Argumentation Update in YALLA (Yet Another Logic Language for Argumentation)

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Abstract

This article[1] proposes a complete framework for handling the dynamics of an abstract argumentation system. This frame can encompass several belief bases under the form of several argumentation systems, more precisely it is possible to express and study how an agent who has her own argumentation system can interact on a target argumentation system (that may represent a state of knowledge at a given stage of a debate). The two argumentation systems are defined inside a reference argumentation system called the universe which constitutes a kind of "common language". This paper establishes three main results. First, we show that change in argumentation in such a framework can be seen as a particular case of belief update. Second, we have introduced a new logical language called YALLA in which the structure of an argumentation system can be encoded, enabling to express all the basic notions of argumentation theory (defense, conflict-freeness, extensions) by formulae of YALLA. Third, due to previous works about dynamics in argumentation we have been in position to provide a set of new properties that are specific for argumentation update.

[1]This report has been submitted as a research article to the International Journal of Approximate Reasoning. It has been accepted with minor revisions in March 2016.

Contents

1 Introduction

Argumentation is commonly used in everyday life where we share and confront our opinions. We use arguments to sustain an idea, we use counter-arguments in order to attack another idea. Argumentation can be viewed as a process done in order to exchange information together with some justification with the aim to obtain well-justified knowledge, or with the aim to increase or decrease the approval of a point of view, or to confront and combine different views. When several people are sharing arguments the argumentation is pervaded by changes. Those changes may concern the public state of a debate or the agent's representation of the world. Each time an argument is uttered, the listener might change her view of the world. This is why it is important to analyze the link between argumentation and change. Let us notice that argumentation does not necessarily require an audience, since it can be done by only one agent (and in that case it is well adapted for reasoning and decision making).

In artificial intelligence, argumentation has been defined for formalizing argumentative reasoning and in order to automatize some forms of dialog (identified in [55]). As done in many other approaches we place ourselves in the framework of abstract argumentation theory (where arguments are not precisely defined; see [31]) which handles *argumentation systems* that are graphs whose vertices are arguments and edges are attacks between those arguments. More generally, argumentation theory aims at computing the acceptability of arguments [31, 11, 3]. A natural development of this theory is called *enforcement* [8] and consists in finding a set of arguments to add to an argumentation system in order to make accepted a particular set of arguments.

For instance, let us consider the argumentation system containing only the two arguments "a_0: Mr. X is innocent of the murder of Mrs. X" and "a_1: Mr. X is guilty of the murder of Mrs. X"[2]*. These arguments cannot be accepted together since they are mutually exclusive, and it is difficult to decide which argument should be accepted. To enforce a_0 to be accepted it is possible to add Argument "a_4: Mr. X loves his wife, a man that loves his wife cannot murder her" which depreciates a_1 and leads to accept $\{a_0, a_4\}$.*

In this paper we want to propose a framework that generalizes this type of example (change in argumentation) and allows us to reason about this

[2]This example is borrowed from Bisquert et al. [12] and is described in more details inside the paper.

change. In such a framework, it will be possible to describe several argumentation systems (*e.g.* for representing an agent's mind or the current state of a dialog), to express their properties (for instance their structure, their sets of acceptable arguments), and to reason about their evolution. Our approach uses notions of belief change theory and is justified by the following remarks.

Dynamics of argumentation covers more than enforcement. In the literature, enforcement is classically considered as typical of a subfield called "dynamics in argumentation" which has already been studied broadly (*e.g.* [50, 43, 14, 24]). Nevertheless, the topic of "dynamics in argumentation" is more general than enforcement since it aims at *reasoning about change* in an argumentation system. For instance some underlying questions can be "In what extent the arrival of an argument modifies the accepted arguments?", "What is the impact of a change in an argumentation system?", "Which change is desirable and why?".

Moreover, the notion of enforcement can also be generalized in order to consider removal of arguments [12, 13] and addition/removal of attacks [24, 59] and not only addition of arguments.

This is why our framework enables us to study *generalized enforcement* operators and their associated change properties.

Links with planning. From the point of view of an agent that aims at enforcing the acceptation of a set of arguments, Mr. X's example is a one-step planning problem. Indeed, planning (see *e.g.* [33]) aims at building a strategy (sequence of actions) to perform in order to solve a task and so it can be used for handling change in artificial intelligence. In enforcement, the problem is to find only one action that leads to satisfy the goal to make accepted a given set of arguments.

Links with persuasion. Studying if a listener has been persuaded by a change is a natural application of dynamics in argumentation. Indeed, the persuasion process studied in the literature is a particular planning process which consists in uttering some arguments (hence producing some actions) in order to justify a fact or a decision [19, 38, 32, 4] (*i.e.* to achieve some enforcements).[3] Usually a persuasion dialog involves several agents that have opposite views on a subject and that aim at

[3] Moreover argumentation theory has also been used in order to analyze persuasion dialogs (see *e.g.* [9, 48, 2, 20, 35]).

persuading each other to change their opinion. It is possible to distinguish two kinds of persuasion settings depending on the outcome evaluation: either one agent has changed her opinion, we call it "private persuasion", or it is publicly proven that the opinion of one agent is not acceptable, called "public persuasion".[4]. In this paper, we situate ourselves in a context where an agent wants to reason about the changes that occur on a target argumentation system that represents the current state of the dialog.

Clearly, our framework is related to *planning* and *persuasion*.

Incomplete knowledge and restrictions on operations. Public persuasion implies the existence of several agents. In this paper we consider that the agents may have (1) an incomplete knowledge and (2) that their actions are restricted with regard to their available knowledge.

For instance, suppose that the two arguments a_0 and a_1 have been uttered during a trial. The possibility that a_0 is accepted publicly after the lawyer intervention depends on her awareness of the existence of an argument defeating a_1. If the lawyer does not know a_4 or that a_4 attacks a_1 then she cannot utter it, hence whatever she may say, a_0 will not be enforced in the public state of the dialog.

Thus the agents may have restricted possibilities according to their (*incomplete*) knowledge and according to the current state of the dialog. The private knowledge of an agent as well as the target being represented by argumentation systems, the restricted possibilities of agents are represented by constraints on the possible changes that they are authorized to perform on the target system. *Enabling constraints* is also a generalization of the enforcement framework, these constraints are restricting the possible changes that can occur to the target.

Let us assume now that Mr. X was not very vigilant at the beginning of the audience, he knows that the arguments a_0, a_1 and a_4 have been presented (a_4 for attacking a_1). Moreover, he knows that a_4 can be attacked by "a_7: Mr. X is known to be venal and cannot love sincerely his wife", but he does not know whether the prosecutor had given or not a_7. Then Mr. X may hesitate between several systems that may

[4]Formally, in private persuasion, it can be checked that the argument representing the subject of the dialog has changed its status in the argumentation system of one agent. Public persuasion uses a public argumentation system on which each agent may act by participating to the dialog; this public argumentation system is the target of every agent and represents the current state of the dialog.

represent the current state of the debate about his own guiltiness: there is a possible state of the debate in which a_0 is accepted and another one in which it is rejected depending on the fact that a_7 is present or not.

This situation is another case of *incomplete knowledge*, this time it concerns the target argumentation system. The study of change in case of incomplete knowledge relates to the domain of belief change theory developed below.

Belief change in argumentation: revision *vs* update. The classical approaches [1, 57, 40, 42] of belief change theory are defining different ways to take into account the arrival of a new piece of information and study the properties of change according to these differences. A main result of our work is to show that when someone utters publicly an argument, we are facing a change that is called "update" in belief change theory, since it involves an action that modifies the part of the external world relating to the current debate. This is very important since many works about enforcement (*e.g.* [44, 25, 28]) are using the term "revision" which is not always appropriate for the study of public debates. Indeed, in public argumentation, the public state of the debate is evolving through the addition (or removal) of arguments hence there is a change in the world. It means that the agent's beliefs (represented by an argumentation system) about the world and more precisely about the part of the world relating to the debate, should be updated. In order to better explain the difference between the two concepts, let us come back to our example.

When Mr. X hears the lawyer uttering Argument a_4 then he knows that a change has occurred to the current state of the debate, hence the reasoning process amounts to update *the representation of the debate state by the fact that a_4 is present in it: it will consist in making a change on every possible argumentation system that can represent the current state of the debate such that in every resulting state, a_4 is present. Mr. X does not use the same reasoning about change when he learns that he is considered as innocent (i.e., that a_0 is accepted), since in that case the dialog current state has not changed. It is Mr. X's awareness about the dialog state that has changed, the belief change operation done in that case is called* revision*.*

As the previous example shows, we do not consider revision as useless within the context of a debate. For instance, the public or private enunciation of an argument might lead to a revision of a hearer's beliefs about the discussed topic. However, we focus on the beliefs that an agent has about the current state of a debate and we distinguish it from her beliefs about the world representing the discussed topic (*e.g.* whether Mr. X actually killed his wife or not). Hence, in this paper, our study focusses on *update operations*.

More than planning and persuasion. Belief change theory gives tools for *reasoning about change*. This reasoning may be done by a passive agent who wants to understand what is the state of the debate after the occurrence of an external change (*e.g.* the change is caused by another agent who has uttered an argument). So it is not a planning problem.

For instance, assume that the lawyer has done an objection about one argument uttered by the prosecutor. Mr. X may want to know if several different points of view may still exist in every possible argumentation system representing the current state of the dialog. This knowledge could inform him about the remaining time left before a conclusion will be reached.

In this example, we see that Mr. X is not necessarily interested in knowing what arguments are accepted but rather in knowing if it is possible that the target argumentation system has only one set of acceptable arguments. Moreover he may not be interested in learning how to do that but only if it has occurred. It is worth noticing that we are not specifically focusing on persuasion, since these properties do not only concern acceptability of arguments. Hence, works such as [47, 52, 46, 49, 34] that take a game-theoretic and strategical approach of persuasion are particular cases of our framework but which are more focused on the planning aspect of the problem. This is why those approaches are not directly related to our work since our aim is to establish rational properties of argumentation and change.

Necessity of a new language. In belief change theory, change properties are expressed under the form of logical formulae. In order to benefit from the results coming from belief change theory we have defined a specific logical language called YALLA. YALLA is not only able to express information about the structure of an argumentation system but also the relations between sets of arguments and the principles underlying the usual semantics. As far as we know, the languages proposed in the literature do not provide the same expressivity thus they would not have allowed us to obtain the new results presented in this paper, namely the properties concerning acceptance and structure changes.

The paper is organized as follows: in the next section we first explain the basic assumptions governing our approach, then we describe a framework based on abstract argumentation, in which we introduce the notion of universe, the notion of argumentation graph and the notion of operation that an agent can execute on a target argumentation graph. Then we recall the results obtained for characterizing change. In Section 3 we recall the principles of belief change theory and extend classical update postulates in order to be able to constrain transitions. In Section 4, we introduce a new logical language for abstract argumentation in order to describe them, their properties and their evolution (the YALLA language). Then, in Section 5, we show that enforcement is a specific kind of update and we provide particular update properties for change in argumentation that are based on the characterization results. We end the paper by developing an example in Section 6, and by comparing our approach to related works in Section 7.

Note that this paper significantly improves the conference paper [15]; more precisely, a formalization using a more complete logical language is proposed, specific properties playing the role of *postulates for argumentation update* are established and additional results are given together with the proofs of all the propositions (the proofs can be found in Appendix A).

2 Framework

2.1 Working assumptions

In order to avoid any ambiguity, the list below presents the assumptions made in this work.

(a). We consider that every agent has an argumentation system representing her knowledge. We do not consider how this system is obtained.

(b). We place ourselves in the context of "public argumentation", that is an argumentative process in which several agents are exchanging arguments and are observed by an audience that will make the final decision based on the exchanged arguments. In particular, agents are not in position to influence directly other agents (since they do not have access to the argumentation systems of other agents), they are just able to utter arguments publicly[5] and to reason about the change operations done by the other agents on the public state of the debate.

(c). By arguing, the agents modify directly a special argumentation system that represents the state of the dialog, called *target system*. This system is empty at the beginning of the dialog and is iteratively modified by the actions of the agents.

(d). Public argumentation may imply restrictions about what agents are allowed to do (*e.g.* in the context of a supervised dialog). Moreover agents may have preferences over their possible actions.

(e). The agents have *only* access to two kinds of information: information about the target system and knowledge about their own system. They do not know the system of other agents. However, they have access to what other agents have said through the target system.

(f). The knowledge of an agent about the target system may be partial since the agents may have doubts about what the other agents might have said.

(g). We do not consider how the agents revise their own system when they hear new arguments. In particular, we do not address the issue whether an agent is actually persuaded or not.

(h). We are not restricting ourselves to standard persuasion where each agent has a goal to make a particular argument accepted. Instead, we study the properties that hold after the modification of an argumentation system. Such a study encompasses more properties than acceptability of one argument, since it may concern any structural modification of the target system or any evolution of its acceptable sets of arguments.

(i). We are not focusing on the practical aspect of how an agent can achieve her goal about the target system, which is related to a planning approach. We propose a study of the properties induced by the operations that can be performed on a target system while respecting some constraints coming from another argumentation system (the one of the

[5]However, they may, or may not, persuade other agents indirectly, as the other agents may revise their argumentation system accordingly to what they observe in the target system (this point is out of the scope of the paper, see item (g) of the Working assumptions).

agent that has performed the operation). Our aim is to reason about the impact of this external change which is related to a belief change approach.

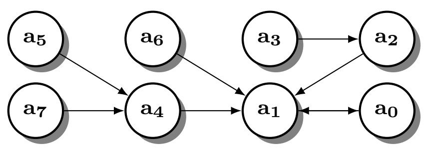
In the remainder of this work, such a setting will typically be exemplified by a trial where two agents, the prosecutor and the lawyer, are modifying a target system representing the current state of a debate. We consider that the jury will use the final state of the debate to decide the culpability of the defendant. This example, together with the framework of abstract argumentation, are presented in the next section.

2.2 Abstract argumentation

Let us consider a set $\mathcal{A}_{\mathbf{U}}$ of symbols (denoted by lower case letters) representing a set of arguments and a relation $\mathcal{R}_{\mathbf{U}}$ on $\mathcal{A}_{\mathbf{U}} \times \mathcal{A}_{\mathbf{U}}$. The pair $(\mathcal{A}_{\mathbf{U}}, \mathcal{R}_{\mathbf{U}})$, called *universe*, allows us to represent the set of possible arguments together with their interactions, and can be represented graphically. More precisely, $\mathcal{A}_{\mathbf{U}}$ represents a set of arguments usable in a given domain. For instance, if the domain is a knowledge base then $\mathcal{A}_{\mathbf{U}}$ and $\mathcal{R}_{\mathbf{U}}$ are the set of all arguments and interactions that may be built from the formulae of the base. In the remainder of this work, we will consider only finite universes, as in the following example borrowed from [12] where we assume that $\mathcal{A}_{\mathbf{U}}$ and $\mathcal{R}_{\mathbf{U}}$ are explicitly provided.

