseteq Acc(\mathfrak{A})$. Then, since $\mathfrak{A} \sqsubseteq \mathfrak{A}'$, Lem. A.3 implies that

$$(S' \cup \Gamma') \;=\; (S \cup \Gamma \cup \{x\}) \;\subseteq\; Acc(\mathfrak{A}) \;\subseteq\; Acc(\mathfrak{A}')$$

and thus, that $\mathfrak{A}'$ is admissible and $y \in Acc(\mathfrak{A}')$. □

Lemma A.8. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some non-recursive framework and* $\mathfrak{A}_0 \sqsubseteq \mathfrak{A}_1 \sqsubseteq \ldots$ *be some sequence of conflict-free structures such that* $\mathfrak{A}_i = \langle S_i, \Gamma_i \rangle$*. Let us also define* $\mathfrak{A} = \langle \bigcup_{0 \leq i} S_i, \bigcup_{0 \leq i} \Gamma_i \rangle$*. Then,* $\mathfrak{A}$ *is conflict-free.* □

Proof. Suppose, for the sake of contradiction, that $\mathfrak{A}$ is not conflict-free. Then, either $(S \cap Def(\mathfrak{A})) \neq \varnothing$ or $(\Gamma \cap Inh(\mathfrak{A})) \neq \varnothing$ (with $S = \bigcup_{0 \leq i} S_i$ and $\Gamma = \bigcup_{0 \leq i} \Gamma_i$). Pick any argument $x \in (S \cap Def(\mathfrak{A}))$ (resp. attack $x \in \Gamma \cap Inh(\mathfrak{A})$)). Then, $x \in Def(\mathfrak{A})$ (resp. $x \in Inh(\mathfrak{A})$) implies that there is $\alpha \in \Gamma$ such that $\mathbf{t}(\alpha) = x$ and $\mathbf{s}(\alpha) \in S$. Hence, there is $0 \leq i$ such that $\alpha \in \Gamma_i$ and $0 \leq j$ such that $\mathbf{s}(\alpha) \in S_j$. Let $k = \max\{i, j\}$. Then, $\alpha \in \Gamma_k$ and $\mathbf{s}(\alpha) \in S_k$ which means that $x \in Def(\mathfrak{A}_k)$ (resp. $x \in Inh(\mathfrak{A}_k)$). Moreover, there is $0 \leq l$ such that $x \in S_l$ (resp. $x \in \Gamma_l$). Let $m = \max\{k, l\}$. Then, $x \in S_m$ (resp. $x \in \Gamma_m$), and from Lem. A.2, we have that $Def(\mathfrak{A}_k) \subseteq Def(\mathfrak{A}_m)$ (resp. $Inh(\mathfrak{A}_k) \subseteq Inh(\mathfrak{A}_m)$). That is in contradiction with the fact that $\mathfrak{A}_m$ is conflict-free.

Hence, $\mathfrak{A}$ must be conflict-free. □

Proof of Proposition 1. First note that $\langle \varnothing, \varnothing \rangle$ is always admissible and that $\langle \varnothing, \varnothing \rangle \sqsubseteq \mathfrak{A}$ for any structure $\mathfrak{A}$. Furthermore, for every chain $\mathfrak{A}_0 \sqsubseteq \mathfrak{A}_1 \sqsubseteq \ldots$ with $\mathfrak{A}_i = \langle S_i, \Gamma_i \rangle$, it follows that $\mathfrak{A}_i \sqsubseteq \mathfrak{A}$ with $\mathfrak{A} = \langle S, \Gamma \rangle$ and $S = \bigcup_{0 \leq i} S_i$ and $\Gamma = \bigcup_{0 \leq i} \Gamma_i$. From Lem. A.8, it follows that $\mathfrak{A}$ is conflict-free. Let us show now that $\mathfrak{A}$ is admissible that is, that every element in $\mathfrak{A}$ is acceptable. Pick $x \in (\Gamma \cup S)$ and any attack $\beta \in \mathbf{K}$ with $\mathbf{t}(\beta) = x$. Then, $x \in (\Gamma_i \cup S_i)$ for some

$0 \leq i$. Since $\mathfrak{A}_i$ is admissible, this implies that $x \in Acc(\mathfrak{A}_i)$ and, thus, there is $\gamma \in \Gamma_i \subseteq \Gamma$ such that $\mathbf{s}(\gamma) \in S_i \subseteq S$ and $\mathbf{t}(\gamma) = \beta$. Hence, $x \in Acc(\mathfrak{A})$ and, thus, $\mathfrak{A}$ is admissible.

To show that, for every admissible structure $\mathfrak{A}$, there is some preferred structure $\mathfrak{A}'$ such that $\mathfrak{A} \sqsubseteq \mathfrak{A}'$, suppose, for the sake of contradiction, that there is some admissible structure $\mathfrak{A}$ such that no preferred structure $\mathfrak{A}'$ with $\mathfrak{A} \sqsubseteq \mathfrak{A}'$ exists. Then, there must be some infinite chain $\mathfrak{A} \sqsubseteq \mathfrak{A}_1 \sqsubseteq \mathfrak{A}_2 \sqsubseteq \dots$. However, as shown above, it follows that there is some $\mathfrak{A}'$ such that $\mathfrak{A}_i \sqsubseteq \mathfrak{A}$ for all $\mathfrak{A}_i$ and, thus, $\mathfrak{A}'$ is a preferred structure. □

Proof of Theorem 2.

1. By definition of a complete structure.

2. By definition, every preferred structure $\mathfrak{A} = \langle S, \Gamma \rangle$ is also admissible. Hence, to show that $\mathfrak{A}$ is complete, it enough to prove that $Acc(\mathfrak{A}) \subseteq (S \cup \Gamma)$. Pick any $x \in Acc(\mathfrak{A})$. Then, from Lem. 1 (Fundamental Lemma) it follows that $\mathfrak{A}' = (\mathfrak{A} \cup \{x\})$ is also admissible and that $\mathfrak{A} \sqsubseteq \mathfrak{A}'$. Furthermore, since $\mathfrak{A}$ is preferred, it follows that $\mathfrak{A}$ is a $\sqsubseteq$-maximal admissible structure and, thus, $\mathfrak{A} \sqsubseteq \mathfrak{A}'$ implies that $\mathfrak{A} = \mathfrak{A}'$. Hence, $x \in (S \cup \Gamma)$ holds and, thus, it follows that $Acc(\mathfrak{A}) \subseteq (S \cup \Gamma)$ and that $\mathfrak{A}$ is complete.

3. Assume that $\mathfrak{A}$ is a stable structure. We have to prove that $\mathfrak{A}$ is a $\sqsubseteq$-maximal admissible structure.

   We first prove that $\mathfrak{A}$ is admissible. By definition, $\mathfrak{A}$ is conflict-free and satisfies $S = \overline{Def(\mathfrak{A})}$ and $\Gamma = \overline{Inh(\mathfrak{A})}$. Pick $x \in (\Gamma \cup S)$ and any attack $\beta \in \mathbf{K}$ with $\mathbf{t}(\beta) = x$. As $\mathfrak{A}$ is conflict-free, either $\beta \notin \Gamma$ or $\mathbf{s}(\beta) \notin S$. Hence, either $\beta \in Inh(\mathfrak{A})$ or $\mathbf{s}(\beta) \in Def(\mathfrak{A})$. Thus, it follows that $x \in Acc(\mathfrak{A})$ and that $\mathfrak{A}$ is admissible.

   Now assume $\mathfrak{A}' = \langle S', \Gamma' \rangle$ to be some admissible structure such that $\mathfrak{A} \sqsubseteq \mathfrak{A}'$. Since $\mathfrak{A}'$ is admissible and thus, conflict-free, it follows from Lem. A.7 that $(S' \cup \Gamma') \subseteq Acc(\mathfrak{A}') \subseteq \overline{Def(\mathfrak{A}')} \cup \overline{Inh(\mathfrak{A}')}$. Furthermore, from Lem. A.2, and $\mathfrak{A} \sqsubseteq \mathfrak{A}'$, it follows that

   $$(Def(\mathfrak{A}) \cup Inh(\mathfrak{A})) \quad \subseteq \quad (Def(\mathfrak{A}') \cup Inh(\mathfrak{A}'))$$

   and thus, $\overline{Def(\mathfrak{A}')} \cup \overline{Inh(\mathfrak{A}')} \subseteq \overline{Def(\mathfrak{A})} \cup \overline{Inh(\mathfrak{A})}$. Hence we have $(S' \cup \Gamma') \subseteq \overline{Def(\mathfrak{A})} \cup \overline{Inh(\mathfrak{A})}$. Furthermore, since $\mathfrak{A}$ is stable, it holds that $\overline{Def(\mathfrak{A})} \cup \overline{Inh(\mathfrak{A})} \subseteq (S \cup \Gamma)$ and thus, that $(S' \cup \Gamma') \subseteq (S \cup \Gamma)$ Recall that $\mathfrak{A} \sqsubseteq \mathfrak{A}'$ implies $(S \cup \Gamma) \subseteq (S' \cup \Gamma')$ and thus, $\mathfrak{A} = \mathfrak{A}'$. That is, $\mathfrak{A}$ is a $\sqsubseteq$-maximal admissible structure and, consequently, $\mathfrak{A}$ is a preferred one.

□

A.2 Proofs of Section 4

Lemma A.9. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be a framework and* $\mathfrak{A}$ *be a structure. Then,* $x \in (Def(\mathfrak{A}) \cup Inh(\mathfrak{A}))$ *implies that there is some* $\alpha \in \mathcal{E}_{\mathfrak{A}}$ *such that* α *defeats* x. □

Proof. Since $x \in (Def(\mathfrak{A}) \cup Inh(\mathfrak{A}))$, there is $\alpha \in \Gamma$ s.t. $\mathbf{t}(\alpha) = x$ and $\mathbf{s}(\alpha) \in S$. Note that $\alpha \in \Gamma$ and $\mathbf{s}(\alpha) \in \Gamma$ imply that $\alpha \in \mathcal{E}_{\mathfrak{A}}$ and that $\mathbf{t}(\alpha) = x$ implies that α defeats x. □

Lemma A.10. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathfrak{A}$ *be some structure. Then, every* $a \in (Acc(\mathfrak{A}) \cap \mathbf{A})$ *is AFRA-acceptable w.r.t.* $\mathcal{E}_{\mathfrak{A}}$. □

Proof. Pick any attack $\alpha \in \mathbf{K}$ such that α defeats a. Then, $\mathbf{t}(\alpha) = a$ and, since $a \in Acc(\mathfrak{A})$ it follows that either $\alpha \in Inh(\mathfrak{A})$ or $\mathbf{s}(\alpha) \in Def(\mathfrak{A})$.

If $\alpha \in Inh(\mathfrak{A})$, then there is $\beta \in \Gamma$ such that $\mathbf{s}(\beta) \in S$ and $\mathbf{t}(\beta) = \alpha$. Note that $\beta \in \Gamma$ plus $\mathbf{s}(\beta) \in S$ imply $\beta \in \mathcal{E}_{\mathfrak{A}}$ and that $\mathbf{t}(\beta) = \alpha$ implies that β defeats α. Hence, the fact that a is AFRA-acceptable w.r.t. $\mathcal{E}$ follows.

Otherwise, $\mathbf{s}(\alpha) \in Def(\mathfrak{A})$, and, there is $\beta \in \Gamma$ such that $\mathbf{s}(\beta) \in S$ and $\mathbf{t}(\beta) = \mathbf{s}(\alpha)$. As above, $\beta \in \Gamma$ plus $\mathbf{s}(\beta) \in S$ imply $\beta \in \mathcal{E}_{\mathfrak{A}}$, and $\mathbf{t}(\beta) = \mathbf{s}(\alpha)$ implies that β defeats α. Hence, the fact that a is AFRA-acceptable w.r.t. $\mathcal{E}$ follows.