Example 1. *During a trial concerning a defendant (Mr. X), several arguments can be involved to determine his guilt. The set of arguments* $\mathcal{A}_{\mathbf{U}}$ *and the relation* $\mathcal{R}_{\mathbf{U}}$ *are given below.*

a_0	*Mr. X is not guilty of premeditated murder of Mrs. X, his wife.*
a_1	*Mr. X is guilty of premeditated murder of Mrs. X.*
a_2	*The defendant has an alibi, his business associate has solemnly sworn that he met him at the time of the murder.*
a_3	*The close working business relationships between Mr. X and his associate induce suspicions about his testimony.*
a_4	*Mr. X loves his wife so deeply that he asked her to marry him twice. A man who loves his wife cannot be her killer.*
a_5	*Mr. X has a reputation for being promiscuous.*
a_6	*The defendant had no interest to kill his wife, since he was not the beneficiary of the huge life insurance she contracted.*
a_7	*The defendant is a man known to be venal and his "love" for a very rich woman could be only lure of profit.*



A new definition of argumentation graph[6] derives directly from a universe $(\mathcal{A}_{\mathbf{U}}, \mathcal{R}_{\mathbf{U}})$. It differs slightly from the definition of [31] by the fact that arguments and interactions can be built according to the universe.

Definition 1. *An* argumentation graph $\mathcal{G}$ *on* $(\mathcal{A}_{\mathbf{U}}, \mathcal{R}_{\mathbf{U}})$ *is a pair* $(\mathcal{A}, \mathcal{R})$ *where*

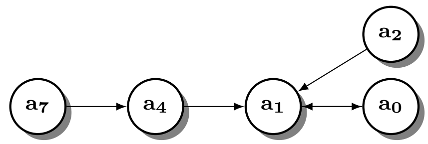
- $\mathcal{A} \subseteq \mathcal{A}_{\mathbf{U}}$ *is the* finite *set of vertices of* $\mathcal{G}$ *called "arguments" and*
- $\mathcal{R} \subseteq \mathcal{R}_{\mathbf{U}} \cap (\mathcal{A} \times \mathcal{A})$ *is its set of edges, called "attacks".*

The set of argumentation graphs that may be built on the universe $(\mathcal{A}_{\mathbf{U}}, \mathcal{R}_{\mathbf{U}})$ *is denoted by* $\Gamma_{\mathbf{U}}$*. In the following,* $x \in \mathcal{G}$*, when* x *is an argument, is a shortcut for* $x \in \mathcal{A}$*.*

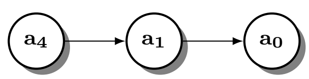
Example 2. *The prosecutor is trying to make accepted the guilt of Mr. X. She is not omniscient and knows only a subset of the arguments of the universe presented in Example 1 (a subset that is not necessarily shared with*

[6]In the literature, two different expressions can be indifferently found: "argumentation system" and "argumentation graph". In the following, we use "argumentation graph".

other agents). Moreover, her knowledge being based on the universe, any argument or attack that does not appear in the universe cannot appear in her graph. Here is her argumentation graph ($\mathcal{G}_{Pro}$):



The prosecutor knows perfectly the current state of the debate, it is represented by the argumentation graph ($\mathcal{G}_D$). Indeed, she had given the argument a_1 against the argument a_0 by making explicit the attack from a_1 to a_0, insisting on the fact that Mr. X is more plausibly guilty than innocent inducing a preference on the attack from a_1 to a_0 over the one from a_0 to a_1 (which is thus neglected). The lawyer had answered by uttering a_4 attacking this suspicion of guiltiness:



Notation 1. *We denote by $\mathcal{R}_\mathcal{A} = \mathcal{R}_\mathbf{U} \cap (\mathcal{A} \times \mathcal{A})$ the restriction of $\mathcal{R}_\mathbf{U}$ on $\mathcal{A}$, i.e. all the attacks concerning arguments of $\mathcal{A}$ that are present in the universe* $(\mathcal{A}_\mathbf{U}, \mathcal{R}_\mathbf{U})$[7].

The acceptable sets of arguments ("extensions") are computed using "semantics" based on the following notions:

Definition 2. *Given an argumentation graph $(\mathcal{A}, \mathcal{R})$, let $a \in \mathcal{A}$ and $\mathcal{S} \subseteq \mathcal{A}$*

- *$\mathcal{S}$ attacks a if and only if $\exists x \in \mathcal{S}$ such that $x\mathcal{R}a$.*
- *$\mathcal{S}$ is* conflict-free *if and only if $\nexists a, b \in \mathcal{S}$ such that $a\mathcal{R}b$.*
- *$\mathcal{S}$* defends *an argument a if and only if $\mathcal{S}$ attacks every argument attacking a. The set of the arguments defended by $\mathcal{S}$ is denoted by $\mathcal{F}(\mathcal{S})$; $\mathcal{F}$ is called the* characteristic function *of $(\mathcal{A}, \mathcal{R})$. More generally, $\mathcal{S}$* indirectly defends *a if and only if $a \in \bigcup_{i \geq 1} \mathcal{F}^i(\mathcal{S})$.*
- *$\mathcal{S}$ is an* admissible set *if and only if it is both conflict-free and defends all its elements.*

[7]Note that in Definition 1, $\mathcal{R}$ is not necessarily equal to $\mathcal{R}_\mathcal{A}$.

In this article, we only consider the traditional semantics proposed by [31]:

Definition 3 (Acceptability semantics). *Given an argumentation graph $(\mathcal{A}, \mathcal{R})$, let $\mathcal{E} \subseteq \mathcal{A}$,*

- *$\mathcal{E}$ is a* complete extension *of $(\mathcal{A}, \mathcal{R})$ if and only if $\mathcal{E}$ is an admissible set and every argument which is defended by $\mathcal{E}$ belongs to $\mathcal{E}$.*

- *$\mathcal{E}$ is a* preferred extension *of $(\mathcal{A}, \mathcal{R})$ if and only if $\mathcal{E}$ is a maximal (with respect to set-inclusion $\subseteq$) admissible set.*

- *$\mathcal{E}$ is* the only grounded extension *of $(\mathcal{A}, \mathcal{R})$ if and only if $\mathcal{E}$ is a minimal (with respect to $\subseteq$) complete extension.*

- *$\mathcal{E}$ is a* stable extension *of $(\mathcal{A}, \mathcal{R})$ if and only if $\mathcal{E}$ is conflict-free and attacks every argument not belonging to $\mathcal{E}$.*

The status of an argument is determined by its presence in the extensions of the selected semantics. For example, an argument is “skeptically accepted” (resp. “credulously”) if it belongs to all the extensions (resp. at least to one extension) and is “rejected” if it does not belong to any extension.

2.3 Change in argumentation

In this section we recall the framework of dynamics in argumentation. In this subfield, researchers study the impacts of a *change operation* done by an agent on an argumentation graph, called the *target*.

These impacts could involve the arguments, the extensions, the set or the number of all the extensions, the set of extensions containing a particular argument as well as its cardinality. For instance, an agent might want to check if the accepted arguments remain exactly the same after the change, or if the number of extensions increases after the change.

A particular change, called *enforcement*, has already been studied in the literature. The main references about enforcement are [8, 7] that address the following question: *“Is it possible to change a given argumentation graph, by applying change operations, so that a desired set of arguments becomes accepted?”* **(author?)** [7] has specified necessary and sufficient conditions under which enforcements are possible, in the case where change operations are restricted to the addition of new arguments and new attacks.

In another context, Cayrol et al. [22] have distinguished four elementary change operations. An elementary change is either adding/ removing one argument with a set of attacks involving it, or adding/removing one attack. Note that in [22] and in [13], operations are defined without considering the notions of universe, agent's own argumentation graph and target graph.

Let us develop the interest of studying not only addition of arguments but also removal. Although dynamics of argumentation graphs has been largely explored (see for instance [17, 18, 8, 44]), the removal of an argument has scarcely been mentioned. However, there exist practical applications. First of all, a speaker may need *to occult* an argument, in particular when she does not want, or is not able, to present this argument in front of a given audience (due to social norms, secrecy, etc.); it is then necessary to know what would happen in the speaker's argumentation graph without this argument. This can be achieved by a removal in her own argumentation graph. In addition, in the case of an "objection", this same audience may force the speaker *to remove* an argument, in particular when it is regarded as illegal in the context. Moreover, the removal turns out to be useful in order to evaluate *a posteriori* the impact of a precise argument on the output of the argumentation graph. In particular for evaluating the quality of a dialog, it is important to be able to differentiate the unnecessarily uttered arguments from the decisive ones (see [2]: an argument is decisive if its removal makes it possible to change the conclusion of the dialog).

Note that the removal of an argument a cannot always be reduced to the addition of a new argument b attacking a, in particular because it may happen that an attacked argument remains acceptable thanks to the defense mechanism. Hence, it is more economic to remove an argument rather than to add one, which might progressively overload the system.

As a consequence, we propose to extend the approach of [22] and [13] by taking into account the idea of forcing some properties to hold after an operation in order to obtain a kind of "generalized enforcement". More precisely, we introduce a framework where an agent may act on a target argumentation graph. This agent should follow some constraints about the actions she has the right to do. For instance, an agent can only advance arguments that she knows. Hence some restrictions are added on the possible changes that may take place on the graph. These constraints are represented by the notion of *executable operation*.

In Definition 4, we first refine the notion of *elementary operation* within the meaning of [22] in four points:

(a). we give a new syntax for taking into account the universe;

(b). we define an *allowed operation* with regard to a given agent's knowledge;

(c). we restrict this notion with regard to its feasibility on the target graph (it is not possible to add an already present argument or to remove an argument which is not in the graph); it leads to the notion of *executable operation*;

(d). and we study the impact of an operation on an argumentation graph.

Note that considering only elementary operations does not result in a loss of generality since any change can be translated into a sequence of elementary operations.

For the following definitions, we consider a particular universe $\mathbf{U} = (\mathcal{A}_{\mathbf{U}}, \mathcal{R}_{\mathbf{U}})$.

Definition 4. *Let k be an agent and $\mathcal{G}_k = (\mathcal{A}_k, \mathcal{R}_{\mathcal{A}_k})$ be her argumentation graph and let $\mathcal{G} = (\mathcal{A}, \mathcal{R})$ be an argumentation graph.*

- *An* operation *is a tuple o of the form $\langle \oplus, \mathcal{Z}, \mathcal{R}_{\mathcal{Z}} \rangle$ or $\langle \ominus, \mathcal{Z}, \varnothing \rangle$ where $\mathcal{Z} \subseteq \mathcal{A}_{\mathbf{U}}$ and $\mathcal{R}_{\mathcal{Z}} \subseteq \mathcal{R}_{\mathbf{U}}$ and $\forall (x, y) \in \mathcal{R}_{\mathcal{Z}}$, $(x \neq y)$ and ($x \in \mathcal{Z}$ or $y \in \mathcal{Z}$).*

- *An* elementary operation *is an operation o of the form $\langle \oplus, \mathcal{Z}, \mathcal{R}_{\mathcal{Z}} \rangle$ or $\langle \ominus, \mathcal{Z}, \varnothing \rangle$ where $card(\mathcal{Z}) = 1$. With a slight abuse of notation, an elementary operation will be noted $\langle \oplus, z, \mathcal{R}_z \rangle$ or $\langle \ominus, z, \varnothing \rangle$ where $z \in \mathcal{A}_{\mathbf{U}}$.*

- *An elementary operation $\langle \oplus, z, \mathcal{R}_z \rangle$ is* allowed for *k if and only if $z \in \mathcal{A}_k$ and $\mathcal{R}_z \subseteq \mathcal{R}_{\mathcal{A}_k}$.*[8] *An elementary operation $\langle \ominus, z, \varnothing \rangle$ is* allowed for *k if $z \in \mathcal{A}_k$.*

- *An operation $\langle \oplus, z, \mathcal{R}_z \rangle$ is* executable by *k* on *$\mathcal{G}$ if it is an operation allowed for k such that $z \notin \mathcal{A}$ and ($\forall (x, y) \in \mathcal{R}_z, x \in \mathcal{A}$ or $y \in \mathcal{A}$). An operation $\langle \ominus, z, \varnothing \rangle$ is* executable by *k* on *$\mathcal{G}$ if it is an operation allowed for k such that $z \in \mathcal{A}$.*

- *An operation $o = \langle \oplus, z, \mathcal{R}_z \rangle$ executable by k on $\mathcal{G}$ provides a new argumentation graph $\mathcal{G}' = o(\mathcal{G}) = (\mathcal{A} \cup \{z\}, \mathcal{R} \cup \mathcal{R}_z)$, an operation $o = \langle \ominus, z, \varnothing \rangle$ executable by k on $\mathcal{G}$ provides a new argumentation graph $\mathcal{G}' = o(\mathcal{G}) = (\mathcal{A} \setminus \{z\}, \mathcal{R} \setminus \{(x, y) \in \mathcal{R} \mid x = z \text{ or } y = z\})$.*

[8] Note that in the case of an argument addition, it is possible to add an argument with only a part of the known attacks and therefore to "lie by omission".

An extension of $\mathcal{G}'$ will be denoted by $\mathcal{E}'$.

The first item of Definition 4 implies that z cannot be a self-attacking argument. However, handling this kind of argument may be useful for instance for modeling a situation where an agent uses a questionable argument in order to make another argument undecided. Nevertheless, in the literature of the domain, forbidding self-attacking arguments is usually considered as a realistic constraint: a self-attacking argument is the symptom of a nonsense.[9]

Example 3. *From $\mathcal{A}_{\mathbf{U}}$ and $\mathcal{R}_{\mathbf{U}}$ given in Example 1, several elementary operations can be given, e.g., $\langle \oplus, a_2, \{(a_2, a_1)\}\rangle$ and $\langle \ominus, a_4, \varnothing\rangle$.*

Among the elementary operations, the prosecutor is only allowed to use those concerning arguments she knows (with regard to $\mathcal{G}_{Pro}$ given in Example 2). For instance, she is allowed to use $\langle \oplus, a_2, \{(a_2, a_1)\}\rangle$, but she is not allowed to use $\langle \oplus, a_5, \{(a_5, a_4)\}\rangle$.

Finally, the prosecutor may execute $\langle \oplus, a_2, \{(a_2, a_1)\}\rangle$ on $\mathcal{G}_D$, but she may not execute $\langle \ominus, a_7, \varnothing\rangle$ on $\mathcal{G}_D$, since a_7 is not in $\mathcal{G}_D$.

2.4 Change characterizations, planning and belief change

A *planning problem in argumentation* could consider the goals of the agent and how she can act (*i.e.* produce a change) on an argumentation graph. If we want to decide which operation an agent has to perform so that a particular property is satisfied in the resulting graph, a *naive* approach would:

- compute all the operations that are executable by the agent,
- compute the extensions of the argumentation graphs obtained by each of these operations and
- check if the property is satisfied in each graph.

The naive approach is expensive, since it requires to compute the extensions of every graph obtained by an executable operation. Some works aim at addressing this efficiency issue by providing incremental computation of

[9]Note that this restriction is not required for establishing Theorem 3 nor for Propositions 9 and 10. It has been done because all the characterizations that are referred in [14] concern graphs with no self-attacking arguments. Hence an additional study should be necessary in order to propose appropriate characterizations integrating self-attacking arguments.

extensions [5, 6] or by defining strong equivalence classes of argumentation graphs that have the same extensions for any addition of arguments [45].

In this paper, we are interested in *reasoning about change i.e.* knowing the properties that are linked to the modifications of an argumentation graph. To be more efficient this study should find ways to reason about the impact of the operations without computing all the extensions. This can be done by using *change characterizations* which were studied in [22] and [14].[10] In these papers, a typology to classify the different properties describing a change operation has been introduced. This typology considers change at three levels:

- the set of extensions in Dung's sense (*e.g.*, the set of extensions is empty before the change and not empty after the change),
- the sets of accepted arguments (*e.g.*, all the arguments skeptically accepted before the change are still skeptically accepted after the change) and
- the status of some given argument (*e.g.*, an accepted argument may become rejected after the change).

Then [22, 14] provide characterizations for these properties: *i.e.* conditions on the argumentation graph and on the change operation that are necessary and/or sufficient to guarantee that the properties are satisfied. The results are twofold: they can be considered as a guide for selecting the change operation to perform in order to obtain a desired property on an argumentation graph (*e.g.* in a planning perspective) and they may also be used as a tool for predicting the result of a change operation in a given context, or for inferring properties about an initial state given information about the operation performed and the result obtained (in a perspective of reasoning about change).

Since a characterization gives necessary and/or sufficient conditions about change properties with regard to a kind of operation and a given semantics, the characterizations can be used to propose another approach which can be applied both for planning (to achieve a given goal represented by a property) or for reasoning about change (to check if a property holds after an unspecified operation done by an external agent). This approach is able to:

- compute all the operations that are executable by the agent,

[10] See [16] for a tool that uses these characterizations.

- select characterizations that concern the considered property,
- check if one of the executable operations satisfies the conditions of one of the selected characterizations.

If an executable operation satisfies the conditions of a characterization, then it is certain that the associated property is satisfied. In [14], we provide many results concerning the grounded, stable and preferred semantics. Below, we give four examples of characterizations, all concerning the grounded semantics.

Characterization 1 ([14]). *Let $(\mathcal{A}, \mathcal{R})$ be an argumentation graph represented by $\mathcal{G}$, and $\mathcal{E}$ its grounded extension. For any operation o of the form $\langle \oplus, z, \mathcal{R}_z \rangle$ executable by an agent on $\mathcal{G}$, if*

- *$\forall x \in \mathcal{A}$ such that $(x, z) \in \mathcal{R}_z$, $\exists y \in \mathcal{E}$ such that $(y, x) \in \mathcal{R}$ and*
- *$\nexists y \in \mathcal{A}$ such that $(z, y) \in \mathcal{R}_z$*

then $\mathcal{E}' = \mathcal{E} \cup \{z\}$ where $\mathcal{E}'$ denotes the grounded extension of $\mathcal{G}' = o(\mathcal{G})$.

The property involved in this first characterization concerns a kind of "monotony" of the extension. Indeed, if an operation adds an argument z under the grounded semantics, such that z is defended by the current extension and z does not attack any argument, then we know (see Proposition 12.3 of [14]), without any computation, that z will become accepted while being certain that the previously accepted arguments remain accepted after the change.

Characterization 2 ([14]). *Let $(\mathcal{A}, \mathcal{R})$ be an argumentation graph represented by $\mathcal{G}$, and $\mathcal{E}$ its grounded extension. For any operation o of the form $\langle \oplus, z, \mathcal{R}_z \rangle$ executable by an agent on $\mathcal{G}$, for any argument $x \in \mathcal{A} \cup \{z\}$, if*

- *$\nexists y \in \mathcal{A}$ such that $(y, z) \in \mathcal{R}_z$ and*
- *$\{z\}$ indirectly defends x and*
- *$x \notin \mathcal{E}$*

then $x \in \mathcal{E}'$ where $\mathcal{E}'$ denotes the grounded extension of $\mathcal{G}' = o(G)$.

This characterization (established by Proposition 15 in [14] [11]) concerns "enforcement" of an argument x, as seen in Section 2.3. Like Characterization 1, this characterization also involves the operation of addition of an

[11]Here we extend the proposition to the case where $x = z$.

argument z. Thanks to this characterization, we know (without requiring a new computation of the extensions) that if an operation adds an argument z under the grounded semantics, such that z is not attacked and indirectly defends another argument x which was not accepted, then x will become accepted.

Characterization 3 ([14])**.** *Let* $(\mathcal{A}, \mathcal{R})$ *be an argumentation graph represented by* $\mathcal{G}$*, and* $\mathcal{E}$ *its grounded extension. For any operation* $o = \langle \ominus, z, \varnothing \rangle$ *executable by an agent on* $\mathcal{G}$*, if*

- $\mathcal{E} \neq \varnothing$ *and*
- $\exists x \in \mathcal{A}$ *such that* $(x, z) \in \mathcal{R}$

then $\mathcal{E}' \neq \varnothing$*, where* $\mathcal{E}'$ *denotes the grounded extension of* $\mathcal{G}' = o(\mathcal{G})$*.*

This characterization (established by Proposition $10.1^{\ominus}$ in [14]) enables us to know, without any computation, that if an operation removes an argument z such that z is attacked by at least one argument of $\mathcal{G}$ and knowing that the extension was not empty before the change, then the extension obtained after the change will not be empty. This can be useful when one wants either to act in order to make sure that the discussion will not be fruitless (planning perspective), or in a perspective of reasoning about change if one wants to know if the discussion is still opened after observing a given operation performed by another agent.

Characterization 4 ([14])**.** *Let* $(\mathcal{A}, \mathcal{R})$ *be an argumentation graph represented by* $\mathcal{G}$*, and* $\mathcal{E}$ *its grounded extension. For any operation* $o = \langle \oplus, z, \mathcal{R}_z \rangle$ *executable by an agent on* $\mathcal{G}$*, if*

- $\nexists x \in \mathcal{E}$ *such that* $(z, x) \in \mathcal{R}_z$ *and*
- $\exists y \in \mathcal{A}$ *such that* $(y, z) \in \mathcal{R}_z$ *and* $\nexists x \in \mathcal{E}$ *such that* $(x, y) \in \mathcal{R}$

then $\mathcal{E}' = \mathcal{E}$*, where* $\mathcal{E}'$ *denotes the grounded extension of* $\mathcal{G}' = o(G)$*.*

This last characterization (coming from Proposition 12.1 in [14]) concerns the non evolution of the conclusion. Thanks to it, we know (without requiring a new computation of the extensions) that if an operation adds an argument z under the grounded semantics, such that z does not attack any argument of the grounded extension while not being defended by this extension, then the extension will remain the same after the change. In a planning perspective, this result could be useful in situations where it is important to speak but without changing the direction of the debate (for a politician, for instance).

Example 4. *From Example 2, we know that the prosecutor is trying to make accepted the argument a_1. So she wants to have $a_1 \in \mathcal{E}'$ and she can use Characterization 2. From Example 3, several executable operations are available and among the executable operations, $\langle \oplus, a_7, \{(a_7, a_4)\} \rangle$ satisfies the conditions of Characterization 2. Hence, the prosecutor is certain to achieve her goal if she executes this operation.*

Mr. X can also use Characterization 2 after having heard the prosecutor uttering a_7 and attacking a_4 (that had been said previously in the debate). This characterization will help Mr. X to understand that at this stage of the debate his guiltiness is accepted.

The fact that an agent may reason about operations done on an argumentation graph ensuring that a property holds on this graph recalls belief change theory, where an agent has to change her beliefs in order to take into account a new piece of information. Moreover, characterizations are properties about elementary change operations performed on an argumentation system. On the other hand, belief change postulates are properties about a belief change operator applied to a knowledge base. Hence, it seems interesting to draw a parallel between these two domains and to try to use the characterizations as supplementary properties playing the role of additional postulates for an argumentation-specific belief change operator.

To explain this parallel, we briefly recall background on belief change theory in the next section.

3 Belief change theory

In the field of belief change theory, the paper of AGM ([1]) has introduced the concept of "belief revision". Belief revision aims at defining how to integrate a new piece of information into a set of initial beliefs while preserving consistency. Beliefs are represented by sentences of a formal language.

A very important distinction between belief revision and belief update was first established in [56]. The difference is in the nature of the new piece of information: either it is completing the knowledge of the world or it informs that there is a change in the world. More precisely, update is a process which takes into account a physical evolution of the world while revision is a process taking into account an epistemic evolution (*i.e.,* it is the knowledge about the world that is evolving).

Considering change in argumentation, we rather face an update problem, since we are studying properties concerning the evolution of an argumentation system (specifically, the target system) due to a change operation executed on it by an agent. Graphs are playing the role of states of the world.[12]

3.1 Classical update postulates

We need to recall some background on belief update. An update operator ([56, 39]) is a function mapping a knowledge base φ, expressed in a propositional logic $\mathscr{L}$ based on a vocabulary V, and a new piece of information $\alpha \in \mathscr{L}$, to a new knowledge base $\varphi \diamond \alpha \in \mathscr{L}$. Here, φ represents knowledge about a system in an initial state and $\varphi \diamond \alpha \in \mathscr{L}$ represents the system after this evolution. Ω denotes the set of all interpretations of $\mathscr{L}$ (states of the world), $[\varphi]$ denotes the set of models of the formula φ ($[\varphi] \subseteq \Omega$). Each state of the world can be described unequivocally thanks to a characteristic formula:

Definition 5. *A function $f : \Omega \to \mathscr{L}$ such that $\forall \omega \in \Omega, [f(\omega)] = \{\omega\}$ is a* characteristic function *for $\mathscr{L}$. $f(\omega)$ is the* characteristic formula *associated to ω by f.*

Remark 1. *The function Φ s.t. $\forall \omega \in \Omega$, $\Phi(\omega) = \bigwedge_{v \in V, \omega \models v} v \wedge \bigwedge_{v \in V, \omega \models \neg v} \neg v$ is a characteristic function.*

The characteristic function mentioned in the above remark will be used in the remainder of this paper.

In belief update, the input α should be interpreted as the projection of the expected effects of some "explicit change", or more precisely, the expected effect of the action "make α true". The key property of belief update is Katsuno and Mendelzon's Postulate **U8** which tells that models of φ are updated independently (contrarily to belief revision). We recall here the postulates of Katsuno and Mendelzon [39]: $\forall \varphi, \psi, \alpha, \beta \in \mathscr{L}$,

U1: $\varphi \diamond \alpha \models \alpha$

U2: $\varphi \models \alpha \implies [\varphi \diamond \alpha] = [\varphi]$

[12] A revision approach would apply to situations in which the agent learns some information about the initial argumentation graph and wants to correct her knowledge about it. This would mean that the argumentation graph has not changed but the awareness of the agent has evolved. See for instance Mr. X's example given in Section 1.

U3: $[\varphi] \neq \varnothing$ and $[\alpha] \neq \varnothing \implies [\varphi \diamond \alpha] \neq \varnothing$

U4: $[\varphi] = [\psi]$ and $[\alpha] = [\beta] \implies [\varphi \diamond \alpha] = [\psi \diamond \beta]$

U5: $(\varphi \diamond \alpha) \wedge \beta \models \varphi \diamond (\alpha \wedge \beta)$

U8: $[(\varphi \vee \psi) \diamond \alpha] = [(\varphi \diamond \alpha) \vee (\psi \diamond \alpha)]$

U9: if $card([\varphi]) = 1$ then $[(\varphi \diamond \alpha) \wedge \beta] \neq \varnothing \implies \varphi \diamond (\alpha \wedge \beta) \models (\varphi \diamond \alpha) \wedge \beta$ (where $card(E)$ denotes the cardinality of the set E)[13]

These postulates allow Katsuno and Mendelzon to write the following representation theorem concerning update, namely, an operator satisfying these postulates can be defined by means of a ternary preference relation on states of the world (see [39]).

Definition 6. *A* faithful assignment *is a function that associates with each $\omega \in \Omega$ a complete preorder $\preceq_\omega$ such that*[14] $\forall \omega_1 \neq \omega \in \Omega$, $\omega \prec_\omega \omega_1$.

Theorem 1 ([39])**.** *There is an operator $\diamond : \mathscr{L} \times \mathscr{L} \to \mathscr{L}$ satisfying **U1**, **U2**, **U3**, **U4**, **U5**, **U8**, **U9** if and only if there is a* faithful *assignment that associates with each $\omega \in \Omega$ a complete preorder*[15]*, denoted by $\preceq_\omega$ such that $\forall \varphi, \alpha \in \mathscr{L}$, $[\varphi \diamond \alpha] = \bigcup_{\omega \in [\varphi]} \{\omega' \in [\alpha]$ such that $\forall \omega'' \in [\alpha], \omega' \preceq_\omega \omega''\}$.*

This set of postulates has already been broadly discussed in the literature (see *e.g.*, [37, 36, 30]). Two postulates have been particularly criticized: Postulate **U2** imposes inertia (which is not always suitable) and **U3** imposes that any update is always possible (which could also be viewed as a too strong assertion).

In order to answer to these critics, a natural extension is to restrain the possible changes that can be done on a state of the world, and hence give a different definition of update which can take into account some constraints about the possible transitions. Moreover, this extension becomes necessary for us in order to take into account the notion of executable operation in the argumentation framework (let us recall that some operations are not authorized on some argumentation graphs; executable operations are the ones

[13] Note that $card([\varphi]) = 1$ if and only if $\exists \omega \in \Omega$ such that $[\varphi] = \{\omega\}$.

[14] Here, and in the rest of this article, $\prec$ is defined classically from $\preceq$ as follows: $x \prec y$ iff ($x \preceq y$ and $y \not\preceq x$)

[15] Postulates **U6** and **U7** are not considered here since the set **U1**-**U8** is only related to a family of partial preorders while replacing **U6**-**U7** by **U9** ensures a family of complete preorders.

that are authorized). Hence, we have chosen to introduce a set of authorized transitions $\mathcal{T}$ which restricts accessible states of the world. Consequently, we have adapted update postulates in order to restrict possible transitions, and to define a representation theorem that allows us to have a complete preorder over the states of the world.

Note that the idea to define an update operator based on a set of authorized transitions was first introduced by Cordier and Siegel [23]. Their proposal goes beyond our idea since they allow for a greater expressivity by using prioritized transition constraints. However, this proposal is only defined at a semantic level (in terms of preorders between states of the world), hence they do not provide postulates nor representation theorem associated with their update operator. In addition, our work generalizes Herzig's approach [36] which proposes to restrict possible updates by taking into account integrity constraints (*i.e.*, formulae that should hold before and after update) since integrity constraints can be encoded with a set of authorized transitions (but the converse is not possible).

3.2 Update postulates respecting transition constraints

In this section, we define new postulates considering constraints on transitions. For that purpose, we first define a new update operator taking into account a set $\mathcal{T}$ of authorized transitions between states of the world.

Definition 7 (**Update operator related to** $\mathcal{T}$)**.** *Given* $\mathcal{T} \subseteq \Omega \times \Omega$*,*

- $\forall \varphi, \psi \in \mathscr{L}$*, the pair* (φ, ψ) *satisfies* $\mathcal{T}$*, denoted by* $(\varphi, \psi) \models \mathcal{T}$*, iff* $([\varphi] \neq \varnothing$ *and* $\forall \omega \in [\varphi]$*,* $\exists \omega' \in [\psi]$*,* $(\omega, \omega') \in \mathcal{T})$*.*

- *An* update operator related to $\mathcal{T}$*, denoted* $\diamondsuit_{\mathcal{T}}$*, is a mapping from* $\mathscr{L} \times \mathscr{L}$ *to* $\mathscr{L}$ *which associates with any formula* φ *and any formula* α *a formula, denoted by* $\varphi \diamondsuit_{\mathcal{T}} \alpha$*, such that* $(\varphi, \varphi \diamondsuit_{\mathcal{T}} \alpha) \models \mathcal{T}$*.*

In other words, if φ gives information about an initial state of the world then $\varphi \diamondsuit_{\mathcal{T}} \alpha$ characterizes the states of the world that can be obtained from states satisfying φ by a change belonging to $\mathcal{T}$.