In consequence, it holds that a is AFRA-acceptable w.r.t. $\mathcal{E}$. □

Lemma A.11. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be some structure. Then, every* $\alpha \in (Acc(\mathfrak{A}) \cap \mathbf{K})$ *that satisfies* $\mathbf{s}(\alpha) \in Acc(\mathfrak{A})$*, is also AFRA-acceptable w.r.t.* $\mathcal{E}_{\mathfrak{A}}$. □

Proof. Pick any attack $\beta \in \mathbf{K}$ such that β defeats α. Then, either $\mathbf{t}(\beta) = \alpha$ or $\mathbf{t}(\beta) = \mathbf{s}(\alpha)$.

If the latter, then Lem. A.10 plus $\mathbf{s}(\alpha) \in Acc(\mathfrak{A})$ imply that $\mathbf{s}(\alpha)$ is AFRA-acceptable w.r.t. $\mathcal{E}_{\mathfrak{A}}$ and, thus, that there is some $\gamma \in \mathcal{E}_{\mathfrak{A}}$ that defeats β.

If the former, $\alpha \in Acc(\mathfrak{A})$ implies that either $\beta \in Inh(\mathfrak{A})$ or $\mathbf{s}(\beta) \in Def(\mathfrak{A})$. Assume $\beta \in Inh(\mathfrak{A})$. Then, there is $\gamma \in \Gamma$ such that $\mathbf{s}(\gamma) \in S$ and $\mathbf{t}(\gamma) = \beta$. Note that $\gamma \in \Gamma$ plus $\mathbf{s}(\gamma) \in S$ imply $\gamma \in \mathcal{E}_{\mathfrak{A}}$ and that $\mathbf{t}(\gamma) = \beta$ implies that γ defeats β.

Otherwise, $\mathbf{s}(\beta) \in Def(\mathfrak{A})$ and there is $\gamma \in \Gamma$ such that $\mathbf{s}(\gamma) \in S$ and $\mathbf{t}(\gamma) = \mathbf{s}(\beta)$. As above, $\gamma \in \Gamma$ plus $\mathbf{s}(\gamma) \in S$ imply $\gamma \in \mathcal{E}_{\mathfrak{A}}$ and $\mathbf{t}(\gamma) = \mathbf{s}(\beta)$ implies that γ defeats β.

Hence, for any attack $\beta \in \mathbf{K}$ that defeats α, there is some attack $\gamma \in \mathcal{E}_{\mathfrak{A}}$ that defeats β. That is, the fact that α is AFRA-acceptable w.r.t. $\mathcal{E}_{\mathfrak{A}}$ follows. □

Lemma A.12. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathfrak{A}$ *be some conflict-free (resp. admissible) structure. Then, it follows that* $\mathcal{E}_{\mathfrak{A}}$ *is AFRA-conflict-free (resp. AFRA-admissible) extension.* □

Proof. Let $\mathfrak{A} = \langle S, \Gamma \rangle$ be a conflict-free structure and suppose that $\mathcal{E}_{\mathfrak{A}}$ is is not an AFRA-conflict-free extension. Then, there are $\alpha, x \in \mathcal{E}_{\mathfrak{A}}$ s.t. α defeats x and, thus, $\alpha \in \Gamma'$ with $\Gamma' = \{ \alpha \in \Gamma \mid \mathbf{s}(\alpha) \in S \}$. That is, $\alpha \in \Gamma$ and $\mathbf{s}(\alpha) \in S$.

If $x \in S$, then we have that $\mathbf{t}(\alpha) = x \in \mathit{Def}(\mathfrak{A})$ which is a contradiction with the fact that $S \cap (\mathit{Def}(\mathfrak{A})) \neq \varnothing$ holds because $\mathfrak{A}$ is conflict-free.

Otherwise, $x \notin S$ implies $x \in \mathbf{K}$. Then, $x \in \mathcal{E}_{\mathfrak{A}}$ implies that $x \in \Gamma'$ and, thus, that $x \in \Gamma$ and $\mathbf{s}(x) \in S$. Furthermore, α defeats x implies that either $\mathbf{t}(\alpha) = x$ or $\mathbf{t}(\alpha) = \mathbf{s}(x)$ holds. The former, $\mathbf{t}(\alpha) = x$, implies that $x \in \mathit{Inh}(\mathfrak{A})$ which is a contradiction with the fact that $(\Gamma \cap \mathit{Inh}(\mathfrak{A})) \neq \varnothing$ follows from $\mathfrak{A}$ being conflict-free. The latter, $\mathbf{t}(\alpha) = \mathbf{s}(x)$, is a contradiction with the fact that $\mathbf{s}(x) \in S$ and the fact that $(S \cap \mathit{Def}(\mathfrak{A})) \neq \varnothing$ follows from $\mathfrak{A}$ being conflict-free.

If, in addition, $\mathfrak{A}$ is admissible, then $(S \cup \Gamma) \subseteq \mathit{Acc}(\mathfrak{A})$ and, from Lem. A.10, it follows that every argument $a \in (\mathcal{E}_{\mathfrak{A}} \cap \mathbf{A})$ is AFRA-acceptable w.r.t. $\mathcal{E}_{\mathfrak{A}}$. Furthermore, by construction, every attack $\alpha \in (\mathcal{E}_{\mathfrak{A}} \cap \mathbf{K})$ satisfies that $\mathbf{s}(\alpha) \in S$. Hence, both α and $\mathbf{s}(\alpha)$ are acceptable w.r.t. $\mathfrak{A}$ and, from Lem. A.11, it follows that α is AFRA-acceptable w.r.t. $\mathcal{E}_{\mathfrak{A}}$. Consequently, if $\mathfrak{A}$ is admissible, it implies that $\mathcal{E}_{\mathfrak{A}}$ is AFRA-admissible. □

Lemma A.13. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathfrak{A}$ *be some structure. Then,* $\alpha \in (\mathit{Acc}(\mathfrak{A}) \cap \mathbf{K})$ *and* $\mathbf{s}(\alpha) \notin S$ *imply* $\alpha \in \Gamma_{\mathcal{E}_{\mathfrak{A}}}$. □

Proof. Let $\mathfrak{A}' = \langle S, \Gamma_{\mathcal{E}_{\mathfrak{A}}} \rangle$ and let us show that $\alpha \in \mathit{Acc}(\mathfrak{A}')$. Pick any $\beta \in \mathbf{K}$ such that $\mathbf{t}(\beta) = \alpha$. Since $\alpha \in \mathit{Acc}(\mathfrak{A})$, it follows that either $\beta \in \mathit{Inh}(\mathfrak{A})$ or $\mathbf{s}(\beta) \in \mathit{Def}(\mathfrak{A})$. If the former, there is $\gamma \in \Gamma$ such that $\mathbf{t}(\gamma) = \beta$ and $\mathbf{s}(\gamma) \in S$. Note that $\gamma \in \Gamma$ plus $\mathbf{s}(\gamma) \in S$ imply that $\gamma \in \mathcal{E}_{\mathfrak{A}}$ and, thus, that $\gamma \in \Gamma_{\mathcal{E}_{\mathfrak{A}}}$ and that $\beta \in \mathit{Inh}(\mathfrak{A}')$. Similarly, $\mathbf{s}(\beta) \in \mathit{Def}(\mathfrak{A})$ implies that there is $\gamma \in \Gamma$ such that $\mathbf{t}(\gamma) = \mathbf{s}(\beta)$ and $\mathbf{s}(\gamma) \in S$ and, thus, $\mathbf{s}(\beta) \in \mathit{Def}(\mathfrak{A}')$. Hence, any $\beta \in \mathbf{K}$ with $\mathbf{t}(\beta) = \alpha$ satisfies either $\beta \in \mathit{Inh}(\mathfrak{A}')$ or $\mathbf{s}(\beta) \in \mathit{Def}(\mathfrak{A}')$. That is, $\alpha \in \mathit{Acc}(\mathfrak{A}')$. Hence, to show that $\alpha \in \Gamma_{\mathcal{E}_{\mathfrak{A}}}$ it is enough to prove that $\mathbf{s}(\alpha) \notin \mathcal{E}_{\mathfrak{A}}$ which directly follows from the fact that $\mathbf{s}(\alpha) \notin S$. □

Lemma A.14. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be an framework and* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be some admissible structure. Then,* $S = S_{\mathcal{E}_{\mathfrak{A}}}$ *and* $\Gamma \subseteq \Gamma_{\mathcal{E}_{\mathfrak{A}}}$ *hold.* □

Proof. Note that, by definition, it follows that $S_{\mathcal{E}_{\mathfrak{A}}} = (\mathcal{E}_{\mathfrak{A}} \cap \mathbf{A}) = S$.

Then, to show that $\Gamma \subseteq \Gamma_{\mathcal{E}_{\mathfrak{A}}}$ holds, pick any attack $\alpha \in \Gamma$. If $\mathbf{s}(\alpha) \in S$, then $\alpha \in (\mathcal{E}_{\mathfrak{A}} \cap \mathbf{K})$ and thus, $\alpha \in \Gamma_{\mathcal{E}_{\mathfrak{A}}}$. Otherwise, $\mathbf{s}(\alpha) \notin S$ and $\alpha \in (\mathit{Acc}(\mathfrak{A}) \cap \mathbf{K})$ as $\mathfrak{A} = \langle S, \Gamma \rangle$ is admissible. So, from Lem. A.13, it follows that $\alpha \in \Gamma_{\mathcal{E}_{\mathfrak{A}}}$. □

Lemma A.15. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathfrak{A}$ *be a complete structure. Then, it follows that* $\mathit{Acc}(\mathfrak{A}) \subseteq (S_{\mathcal{E}_{\mathfrak{A}}} \cup \Gamma_{\mathcal{E}_{\mathfrak{A}}})$. □

Proof. Let $\mathfrak{A} = \langle S, \Gamma \rangle$. Note that, since $\mathfrak{A}$ is complete, it follows that $\mathit{Acc}(\mathfrak{A}) \subseteq (S \cup \Gamma)$ and, thus, the result follows directly from Lem. A.14. □

Lemma A.16. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathfrak{A}$ *be a complete structure. Then,* $\mathfrak{A}_{\mathcal{E}_{\mathfrak{A}}} = \mathfrak{A}$. □

Proof. Let $\mathfrak{A} = \langle S, \Gamma \rangle$. From Lem. A.14, it follows that $S = S_{\mathcal{E}_\mathfrak{A}}$ and $\Gamma \subseteq \Gamma_{\mathcal{E}_\mathfrak{A}}$ hold. Hence, it remains to be shown that $\Gamma_{\mathcal{E}_\mathfrak{A}} \subseteq \Gamma$ also holds.

Let $\Gamma' = (\mathcal{E}_\mathfrak{A} \cap \mathbf{K})$ and $\mathfrak{A}' = \langle S, \Gamma' \rangle$. Then, it follows that $\alpha \in \Gamma_{\mathcal{E}_\mathfrak{A}}$ implies that either $\alpha \in \Gamma' \subseteq \Gamma$ or both $\mathbf{s}(\alpha) \notin \mathcal{E}_\mathfrak{A}$ and $\alpha \in \mathit{Acc}(\langle S_{\mathcal{E}_\mathfrak{A}}, (\mathcal{E}_\mathfrak{A} \cap \mathbf{K}) \rangle)$. Furthermore, the latter plus $S_{\mathcal{E}_\mathfrak{A}} = S$ imply that $\alpha \in \mathit{Acc}(\mathfrak{A}')$. Note that, from Lem. A.3, this plus $\mathfrak{A}' \sqsubseteq \mathfrak{A}$ imply $\alpha \in \mathit{Acc}(\mathfrak{A})$ and, since $\mathfrak{A}$ is complete, this implies that $\alpha \in \Gamma$. Therefore, $\Gamma_{\mathcal{E}_\mathfrak{A}} \subseteq \Gamma$ holds. □

Lemma A.17. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ *be some closed AFRA-conflict-free extension. Then, every AFRA-acceptable element* x *w.r.t.* $\mathcal{E}$ *satisfies* $x \in \mathit{Acc}(\mathfrak{A}_\mathcal{E})$. □

Proof. Let $x \in (\mathbf{A} \cup \mathbf{K})$ be an AFRA-acceptable element w.r.t. $\mathcal{E}$ and pick any attack $\alpha \in \mathbf{K}$ such that $\mathbf{t}(\alpha) = x$. Since x is AFRA-acceptable w.r.t. $\mathcal{E}$, there is $\beta \in \mathcal{E}$ such that β defeats α. Note that $\beta \in \mathcal{E}$ implies that $\beta \in \Gamma_\mathcal{E}$ and that β defeats α implies that either $\mathbf{t}(\beta) = \alpha$ or $\mathbf{t}(\beta) = \mathbf{s}(\alpha)$ holds.

Furthermore, by Lem. hypothesis, $\beta \in \mathcal{E}$ implies that $\mathbf{s}(\beta) \in \mathcal{E}$. Then, this implies $\mathbf{s}(\beta) \in S_\mathcal{E}$ and, thus, that $\mathbf{t}(\beta) \in (\mathit{Inh}(\mathfrak{A}_\mathcal{E}) \cup \mathit{Def}(\mathfrak{A}_\mathcal{E}))$ holds.

Hence, the fact that either $\mathbf{t}(\beta) = \alpha$ or $\mathbf{t}(\beta) = \mathbf{s}(\alpha)$ holds implies that either $\alpha \in \mathit{Inh}(\mathfrak{A}_\mathcal{E})$ or $\mathbf{s}(\alpha) \in \mathit{Def}(\mathfrak{A}_\mathcal{E})$ must also hold and thus, $x \in \mathit{Acc}(\mathfrak{A}_\mathcal{E})$. □

Lemma A.18. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework,* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be a complete structure and* x *be some AFRA-acceptable element w.r.t.* $\mathcal{E}_\mathfrak{A}$*. Then,* $x \in (S \cup \Gamma)$ *holds.* □

Proof. From Lem. A.17, the hypothesis implies that $x \in \mathit{Acc}(\mathfrak{A}_{\mathcal{E}_\mathfrak{A}})$. Note that, since $\mathfrak{A}$ is complete, Lem. A.16, implies that $\mathfrak{A}_{\mathcal{E}_\mathfrak{A}} = \mathfrak{A}$ and, thus, that $x \in \mathit{Acc}(\mathfrak{A})$ and that $x \in (S \cup \Gamma)$ (recall that $\mathfrak{A}$ is complete). □

Lemma A.19. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be a complete structure. Then,* $\mathcal{E}_\mathfrak{A}$ *is AFRA-complete.* □

Proof. Since $\mathfrak{A}$ is a complete structure, it is admissible and, in addition, it satisfies $(S \cup \Gamma) = \mathit{Acc}(\mathfrak{A})$. From Lem. A.12, the former implies that $\mathcal{E}_\mathfrak{A}$ is AFRA-admissible. Hence, to show that $\mathcal{E}_\mathfrak{A}$ is AFRA-complete, it is enough to prove that every acceptable element x w.r.t. $\mathcal{E}_\mathfrak{A}$ belongs to $\mathcal{E}_\mathfrak{A}$.

Pick any AFRA-acceptable element $x \in (\mathbf{A} \cup \mathbf{K})$ w.r.t. $\mathcal{E}_\mathfrak{A}$. From Lem. A.18, this implies that $x \in (S \cup \Gamma)$. Note that, by construction, we have that $S \subseteq \mathcal{E}_\mathfrak{A}$. Furthermore, if $x \in \Gamma$, then Lem. 1 in [4] plus the fact that x is AFRA-acceptable w.r.t. $\mathcal{E}_\mathfrak{A}$, imply that $\mathbf{s}(x)$ is AFRA-acceptable w.r.t. $\mathcal{E}_\mathfrak{A}$ and, from Lem. A.18 again, this implies that $\mathbf{s}(x) \in S$. By definition, $x \in \Gamma$ plus $\mathbf{s}(x) \in S$ imply $x \in \mathcal{E}_\mathfrak{A}$ and, thus, that $\Gamma \subseteq \mathcal{E}_\mathfrak{A}$. Therefore, we have that every AFRA-acceptable element x w.r.t. $\mathcal{E}_\mathfrak{A}$ belongs to $\mathcal{E}_\mathfrak{A}$ and, thus, that $\mathcal{E}_\mathfrak{A}$ is an AFRA-complete extension. □

Lemma A.20. *Let* **RAF** $= \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ *be an AFRA-conflict-free extension. Then, it follows that* $\mathfrak{A}_{\mathcal{E}}$ *is a conflict-free structure.* □

Proof. Let $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ be a conflict-free structure and $\mathfrak{A}_{\mathcal{E}} = \langle S_{\mathcal{E}}, \Gamma_{\mathcal{E}} \rangle$, and pick any $a \in \mathit{Def}(\mathfrak{A}_{\mathcal{E}})$. Then, from Lem. A.9, there is some $\alpha \in \mathcal{E}_{\mathfrak{A}}$ such that α defeats a. Furthermore, since $\mathcal{E}$ is AFRA-conflict-free, $\alpha \in (\mathcal{E} \cap \mathbf{K})$ implies that $a \notin \mathcal{E}$ and, by definition, this implies that $a \notin S_{\mathcal{E}}$. Then, since a is an arbitrary element of $\mathit{Def}(\mathfrak{A}_{\mathcal{E}})$, it follows that $(S_{\mathcal{E}} \cap \mathit{Def}(\mathfrak{A}_{\mathcal{E}})) = \varnothing$.

Similarly, pick any $\alpha \in \mathit{Inh}(\mathfrak{A}_{\mathcal{E}})$. Then, there exists some attack $\beta \in \Gamma_{\mathcal{E}}$ such that $\mathbf{s}(\beta) \in S_{\mathcal{E}}$ and $\mathbf{t}(\beta) = \alpha$. As above, this implies that β (directly) defeats α and, since $\beta \in \Gamma_{\mathcal{E}}$ and $\mathbf{s}(\beta) \in S_{\mathcal{E}} \subseteq \mathcal{E}$, it follows that $\beta \in (\mathcal{E} \cap \mathbf{K})$. Moreover, since $\mathcal{E}$ is an AFRA-conflict-free extension, this implies that $\alpha \notin \mathcal{E}$.

Then, to show that $\alpha \notin \Gamma_{\mathcal{E}}$, it is enough to prove that $\alpha \notin \mathit{Acc}(\langle S_{\mathcal{E}}, (\mathcal{E} \cap \mathbf{K}) \rangle)$. Note that $\beta \in \mathcal{E}$ plus the fact that $\mathcal{E}$ is AFRA-conflict-free imply that there is no $\gamma \in \mathcal{E}$ that defeats β. Hence, every $\gamma \in (\mathcal{E} \cap \mathbf{K})$ satisfies both $\mathbf{t}(\gamma) = \beta$ and $\mathbf{t}(\gamma) = \mathbf{s}(\beta)$. Then, the former implies $\alpha \notin \mathit{Acc}(\langle S_{\mathcal{E}}, (\mathcal{E} \cap \mathbf{K}) \rangle)$. Hence, we obtain $(\Gamma_{\mathcal{E}} \cap \mathit{Inh}(\mathfrak{A}_{\mathcal{E}})) = \varnothing$, which together with $(S_{\mathcal{E}} \cap \mathit{Def}(\mathfrak{A}_{\mathcal{E}})) = \varnothing$, implies that $\mathfrak{A}_{\mathcal{E}}$ is conflict-free. □

Lemma A.21. *Let* **RAF** $= \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ *be a closed AFRA-admissible extension. Then,* $\mathfrak{A}_{\mathcal{E}}$ *is an admissible structure.* □

Proof. First note that, since $\mathcal{E}$ is an AFRA-admissible extension, it is also AFRA-conflict-free and, from Lem. A.20, it follows that $\mathfrak{A}_{\mathcal{E}}$ is a conflict-free structure. Furthermore, since $\mathcal{E}$ is AFRA-admissible, then every element belonging to $\mathcal{E}$ is AFRA-acceptable w.r.t. $\mathcal{E}$. Let $\Gamma' = (\mathcal{E} \cap \mathbf{K})$ and $\mathfrak{A}' = \langle S_{\mathcal{E}}, \Gamma' \rangle$. Since $(S_{\mathcal{E}} \cup \Gamma') \subseteq \mathcal{E}$, every element belonging to $(S_{\mathcal{E}} \cup \Gamma')$ is AFRA-acceptable w.r.t. $\mathcal{E}$. Moreover, since $\mathcal{E}$ is closed, from Lem. A.17, this implies $(S_{\mathcal{E}} \cup \Gamma') \subseteq \mathit{Acc}(\mathfrak{A}_{\mathcal{E}})$. Note also that, by definition, $(\Gamma_{\mathcal{E}} \backslash \Gamma') \subseteq \mathit{Acc}(\mathfrak{A}')$ and that $\mathfrak{A}' \sqsubseteq \mathfrak{A}_{\mathcal{E}}$. Then, from Lem. A.3, it follows that $(\Gamma_{\mathcal{E}} \backslash \Gamma') \subseteq \mathit{Acc}(\mathfrak{A}') \subseteq \mathit{Acc}(\mathfrak{A}_{\mathcal{E}})$ and, thus, that $(S_{\mathcal{E}} \cup \Gamma_{\mathcal{E}}) \subseteq \mathit{Acc}(\mathfrak{A}_{\mathcal{E}})$ holds. Consequently, $\mathfrak{A}_{\mathcal{E}}$ is admissible. □

Lemma A.22. *Let* **RAF** $= \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ *be a closed set. Then, it follows that* $\mathcal{E}_{\mathfrak{A}_{\mathcal{E}}} = \mathcal{E}$. □

Proof. Note that, by definition, it follows that

$$(\mathcal{E}_{\mathfrak{A}_{\mathcal{E}}} \cap \mathbf{A}) \;=\; S_{\mathcal{E}} \;=\; (\mathcal{E} \cap \mathbf{A})$$

It remains to be shown that $(\mathcal{E}_{\mathfrak{A}_{\mathcal{E}}} \cap \mathbf{K}) = (\mathcal{E} \cap \mathbf{K})$. By definition, it follows that $\Gamma_{\mathcal{E}} \supseteq (\mathcal{E} \cap \mathbf{K})$. Therefore, $\alpha \in \mathcal{E}$ implies that $\alpha \in \Gamma_{\mathcal{E}}$ and, since $\mathcal{E}$ is closed, this implies that $\mathbf{s}(\alpha) \in (\mathcal{E} \cap \mathbf{A}) = S_{\mathcal{E}}$ and, thus, that $\alpha \in (\mathcal{E}_{\mathfrak{A}_{\mathcal{E}}} \cap \mathbf{K})$. In its turn, this implies $(\mathcal{E}_{\mathfrak{A}_{\mathcal{E}}} \cap \mathbf{K}) \supseteq (\mathcal{E} \cap \mathbf{K})$.