Now, we define a set of rational postulates for $\diamondsuit_{\mathcal{T}}$. These postulates aim at translating the idea of update under authorized transitions. Some postulates coming from update are suitable, namely **U1**, since it ensures that after an update the constraints imposed by α hold. **U2** postulate is optional, it imposes that if α already holds in a state of the world then updating α means no change. This postulate imposes inertia as a preferred change, this may not be desirable in all situations. For instance, Dubois et al. [30] proposes to not impose inertia and to allow for update failure even if the formulae are consistent; this has been done by introducing an unreachable world in order to dispose of an upper bound of the proximity from a current world to an unreachable world. **U3** imposes that if a formula holds for some states of the world and if the update piece of information also holds for some state then the result of update should give a non empty set of states. Here, we do not want to impose that any update is always possible since some state of the world may be unreachable from others. So we propose to replace **U3** by a postulate called **E3** based on the set of authorized transitions $\mathcal{T}$: $\forall \varphi, \psi, \alpha, \beta \in \mathscr{L}$

E3: $[\varphi \diamondsuit_{\mathcal{T}} \alpha] \neq \varnothing$ if and only if $(\varphi, \alpha) \models \mathcal{T}$.

Due to the definition of $(\varphi, \alpha) \models \mathcal{T}$, **E3** handles two cases of update impossibility: no possible transition and no state of the world satisfying φ or α, as it will be shown in Proposition 3.

U4 is suitable in our setting since update operators are defined semantically.

U5 is also suitable for update since it says that states of the world updated by α in which β already holds are states in which the constraints α and β are updated. Due to the fact that we wanted to allow for update failure, this postulate has been restricted to "complete" formulae (*i.e.,* such that $card([\varphi]) = 1$):

E5: if $card([\varphi]) = 1$ then $(\varphi \diamondsuit_{\mathcal{T}} \alpha) \wedge \beta \models \varphi \diamondsuit_{\mathcal{T}} (\alpha \wedge \beta)$.

U8 captures the decomposability of update with respect to a set of possible input states of the world. We slightly change this postulate in order to take into account the possibility of failure, namely if updating something is impossible then updating it on a larger set of states is also impossible, else the update can be decomposable:

E8: if $([\varphi] \neq \varnothing$ and $[\varphi \diamondsuit_{\mathcal{T}} \alpha] = \varnothing)$ or $([\psi] \neq \varnothing$ and $[\psi \diamondsuit_{\mathcal{T}} \alpha] = \varnothing)$
then $[(\varphi \vee \psi) \diamondsuit_{\mathcal{T}} \alpha] = \varnothing$
else $[(\varphi \vee \psi) \diamondsuit_{\mathcal{T}} \alpha] = [(\varphi \diamondsuit_{\mathcal{T}} \alpha) \vee (\psi \diamondsuit_{\mathcal{T}} \alpha)]$.

Postulate **U9** is a kind of converse of **U5**, restricted to a "complete" formula φ. This restriction is required in the proof of Theorem 1 as well as in Theorem 2.

Note that the presence of **U1** in the set of postulates characterizing an update operator is not necessary since **U1** can be derived from **E3**, **E5** and **E8**.

Proposition 1. ***U1*** *is implied by* ***E3****,* ***E5*** *and* ***E8****.*

Using Proposition 1, the following proposition establishes the fact that five postulates are necessary and sufficient to define an update operator, namely **E3**, **U4**, **E5**, **E8** and **U9**.

Proposition 2. ***E3****,* ***U4****,* ***E5****,* ***E8****,* ***U9*** *constitute a minimal set: no postulate of this set can be derived from the others.*

Hence, we get a new set of postulates for update respecting transition constraints:

E3: $[\varphi \diamond_{\mathcal{T}} \alpha] \neq \varnothing$ if and only if $(\varphi, \alpha) \models \mathcal{T}$

U4: $[\varphi] = [\psi]$ and $[\alpha] = [\beta] \implies [\varphi \diamond_{\mathcal{T}} \alpha] = [\psi \diamond_{\mathcal{T}} \beta]$

E5: if $card([\varphi]) = 1$ then $(\varphi \diamond_{\mathcal{T}} \alpha) \wedge \beta \models \varphi \diamond_{\mathcal{T}} (\alpha \wedge \beta)$

E8: if $([\varphi] \neq \varnothing$ and $[\varphi \diamond_{\mathcal{T}} \alpha] = \varnothing)$ or $([\psi] \neq \varnothing$ and $[\psi \diamond_{\mathcal{T}} \alpha] = \varnothing)$
then $[(\varphi \vee \psi) \diamond_{\mathcal{T}} \alpha] = \varnothing$
else $[(\varphi \vee \psi) \diamond_{\mathcal{T}} \alpha] = [(\varphi \diamond_{\mathcal{T}} \alpha) \vee (\psi \diamond_{\mathcal{T}} \alpha)]$

U9: if $card([\varphi]) = 1$ then $[(\varphi \diamond_{\mathcal{T}} \alpha) \wedge \beta] \neq \varnothing \implies \varphi \diamond_{\mathcal{T}} (\alpha \wedge \beta) \models (\varphi \diamond_{\mathcal{T}} \alpha) \wedge \beta$

These postulates allow us to write the following representation theorem concerning update, namely, an update operator satisfying these postulates can be defined by means of the definition of a family of preorders on states of the world.

Definition 8. *Given a set $\mathcal{T} \subseteq \Omega \times \Omega$ of authorized transitions, an* assignment respecting $\mathcal{T}$ *is a function that associates with each $\omega \in \Omega$ a complete preorder $\preceq_\omega$ such that $\forall \omega_1, \omega_2 \in \Omega$, if $(\omega, \omega_1) \in \mathcal{T}$ and $(\omega, \omega_2) \notin \mathcal{T}$ then $\omega_1 \prec_\omega \omega_2$.*

Theorem 2. *Given a set $\mathcal{T} \subseteq \Omega \times \Omega$ of authorized transitions, there is an operator $\diamond_{\mathcal{T}} : \mathscr{L} \times \mathscr{L} \to \mathscr{L}$ satisfying* ***E3****,* ***U4****,* ***E5****,* ***E8****,* ***U9*** *if and only if there is an assignment respecting $\mathcal{T}$ such that $\forall \omega \in \Omega$, $\forall \varphi, \alpha \in \mathscr{L}$,*

(1) $[\varphi \diamond_{\mathcal{T}} \alpha] = \varnothing$ *if* $\exists \omega \in [\varphi]$ *such that* $[\Phi(\omega) \diamond_{\mathcal{T}} \alpha] = \varnothing$

(2) $[\varphi \diamond_{\mathcal{T}} \alpha] = \bigcup_{\omega \in [\varphi]} [\Phi(\omega) \diamond_{\mathcal{T}} \alpha]$ *otherwise*

(3) $[\Phi(\omega) \diamond_{\mathcal{T}} \alpha] = \left\{ \omega_1 \in \Omega \;\middle|\; \begin{array}{l} \omega_1 \in [\alpha] \textit{ and} \\ (\omega, \omega_1) \in \mathcal{T} \textit{ and} \\ (\forall \omega_2 \in [\alpha] \textit{ such that } (\omega, \omega_2) \in \mathcal{T}, \omega_1 \preceq_\omega \omega_2) \end{array} \right\}$

In other words, (1) and (2) allow us to define the update of a formula φ wrt the update of its individual models, with (1) stating that if one of them cannot be updated the whole update fails, and (2) stating that otherwise the whole update corresponds to the union of the individual updates (an individual update being described by (3)).

This result is a significant headway,[16] but as usual for a representation theorem, it gives only a link between the existence of an assignment of preorders and the fact that an update operator satisfies the postulates. It does not give any clue about how to assign these preorders *i.e.*, how to design precisely an update operator.

Example 5. *Let us consider three variables xx, xy and t meaning respectively "Mrs. X is alive", "Mr. X is alive", "Mr. and Mrs. X are together in the same room". Suppose that we know that at a given time point Mr. X is alive (xy), we do not know whether Mrs. X is alive and if they are together. However we know that Mrs. X cannot be alive if they are together, this can be expressed by $\varphi = xy \wedge (t \rightarrow \neg xx)$. It means that among the eight worlds: $w_1 = (xx, xy, t)$, $w_2 = (xx, xy, \overline{t})$, $w_3 = (xx, \overline{xy}, t)$, $w_4 = (xx, \overline{xy}, \overline{t})$, $w_5 = (\overline{xx}, xy, t)$, $w_6 = (\overline{xx}, xy, \overline{t})$, $w_7 = (\overline{xx}, \overline{xy}, t)$, $w_8 = (\overline{xx}, \overline{xy}, \overline{t})$ there are three possible worlds representing the situation: w_2, w_5 and w_6 (i.e., $[\varphi] = \{\omega_2, \omega_5, \omega_6\}$). We know that some transitions are not possible between two consecutive time points: it is impossible that Mrs. X (respectively Mr. X) rises from the dead, i.e., every transition from $(\overline{xx}, .., ..)$ to $(xx, .., ..)$ (respectively $(.., \overline{xy}, ..)$ to $(.., xy, ..)$) does not belong to $\mathcal{T}$.*

A gunshot has been heard and "Mrs. X was found dead". It means that the world has evolved in such a way that xx is false, which can be expressed by $\alpha = \neg xx$. Let us consider a particular assignment satisfying $\mathcal{T}$ with the following preference relations on transitions:

- $\forall i \neq 6, w_6 \prec_{w_5} w_i$: *if Mrs. X is dead and Mr. X is alive in the same*

[16] Note that the condition part in the third item of this theorem could be simplified into: $\omega_1 \in [\alpha]$ and $(\omega, \omega_1) \in \mathcal{T}$ and $\forall \omega_2 \in [\alpha]$, $\omega_1 \preceq_\omega \omega_2$ by taking into account the constraints on $\preceq_\omega$ stated in Definition 8. Nevertheless, the current formulation gives a more explicit result.

room, then it is likely that at the next time point Mr. X has left since he does not like to stay with dead people.

- $\forall i \neq 2, w_2 \prec_{w_2} w_i$*: if Mr. and Mrs. X are alive and not together then it is more plausible that at the next time point it is still the case, otherwise it is more plausible that they met and stay alive than that one of them dies, which in turn is more plausible than both of them died separately and so on*[17]*:* $w_1 \prec_{w_2} \{w_4, w_6\} \prec_{w_2} \{w_3, w_5, w_7, w_8\}$.

- $\forall i \neq 6, w_6 \prec_{w_6} w_i$*: if Mrs. X is dead and Mr. X is alive but elsewhere, then it is more plausible that it is still the case at the next time point.*

In that case $[\varphi \diamondsuit_{\mathcal{T}} \alpha] = [xy \wedge (t \to \neg xx) \diamondsuit_{\mathcal{T}} \neg xx] = \{w_6\}$ *since for every* $w \in \{w_2, w_5, w_6\}$, $w_6 \prec_w w'$ $\forall w'$ *s.t.* $w' \models \neg xx$ *and* $(w, w') \in \mathcal{T}$.[18]

From Theorem 2 we can deduce two simple cases of impossibility: if the initial situation described by φ or the imposed property α are impossible then the update is impossible (this result is a kind of converse of **U3**).

Proposition 3. *If* $\diamondsuit_{\mathcal{T}}$ *satisfies* ***E3****,* ***U4****,* ***E5****,* ***E8*** *and* ***U9*** *then* $([\varphi] = \varnothing$ *or* $[\alpha] = \varnothing) \implies [\varphi \diamondsuit_{\mathcal{T}} \alpha] = \varnothing$.

The following property ensures that if an update is possible then a more general update is also possible.

Proposition 4. *If* $\diamondsuit_{\mathcal{T}}$ *satisfies* ***E3*** *then* $([\varphi] \neq \varnothing$ *and* $[\varphi \diamondsuit_{\mathcal{T}} \alpha] \neq \varnothing) \implies (\forall \gamma,\ [\varphi \diamondsuit_{\mathcal{T}} (\alpha \vee \gamma)] \neq \varnothing)$.

Note that there are some cases where **U2** is not required together with **E3**, **U4**, **E5**, **E8** and **U9**. If **U2** is imposed then the update operator is associated with a preorder in which a given state is always closer to itself than to any other state of the world. This is why **U2** imposes to have a faithful assignment (see Definition 6). In that case, the relation represented by $\mathcal{T}$ should be reflexive.

[17] w_5 is also considered as less plausible since it would require two steps for passing from w_2 to w_5 namely killing Mrs. X and gathering Mr. X and Mrs. X; moreover we know that Mr. X does not like to be in such an equivocal situation.

[18] Note that the reasoning process would have been different if the coroner had discovered that Mrs. X was already dead before the gunshot. This process is a *revision* and would have amount to complete the initial knowledge hence to deduce that at the initial time, only two worlds were possible w_5 and w_6, denoted $[xy \wedge (t \to \neg xx) \star \neg xx] = \{w_5, w_6\}$.

Proposition 5. *Given a* reflexive *relation $\mathcal{T} \subseteq \Omega \times \Omega$ of authorized transitions, there is an operator $\diamondsuit_{\mathcal{T}} : \mathscr{L} \times \mathscr{L} \to \mathscr{L}$ satisfying* ***E3**, **U4**, **E5**, **E8**, **U9** that satisfies **U2** if and only if there is a faithful assignment respecting $\mathcal{T}$ defined as in Theorem 2.*

If we remove the restriction about authorized transitions then we recover Katsuno and Mendelzon theorem, namely:

Proposition 6. *If $\mathcal{T} = \Omega \times \Omega$ then $\diamondsuit_{\mathcal{T}}$ satisfies **U2**, **E3**, **U4**, **E5**, **E8**, **U9** if and only if $\diamondsuit_{\mathcal{T}}$ satisfies **U1**, **U2**, **U3**, **U4**, **U5**, **U8** and **U9**.*

We have seen in the previous sections a framework in which an agent can change an argumentation graph. We have also seen that these notions are close to belief change theory, in particular since we now consider authorized transitions in the context of belief update. To make concrete the parallel between these two domains, in the next section we define a logical language able to express classical notions of abstract argumentation. This new language is introduced in order to be in position to propose an update operator and a set of properties specific to change in abstract argumentation in Section 5.

4 YALLA: logical theory for abstract argumentation graphs

Our purpose is to build a first-order logical theory capable of describing abstract argumentation graphs, especially the attack relation between sets of arguments.

So, we first define a logical language with specific axioms and we show how formulae of the language can be interpreted by argumentation graphs. Then, we slightly modify this logical theory for describing argumentation graphs built on a given universe. Lastly we provide formulae that express the criteria underlying the traditional argumentation semantics.

4.1 The basic language YALLA

The signature of a first-order logical language lays down the set of individual constants (V_{const}), the set of function symbols (V_f) and the set of predicate symbols (V_P). The signature Σ of our basic language YALLA is defined as follows:[19]

[19] Each function or predicate symbol has an arity which is indicated by an exponent attached to the symbol.