On the other hand, every $\alpha \in (\mathcal{E}_{\mathfrak{A}_{\mathcal{E}}} \cap \mathbf{K})$ satisfies $\alpha \in \Gamma_{\mathcal{E}}$ and $\mathbf{s}(\alpha) \in S_{\mathcal{E}} \subseteq \mathcal{E}$. Together, these two facts imply $\alpha \in (\mathcal{E} \cap \mathbf{K})$. Hence, $(\mathcal{E}_{\mathfrak{A}_{\mathcal{E}}} \cap \mathbf{K}) = (\mathcal{E} \cap \mathbf{K})$ holds and, thus, $\mathcal{E}_{\mathfrak{A}_{\mathcal{E}}} = \mathcal{E}$ follows. □

Lemma A.23. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathcal{E}$ *be some AFRA-complete extension. Then,* $\mathcal{E}$ *is closed.* □

Proof. Pick any attack $\alpha \in (\mathcal{E} \cap \mathbf{K})$. Then, since $\mathcal{E}$ is AFRA-complete, this implies that α is AFRA-acceptable w.r.t. $\mathcal{E}$ and, from Lem. 1 in [4], this implies that $\mathbf{s}(\alpha)$ is also AFRA-acceptable w.r.t. $\mathcal{E}$. This plus the fact that $\mathcal{E}$ is AFRA-complete imply that $\mathbf{s}(\alpha) \in \mathcal{E}$. □

Lemma A.24. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ *be a AFRA-complete extension. Then,* $\mathfrak{A}_{\mathcal{E}}$ *is a complete structure.* □

Proof. By definition, every AFRA-complete extension is also AFRA-admissible. Furthermore, from Lem. A.23, every AFRA-complete extension is also closed, thus, Lem. A.21 implies that $\mathfrak{A}_{\mathcal{E}}$ is an admissible structure. Then, to show that $\mathfrak{A}_{\mathcal{E}}$ is a complete structure, it is enough to prove the following inclusion: $Acc(\mathfrak{A}_{\mathcal{E}}) \subseteq (S_{\mathcal{E}} \cup \Gamma_{\mathcal{E}})$. Let us recall that, from Lem. A.15, it follows that

$$Acc(\mathfrak{A}_{\mathcal{E}}) \quad \subseteq \quad (S_{\mathcal{E}_{\mathfrak{A}_{\mathcal{E}}}} \cup \Gamma_{\mathcal{E}_{\mathfrak{A}_{\mathcal{E}}}})$$

and that, from Lem. A.22, it follows $\mathcal{E}_{\mathfrak{A}_{\mathcal{E}}} = \mathcal{E}$. As a result, $Acc(\mathfrak{A}_{\mathcal{E}}) \subseteq (S_{\mathcal{E}} \cup \Gamma_{\mathcal{E}})$ holds and, thus, $\mathfrak{A}_{\mathcal{E}}$ is complete. □

Lemma A.25. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathcal{E} \subseteq \mathcal{E}' \subseteq (\mathbf{A} \cup \mathbf{K})$ *be two AFRA-complete extensions. Then,* $\mathfrak{A}_{\mathcal{E}} \sqsubseteq \mathfrak{A}_{\mathcal{E}'}$. □

Proof. First, note that

$$S_{\mathcal{E}} \;=\; (\mathcal{E} \cap \mathbf{A}) \;\subseteq\; (\mathcal{E}' \cap \mathbf{A}) \;=\; S_{\mathcal{E}'}$$

Let $S = S_{\mathcal{E}}$, $S' = S_{\mathcal{E}'}$, $\Gamma = (\mathcal{E} \cap \mathbf{K})$ and $\Gamma' = (\mathcal{E}' \cap \mathbf{K})$. Let also $\mathfrak{A} = \langle S, \Gamma \rangle$ and $\mathfrak{A}' = \langle S', \Gamma' \rangle$. Then,

$$\begin{array}{lcl} \Gamma_{\mathcal{E}} & = & (\mathcal{E} \cap \mathbf{K}) \cup \{\, \alpha \in Acc(\mathfrak{A}) \;\big|\; \mathbf{s}(\alpha) \notin S \,\} \\ \Gamma_{\mathcal{E}'} & = & (\mathcal{E}' \cap \mathbf{K}) \cup \{\, \alpha \in Acc(\mathfrak{A}') \;\big|\; \mathbf{s}(\alpha) \notin S' \,\} \end{array}$$

Hence, to show $\Gamma_{\mathcal{E}} \subseteq \Gamma_{\mathcal{E}'}$, it is enough to prove

$$\{\, \alpha \in Acc(\mathfrak{A}) \;\big|\; \mathbf{s}(\alpha) \notin S \,\} \subseteq \mathcal{E}' \cup \{\, \alpha \in Acc(\mathfrak{A}') \;\big|\; \mathbf{s}(\alpha) \notin S' \,\}$$

Furthermore, from Lem. A.3 and the fact that $\mathfrak{A} \sqsubseteq \mathfrak{A}'$, it follows that $Acc(\mathfrak{A}) \subseteq Acc(\mathfrak{A}')$ and, thus, it is enough to show that every $\alpha \in Acc(\mathfrak{A})$ satisfies that: $\mathbf{s}(\alpha) \notin S$ implies that either $\mathbf{s}(\alpha) \notin S'$ or $\alpha \in \mathcal{E}'$.

Suppose, for the sake of contradiction, that there is some $\alpha \in (Acc(\mathfrak{A}) \backslash \mathcal{E}')$ that satisfies $\mathbf{s}(\alpha) \notin S$ and $\mathbf{s}(\alpha) \in S'$. Since by hypothesis $\mathcal{E}'$ is AFRA-complete, $\alpha \notin \mathcal{E}'$ implies that α is not AFRA-acceptable w.r.t. $\mathcal{E}'$ and, thus, there is some $\beta \in \mathbf{K}$ that defeats α and is not defeated by any $\gamma \in \mathcal{E}'$. That is, either $\mathbf{t}(\beta) = \alpha$ or $\mathbf{t}(\beta) = \mathbf{s}(\alpha)$. If the latter, then β also defeats $\mathbf{s}(\alpha)$. But, then $\mathbf{s}(\alpha) \in S'$ implies $\mathbf{s}(\alpha) \in \mathcal{E}'$ which, in its turn, implies that $\mathbf{s}(\alpha)$ is AFRA-acceptable w.r.t. $\mathcal{E}'$ and, thus, that β is defeated by some $\gamma \in \mathcal{E}'$ which is a contradiction with the above. Hence, it must be that $\mathbf{s}(\beta) = \alpha$ and, thus, that $\alpha \notin Acc(\mathfrak{A})$ which is a contradiction with the assumption. Hence, $\Gamma \subseteq \Gamma'$ holds. □

Lemma A.26. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be a preferred structure. Then,* $\mathcal{E}_{\mathfrak{A}}$ *is AFRA-preferred.* □

Proof. Since $\mathfrak{A}$ is a preferred structure, it is admissible and, in addition, there is no admissible structure $\mathfrak{A}$ such that $\mathfrak{A} \sqsubseteq \mathfrak{A}'$. From Lem. A.12, the former implies that $\mathcal{E}_{\mathfrak{A}}$ is AFRA-admissible. Hence, to show that $\mathcal{E}_{\mathfrak{A}}$ is AFRA-preferred, it is enough to prove that there does not exist any AFRA-admissible extension $\mathcal{E}$ such that $\mathcal{E}_{\mathfrak{A}} \subset \mathcal{E}$.

Suppose, for the sake of contradiction, that there exists any AFRA-admissible extension $\mathcal{E}$ such that $\mathcal{E}_{\mathfrak{A}} \subset \mathcal{E}$. Since $\mathcal{E}$ is AFRA-admissible, from Theorem 2 in [4], there is some AFRA-preferred extension $\mathcal{E}'$ such that $\mathcal{E}_{\mathfrak{A}} \subset \mathcal{E} \subseteq \mathcal{E}'$. Furthermore, from Lem. 4 in [4], it follows that $\mathcal{E}'$ is also AFRA-complete and, thus, from Lem. A.24, that $\mathfrak{A}_{\mathcal{E}'}$ is a complete structure. Furthermore, since $\mathfrak{A}$ is a preferred structure, from Theorem. 2, it follows that $\mathfrak{A}$ is also complete and thus, from Lem. A.19, that $\mathcal{E}_{\mathfrak{A}}$ is AFRA-complete. From Lem. A.25 and the fact that both $\mathcal{E}_{\mathfrak{A}}$ and $\mathcal{E}'$ are complete, $\mathcal{E}_{\mathfrak{A}} \subseteq \mathcal{E}$ implies that $\mathfrak{A}_{\mathcal{E}_{\mathfrak{A}}} \sqsubseteq \mathfrak{A}_{\mathcal{E}'}$ Moreover, since $\mathfrak{A}$ is complete, from Lem. A.16, it follows that $\mathfrak{A}_{\mathcal{E}_{\mathfrak{A}}} = \mathfrak{A}$ and, thus, that $\mathfrak{A} \sqsubseteq \mathfrak{A}_{\mathcal{E}'}$.

Note that $S \subset S'$ implies $\mathfrak{A} \sqsubset \mathfrak{A}_{\mathcal{E}'}$, which is a contradiction with the assumption that $\mathfrak{A}$ is a preferred structure. Hence, it must be that $S = S'$ holds. Furthermore, since $\mathcal{E}_{\mathfrak{A}} \subset \mathcal{E}'$, there is some element $x \in (\mathcal{E}' \backslash \mathcal{E}_{\mathfrak{A}})$ and, since $S = S'$, it follows that $x \in \mathbf{K}$. From Lem. A.23 and the fact that $\mathcal{E}'$ is AFRA-complete, it follows that $\mathcal{E}'$ is closed and, thus, $x \in \mathcal{E}'$ implies that $\mathbf{s}(x) \in \mathcal{E}'$. This implies $\mathbf{s}(x) \in S$ and, since $S = S'$, that $\mathbf{s}(x) \in S$ and $\mathbf{s}(x) \in \mathcal{E}$. This plus $x \notin \mathcal{E}_{\mathfrak{A}}$ imply that $x \notin \Gamma$ and, thus, that $\Gamma \subset \Gamma'$ and $\mathfrak{A} \sqsubset \mathfrak{A}'$ hold. The latter is a contradiction with the assumption that $\mathfrak{A}$ is a preferred structure. Hence, $\mathcal{E}_{\mathfrak{A}}$ is AFRA-preferred. □

Lemma A.27. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be a stable structure. Then,* $\mathcal{E}_{\mathfrak{A}}$ *is AFRA-stable.* □

Proof. Since $\mathfrak{A}$ is a stable structure, it is conflict-free and, in addition, it satisfies $S = \overline{\mathit{Def}(\mathfrak{A})}$ and $\Gamma = \overline{\mathit{Inh}(\mathfrak{A})}$. From Lem. A.12, the former implies that $\mathcal{E}_{\mathfrak{A}}$ is AFRA-conflict-free. Hence, to show that $\mathcal{E}_{\mathfrak{A}}$ is AFRA-stable, it is enough to prove that, for every $x \in ((\mathbf{A} \cup \mathbf{K}) \backslash \mathcal{E}_{\mathfrak{A}})$, there is $\alpha \in \mathcal{E}_{\mathfrak{A}}$ such that α defeats x.

First, note that $x \in ((\mathbf{A} \cup \mathbf{K}) \backslash \mathcal{E}_{\mathfrak{A}})$ implies that either $x \notin (S \cup \Gamma)$ or $x \in \Gamma$ but $\mathbf{s}(x) \notin S$. Since $\mathfrak{A}$ is stable, the former implies that $x \in (\mathit{Def}(\mathfrak{A}) \cup \mathit{Inh}(\mathfrak{A}))$ and, from Lem. A.9, this implies that there is some $\alpha \in \mathcal{E}_{\mathfrak{A}}$ such that α defeats x. On the other hand, the latter implies that $\mathbf{s}(x) \notin S$ and, thus, that $\mathbf{s}(x) \in \mathit{Def}(\mathfrak{A})$. From Lem. A.9, this implies that there is some $\alpha \in \mathcal{E}_{\mathfrak{A}}$ such that α defeats $\mathbf{s}(x)$ and, thus, that defeats x. □

Proof of Proposition 2. For i) and ii), note that from Lem. A.12, it follows that $\mathfrak{A}$ being conflict-free (resp. admissible) implies that $\mathcal{E}_{\mathfrak{A}}$ is AFRA-conflict-free (resp. AFRA-admissible). Conditions iii), iv) and v) follow directly from Lem. A.19, A.26 and A.27, respectively. □

Lemma A.28. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ *be a closed set. Then, it follows that* $x \in (\overline{Def(\mathfrak{A}_{\mathcal{E}})} \cup \overline{Inh(\mathfrak{A}_{\mathcal{E}})})$ *implies that there is no* $\alpha \in \mathcal{E}$ *such that* α *directly defeats* x. □

Proof. Suppose, for the sake of contradiction, that there is $\alpha \in \mathcal{E}$ such that α directly defeats x. If α directly defeats x, then $\mathbf{t}(\alpha) = x$. Furthermore, since $\mathcal{E}$ is closed, $\alpha \in \mathcal{E}$ implies that $\mathbf{s}(\alpha) \in \mathcal{E}$ and, thus, that $\alpha \in \Gamma_{\mathcal{E}}$ and that $\mathbf{s}(\alpha) \in S_{\mathcal{E}}$. This implies that $x \in (Def(\mathfrak{A}_{\mathcal{E}}) \cup Inh(\mathfrak{A}_{\mathcal{E}}))$ which is a contradiction with the assumption. □

Lemma A.29. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ *be a closed set. Then,* $a \in \overline{Def(\mathfrak{A}_{\mathcal{E}})}$ *implies that there is no* $\alpha \in \mathcal{E}$ *such that* α *defeats* a. □

Proof. $a \in \overline{Def(\mathfrak{A}_{\mathcal{E}})}$ implies that $a \in \mathbf{A}$ and, thus, α defeats a only if α directly defeats a. Then, the result follows directly from Lem. A.28. □

Lemma A.30. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ *be a closed set. Then,* $\alpha \in \overline{Inh(\mathfrak{A}_{\mathcal{E}})}$ *and* $\mathbf{s}(\alpha) \in \overline{Def(\mathfrak{A}_{\mathcal{E}})}$ *imply that there is no* $\beta \in \mathcal{E}$ *such that* β *defeats* α. □

Proof. From Lem. A.28, it follows that there is is no $\beta \in \mathcal{E}$ such that β directly defeats α. Suppose, for the sake of contradiction, that there is $\beta \in \mathcal{E}$ such that β indirectly defeats α. This implies that $\mathbf{t}(\beta) = \mathbf{s}(\alpha)$. Furthermore, since $\mathcal{E}$ is closed, it follows that $\beta \in \mathcal{E}$ implies that $\mathbf{s}(\beta) \in \mathcal{E}$ and, thus, that $\beta \in \Gamma_{\mathcal{E}}$ and that $\mathbf{s}(\beta) \in S_{\mathcal{E}}$. This implies that $\mathbf{s}(\alpha) \in Def(\mathfrak{A}_{\mathcal{E}})$ which is a contradiction with the assumption. □

Lemma A.31. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ *be a closed AFRA-conflict-free set. Then,* $x \in (\overline{Def(\mathfrak{A}_{\mathcal{E}})} \cup \overline{Inh(\mathfrak{A}_{\mathcal{E}})})$ *implies that there is no* $\alpha \in \mathcal{E}$ *such that* α *defeats* x. □

Proof. Pick any $x \in (\overline{Def(\mathfrak{A})} \cup \overline{Inh(\mathfrak{A}_{\mathcal{E}})})$. If $x \in \mathbf{A}$, from Lem. A.29, it follows that there is no $\alpha \in \mathcal{E}$ such that α defeats x. Otherwise, $x \in \mathbf{K}$ and $x \in \overline{Inh(\mathfrak{A}_{\mathcal{E}})}$. Since $\mathcal{E}$ is closed, it follows that $\mathbf{s}(x) \in \mathcal{E}$ and $\mathbf{s}(x) \in S_{\mathcal{E}}$. Furthermore, since $\mathcal{E}$ is conflict-free, Lem. A.20 implies that $\mathfrak{A}_{\mathcal{E}}$ is conflict-free. Then, $\mathbf{s}(x) \in S_{\mathcal{E}}$ implies $\mathbf{s}(x) \in \overline{Def(\mathfrak{A}_{\mathcal{E}})}$. From Lem. A.30, this plus $x \in \overline{Inh(\mathfrak{A}_{\mathcal{E}})}$ imply that there is no $\alpha \in \mathcal{E}$ such that α defeats x. □

Lemma A.32. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework. Then, every AFRA-stable extension is closed.* □

Proof. Note that every AFRA-stable extension is also AFRA-complete (Lem. 4 and 5 in [4]) and, thus, Lem. A.23 implies that every AFRA-stable extension is also closed. □

Lemma A.33. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ *be a AFRA-stable extension. Then, it follows that* $(\overline{Def(\mathfrak{A}_{\mathcal{E}})} \cup \overline{Inh(\mathfrak{A}_{\mathcal{E}})}) \subseteq \mathcal{E}$. □

Proof. By definition every AFRA-stable extension is AFRA-conflict-free. Furthermore, from Lem. A.32, every AFRA-stable extension is closed. Then, Lem. A.31 implies that, for every $x \in (\overline{Def(\mathfrak{A})} \cup \overline{Inh(\mathfrak{A}_{\mathcal{E}})})$, there is no $\alpha \in \mathcal{E}$ such that α defeats x. Then, since $\mathcal{E}$ is AFRA-stable, this implies that $x \in \mathcal{E}$ and, consequently, that $(\overline{Def(\mathfrak{A})} \cup \overline{Inh(\mathfrak{A}_{\mathcal{E}})}) \subseteq \mathcal{E}$ holds. ◻

Lemma A.34. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ *be a AFRA-stable extension. Then,* $\mathfrak{A}_{\mathcal{E}}$ *is a stable structure.* ◻

Proof. Since by definition every AFRA-stable extension is AFRA-conflict-free, it follows that $\mathfrak{A}_{\mathcal{E}}$ is a conflict-free structure (Lem. A.20). Then, to show that $\mathfrak{A}_{\mathcal{E}}$ is stable, it is enough to prove $S_{\mathcal{E}} = \overline{Def(\mathfrak{A}_{\mathcal{E}})}$ and $\Gamma_{\mathcal{E}} = \overline{Inh(\mathfrak{A}_{\mathcal{E}})}$. Note that, since $\mathfrak{A}_{\mathcal{E}}$ is conflict-free, it follows that $S \subseteq \overline{Def(\mathfrak{A}_{\mathcal{E}})}$ and $\Gamma \subseteq \overline{Inh(\mathfrak{A}_{\mathcal{E}})}$ hold. Furthermore, from Lem. A.33, it follows that

$$(\overline{Def(\mathfrak{A}_{\mathcal{E}})} \cup \overline{Inh(\mathfrak{A}_{\mathcal{E}})}) \quad \subseteq \quad \mathcal{E} \quad \subseteq \quad (S_{\mathcal{E}} \cup \Gamma_{\mathcal{E}})$$

and, thus, that $S = \overline{Def(\mathfrak{A}_{\mathcal{E}})}$ and $\Gamma = \overline{Inh(\mathfrak{A}_{\mathcal{E}})}$ hold. Consequently, $\mathfrak{A}_{\mathcal{E}}$ is a stable structure. ◻

Lemma A.35. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework. Every AFRA-preferred extension is closed.* ◻

Proof. Note that every AFRA-preferred extension is also AFRA-complete (Lem. 4 in [4]) and, thus, Lem. A.23 implies that every AFRA-preferred extension is also closed. ◻

Lemma A.36. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathfrak{A} \sqsubseteq \mathfrak{A}'$ *be two structures. Then,* $\mathcal{E}_{\mathfrak{A}} \subseteq \mathcal{E}_{\mathfrak{A}'}$*.* ◻

Proof. Let $\mathfrak{A} = \langle S, \Gamma \rangle$ and $\mathfrak{A}' = \langle S', \Gamma' \rangle$. Then,

$$(\mathcal{E}_{\mathfrak{A}} \cap \mathbf{A}) \;=\; S \;\subseteq\; S' \;=\; (\mathcal{E}_{\mathfrak{A}'} \cap \mathbf{A})$$

Furthermore,

$$\begin{array}{rcl} (\mathcal{E}_{\mathfrak{A}} \cap \mathbf{K}) & = & \{ \, \alpha \in \Gamma \;\big|\; \mathbf{s}(\alpha) \in S \, \} \\ (\mathcal{E}_{\mathfrak{A}} \cap \mathbf{K}) & = & \{ \, \alpha \in \Gamma' \;\big|\; \mathbf{s}(\alpha) \in S' \, \} \end{array}$$

and, thus, $(\mathcal{E}_{\mathfrak{A}} \cap \mathbf{K}) \subseteq (\mathcal{E}_{\mathfrak{A}} \cap \mathbf{K})$. These two facts together imply $\mathcal{E}_{\mathfrak{A}} \subseteq \mathcal{E}_{\mathfrak{A}'}$. ◻

Lemma A.37. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ *be a set. Then,* $\mathcal{E}_{\mathfrak{A}_{\mathcal{E}}} \subseteq \mathcal{E}$*. Furthermore, if* $\mathcal{E}$ *is closed, then* $\mathcal{E}_{\mathfrak{A}_{\mathcal{E}}} = \mathcal{E}$*.* ◻

Proof. First, note that by definition $S_{\mathcal{E}} = (\mathcal{E} \cap \mathbf{A})$ and, therefore, it holds that $(\mathcal{E}_{\mathfrak{A}_{\mathcal{E}}} \cap \mathbf{A}) = (\mathcal{E} \cap \mathbf{A})$. Furthermore, $\alpha \in (\mathcal{E}_{\mathfrak{A}_{\mathcal{E}}} \cap \mathbf{K})$ satisfies that $\alpha \in \Gamma_{\mathcal{E}}$ and $\mathbf{s}(\alpha) \in S_{\mathcal{E}}$. Moreover, note that $\alpha \in \Gamma_{\mathcal{E}}$ implies that either $\alpha \in (\mathcal{E} \cap \mathbf{K}) \subseteq \mathcal{E}$ or $\mathbf{s}(\alpha) \notin \mathcal{E}$. However, the latter is a contradiction with the facts that $\mathbf{s}(\alpha) \in S_{\mathcal{E}}$ and $S_{\mathcal{E}} = (\mathcal{E} \cap \mathbf{A}) \subseteq \mathcal{E}$. Hence, it follows that $(\mathcal{E}_{\mathfrak{A}_{\mathcal{E}}} \cap \mathbf{K}) \subseteq (\mathcal{E} \cap \mathbf{K})$ holds. This plus $(\mathcal{E}_{\mathfrak{A}_\mathcal{E}} \cap \mathbf{A}) = (\mathcal{E} \cap \mathbf{A})$ imply $(\mathcal{E}_{\mathfrak{A}_\mathcal{E}} \subseteq \mathcal{E})$.

Assume now that $\mathcal{E}$ is closed and pick $\alpha \in (\mathcal{E} \cap \mathbf{K})$. By definition, it follows that $\alpha \in \Gamma_\mathcal{E}$. Furthermore, since $\alpha \in \mathcal{E}$ and $\mathcal{E}$ is closed, it follows that $\mathbf{s}(\alpha) \in \mathcal{E}$ and, thus, that $\mathbf{s}(\alpha) \in S_\mathcal{E}$. Consequently, it follows that $\alpha \in \mathcal{E}_{\mathfrak{A}_\mathcal{E}}$ and that $\mathcal{E}_{\mathfrak{A}_\mathcal{E}} = \mathcal{E}$ holds. □

Lemma A.38. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ *be a AFRA-preferred extension. Then,* $\mathfrak{A}_\mathcal{E}$ *is a preferred structure.* □

Proof. By definition every AFRA-preferred extension is AFRA-admissible. Furthermore, from Lem. A.35, it follows that AFRA-preferred extensions are closed. Then, from Lem. A.21, it follows that $\mathfrak{A}_\mathcal{E}$ is an admissible structure. Hence, to show that $\mathfrak{A}_\mathcal{E}$ is preferred, it is enough to show that there does not exist any admissible structure $\mathfrak{A}'$ such that $\mathfrak{A}_\mathcal{E} \sqsubset \mathfrak{A}'$.