Definition 9 (**Signature**). *$\Sigma = (V_{const}, V_f, V_P)$ where $V_{const} = \{c_\perp\}$, $V_f = \{union^2\}$ and $V_P = \{\rhd^2, \subseteq^2\}$.*

The terms and formulae of YALLA are built up from the logical symbols (logical connectives, quantifiers and variables) and the symbols in the signature, thanks to the syntactic rules of first-order logic.[20]

4.1.1 Structure over Σ

We define the semantics of YALLA thanks to a structure, on which terms and formulae will be interpreted. A structure is associated with an argumentation graph $(\mathcal{A}, \mathcal{R})$. So, the domain of the structure is $\mathcal{D} = 2^{\mathcal{A}}$, the set of sets of arguments, which contains at least the empty set.

Definition 10 (**Structure**). *A* structure *$\mathcal{M}$ of signature Σ, associated with $(\mathcal{A}, \mathcal{R})$, is a pair $(\mathcal{D}, \mathcal{I})$ where $\mathcal{D} = 2^{\mathcal{A}}$ is the* domain of the structure *and $\mathcal{I}$ is an* interpretation function *associating:*

- *the empty set to the constant symbol $c_\perp$,*
- *the binary set theoretic union operator (function from $\mathcal{D}^2$ to $\mathcal{D}$) to the function symbol union,*
- *the binary set theoretic inclusion relation (binary relation on $\mathcal{D}^2$) to the predicate symbol $\subseteq$,*
- *the binary relation of attack between sets of arguments induced by $\mathcal{R}$,[21] and defined by $\mathcal{S}_1 \mathcal{R} \mathcal{S}_2$ if and only if $\exists x_1 \in \mathcal{S}_1$, $\exists x_2 \in \mathcal{S}_2$, $(x_1 \mathcal{R} x_2)$, to the predicate symbol $\rhd$.*

Note that the first two items allow to interpret variable-free terms, and the last two allow to interpret variable-free atomic formulae on a given structure $\mathcal{M}$. Then, we rely on the Tarski truth definition rules (see for instance [51]) to interpret any closed formula.[22] For instance, a formula $\forall x\ \varphi$ (resp. $\exists x\ \varphi$) is interpreted by True on the structure $\mathcal{M}$ if and only if φ is interpreted by True on $\mathcal{M}$ for each (resp. at least one) assignment of an element of $\mathcal{D}$ to the variable x.

[20] The reader is referred to [51] for instance, for a detailed presentation of mathematical logic.

[21] For sake of simplicity, the attack relation between sets of arguments is denoted with the same symbol as the attack relation between arguments.

[22] A formula is closed if each variable x of the formula appears in a subformula of the form $\forall x\ (\varphi)$ or $\exists x\ (\varphi)$.

Definition 11 (Model). *Let φ be a closed formula of the language. A structure $\mathcal{M}$ is a model of φ, denoted by $\mathcal{M} \models \varphi$, if and only if φ is interpreted by True on the structure $\mathcal{M}$.*

In the following, we will write $(\mathcal{A}, \mathcal{R}) \models \varphi$ for $\mathcal{M} \models \varphi$, when $\mathcal{M}$ is associated with $(\mathcal{A}, \mathcal{R})$. Indeed, in that case, the structure $\mathcal{M}$ is entirely determined by $(\mathcal{A}, \mathcal{R})$.

4.1.2 Specific axioms

Among all the formulae that can be built on the signature Σ, some formulae characterize the structures defined above. We will consider these formulae as the specific axioms of the logical theory describing an abstract argumentation graph. Let x, y, z be variables of YALLA:

Axioms for set inclusion

- $\forall x\ (c_\perp \subseteq x)$
- $\forall x\ (x \subseteq x)$
- $\forall x, y, z\ ((x \subseteq y \wedge y \subseteq z) \implies x \subseteq z)$.

Axioms for set operators

- $\forall x, y\ ((x \subseteq union(x, y))$
- $\forall x, y\ ((y \subseteq union(x, y))$
- $\forall x, y, z\ (((x \subseteq z) \wedge (y \subseteq z)) \implies (union(x, y) \subseteq z)))$

Axioms combining set-inclusion and attack relation

- $\forall x, y, z\ (((x \rhd y) \wedge (x \subseteq z)) \implies (z \rhd y))$
- $\forall x, y, z\ (((x \rhd y) \wedge (y \subseteq z)) \implies (x \rhd z))$
- $\forall x, y, z\ ((union(x, y) \rhd z) \implies ((x \rhd z) \vee (y \rhd z)))$
- $\forall x, y, z\ ((x \rhd union(y, z)) \implies ((x \rhd y) \vee (x \rhd z)))$

It is easy to prove that the above axioms hold in any structure $\mathcal{M}$ of signature Σ. For instance, let φ denote the first axiom combining set-inclusion and attack relation. Let $\mathcal{M}$ be any structure of signature Σ. We have to prove that φ is interpreted by True on $\mathcal{M}$. It holds since for any t_1, t_2, t_3 subsets of $2^{\mathcal{A}}$, if $t_1 \mathcal{R} t_2$ and $t_1 \subseteq t_3$ then $t_3 \mathcal{R} t_2$.

4.2 The language $\text{YALLA}_{\mathbf{U}}$

Now, we propose a first-order logical theory capable of describing abstract argumentation graphs built on a given *finite* universe $(\mathcal{A}_{\mathbf{U}}, \mathcal{R}_{\mathbf{U}})$, where $\mathcal{A}_{\mathbf{U}} = \{a_1, a_2, \cdots, a_k\}$ with $k = card(\mathcal{A}_{\mathbf{U}})$.

For that purpose, we have to enrich the language. First, the signature must include as many symbols as elements in $2^{\mathcal{A}_{\mathbf{U}}}$. As a consequence, the domain of a structure must enable to interpret all these symbols. Then, we must be able to distinguish between all the terms those which can be interpreted on a given argumentation graph based on the universe $(\mathcal{A}_{\mathbf{U}}, \mathcal{R}_{\mathbf{U}})$. That is the reason why we add the predicate symbol *on*.

The new signature $\Sigma_{\mathbf{U}}$ is defined as follows:

Definition 12 (**Signature**). $\Sigma_{\mathbf{U}} = (V_{const}, V_f, V_P)$ *where* $V_{const} = \{c_\perp, c_1, \ldots, c_p\}$ *with* $p = 2^k - 1$, $V_f = \{union^2\}$ *and* $V_P = \{on^1, \rhd^2, \subseteq^2\}$.

4.2.1 Structure over $\Sigma_{\mathbf{U}}$

The semantics of $\text{YALLA}_{\mathbf{U}}$ is defined thanks to a structure over $\Sigma_{\mathbf{U}}$. Such a structure is associated with an argumentation graph $(\mathcal{A}, \mathcal{R})$ built on the universe $(\mathcal{A}_{\mathbf{U}}, \mathcal{R}_{\mathbf{U}})$. We have $\mathcal{A} \subseteq \mathcal{A}_{\mathbf{U}}$ and $\mathcal{R} \subseteq \mathcal{R}_{\mathbf{U}} \cap (\mathcal{A} \times \mathcal{A})$. So, the domain of the structure is $\mathcal{D} = 2^{\mathcal{A}_{\mathbf{U}}}$, which is not empty.

Definition 13 (**Structure**). *A* structure $\mathcal{M}$ *of signature* $\Sigma_{\mathbf{U}}$*, associated with* $(\mathcal{A}, \mathcal{R})$*, is a pair* $(\mathcal{D}, \mathcal{I})$ *where* $\mathcal{D} = 2^{\mathcal{A}_{\mathbf{U}}}$ *is the* domain of the structure *and* $\mathcal{I}$ *is an* interpretation function *associating:*

(a). a unique element of $\mathcal{D}$ *to each constant symbol* c_i *(in particular the empty set is associated with the constant symbol* $c_\perp$*),*

(b). the binary set theoretic union operator (function from $\mathcal{D}^2$ *to* $\mathcal{D}$*) to the function symbol union,*

(c). the characterization of the subsets of $\mathcal{A}$ *to the predicate symbol on:* $on(\mathcal{S})$ *if and only if* $\mathcal{S} \subseteq \mathcal{A}$

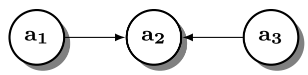
(d). the binary set theoretic inclusion relation (binary relation on $\mathcal{D}^2$*) to the predicate symbol* $\subseteq$*,*

(e). the binary relation of attack between sets of arguments induced by $\mathcal{R}$*, and defined by* $\mathcal{S}_1 \mathcal{R} \mathcal{S}_2$ *if and only if* $\mathcal{S}_1 \subseteq \mathcal{A}, \mathcal{S}_2 \subseteq \mathcal{A}$ *and* $\exists x_1 \in \mathcal{S}_1, \exists x_2 \in \mathcal{S}_2, (x_1 \mathcal{R} x_2)$*, to the predicate symbol* $\rhd$.

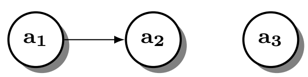
As for the language YALLA, a structure $\mathcal{M}$ over $\Sigma_{\mathbf{U}}$ is entirely determined by $(\mathcal{A}, \mathcal{R})$ built on $\mathcal{A}_{\mathbf{U}}$.

For short, in the following, we identify t with the subset of $\mathcal{A}$ which interprets t when t denotes a term of $\text{YALLA}_{\mathbf{U}}$.

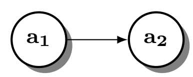
Example 6. *Let us consider the universe* $\mathcal{A}_{\mathbf{U}} = \{a_1, a_2, a_3\}$, $\mathcal{R}_{\mathbf{U}} = \{(a_1, a_2), (a_3, a_2)\}$. *Let* $(\mathcal{A}_1, \mathcal{R}_1)$ *and* $(\mathcal{A}_2, \mathcal{R}_2)$ *be the two argumentation graphs built on the universe* $(\mathcal{A}_{\mathbf{U}}, \mathcal{R}_{\mathbf{U}})$ *defined by:* $\mathcal{A}_1 = \{a_1, a_2, a_3\}$, $\mathcal{R}_1 = \{(a_1, a_2)\}$, $\mathcal{A}_2 = \{a_1, a_2\}$, $\mathcal{R}_2 = \{(a_1, a_2)\}$.



(a) *Universe* $(\mathcal{A}_{\mathbf{U}}, \mathcal{R}_{\mathbf{U}})$.



(b) *Argumentation graph* $(\mathcal{A}_1, \mathcal{R}_1)$.



(c) *Argumentation graph* $(\mathcal{A}_2, \mathcal{R}_2)$.

Figure 1: *Examples of argumentation graphs*

Let φ_1 *be the formula* $on(\{a_1, a_2, a_3\}) \wedge (\{a_1\} \rhd \{a_2\})$ *and* φ_2 *the formula* $on(\{a_1, a_2\}) \wedge (\{a_1\} \rhd \{a_2\})$.

We have $(\mathcal{A}_1, \mathcal{R}_1) \models \varphi_1$, $(\mathcal{A}_1, \mathcal{R}_1) \models \varphi_2$, $(\mathcal{A}_2, \mathcal{R}_2) \models \varphi_2$. *However* $(\mathcal{A}_2, \mathcal{R}_2)$ *is not a model of* φ_1, *since* $\{a_1, a_2, a_3\}$ *is not a subset of* $\mathcal{A}_2$.

Let φ'_2 *be the formula* $on(\{a_1, a_2\})$. *We also have* $(\mathcal{A}_2, \mathcal{R}_2) \models \varphi'_2$.

4.2.2 Specific axioms

For describing the structures over $\Sigma_{\mathbf{U}}$ we need all the axioms presented in Section 4.1.2 augmented by the following ones: let x, y, z be variables of $\text{YALLA}_{\mathbf{U}}$,

Axioms for the predicate *on*

- $on(c_\perp)$
- $\forall x, y \ ((on(x) \wedge (y \subseteq x)) \implies on(y))$
- $\forall x, y \ ((on(x) \wedge on(y)) \implies on(union(x, y))$
- $\forall x, y \ ((x \rhd y) \implies (on(x) \wedge on(y)))$

So far, we have proposed a first-order logical theory for describing argumentation graphs built on a given universe $\mathcal{A}_{\mathbf{U}}$. We have proposed an axiomatisation and a semantics. Let $AX_{\mathbf{U}}$ denote the set of the specific axioms given in 4.1.2 expanded by those given above, that is to say the axioms for the predicate *on*, the axioms for set inclusion, the axioms for set operators and the axioms combining set-inclusion and attack relation.

The following results hold:

- The axiomatisation is sound: it is easy to prove that each structure is a model of the axioms

- The axiomatisation is complete in the following sense: Let φ be a formula of the language YALLA$_{\mathbf{U}}$. Let *Sys* be a sound and complete axiomatic system for predicate calculus, with its logical axioms and inference rules. We denote by $AX_{\mathbf{U}} \vdash_{Sys} \varphi$ the fact that φ is provable in *Sys* augmented by the specific axioms of $AX_{\mathbf{U}}$. We recall that $AX_{\mathbf{U}} \models \varphi$ means that φ logically follows from $AX_{\mathbf{U}}$ (or equivalently each structure which is a model of $AX_{\mathbf{U}}$ is also a model of φ). Then it holds[23] that $AX_{\mathbf{U}} \models \varphi$ if and only if $AX_{\mathbf{U}} \vdash_{Sys} \varphi$.

4.2.3 Examples of formulae in YALLA$_{\mathbf{U}}$

In the following, the constant symbols of the language YALLA$_{\mathbf{U}}$ will be denoted by the elements of $2^{\mathcal{A}_{\mathbf{U}}}$.

We present several examples illustrating the expressive power of YALLA$_{\mathbf{U}}$. In particular, we show that we can precisely describe an argumentation graph by its characteristic formula. Moreover, we are able to express knowledge held by an agent about an incompletely known argumentation graph.

Definition 14 (**Formula describing an argumentation graph**). *The function $\Phi_{\mathbf{U}}$ associated with YALLA$_{\mathbf{U}}$ is defined by:*

$$\begin{array}{llll} \Phi_{\mathbf{U}}: & \Gamma_{\mathbf{U}} & \rightarrow & YALLA_{\mathbf{U}} \\ & (\mathcal{A},\mathcal{R}) & \mapsto & on(\mathcal{A}) \quad \wedge \quad \bigwedge_{x\in\mathcal{A}_{\mathbf{U}}\setminus\mathcal{A}} \neg on(\{x\}) \wedge \\ & & & \bigwedge_{(x,y)\in\mathcal{R}} (\{x\} \rhd \{y\}) \quad \wedge \quad \bigwedge_{(x,y)\in\mathcal{R}_{\mathbf{U}}\setminus\mathcal{R}} \neg(\{x\} \rhd \{y\}) \end{array}$$

$\Phi_{\mathbf{U}}(\mathcal{A},\mathcal{R})$ *is called the* characteristic formula[24] of the graph $(\mathcal{A},\mathcal{R})$.

Proposition 7. $(\mathcal{A},\mathcal{R})$ *is the unique model of* $\Phi_{\mathbf{U}}(\mathcal{A},\mathcal{R})$.

[23] It is a consequence of the Godel's completeness theorem [51, 41].

[24] Note that it should not be confused with the characteristic function of an argumentation graph that is used, for instance, for defining the grounded semantics (see Definition 2).