Suppose that such admissible structure exists. Then, from Prop. 1, there is some preferred structure $\mathfrak{A}''$ such that $\mathfrak{A}_\mathcal{E} \sqsubset \mathfrak{A}' \sqsubseteq \mathfrak{A}''$. From Lem. A.36, this implies that $\mathcal{E}_{\mathfrak{A}_\mathcal{E}} \subseteq \mathcal{E}_{\mathfrak{A}''}$. Note also that, since $\mathcal{E}$ is closed, Lem. A.37 implies that $\mathcal{E}_{\mathfrak{A}_\mathcal{E}} = \mathcal{E}$ and, thus, $\mathcal{E} \subseteq \mathcal{E}_{\mathfrak{A}''}$. Since $\mathcal{E}$ is AFRA-preferred, this implies that $\mathcal{E} \supseteq \mathcal{E}_{\mathfrak{A}''}$ also holds. From Lem. A.25, this implies that $\mathfrak{A}_\mathcal{E} \sqsupseteq \mathfrak{A}_{\mathcal{E}_{\mathfrak{A}''}}$. Finally, note that, since $\mathfrak{A}''$ is a preferred structure, it is also a complete one (Theorem. 2) and, thus, $\mathfrak{A}_{\mathcal{E}_{\mathfrak{A}''}} = \mathfrak{A}''$ (Lem. A.16). That is, we have that $\mathfrak{A}_\mathcal{E} \sqsupseteq \mathfrak{A}''$. However, this is a contradiction with the fact that $\mathfrak{A}_\mathcal{E} \sqsubset \mathfrak{A}''$ and, thus, it must be that $\mathfrak{A}_\mathcal{E}$ is preferred. □

Proof of Proposition 3. For i), note that from Lem. A.20, it follows that the fact of $\mathcal{E}$ being AFRA-conflict-free implies that $\mathfrak{A}_\mathcal{E}$ is a conflict-free structure. Conditions iii), iv) and v) follow directly from Lem. A.24, A.38 and A.34, respectively. □

Proof of Proposition 4. This is just a rephrasing of Lem. A.21 in order to keep the order of presentation. □

Proof of Proposition 5. Condition i) follows directly from Lem. A.37 and A.23. Condition ii) follows from Lem. A.16. □

A.3 Proofs of Section 5

Proof of Observation 3. Pick any $\alpha \in \mathbf{K}$. Since **RAF** is non-recursive, there is no $\beta \in \mathbf{K}$ s.t. $\mathbf{t}(\beta) = \alpha$ and thus $\alpha \in Acc(\mathfrak{A})$. As $\mathfrak{A}$ is a d-structure, $\alpha \in \Gamma$. □

Proposition 7. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some non-recursive framework and* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be some d-structure. Then, any argument* $a \in \mathbf{A}$ *satisfies:* $a \in Def(\mathfrak{A})$ *iff it is defeated w.r.t.* S *and* $\mathbf{RAF}^D$ *(Definition 2).* □

Proof. Recall that, by definition, it follows that $\mathbf{RAF}^D = \langle \mathbf{A}, \mathbf{R_{RAF}} \rangle$. Then, from Observation 3, it follows that $\mathbf{R}_{\mathfrak{A}} = \mathbf{R_{RAF}}$ for every d-structure $\mathfrak{A}$. Then, the result follows by observing that the definition of $Def(\mathfrak{A})$ (Equation (1)) is obtained from the defeated definition (Def. 2) by just replacing $\mathbf{R_{RAF}}$ by $\mathbf{R}_{\mathfrak{A}}$. □

Proposition 8. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some non-recursive framework and* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be some conflict-free d-structure. Then, any argument* $a \in \mathbf{A}$ *satisfies:* $a \in Acc(\mathfrak{A})$ *iff* a *is acceptable w.r.t.* S *and* $\mathbf{RAF}^D$ *(Definition 2).* □

Proof. First note that, since **RAF** is non recursive, it follows that $Inh(\mathfrak{A}) = \varnothing$ and, thus, that $\overline{Inh(\mathfrak{A})} = \mathbf{K}$ holds. Furthermore, from Observation 3, it also follows that $\Gamma = \mathbf{K}$. Hence, we may rewrite the definition of acceptability as follows:
$a \in \mathbf{A}$ is acceptable with respect to some d-structure $\mathfrak{A}$
iff every $\alpha \in \mathbf{K}$ with $\mathbf{t}(\alpha) = a$ satisfies $\mathbf{s}(\alpha) \in Def(\mathfrak{A})$
iff for every $b \in \mathbf{A}$, $(b, a) \in \mathbf{R}_{\mathfrak{A}}$ implies $b \in Def(\mathfrak{A})$
iff for every $b \in \mathbf{A}$, $(b, a) \in \mathbf{R_{RAF}}$ implies $b \in Def(\mathfrak{A})$
iff a is acceptable w.r.t. S and $\mathbf{RAF}^D$ (Definition 2). □

Proof of Theorem 4. First note that due to Observation 3, S is a conflict-free (resp. admissible, complete, preferred or stable) extension of some non-recursive **RAF** iff $\mathfrak{A} = \langle S, \mathbf{K} \rangle$ is a conflict-free (resp. admissible, complete, preferred or stable) structure. Then,

1. Conflict-free: $\mathfrak{A} = \langle S, \mathbf{K} \rangle$ is a conflict-free structure
   iff $S \cap Def(\mathfrak{A}) = \varnothing$ and $\mathbf{K} \cap Inh(\mathfrak{A}) = \varnothing$
   iff $S \cap Def(\mathfrak{A}) = \varnothing$ (note that $Inh(\mathfrak{A}) = \varnothing$)
   iff $S \cap Def(S) = \varnothing$ (Prop. 7)
   iff S is a conflict-free extension of $\mathbf{RAF}^D$.

2. Admissible: $\mathfrak{A} = \langle S, \mathbf{K} \rangle$ is an admissible structure
   iff $\mathfrak{A}$ is conflict-free and $(S \cup \mathbf{K}) \subseteq Acc(\mathfrak{A})$
   iff S is conflict-free and $(S \cup \mathbf{K}) \subseteq Acc(\mathfrak{A})$
   iff S is conflict-free and $S \subseteq Acc(\mathfrak{A})$ (since no attack is attacked)
   iff S is conflict-free and $S \subseteq Acc(S)$ (Prop. 8)
   iff S is an admissible extension of $\mathbf{RAF}^D$.

3. Complete: $\mathfrak{A} = \langle S, \mathbf{K} \rangle$ is a complete structure
   iff $\mathfrak{A}$ is admissible and $Acc(\mathfrak{A}) \subseteq (S \cup \mathbf{K})$
   iff S is admissible and $Acc(\mathfrak{A}) \subseteq (S \cup \mathbf{K})$

iff S is admissible and $(Acc(\mathfrak{A}) \cap \mathbf{A}) \subseteq S$
iff S is admissible and $Acc(S) \subseteq S$ (Prop. 8)
iff S is a complete extension of $\mathbf{RAF}^D$.

4. Preferred: $\mathfrak{A} = \langle S, \mathbf{K} \rangle$ is a preferred structure
iff $\mathfrak{A}$ is admissible and $\nexists \mathfrak{A}' = \langle S', \Gamma' \rangle$ admissible structure s.t. $(S \cup \mathbf{K}) \subset (S' \cup \Gamma')$
iff $\mathfrak{A}$ is admissible and $\nexists \mathfrak{A}' = \langle S', \mathbf{K} \rangle$ admissible structure s.t. $S \subset S'$
iff S is admissible and $\nexists \mathfrak{A}' = \langle S', \mathbf{K} \rangle$ admissible structure s.t. $S \subset S'$
iff S is admissible and $\nexists S'$ admissible s.t. $S \subset S'$
iff S is a preferred extension of $\mathbf{RAF}^D$.

5. Stable: $\mathfrak{A} = \langle S, \mathbf{K} \rangle$ is a stable structure
iff $\mathfrak{A}$ is conflict-free, $S = \overline{Def(\mathfrak{A})}$ and $\mathbf{K} = \overline{Inh(\mathfrak{A})}$
iff S is conflict-free, $S = \overline{Def(\mathfrak{A})}$ and $\mathbf{K} = \overline{Inh(\mathfrak{A})}$
iff S is conflict-free and $S = \overline{Def(\mathfrak{A})}$ (no attack is attacked)
iff S is conflict-free and $S = \overline{Def(S)}$ (Prop. 7)
iff S is a stable extension of $\mathbf{RAF}^D$.

□

A.4 Proofs of Section 6

By $\downarrow\alpha = \{ \beta \in \mathbf{K} \mid \beta \preceq \alpha \}$ we denote the down set generated by α. Furthermore, for some argumentation framework $\mathbf{RAF}$ and structure $\mathfrak{A}$, by $Def(\mathbf{RAF}, \mathfrak{A})$ and $Inh(\mathbf{RAF}, \mathfrak{A})$ we respectively denote the defeated arguments and inhibited attacks w.r.t. $\mathbf{RAF}$ and $\mathfrak{A}$. This allows us to relate defeated arguments (resp. inhibited attacks) w.r.t. different argumentation framewokrs.

Lemma A.39. *Let $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ be some framework and $\alpha, \beta \in \mathbf{K}$ be be two attacks and $x \in (\mathbf{A} \cup \mathbf{K})$ be some argument or attack. Then, $\alpha \neq \beta$, $\mathbf{t}(\beta) = x$ and $x \notin\downarrow\alpha$ imply $\beta \notin\downarrow\alpha$.*

Proof. Suppose, for the sake of contradiction, that $\beta \in\downarrow\alpha$. Then, since $\alpha \neq \beta$, it follows that there is some chain $\delta_0, \delta_1, \delta_2, \delta_n$ such that $\mathbf{t}(\delta_i) = \delta_{i+1}$ and $\delta_0 = \beta$ and $\delta_n = \alpha$. But $\delta_0 = \beta$ plus $\mathbf{t}(\beta) = x$ imply that $\delta_1 = x$ and, thus, that $x \in\downarrow\alpha$. This is a contradiction with the assumption. Consequently, it must be that $\beta \notin\downarrow\alpha$. □

Lemma A.40. *Let $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ be some framework, $\mathfrak{A}$ be some structure and $\alpha \in \mathbf{K}$ be some attack. Then, $Def(\mathbf{RAF}, \mathfrak{A}) \supseteq Def(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$.* □

Proof. Let $\mathfrak{A} = \langle S, \Gamma \rangle$ and $\mathfrak{A}^{-\alpha} = \langle S, \Gamma^{-\alpha} \rangle$. Pick any $a \in Def(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$. Then, there is some $\beta \in \Gamma^{-\alpha}$ such that $\mathbf{t}(\beta) = a$ and $\mathbf{s}(\beta) \in S$. Furthermore, $\beta \in \Gamma^{-\alpha}$ plus $\Gamma' \subseteq \Gamma$ imply $\beta \in \Gamma$ which, in its turn, implies that $a \in Def(\mathbf{RAF}, \mathfrak{A})$. □

Lemma A.41. *Let $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ be some framework, $\mathfrak{A}$ be some structure and $\alpha \in \mathbf{K}$ be some attack. Then, $Inh(\mathbf{RAF}, \mathfrak{A})^{-\alpha} \supseteq Inh(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$.* □

Proof. Let $\mathfrak{A} = \langle S, \Gamma \rangle$ and $\mathfrak{A}^{-\alpha} = \langle S, \Gamma^{-\alpha} \rangle$. Pick any $\beta \in \mathit{Inh}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$. Then, there is some $\gamma \in \Gamma^{-\alpha}$ such that $\mathbf{t}(\gamma) = \beta$ and $\mathbf{s}(\gamma) \in S$. Furthermore, $\gamma \in \Gamma^{-\alpha}$ plus $\Gamma^{-\alpha} \subseteq \Gamma$ imply $\gamma \in \Gamma$ which, in its turn, implies that $\beta \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})$. Furthermore, $\beta \in \mathit{Inh}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$ implies $\beta \not\leq \alpha$. Hence, $\beta \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})^{-\alpha}$. □

Lemma A.42. *Let $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ be some framework, $\mathfrak{A}$ be some conflict-free structure w.r.t. $\mathbf{RAF}$ and $\alpha \in \mathbf{K}$ be some attack. Then, $\mathfrak{A}^{-\alpha}$ is conflict-free w.r.t. $\mathbf{RAF}^{-\alpha}$.* □

Proof. Let $\mathfrak{A} = \langle S, \Gamma \rangle$ and $\mathfrak{A}^{-\alpha} = \langle S, \Gamma^{-\alpha} \rangle$. Pick $a \in S$. Then, since $\mathfrak{A}$ is conflict-free, it follows that $a \notin \mathit{Def}(\mathbf{RAF}, \mathfrak{A})$ and, from Lem. A.40, that $a \notin \mathit{Def}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$. Similarly, $\beta \in \Gamma^{-\alpha}$ implies $\beta \notin \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})$. From Lem. A.41, this implies $\beta \notin \mathit{Inh}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$. Hence, $\mathfrak{A}^{-\alpha}$ is conflict-free w.r.t. $\mathbf{RAF}^{-\alpha}$. □

Lemma A.43. *Let $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ be some framework, $\mathfrak{A}$ be some structure and $\alpha \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})$ be some inhibited attack. Then, $\mathit{Acc}(\mathbf{RAF}, \mathfrak{A})^{-\alpha} \supseteq \mathit{Acc}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$.* □

Proof. Let $\mathfrak{A} = \langle S, \Gamma \rangle$ and $\mathfrak{A}^{-\alpha} = \langle S, \Gamma^{-\alpha} \rangle$. Pick any $x \in \mathit{Acc}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$ and $\gamma \in \mathbf{K}$ such that $\mathbf{t}(\gamma) = x$. Then, $x \in \mathit{Acc}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$ implies $x \in (\mathbf{A} \cup \mathbf{K}^{-\alpha})$ and, thus, that $x \notin\downarrow\alpha$. From Lem. A.39, this plus $\mathbf{t}(\gamma) = x$ implies that either $\gamma = \alpha$ or $\gamma \notin\downarrow\alpha$. On the one hand, by hypothesis we have that $\alpha \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})$ and, thus, the former implies $\gamma \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})$. On the other hand, the latter implies $\gamma \in \mathbf{K}^{-\alpha}$ and, thus, $x \in \mathit{Acc}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$ implies that $\gamma \in \mathit{Inh}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$ or $\mathbf{s}(\gamma) \in \mathit{Def}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$. From Lem. A.41 and A.40, this implies that $\gamma \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})$ or $\mathbf{s}(\gamma) \in \mathit{Def}(\mathbf{RAF}, \mathfrak{A})$. Hence, $x \in \mathit{Acc}(\mathbf{RAF}, \mathfrak{A})$. Finally, we have that $x \notin\downarrow\alpha$ implies $x \in \mathit{Acc}(\mathbf{RAF}, \mathfrak{A})^{-\alpha}$. □

Lemma A.44. *Let $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ be some framework, $\mathfrak{A}$ be some structure and $\alpha \in (\mathit{Inh}(\mathbf{RAF}, \mathfrak{A}) \backslash \Gamma)$ be some inhibited attack w.r.t. $\mathfrak{A}$. Then, it follows that $\mathit{Def}(\mathbf{RAF}, \mathfrak{A}) = \mathit{Def}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$.* □

Proof. Let $\mathfrak{A} = \langle S, \Gamma \rangle$ and $\mathfrak{A}^{-\alpha} = \langle S, \Gamma^{-\alpha} \rangle$. From Lem. A.40, it follows that $\mathit{Def}(\mathbf{RAF}, \mathfrak{A}) \supseteq \mathit{Def}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$. Pick now any argument $a \in \mathit{Def}(\mathbf{RAF}, \mathfrak{A})$. Then, there is some $\beta \in \Gamma$ such that $\mathbf{t}(\beta) = a$ and $\mathbf{s}(\beta) \in S$. Furthermore, since $\alpha \notin \Gamma$, it follows that $\beta \neq \alpha$. Moreover, every $\gamma \in\downarrow\alpha$ satisfies either $\gamma = \alpha$ or $\mathbf{t}(\gamma) \in \mathbf{K}$ and, thus, that $\beta \notin\downarrow\alpha$. This implies that $\beta \in \Gamma^{-\alpha}$ and, thus, that $a \in \mathit{Def}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$ □

Lemma A.45. *Let $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ be some framework, $\mathfrak{A} = \langle S, \Gamma \rangle$ be some structure and $\alpha \in (\mathit{Inh}(\mathbf{RAF}, \mathfrak{A}) \backslash \Gamma)$ be some inhibited attack w.r.t. $\mathfrak{A}$. Then, it follows that $\mathit{Inh}(\mathbf{RAF}, \mathfrak{A})^{-\alpha} = \mathit{Inh}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$.* □

Proof. First note that, from Lem. A.41, it follows that

$$\mathit{Inh}(\mathbf{RAF}, \mathfrak{A})^{-\alpha} \supseteq \mathit{Inh}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$$

Let $\mathfrak{A} = \langle S, \Gamma \rangle$ and $\mathfrak{A}^{-\alpha} = \langle S, \Gamma^{-\alpha} \rangle$. Pick now any $\beta \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})^{-\alpha}$.

Then, there is some $\gamma \in \Gamma$ such that $\mathbf{t}(\gamma) = \beta$ and $\mathbf{s}(\gamma) \in S$. Furthermore, since $\alpha \notin \Gamma$, it follows that $\gamma \neq \alpha$. Moreover, $\beta \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})^{-\alpha}$ implies that $\beta \not\preceq \alpha$. Then, since $\gamma \prec \beta$, that $\gamma \not\preceq \alpha$. Hence, it follows that $\gamma \in \Gamma^{-\alpha}$ and $\beta \in \mathit{Inh}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$. □

Lemma A.46. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework,* $\mathfrak{A}$ *be some structure and* $\alpha \in (\mathit{Inh}(\mathbf{RAF}, \mathfrak{A}) \backslash \Gamma)$ *be some inhibited attack w.r.t.* $\mathfrak{A}$.
Then, $(\mathit{Acc}(\mathbf{RAF}, \mathfrak{A})^{-\alpha}) = \mathit{Acc}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$. □

Proof. First note that, from Lem. A.43, it follows that

$$\mathit{Acc}(\mathbf{RAF}, \mathfrak{A})^{-\alpha} \supseteq \mathit{Acc}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$$

Let $\mathfrak{A} = \langle S, \Gamma \rangle$ and $\mathfrak{A}^{-\alpha} = \langle S, \Gamma^{-\alpha} \rangle$. Pick any $\beta \in \mathit{Acc}(\mathbf{RAF}, \mathfrak{A})^{-\alpha}$ and $\gamma \in \mathbf{K}^{-\alpha} \subseteq \mathbf{K}$ such that $\mathbf{t}(\gamma) = \beta$. Since $\beta \in \mathit{Acc}(\mathbf{RAF}, \mathfrak{A})$, it follows that either $\gamma \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})$ or $\mathbf{s}(\gamma) \in \mathit{Def}(\mathbf{RAF}, \mathfrak{A})$. Furthermore, $\gamma \in \mathbf{K}^{-\alpha}$ implies $\gamma \not\preceq \alpha$ and, from Lem. A.45 and A.44, this implies that either $\gamma \in \mathit{Inh}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}')$ or $\mathbf{s}(\gamma) \in \mathit{Def}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}')$. Hence, $\beta \in \mathit{Acc}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}')$ and, thus, $\mathit{Acc}(\mathbf{RAF}, \mathfrak{A})^{-\alpha}) = \mathit{Acc}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$. □

Lemma A.47. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework,* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be some admissible structure w.r.t.* $\mathbf{RAF}$ *and* $\alpha \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})$ *be some inhibited attack. Then, it follows that* $\mathfrak{A}^{-\alpha}$ *is admissible w.r.t.* $\mathbf{RAF}^{-\alpha}$. □

Proof. Since $\mathfrak{A}$ is an admissible structure w.r.t. $\mathbf{RAF}$, it is conflict-free and, from Lem. A.42, this implies that $\mathfrak{A}^{-\alpha}$ is conflict-free w.r.t. $\mathbf{RAF}^{-\alpha}$. Furthermore, since $\mathfrak{A}$ is admissible, it follows that $(S \cup \Gamma) \subseteq \mathit{Acc}(\mathbf{RAF}, \mathfrak{A})$. From Lem. A.46, this implies $(S \cup \Gamma^{-\alpha}) \subseteq \mathit{Acc}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$ and, thus, that $\mathfrak{A}^{-\alpha}$ is admissible w.r.t. $\mathbf{RAF}^{-\alpha}$. □

Lemma A.48. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework,* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be some complete structure w.r.t.* $\mathbf{RAF}$ *and* $\alpha \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})$ *be some inhibited attack. Then, it follows that* $\mathfrak{A}^{-\alpha}$ *is complete w.r.t.* $\mathbf{RAF}^{-\alpha}$. □

Proof. Since $\mathfrak{A}$ is a complete structure w.r.t. $\mathbf{RAF}$, it follows that it is admissible and, from Lem. A.47, this implies that $\mathfrak{A}^{-\alpha}$ is admissible w.r.t. $\mathbf{RAF}^{-\alpha}$. Furthermore, since $\mathfrak{A}$ is complete, it follows that $(S \cup \Gamma) \supseteq \mathit{Acc}(\mathbf{RAF}, \mathfrak{A})$. From Lem. A.46, this implies $(S \cup \Gamma^{-\alpha}) \supseteq \mathit{Acc}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$ and, thus, that $\mathfrak{A}^{-\alpha}$ is complete w.r.t. $\mathbf{RAF}^{-\alpha}$. □

Lemma A.49. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework,* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be some conflict-free structure w.r.t.* $\mathbf{RAF}$ *and* $\alpha \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})$ *be some inhibited attack. Let* $\mathfrak{A}' = \langle S', \Gamma' \rangle$ *some conflict-free structure w.r.t.* $\mathbf{RAF}^{-\alpha}$ *such that* $\Gamma^{-\alpha} \subseteq \Gamma'$. *Then, the structure* $\mathfrak{A}'' = \langle S', \Gamma \cup \Gamma' \rangle$ *is conflict-free w.r.t.* $\mathbf{RAF}$. □

Proof. Let $\Gamma'' = \Gamma \cup \Gamma'$. Pick first $a \in S'$ and any $\beta \in \Gamma''$ such that $\mathbf{t}(\beta) = a$. Then, since $\mathbf{t}(\beta) = a$, it follows that $\beta \not\preceq \alpha$ and, thus, that $\beta \in \mathbf{K}^{-\alpha}$. Hence, $\beta \in \Gamma$ implies $\beta \in \Gamma^{-\alpha} \subseteq \Gamma'$. Then, since $\mathfrak{A}'$ is conflict-free, it follows that

$\mathbf{s}(\beta) \notin S'$. This implies that every $\beta \in \Gamma''$ with $\mathbf{t}(\beta) = a$ satisfies $\mathbf{s}(\beta) \notin S'$ and, thus, that $a \notin \mathit{Def}(\mathbf{RAF}, \mathfrak{A}'')$.