Example 7. *Let us consider the argumentation graphs* $(\mathcal{A}_1, \mathcal{R}_1)$ *and* $(\mathcal{A}_2, \mathcal{R}_2)$ *from Example 6. Following Definition 14, we have:*[25]

$$\Phi_{\mathbf{U}}(\mathcal{A}_1, \mathcal{R}_1) = on(\{a_1, a_2, a_3\}) \wedge (\{a_1\} \rhd \{a_2\}) \wedge \neg(\{a_3\} \rhd \{a_2\})$$
$$\Phi_{\mathbf{U}}(\mathcal{A}_2, \mathcal{R}_2) = on(\{a_1, a_2\}) \wedge \neg(on(\{a_3\})) \wedge (\{a_1\} \rhd \{a_2\}) \wedge \neg(\{a_3\} \rhd \{a_2\})$$

YALLA$_{\mathbf{U}}$ allows us to express incomplete knowledge about $(\mathcal{A}_{\mathbf{U}}, \mathcal{R}_{\mathbf{U}})$:

Example 8. *Let us consider the universe* $(\mathcal{A}_{\mathbf{U}}, \mathcal{R}_{\mathbf{U}})$ *given in Figure 2.*

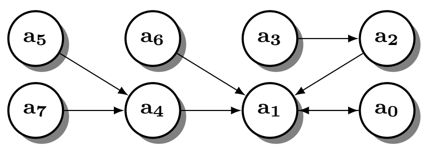
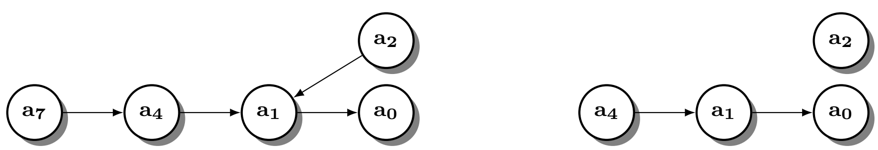


Figure 2: *Universe* $(\mathcal{A}_{\mathbf{U}}, \mathcal{R}_{\mathbf{U}})$.

Assume that Agent Ag_a *has only a partial knowledge about the argumentation graph built by Agent* Ag_b*. Indeed,* Ag_a *hesitates between two possible situations for* Ag_b*'s graph, namely* $(\mathcal{A}_1, \mathcal{R}_1)$ *and* $(\mathcal{A}_2, \mathcal{R}_2)$ *given in Figures 3a and 3b.*



(a) *Argumentation graph* $(\mathcal{A}_1, \mathcal{R}_1)$. **(b)** *Argumentation graph* $(\mathcal{A}_2, \mathcal{R}_2)$.

Figure 3: *Two possible cases for the argumentation graph of Agent* Ag_b

The knowledge held by Ag_a *can be expressed by the following formula of*

[25] Note that the absence of an attack is expressed only if this attack is in the universe: $\neg(\{a_3\} \rhd \{a_2\})$ is in $\Phi_{\mathbf{U}}(\mathcal{A}_1, \mathcal{R}_1)$ since a_3 attacks a_2 in $\mathbf{U}$, whereas $\neg(\{a_2\} \rhd \{a_1\})$ is not in $\Phi_{\mathbf{U}}(\mathcal{A}_1, \mathcal{R}_1)$ since a_2 does not attack a_1 in $\mathbf{U}$.

YALLA$_{\mathbf{U}}$:[26]

$$
\begin{array}{rcccccccc}
\varphi & = & & & on(\{a_0, a_1, a_2, a_4\}) & \wedge & & \\
& & \neg(on(\{a_3\})) & \wedge & \neg(on(\{a_5\})) & \wedge & \neg(on(\{a_6\})) & \wedge \\
& & (\{a_4\} \rhd \{a_1\}) & \wedge & (\{a_1\} \rhd \{a_0\}) & \wedge & & \\
& & \neg(\{a_0\} \rhd \{a_1\}) & \wedge & \neg(\{a_3\} \rhd \{a_2\}) & \wedge & \neg(\{a_5\} \rhd \{a_4\}) & \wedge \\
& & \neg(\{a_6\} \rhd \{a_1\}) & \wedge & & & & \\
& \multicolumn{7}{l}{\left(\begin{array}{ccccc}
(on(\{a_7\}) & \wedge & (\{a_7\} \rhd \{a_4\}) & \wedge & (\{a_2\} \rhd \{a_1\})) \\
& & \vee & & \\
(\neg(on(\{a_7\})) & \wedge & \neg(\{a_7\} \rhd \{a_4\}) & \wedge & \neg(\{a_2\} \rhd \{a_1\}))
\end{array}\right)}
\end{array}
$$

φ has only two models which are exactly the argumentation graphs $(\mathcal{A}_1, \mathcal{R}_1)$ and $(\mathcal{A}_2, \mathcal{R}_2)$.

Note that $\varphi \equiv \Phi_{\mathbf{U}}(\mathcal{A}_1, \mathcal{R}_1) \vee \Phi_{\mathbf{U}}(\mathcal{A}_2, \mathcal{R}_2)$.

4.3 Encoding argumentation semantics

Since we want to use YALLA$_{\mathbf{U}}$ for encoding argumentation semantics, some formulae will be found very often, we propose useful notations for them.

Definition 15. *Let t_1 and t_2 be terms of YALLA$_{\mathbf{U}}$. We define:*

$$
\begin{array}{rcl}
t_1 = t_2 & \stackrel{def}{\equiv} & (t_1 \subseteq t_2) \wedge (t_2 \subseteq t_1). \\
t_1 \neq t_2 & \stackrel{def}{\equiv} & \neg(t_1 = t_2). \\
singl(t_1) & \stackrel{def}{\equiv} & (t_1 \neq c_\perp) \wedge \forall t_2 \ (((t_2 \neq c_\perp) \wedge (t_2 \subseteq t_1)) \implies (t_1 \subseteq t_2)).
\end{array}
$$

Obviously, $(\mathcal{A}, \mathcal{R}) \models singl(t)$ if and only if the term t is interpreted by a singleton of $\mathcal{A}$.

The next step is to provide formulae that express the criteria underlying the traditional argumentation semantics. Traditional semantics are defined by lists of criteria, based on notions such as conflict-freeness, defense, admissibility for instance. In the following, we encode these notions by appropriate formulae of YALLA$_{\mathbf{U}}$.

[26]It would be possible to find a simpler and logically equivalent formula. Nevertheless, we keep this formulation since it seems more natural and closer to an automatic translation of the graph.

Proposition 8. *Let $\mathcal{A}_{\mathbf{U}}$ be a set of arguments and $(\mathcal{A}, \mathcal{R})$ be an argumentation graph such that $\mathcal{A} \subseteq \mathcal{A}_{\mathbf{U}}$ and $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$. Let t, t_1, t_2, t_3 be terms of YALLA$_{\mathbf{U}}$. We have:*

- *t is conflict-free in $(\mathcal{A}, \mathcal{R})$ if and only if $(\mathcal{A}, \mathcal{R}) \models on(t) \wedge (\neg(t \rhd t))$. The latter formula is denoted by $F(t)$.*

- *t_1 defends each element of t_2 in $(\mathcal{A}, \mathcal{R})$ if and only if $(\mathcal{A}, \mathcal{R}) \models (\forall t_3\ ((singl(t_3) \wedge (t_3 \rhd t_2)) \implies (t_1 \rhd t_3)))$, which is denoted by $(\mathcal{A}, \mathcal{R}) \models t_1 \ \rhd\!\!\!\rhd t_2$.*

- *If $(\mathcal{A}, \mathcal{R}) \models singl(t_2)$, then t_1 indirectly defends the unique element of t_2 in $(\mathcal{A}, \mathcal{R})$ (which is denoted by $(\mathcal{A}, \mathcal{R}) \models t_1 \rhd\!\!\leadsto\!\!\rhd t_2$) if and only if $(\mathcal{A}, \mathcal{R}) \models (t_1 \ \rhd\!\!\!\rhd t_2) \vee (\exists y\ ((t_1 \ \rhd\!\!\!\rhd y) \wedge (y \rhd\!\!\leadsto\!\!\rhd t_2)))$.*

- *t is admissible in $(\mathcal{A}, \mathcal{R})$ if and only if $(\mathcal{A}, \mathcal{R}) \models (F(t) \wedge (t \ \rhd\!\!\!\rhd t))$, which is denoted by $(\mathcal{A}, \mathcal{R}) \models A(t)$.*

- *t is a complete extension of $(\mathcal{A}, \mathcal{R})$ if and only if $(\mathcal{A}, \mathcal{R}) \models (A(t) \wedge \forall t_2\ ((singl(t2) \wedge (t \ \rhd\!\!\!\rhd t_2)) \implies (t_2 \subseteq t)))$, which is denoted by $(\mathcal{A}, \mathcal{R}) \models C(t)$.*

- *t is the grounded extension of $(\mathcal{A}, \mathcal{R})$ if and only if $(\mathcal{A}, \mathcal{R}) \models (C(t) \wedge \forall t_2\ (C(t_2) \implies (t \subseteq t_2)))$, which is denoted by $(\mathcal{A}, \mathcal{R}) \models G(t)$.*

- *t is a stable extension of $(\mathcal{A}, \mathcal{R})$ if and only if $(\mathcal{A}, \mathcal{R}) \models (F(t) \wedge \forall t_2\ ((singl(t_2) \wedge \neg(t_2 \subseteq t)) \implies (t \rhd t_2)))$, which is denoted by $(\mathcal{A}, \mathcal{R}) \models S(t)$.*

- *t is a preferred extension of $(\mathcal{A}, \mathcal{R})$ if and only if $(\mathcal{A}, \mathcal{R}) \models (A(t) \wedge \forall t_2\ (((t_2 \neq t) \wedge (t \subseteq t_2)) \implies \neg A(t_2)))$, which is denoted by $(\mathcal{A}, \mathcal{R}) \models P(t)$.*

Now, using YALLA$_{\mathbf{U}}$, we apply belief update concepts to argumentation.

5 Belief update and argumentation

We have seen in Section 3 that a change operation in argumentation is close to a transition in belief change: an argumentation graph corresponds to a state of the world and an executable operation to an authorized transition. In this section, we propose an update operator and a set of properties specific to change in abstract argumentation.

Note that the belief update framework is defined for a *propositional logic*, whereas $\mathrm{YALLA}_{\mathbf{U}}$ is a *first-order language*. Nevertheless, as the universe $(\mathcal{A}_{\mathbf{U}}, \mathcal{R}_{\mathbf{U}})$ is *finite*, each structure that interprets formulae of $\mathrm{YALLA}_{\mathbf{U}}$ has a finite domain, so each formula of $\mathrm{YALLA}_{\mathbf{U}}$ is *equivalent to a propositional combination of variable-free atomic formulae*, using only the logical connectives. In the following, quantifiers are kept for the sake of shortness of the notations.

First, we recall some notions and definitions.

- The set of argumentation graphs built on the universe $(\mathcal{A}_{\mathbf{U}}, \mathcal{R}_{\mathbf{U}})$ is denoted by $\Gamma_{\mathbf{U}}$.
- Moreover, for any formula φ of $\mathrm{YALLA}_{\mathbf{U}}$, we denote by $[\varphi]$ the set of models of φ, *i.e.* the set of argumentation graphs such that φ is true in these graphs (see Definition 11). Hence, $[\varphi]$ can be written as $\{\mathcal{G} \in \Gamma_{\mathbf{U}} \mid \mathcal{G} \models \varphi\}$.
- $[\Phi_{\mathbf{U}}(\mathcal{G})] = \{\mathcal{G}\}$, where $\Phi_{\mathbf{U}}(\mathcal{G})$ is the characteristic formula of the argumentation graph $\mathcal{G}$ (see Proposition 7).
- Finally, $\varphi \models \psi$ means $\forall \mathcal{G} \in \Gamma_{\mathbf{U}}$, if $\mathcal{G} \models \varphi$ then $\mathcal{G} \models \psi$, or equivalently $[\varphi] \subseteq [\psi]$.

5.1 Classical belief update in abstract argumentation

Classical belief update (*à la* Katsuno-Mendelzon) in an argumentation framework amounts to consider:

- a formula φ in $\mathrm{YALLA}_{\mathbf{U}}$ representing a current state of knowledge about exchanged arguments (*i.e.*, it may encompass several possible argumentation systems),
- and a new piece of information α stating that the debate has evolved in such a way that α now holds (*i.e.*, the current state of the debate is inside a set of argumentation systems satisfying α).

Updating φ by α gives a formula $\varphi \diamond \alpha$ that represents the set of argumentation systems that corresponds to an evolution of the debate where a change has been done imposing α. Using Theorem 1, we know that this formula corresponds to some argumentation systems which can be computed thanks to a faithful pre-order.

In the framework described in Section 2.3, an argumentation system can only evolve by an allowed operation made by an agent (according to the agent's own argumentation system and according to her target). This means that some transitions are not allowed. So the update operator given by Definition 7 is required for reasoning about change in this framework.

5.2 Belief update postulates adapted to our argumentation framework

Let $\mathcal{T} \subseteq \Gamma_{\mathbf{U}} \times \Gamma_{\mathbf{U}}$ representing a set of authorized transitions (for instance, the set of authorized transitions can be defined as in Definition 4). Using YALLA$_{\mathbf{U}}$, we can express a change respecting $\mathcal{T}$:

Definition 16 (**Change respecting** $\mathcal{T}$)**.** *For any φ, ψ in YALLA$_{\mathbf{U}}$, a change from a set of argumentation graphs satisfying φ to a set of argumentation graphs satisfying ψ* is respecting *$\mathcal{T}$, denoted by $(\varphi, \psi) \models \mathcal{T}$, if and only if $([\varphi] \neq \varnothing$ and $\forall \mathcal{G} \in [\varphi], \exists \mathcal{G}' \in [\psi], (\mathcal{G}, \mathcal{G}') \in \mathcal{T})$.*

In other words, the change is possible if there exists a change from each graph of the first set (which must be non empty) to at least one graph of the second set.

Now, we can define a generalized enforcement operator:

Definition 17 (**Generalized enforcement operator related to** $\mathcal{T}$)**.** *A generalized enforcement operator $\diamondsuit_{\mathcal{T}}$ related to $\mathcal{T}$ is a mapping from YALLA$_{\mathbf{U}}$ $\times$ YALLA$_{\mathbf{U}}$ to YALLA$_{\mathbf{U}}$ which associates with any formula φ and any formula α a formula, denoted by $\varphi \diamondsuit_{\mathcal{T}} \alpha$, such that $(\varphi, \varphi \diamondsuit_{\mathcal{T}} \alpha) \models \mathcal{T}$.*

In other words, $\varphi \diamondsuit_{\mathcal{T}} \alpha$ characterizes the argumentation graphs obtained from argumentation graphs satisfying φ through a change respecting $\mathcal{T}$. The postulates[27] defined in Section 3.2 allow us to write the following representation theorem concerning enforcement, namely, an enforcement operator satisfying these postulates can be defined by means of the definition of a family of preorders on argumentation graphs.

[27]Reminder of the postulates defined in Section 3.2:

E3: $[\varphi \diamondsuit_{\mathcal{T}} \alpha] \neq \varnothing$ iff $(\varphi, \alpha) \models \mathcal{T}$.

U4: $[\varphi] = [\psi]$ and $[\alpha] = [\beta] \implies [\varphi \diamondsuit_{\mathcal{T}} \alpha] = [\psi \diamondsuit_{\mathcal{T}} \beta]$.

E5: if $card([\varphi]) = 1$, then $(\varphi \diamondsuit_{\mathcal{T}} \alpha) \wedge \beta \models \varphi \diamondsuit_{\mathcal{T}} (\alpha \wedge \beta)$.

E8: if $([\varphi] \neq \varnothing$ and $[\varphi \diamondsuit_{\mathcal{T}} \alpha] = \varnothing)$ or $([\psi] \neq \varnothing$ and $[\psi \diamondsuit_{\mathcal{T}} \alpha] = \varnothing)$,
then $[(\varphi \vee \psi) \diamondsuit_{\mathcal{T}} \alpha] = \varnothing$
else $[(\varphi \vee \psi) \diamondsuit_{\mathcal{T}} \alpha] = [(\varphi \diamondsuit_{\mathcal{T}} \alpha) \vee (\psi \diamondsuit_{\mathcal{T}} \alpha)]$.

U9: if $card([\varphi]) = 1$, then $[(\varphi \diamondsuit_{\mathcal{T}} \alpha) \wedge \beta] \neq \varnothing \implies \varphi \diamondsuit_{\mathcal{T}} (\alpha \wedge \beta) \models (\varphi \diamondsuit_{\mathcal{T}} \alpha) \wedge \beta$.

Definition 18. *Given $\mathcal{T} \subseteq \Gamma_{\mathbf{U}} \times \Gamma_{\mathbf{U}}$ a set of authorized transitions, an assignment respecting $\mathcal{T}$ is a function that associates with each $\mathcal{G} \in \Gamma_{\mathbf{U}}$ a complete preorder $\preceq_{\mathcal{G}}$ such that $\forall \mathcal{G}_1, \mathcal{G}_2 \in \Gamma_{\mathbf{U}}$ if $(\mathcal{G}, \mathcal{G}_1) \in \mathcal{T}$ and $(\mathcal{G}, \mathcal{G}_2) \notin \mathcal{T}$ then $\mathcal{G}_1 \prec_{\mathcal{G}} \mathcal{G}_2$.*

Theorem 3. *Given $\mathcal{T} \subseteq \Gamma_{\mathbf{U}} \times \Gamma_{\mathbf{U}}$ a set of authorized transitions, there exists an operator $\diamondsuit_{\mathcal{T}}$ satisfying **E3**, **U4**, **E5**, **E8**, **U9** if and only if there is an assignment respecting $\mathcal{T}$ such that $\forall \mathcal{G} \in \Gamma_{\mathbf{U}}, \forall \varphi, \alpha \in$ YALLA$_{\mathbf{U}}$,*

(1) $[\varphi \diamondsuit_{\mathcal{T}} \alpha] = \varnothing$ *if* $\exists \mathcal{G} \in [\varphi]$ *such that* $[\Phi_{\mathbf{U}}(\mathcal{G}) \diamondsuit_{\mathcal{T}} \alpha] = \varnothing$

(2) $[\varphi \diamondsuit_{\mathcal{T}} \alpha] = \bigcup_{\mathcal{G} \in [\varphi]} [\Phi_{\mathbf{U}}(\mathcal{G}) \diamondsuit_{\mathcal{T}} \alpha]$ *otherwise*

(3) $$[\Phi_{\mathbf{U}}(\mathcal{G}) \diamondsuit_{\mathcal{T}} \alpha] = \left\{ \mathcal{G}_1 \in \Gamma_{\mathbf{U}} \;\middle|\; \begin{array}{l} \mathcal{G}_1 \in [\alpha] \textit{ and} \\ (\mathcal{G}, \mathcal{G}_1) \in \mathcal{T} \textit{ and} \\ (\forall \mathcal{G}_2 \in [\alpha] \textit{ such that } (\mathcal{G}, \mathcal{G}_2) \in \mathcal{T}, \mathcal{G}_1 \preceq_{\mathcal{G}} \mathcal{G}_2) \end{array} \right\}$$

Following this theorem, given $\mathcal{G}$, α a formula of YALLA$_{\mathbf{U}}$, and an assignment respecting $\mathcal{T}$, the formula $\Phi_{\mathbf{U}}(\mathcal{G}) \diamondsuit_{\mathcal{T}} \alpha$ characterizes the closest graphs with regard to $\mathcal{G}$ in which α is satisfied and which are obtained from $\mathcal{G}$ through a transition in $\mathcal{T}$ (where "closest with regard to $\mathcal{G}$" means minimal with regard to the order $\preceq_{\mathcal{G}}$ defined by the assignment).

Note that the above theorem is only a representation theorem (it is the "argumentation" version of Theorem 2). It does not provide a constructive way to build an assignment respecting the transitions. Nevertheless this result is significant in the sense that it justifies the use of a distance relation for finding the results of an argumentation update, and it explains as in Katsuno-Mendelzon Theorem that the computation of the closest argumentation systems (in the sense of this distance) should be done on each argumentation system representing the initial knowledge. Note also that Postulates **E3** to **U9** are general enough to characterize any update expressed in a logical language. Hence, they apply to argumentation dynamics in the case where argumentation notions are expressed in a logical framework as, for instance, YALLA. However, they do not give any specific insight about how the particular concepts of argumentation behave in the presence of change. In the next section, we show that characterizations may be considered as properties that refine update operators in the context of argumentation.

5.3 Characterizations as a way to specialize update operators for argumentation

The characterizations presented in Section 2.4 enable us to learn what properties hold on the target graph when one argument is either added or removed. Thanks to YALLA$_{\mathbf{U}}$ and considering an update operator $\diamondsuit_{\mathcal{T}}$ that *minimizes the changes wrt to addition and removal of arguments*, with $\mathcal{T}$ being a set of authorized transitions for a given user (depending on its own knowledge, *i.e.* its own argumentation graph), we are going to translate the characterizations into update properties specific to the argumentation domain. These properties will play the same role as the postulates in belief change theory since they are going to constrain the update operators and the associated family of pre-orders. However they are not called "postulates" since they are proven results that are coming from the argumentation domain.

The characterizations were obtained by the addition (resp. removal) of a single argument. Therefore, if the "nearest" graphs of $\mathcal{G}$ are precisely those obtained by doing only one elementary operation executable on $\mathcal{G}$ (*i.e.* adding or removing one single argument with its interactions and doing nothing else), then an update operator with an assignment complying with this closeness relation will satisfy the conclusion of the characterization. Note that formally the executability of the change depends on the operating agent system (see Definition 4), but, in this section, in order to simplify the notations we consider that the agent that acts on the target is fixed.

First, we are going to define the elementary transitions corresponding to elementary operations executable by the agent.

Definition 19 (**Executable elementary transition set**)**.** *The* set of executable elementary transitions*, denoted $\mathcal{T}_e$, is defined by $T_e = \mathcal{T}_+ \cup \mathcal{T}_-$ where*

$$\mathcal{T}_+ = \left\{ (\mathcal{G}_1, \mathcal{G}_2) \in \Gamma_{\mathbf{U}} \times \Gamma_{\mathbf{U}} \;\middle|\; \begin{array}{l} \exists z \in \mathcal{A}_{\mathbf{U}} \textit{ and } \mathcal{R}_z \subseteq \mathcal{R}_{\mathbf{U}} \textit{ such that } o = \langle \oplus, z, \mathcal{R}_z \rangle \\ \textit{is an elementary operation executable by the} \\ \textit{agent on } \mathcal{G}_1 \textit{ and } \mathcal{G}_2 = o(\mathcal{G}_1) \end{array} \right\}$$

is the set of transitions corresponding to an elementary executable addition, and

$$\mathcal{T}_- = \left\{ (\mathcal{G}_1, \mathcal{G}_2) \in \Gamma_{\mathbf{U}} \times \Gamma_{\mathbf{U}} \;\middle|\; \begin{array}{l} \exists z \in \mathcal{A}_{\mathbf{U}} \textit{ such that } o = \langle \ominus, z, \varnothing \rangle \textit{ is an elementary} \\ \textit{operation executable by the agent on } \mathcal{G}_1 \textit{ and} \\ \mathcal{G}_2 = o(\mathcal{G}_1) \end{array} \right\}$$

is the set of transitions corresponding to an elementary executable suppression.

The proposition below is a kind of "argumentation update property builder", since for every result already established by a characterization we can build an associated generalized enforcement property provided that the chosen operator always prefers elementary changes to more complex ones.

Proposition 9. *Given a set of authorized transitions $\mathcal{T} \subseteq \Gamma_{\mathbf{U}} \times \Gamma_{\mathbf{U}}$ such that*[28] *$\mathcal{T}_e \subseteq \mathcal{T}$, for any characterization C of the form:*

> *"If $\mathcal{G} \models \gamma_1$ and $\exists z, \mathcal{R}_z$ such that $o = \langle \oplus, z, \mathcal{R}_z \rangle$ is executable by the agent on $\mathcal{G}$ and $o(\mathcal{G}) \models \alpha_z$ then $o(\mathcal{G}) \models \gamma_2$",*

*it holds that every assignment respecting $\mathcal{T}$ and $\mathcal{T}_e$ allows us to define a generalized enforcement operator $\Diamond_{\mathcal{T}}$ in the same way as in Theorem 3 satisfying **E3**, **U4**, **E5**, **E8**, **U9** and the additional property*

$$\begin{array}{ll}
\boldsymbol{A}_C\textit{:} & \forall \mathcal{A} \subseteq \mathcal{A}_{\mathbf{U}}, \forall \mathcal{R}_{\mathcal{A}} \subseteq \mathcal{A} \times \mathcal{A}, \forall z \in \mathcal{A}_{\mathbf{U}} \setminus \mathcal{A}, \\
 & \forall \mathcal{R}_z \subseteq (\{z\} \times \mathcal{A}) \cup (\mathcal{A} \times \{z\}), \\
\quad \textit{let} & \mathcal{G} = (\mathcal{A}, \mathcal{R}_{\mathcal{A}}) \textit{ and } o = \langle \oplus, z, \mathcal{R}_z \rangle, \\
\quad \textit{if} & \Phi_{\mathbf{U}}(\mathcal{G}) \models \gamma_1 \textit{ and } (\mathcal{G}, o(\mathcal{G})) \in \mathcal{T}_e \textit{ and } \Phi_{\mathbf{U}}(o(\mathcal{G})) \models \alpha_z \\
\quad \textit{then} & \Phi_{\mathbf{U}}(\mathcal{G}) \Diamond_{\mathcal{T}} (on(z) \wedge \varphi_{\mathcal{R}_z}) \models \gamma_2.
\end{array}$$

where $\varphi_{\mathcal{R}_z} = \bigwedge_{(x,y) \in \mathcal{R}_z} (x \rhd y) \wedge \bigwedge_{x \in \mathcal{G} | (x,z) \notin \mathcal{R}_z} \neg(x \rhd z) \wedge \bigwedge_{y \in \mathcal{G} | (z,y) \notin \mathcal{R}_z} \neg(z \rhd y)$ is a formula that describes the attacks that are in $\mathcal{R}_z$ and that excludes any other attack concerning z.

Remark 2. *The additional property A_C induces a new constraint on the assignment: each pre-order of the assignment should be refined in order to prefer elementary executable changes (in $\mathcal{T}_e$) in addition to the preferences on transitions satisfying $\mathcal{T}$. More precisely elementary changes should be preferred to non elementary changes and among the non elementary changes those in $\mathcal{T}$ are preferred to those outside of $\mathcal{T}$.*

Let us illustrate this "meta-proposition" with Characterization 1 given in Section 2.4. Characterization 1 states that if p is the grounded extension of $(\mathcal{A}, \mathcal{R})$, then all the graphs which are obtained from $\mathcal{G}$ by adding only z

[28] $\mathcal{T}_e \subseteq \mathcal{T}$ means that the elementary operations that are executable should be authorized, *i.e.* included in the set of authorized transitions. Moreover, the inclusion of $\mathcal{T}$ into $\Gamma_{\mathbf{U}} \times \Gamma_{\mathbf{U}}$ implies that $\mathcal{R}_{\mathcal{A}} \subseteq \mathcal{R}_{\mathbf{U}}$ and $\mathcal{R}_z \subseteq \mathcal{R}_{\mathbf{U}}$. Note however that, since characterizations concern any argumentation graphs, this property would hold with any transition set $\mathcal{T}$ that contains at least all the transitions that add or remove one argument.

and attacks involving z, such that z does not attack $\mathcal{G}$ and p defends z, have $p \cup \{z\}$ as grounded extension.

Using YALLA$_{\mathbf{U}}$, the sufficient conditions expressed in Characterization 1 can be encoded by the following formula related to two terms z and p denoting two subsets of $\mathcal{A}_{\mathbf{U}}$:

$$\Phi^1_{z,p} = (singl(z) \wedge (p \ \mathrel{\rhd\!\!\!\rhd} z) \wedge \neg(\exists t\ on(t) \wedge (z \rhd t)))$$

Moreover, the assertion "the set p is the grounded extension of $\mathcal{G}$" can be expressed by $\Phi_{\mathbf{U}}(\mathcal{G}) \models G(p)$ in YALLA$_{\mathbf{U}}$. Characterization 1 can be written as follows: $\forall p$, if $\mathcal{G} \models G(p)$ and $\exists z, \mathcal{R}_z$ such that $o = \langle \oplus, z, \mathcal{R}_z \rangle$ is executable by the agent on $\mathcal{G}$ and $o(\mathcal{G}) \models \Phi^1_{z,p}$ then $o(\mathcal{G}) \models G(union(p, z))$.

The corresponding update property is:

$$\begin{array}{ll}
\mathbf{A}_{C1}: & \forall \mathcal{A} \subseteq \mathcal{A}_{\mathbf{U}}, \forall \mathcal{R}_{\mathcal{A}} \subseteq \mathcal{A} \times \mathcal{A}, \forall z \in \mathcal{A}_{\mathbf{U}} \setminus \mathcal{A}, \\
 & \forall \mathcal{R}_z \subseteq (\{z\} \times \mathcal{A}) \cup (\mathcal{A} \times \{z\}), \\
\quad \text{let} & \mathcal{G} = (\mathcal{A}, \mathcal{R}_{\mathcal{A}}) \text{ and } o = \langle \oplus, z, \mathcal{R}_z \rangle, \\
\quad \forall p,\ \text{if} & \Phi_{\mathbf{U}}(\mathcal{G}) \models G(p) \text{ and } (\mathcal{G}, o(\mathcal{G})) \in \mathcal{T}_e \text{ and } \Phi_{\mathbf{U}}(o(\mathcal{G})) \models \Phi^1_{z,p} \\
\quad \text{then} & \Phi_{\mathbf{U}}(\mathcal{G}) \diamondsuit_{\mathcal{T}} (on(z) \wedge \varphi_{\mathcal{R}_z}) \models G(union(p, z)).
\end{array}$$

Due to Proposition 9, an operator $\diamondsuit_{\mathcal{T}}$ satisfies the property $\mathbf{A}_{C1}$ if $\diamondsuit_{\mathcal{T}}$ is defined as in Theorem 3 from an assignment such that the closest graphs which are obtained from a graph $\mathcal{G}$ using a transition in $\mathcal{T}$ are exactly the graphs obtained from $\mathcal{G}$ by executing an elementary executable addition.