Pick now $\gamma, \beta \in \Gamma''$ such that $\mathbf{t}(\beta) = \gamma$. Suppose, for the sake of contradiction, that $\gamma \in (\Gamma'' \backslash \Gamma)$ and $\beta \in (\Gamma'' \backslash \Gamma')$. Then, $\gamma \notin (\Gamma'' \backslash \Gamma)$ implies $\gamma \in \Gamma' \subseteq \mathbf{K}^{-\alpha}$ and, thus, it follows that $\gamma \not\preceq \alpha$ and $\beta \not\preceq \alpha$. On the other hand, $\beta \in (\Gamma'' \backslash \Gamma')$ implies $\beta \in \Gamma$ which, since $\beta \not\preceq \alpha$, implies $\beta \in \Gamma^{-\alpha} \subseteq \Gamma'$. This is a contradiction with the assumption. Similarly, suppose that $\gamma \in (\Gamma'' \backslash \Gamma')$ and $\beta \in (\Gamma'' \backslash \Gamma)$. Then, $\gamma \in (\Gamma'' \backslash \Gamma')$ implies and $\gamma \in \Gamma$. Furthermore, since $\Gamma^{-\alpha} \subseteq \Gamma'$, it follows that $\gamma \notin \Gamma'$ implies $\gamma \notin \Gamma^{-\alpha}$ and, this plus $\gamma \in \Gamma$, imply $\gamma \preceq \alpha$. Since $\mathbf{t}(\beta) = \gamma$, the latter implies that $\beta \preceq \alpha$ holds and, thus, that $\beta \notin \Gamma'$. This is a contradiction with the assumption that $\beta \in (\Gamma'' \backslash \Gamma)$. Hence, either $\gamma, \beta \in \Gamma$ or $\gamma, \beta \in \Gamma'$ must hold. In both cases, the fact that $\mathfrak{A}$ and $\mathfrak{A}'$ are conflict-free imply $\mathbf{s}(\beta) \notin S$. This implies that every $\beta \in \Gamma''$ with $\mathbf{t}(\beta) = \gamma$ satisfies $\mathbf{s}(\beta) \notin S$ and, thus, that $\gamma \notin \mathit{Inh}(\mathbf{RAF}, \mathfrak{A}'')$. Consequently, $\mathfrak{A}''$ is conflict-free. □

Lemma A.50. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework,* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be some admissible structure w.r.t.* $\mathbf{RAF}$ *and* $\alpha \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})$ *be some inhibited attack. Let* $\mathfrak{A}' = \langle S', \Gamma' \rangle$ *some admissible structure w.r.t.* $\mathbf{RAF}^{-\alpha}$ *such that* $\mathfrak{A}^{-\alpha} \sqsubseteq \mathfrak{A}'$*. Then, the structure* $\mathfrak{A}'' = \langle S', \Gamma \cup \Gamma' \rangle$ *is admissible w.r.t.* $\mathbf{RAF}$*.* □

Proof. From Lemma A.49, it follows that $\mathfrak{A}''$ is conflict-free.

Furthermore, since $\mathfrak{A}$ is admissible, it follows that $(S \cup \Gamma) \subseteq \mathit{Acc}(\mathbf{RAF}, \mathfrak{A})$. Note also that $\mathfrak{A}^{-\alpha} \sqsubseteq \mathfrak{A}'$ and $\alpha \in \mathbf{K}$ implies that $\Gamma = \Gamma^{-\alpha} \subseteq \Gamma'$ and, thus, that $\mathfrak{A} \sqsubseteq \mathfrak{A}''$. Then, from Lem. A.3, it follows that $(S \cup \Gamma) \subseteq \mathit{Acc}(\mathbf{RAF}, \mathfrak{A}'')$. Pick now any attack $\gamma \in (\Gamma'' \backslash \Gamma)$ and any attack $\beta \in \mathbf{K}$ such that $\mathbf{t}(\beta) = \gamma$. Since $\gamma \in (\Gamma'' \backslash \Gamma)$, it follows that $\gamma \in \Gamma' \subseteq \mathbf{K}^{-\alpha}$ and, thus, that $\gamma \not\preceq \alpha$ and $\beta \not\preceq \alpha$. Hence, $\beta \in \mathbf{K}$ implies $\beta \in \mathbf{K}^{-\alpha}$. Furthermore, since $\mathfrak{A}'$ is admissible w.r.t. $\mathbf{RAF}^{-\alpha}$, this implies that either $\mathbf{s}(\beta) \in \mathit{Def}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$ or $\beta \in \mathit{Inh}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$. From Lem. A.44 and A.45 respectively, this implies that either

$$\mathbf{s}(\beta) \in \mathit{Def}(\mathbf{RAF}, \mathfrak{A})^{-\alpha} \subseteq \mathit{Def}(\mathbf{RAF}, \mathfrak{A})$$

or

$$\beta \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})^{-\alpha} \subseteq \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})$$

holds. Furthermore, since $\mathfrak{A} \sqsubseteq \mathfrak{A}''$, Lem. A.2 implies either $\mathbf{s}(\beta) \in \mathit{Def}(\mathbf{RAF}, \mathfrak{A}'')$ or $\beta \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A}'')$. Hence, every $\gamma \in (\Gamma'' \backslash \Gamma)$ satisfies that $\gamma \in \mathit{Acc}(\mathbf{RAF}, \mathfrak{A}'')$ and, thus, $(S \cup \Gamma'') \subseteq \mathit{Acc}(\mathbf{RAF}, \mathfrak{A}'')$. This means that $\mathfrak{A}''$ is admissible. □

Lemma A.51. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework,* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be some preferred structure w.r.t.* $\mathbf{RAF}$ *and* $\alpha \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})$ *be some inhibited attack. Then,* $\mathfrak{A}^{-\alpha}$ *is preferred w.r.t.* $\mathbf{RAF}^{-\alpha}$*.* □

Proof. Since $\mathfrak{A}$ is a preferred structure w.r.t. $\mathbf{RAF}$, it follows that it is admissible and, from Lem. A.47, this implies that $\mathfrak{A}^{-\alpha}$ is admissible w.r.t. $\mathbf{RAF}^{-\alpha}$. Suppose, for the sake of contradiction, that there is some structure $\mathfrak{A}' = \langle S', \Gamma' \rangle$ which is admissible w.r.t. $\mathbf{RAF}^{-\alpha}$ and that satisfies $\mathfrak{A}^{-\alpha} \sqsubset \mathfrak{A}'$. Then, from

Prop. 1, we may assume without loss of generality that $\mathfrak{A}'$ is also preferred w.r.t. $\mathbf{RAF}^{-\alpha}$.

Let $\Gamma'' = (\Gamma \cup \Gamma')$ and let $\mathfrak{A}'' = \langle S', \Gamma'' \rangle$ be some structure. Then, from Lemma A.50, it follows that $\mathfrak{A}''$ is admissible. Furthermore, note that by construction, $\mathfrak{A}, \mathfrak{A}' \sqsubseteq \mathfrak{A}''$ holds and, thus, the fact that $\mathfrak{A}$ is a preferred structure implies $\mathfrak{A} \sqsupseteq \mathfrak{A}''$. On the other hand, $\mathfrak{A}^{-\alpha} \sqsubset \mathfrak{A}'$ implies that there is some element $x \in ((S' \cup \Gamma') \backslash (S^{-\alpha} \cup \Gamma^{-\alpha})$. Moreover, $x \in (S' \cup \Gamma')$ implies that $x \not\preceq \alpha$ and that $x \in (S' \cup \Gamma'')$. Since $\mathfrak{A} \sqsupseteq \mathfrak{A}''$, the latter implies that $x \in (S \cup \Gamma)$ which, together with $x \not\preceq \alpha$, implies that $x \in (S^{-\alpha} \cup \Gamma^{-\alpha})$. This is a contradiction and, consequently, we have that $\mathfrak{A}^{-\alpha}$ must be preferred. □

Lemma A.52. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework,* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be some stable structure w.r.t.* $\mathbf{RAF}$ *and* $\alpha \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})$ *be some inhibited attack. Then, it follows that* $\mathfrak{A}^{-\alpha}$ *is stable w.r.t.* $\mathbf{RAF}^{-\alpha}$. □

Proof. Since $\mathfrak{A}$ is a stable structure w.r.t. $\mathbf{RAF}$, it follows that it is conflict-free and, from Lem. A.42, this implies that $\mathfrak{A}^{-\alpha}$ is conflict-free w.r.t. $\mathbf{RAF}^{-\alpha}$. Furthermore, since $\mathfrak{A}$ is stable, it follows that $S = \overline{\mathit{Def}(\mathbf{RAF}, \mathfrak{A})}$ and $\Gamma = \overline{\mathit{Inh}(\mathbf{RAF}, \mathfrak{A})}$. From Lem. A.44 and A.45 respectively, this implies that $S = \overline{\mathit{Def}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})}$ and $\Gamma^{-\alpha} = \overline{\mathit{Inh}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})}$. This implies that $\mathfrak{A}^{-\alpha}$ is stable w.r.t. $\mathbf{RAF}^{-\alpha}$. □

Proof of Proposition 6. The fact that $\mathfrak{A}^{-\alpha}$ is conflict-free follows directly from Lem. A.42. Similarly, the fact that $\mathfrak{A}^{-\alpha}$ is admissible, complete, preferred or stable respectively follows from Lem. A.47, Lem. A.48, Lem. A.51 and A.52. □

Theorem 5. *The problem of credulous acceptance w.r.t. the preferred or the stable semantics (whether there exists some preferred or stable structure containing some argument) are* NP*-complete. The problem of sceptical acceptance w.r.t. the preferred (resp. stable) semantics is* Π_2^{P}*-complete (resp.* coNP*-complete).* □

Proof. Let $a \in \mathbf{A}$ be some argument. Then, from Theorem 3 and Prop. 12 in [4], it follows that some structure $\mathfrak{A} = \langle S, \Gamma \rangle$ is preferred (resp. stable) w.r.t. $\mathbf{RAF}$
iff `Afra`($\mathfrak{A}$) is a preferred (resp. stable) extension w.r.t. $\mathbf{RAF}$
iff `Afra`($\mathfrak{A}$) is a preferred (resp. stable) extension w.r.t. $\mathbf{RAF}_{AF}$
with $\mathbf{RAF}_{AF}$ is the corresponding Dung framework of $\mathbf{RAF}$ as given by Def. 19 in [4].

Hence, a is credulous accepted w.r.t. $\mathbf{RAF}$ and the preferred (resp. stable) semantics
iff there is some preferred (resp. stable) structure $\mathfrak{A} = \langle S, \Gamma \rangle$ of $\mathbf{RAF}$ such that $a \in S$
iff there is some preferred (resp. stable) extension $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ of $\mathbf{RAF}$ such that $a \in \mathcal{E}$
iff there is some preferred (resp. stable) extension $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ of $\mathbf{RAF}_{AF}$ such

that $a \in \mathcal{E}$.
iff a is credulous accepted w.r.t. $\mathbf{RAF}_{AF}$ and the preferred (resp. stable) semantics.

Then, since credulous acceptance for Dung's frameworks w.r.t. the preferred and the stable semantics is NP-complete [11] and $\mathbf{RAF}_{AF}$ can be computed in polynomial time, it follows that credulous acceptance for RAFs is in NP. Hardness, follows from the fact that every Dung's framework is also a RAF and that, from Theorem 4, the preferred (resp. stable) semantics for RAFs are conservative generalisations.

Analogously, since sceptical acceptance for Dung's frameworks w.r.t. the preferred (resp. stable) semantics is coNP-complete (resp. Π_2^{P}-complete) [11], it follows that sceptical acceptance for RAFs w.r.t. the preferred (resp. stable) semantics is coNP-complete (resp. Π_2^{P}-complete).

Finally, for the complete semantics, note that from Theorem 2, every preferred structure is also a complete structure and, thus, if an argument is credulous accepted w.r.t. the preferred semantics, it is also credulous accepted w.r.t. the complete semantics. Furthermore, every complete $\mathfrak{A}$ structure is admissible and, from Proposition 1, this implies that there is a preferred structure $\mathfrak{A}'$ such that $\mathfrak{A} \sqsubseteq \mathfrak{A}'$. This implies that, if an argument is credulous accepted w.r.t. the complete semantics, it is also credulous accepted w.r.t. the preferred semantics. □