Proposition 9 enables us to show a similar result concerning Characterization 2 which states that: if an operation adds an argument z, such that z is not attacked and indirectly defends another argument x which was not accepted under the grounded semantics, then x will become accepted under the grounded semantics.

Let $\Phi^2_{z,x} = (singl(z) \wedge \neg(\exists t\ on(t) \wedge (t \rhd z)) \wedge (z \rhd\!\!\leadsto\!\!\rhd x))$.[29] Characterization 2 can be written as follows: $\forall x, \forall p$, if $\mathcal{G} \models G(p) \wedge singl(x) \wedge \neg(x \subseteq p)$ and $\exists z, \mathcal{R}_z$ such that $o = \langle \oplus, z, \mathcal{R}_z \rangle$ is executable by the agent on $\mathcal{G}$ and $o(\mathcal{G}) \models \Phi^2_{z,x}$ then $\exists p'$ such that $o(\mathcal{G}) \models G(p') \wedge (x \subseteq p')$.

[29] Note that $\Phi^2_{z,x} \models on(x)$.

The corresponding update property is:

$$
\begin{array}{ll}
\mathbf{A}_{C2}: & \forall \mathcal{A} \subseteq \mathcal{A}_{\mathbf{U}}, \forall \mathcal{R}_{\mathcal{A}} \subseteq \mathcal{A} \times \mathcal{A}, \forall z \in \mathcal{A}_{\mathbf{U}} \setminus \mathcal{A}, \\
& \forall \mathcal{R}_z \subseteq (\{z\} \times \mathcal{A}) \cup (\mathcal{A} \times \{z\}), \\
\text{let} & \mathcal{G} = (\mathcal{A}, \mathcal{R}_{\mathcal{A}}) \text{ and } o = \langle \oplus, z, \mathcal{R}_z \rangle, \\
\forall p, x, \text{ if} & \Phi_{\mathbf{U}}(\mathcal{G}) \models G(p) \wedge singl(x) \wedge \neg(x \subseteq p) \text{ and } (\mathcal{G}, o(\mathcal{G})) \in \mathcal{T}_e \text{ and} \\
& \Phi_{\mathbf{U}}(o(\mathcal{G})) \models \Phi^2_{z,x} \\
\text{then} & \exists p', \Phi_{\mathbf{U}}(\mathcal{G}) \diamondsuit_{\mathcal{T}} (on(z) \wedge \varphi_{\mathcal{R}_z}) \models G(p') \wedge (x \subseteq p').
\end{array}
$$

Similarly, Characterization 4 given in Section 2.4 can be taken into account. This characterization states that if p is the grounded extension of $\mathcal{G}$ then all the graphs which are obtained from $\mathcal{G}$ by adding only an argument z and a set of attacks involving z, such that z does not attack p and p does not defend z, have p as the grounded extension.

Let $\Phi^4_{z,p} = (singl(z) \wedge \neg(z \rhd p) \wedge \neg(p \;\, \rhd\!\!\!\rhd z))$. Characterization 4 can be translated into YALLA$_{\mathbf{U}}$ as follows: $\forall p$, if $\mathcal{G} \models G(p)$ and $\exists z, \mathcal{R}_z$ such that $o = \langle \oplus, z, \mathcal{R}_z \rangle$ is executable by the agent on $\mathcal{G}$ and $o(\mathcal{G}) \models \Phi^4_{z,p}$ then $o(\mathcal{G}) \models G(p)$.

The above proposition allows us to deduce a new property for a generalized enforcement operator, namely:

$$
\begin{array}{ll}
\mathbf{A}_{C4}: & \forall \mathcal{A} \subseteq \mathcal{A}_{\mathbf{U}}, \forall \mathcal{R}_{\mathcal{A}} \subseteq \mathcal{A} \times \mathcal{A}, \forall z \in \mathcal{A}_{\mathbf{U}} \setminus \mathcal{A}, \\
& \forall \mathcal{R}_z \subseteq (\{z\} \times \mathcal{A}) \cup (\mathcal{A} \times \{z\}), \\
\text{let} & \mathcal{G} = (\mathcal{A}, \mathcal{R}_{\mathcal{A}}) \text{ and } o = \langle \oplus, z, \mathcal{R}_z \rangle, \\
\forall p, \text{ if} & \Phi_{\mathbf{U}}(\mathcal{G}) \models G(p) \text{ and } (\mathcal{G}, o(\mathcal{G})) \in \mathcal{T}_e \text{ and } \Phi_{\mathbf{U}}(o(\mathcal{G})) \models \Phi^4_{z,p} \\
\text{then} & \Phi_{\mathbf{U}}(\mathcal{G}) \diamondsuit_{\mathcal{T}} (on(z) \wedge \varphi_{\mathcal{R}_z}) \models G(p).
\end{array}
$$

Below, we establish a proposition similar to Proposition 9 in the case of removal.

Proposition 10. *Given a set of authorized transitions $\mathcal{T} \subseteq \Gamma_{\mathbf{U}} \times \Gamma_{\mathbf{U}}$ such that $\mathcal{T}_e \subseteq \mathcal{T}$, for any characterization C of the form:*

> *"If $\mathcal{G} \models \gamma_1$ and $\exists z$ such that $o = \langle \ominus, z, \varnothing \rangle$ is executable by the agent on $\mathcal{G}$ and $\mathcal{G} \models \alpha_z$ then $o(\mathcal{G}) \models \gamma_2$",*

*it holds that every assignment respecting $\mathcal{T}$ and $\mathcal{T}_e$ allows us to define a generalized enforcement operator $\diamondsuit_{\mathcal{T}}$ in the same way as in Theorem 3 satisfying **E3**, **U4**, **E5**, **E8**, **U9** and the additional property*

$$
\begin{array}{ll}
\boldsymbol{A}_C\textit{:} & \forall \mathcal{A} \subseteq \mathcal{A}_{\mathbf{U}}, \forall \mathcal{R}_{\mathcal{A}} \subseteq \mathcal{A} \times \mathcal{A}, \forall z \in \mathcal{A}, \\
\quad \textit{let} & \mathcal{G} = (\mathcal{A}, \mathcal{R}_{\mathcal{A}}) \textit{ and } o = \langle \ominus, z, \varnothing \rangle, \\
\quad \textit{if} & \Phi_{\mathbf{U}}(\mathcal{G}) \models \gamma_1 \textit{ and } (\mathcal{G}, o(\mathcal{G})) \in \mathcal{T}_e \textit{ and } \Phi_{\mathbf{U}}(\mathcal{G}) \models \alpha_z \\
\quad \textit{then} & \Phi_{\mathbf{U}}(\mathcal{G}) \diamondsuit_{\mathcal{T}} \neg on(z) \models \gamma_2
\end{array}
$$

Remark 3. *Note that while Proposition 9 and Proposition 10 seem alike, they reflect the difference between the operations of addition and removal of an argument. Thus, the additional property is different in these propositions, which is due in particular to the set of attacks to consider in case of addition. Another difference between Proposition 9 and Proposition 10 is the fact that α_z is not satisfied by the same graph ($\Phi_{\mathbf{U}}(o(\mathcal{G})) \models \alpha_z$ in Proposition 9 and $\Phi_{\mathbf{U}}(\mathcal{G}) \models \alpha_z$ in Proposition 10).*

An example of update property obtained from a removal characterization is the one corresponding to Characterization 3 given in Section 2.4 saying that if an operation removes an argument z such that z is attacked by at least one argument of $\mathcal{G}$ and knowing that the extension was not empty before the change, then the extension obtained after the change will not be empty.

Let $\Phi^3_z = (singl(z) \wedge (\exists t\ on(t) \wedge (t \rhd z))$. Characterization 3 can be translated into YALLA$_{\mathbf{U}}$ as follows: $\forall p$, If $\mathcal{G} \models G(p) \wedge \neg(p \subseteq c_\perp)$ and $\exists z$ such that $o = \langle \ominus, z, \varnothing \rangle$ is executable by the agent on $\mathcal{G}$ and $\mathcal{G} \models \Phi^3_z$ then $o(\mathcal{G}) \models \exists p', G(p') \wedge \neg(p' \subseteq c_\perp)$.

The above proposition allows us to deduce a new property for a generalized enforcement operator, namely:

$$
\begin{array}{ll}
\mathbf{A}_{C3}\text{:} & \forall \mathcal{A} \subseteq \mathcal{A}_{\mathbf{U}}, \forall \mathcal{R}_{\mathcal{A}} \subseteq \mathcal{A} \times \mathcal{A}, \forall z \in \mathcal{A}, \\
\quad \text{let} & \mathcal{G} = (\mathcal{A}, \mathcal{R}_{\mathcal{A}}) \text{ and } o = \langle \ominus, z, \varnothing \rangle, \\
\quad \forall p, \text{ if} & \Phi_{\mathbf{U}}(\mathcal{G}) \models G(p) \wedge \neg(p \subseteq c_\perp) \text{ and } (\mathcal{G}, o(\mathcal{G})) \in \mathcal{T}_e \text{ and } \Phi_{\mathbf{U}}(\mathcal{G}) \models \Phi^3_z \\
\quad \text{then} & \Phi_{\mathbf{U}}(\mathcal{G}) \diamondsuit_{\mathcal{T}} \neg on(z) \models \exists p', G(p') \wedge \neg(p' \subseteq c_\perp).
\end{array}
$$

In this way, all characterizations about elementary changes that had been already established (see for instance [14]) can be written as properties of a generalized enforcement operator. These properties hold for any operator $\diamondsuit_{\mathcal{T}}$ preferring elementary changes (see examples in Section 6).

Remark 4. *Note that these additional properties are not designed specifically for computing the minimal operations to do in order to achieve a given goal but they are properties that are specific for argumentation update operators and that can be used to establish properties about elementary changes. However, even if they were not designed to this end, the characterizations have been used in a tool that computes the operations satisfying a goal (see [16]).*

5.4 Particular cases of generalized enforcement operators

A first idea is to define an operator that minimizes the changes in terms of removal or addition of arguments in the target graph. It corresponds to an operator that minimizes the Hamming distance between the worlds wrt the predicate *on*. The operator $\diamondsuit^{on}$ is the operator associated to the assignment that gives the preorders $\preceq^{on}_{\mathcal{G}}$ such that

$$\mathcal{G}_1 \preceq^{on}_{\mathcal{G}} \mathcal{G}_2 \text{ iff } dist_{on}(\mathcal{G},\mathcal{G}_1) \leq dist_{on}(\mathcal{G},\mathcal{G}_2)$$

where

$$dist_{on}(\mathcal{G},\mathcal{G}') = card(\left\{ x \in \mathcal{A}_{\mathbf{U}} \;\middle|\; \begin{array}{l} \mathcal{G} \models on(x) \text{ and } \mathcal{G}' \models \neg on(x), \\ \text{or } \mathcal{G} \models \neg on(x) \text{ and } \mathcal{G}' \models on(x) \end{array} \right\})$$

Proposition 11. $\diamondsuit^{on}$ *satisfies all the update properties.*

A second idea is to define an operator that minimizes the changes on the attacks.

As in the previous paragraph, we can define the operator $\diamondsuit^{att}$ based on the distance $dist_{att}$ defined by:

$$dist_{att}(\mathcal{G},\mathcal{G}') = card\left\{ (x,y) \in \mathcal{A}_{\mathbf{U}} \times \mathcal{A}_{\mathbf{U}} \;\middle|\; \begin{array}{l} \mathcal{G} \models att(x,y) \text{ and } \mathcal{G}' \models \neg att(x,y), \\ \text{or } \mathcal{G} \models \neg att(x,y) \text{ and } \mathcal{G}' \models att(x,y) \end{array} \right\}$$

with $att(x,y) \equiv singl(x) \wedge singl(y) \wedge (x \rhd y)$ is a shortcut for expressing that the argument x attacks the argument y.

Note that it is a generalized enforcement operator but some characterization properties may not hold for this operator since the assignment of $\preceq^{att}$ is not necessarily preferring elementary changes. This kind of enforcement operator has been used by Baumann [8, 7] with a restricted set of authorized transitions (in Section 7 we show that Baumann's normal enforcement operator is a particular update operator).

Other examples of generalized enforcement operators can be defined on preference relations that take into account preferences on arguments to remove/add or relations that prefer addition wrt to removal; moreover instead of counting the number of attacks or arguments that differ it is possible to define an operator based on set-inclusion. Another operator $\diamondsuit^{dist}$ could be defined on the basis of distances between graphs taking into account the different status that each argument has in each graph. In that case, $\preceq_{\mathcal{G}}^{dist}$ would be based on $dist(\mathcal{G}, \mathcal{G}_1)$ which measures the number of arguments that do not have the same status in $\mathcal{G}$ and $\mathcal{G}_1$ for a given semantics. This operator could represent the reluctance of an agent to change her mind about arguments statuses. So the lawyer could consider that it is less problematic to use many new arguments/attacks to ensure that the argument "Mr. X is not guilty" is accepted, provided that the change on argument acceptance is minimal.

In the next section, we give a practical example in which some generalized enforcement operators are exemplified.

6 Trial example

In this section, we give an example of how YALLA can be used. More precisely, we consider the context of the previous examples, that is to say during a trial; Figure 4 recalls the universe of arguments used by the participants. We are considering the reasoning done by Mr. X while attending the trial concerning his own sad case.

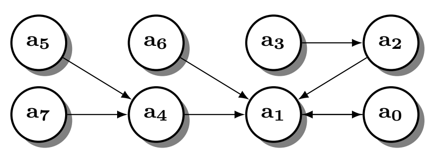


Figure 4: *Universe* $(\mathcal{A}_{\mathbf{U}}, \mathcal{R}_{\mathbf{U}})$.

6.1 Prosecutor turn

First, we are interested in a turn of the prosecutor. The state of the debate is described by the argumentation graph given by Figure 5.

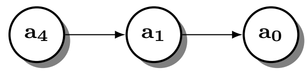


Figure 5: *System representing the debate $\mathcal{G}_D$.*

Formula representing the debate Thanks to YALLA$_{\mathbf{U}}$ and following Definition 14, we are able to represent the debate graph with regard to the universe.

$$\begin{array}{rcrcrcrc} \varphi & = & on(\{a_0, a_1, a_4\}) & \wedge & \neg(on(\{a_2\}) & \wedge & \neg(on(\{a_3\}) & \wedge \\ & & \neg(on(\{a_5\}) & \wedge & \neg(on(\{a_6\}) & \wedge & \neg(on(\{a_7\}) & \\ & & (\{a_4\} \rhd \{a_1\}) & \wedge & (\{a_1\} \rhd \{a_0\}) & \wedge & & \\ & & \neg(\{a_0\} \rhd \{a_1\}) & \wedge & \neg(\{a_3\} \rhd \{a_2\}) & \wedge & \neg(\{a_2\} \rhd \{a_1\}) & \wedge \\ & & \neg(\{a_5\} \rhd \{a_4\}) & \wedge & \neg(\{a_6\} \rhd \{a_1\}) & \wedge & \neg(\{a_7\} \rhd \{a_4\}) & \end{array}$$

Note that φ is exactly $\Phi_{\mathbf{U}}(\mathcal{G}_D)$.

Authorized transitions for the prosecutor As any agent, the prosecutor is not allowed to object against arguments that have not been uttered and she is not able to add arguments that she does not know. Moreover, the judge has allowed very short interventions for each participant. Hence, the prosecutor can only use operations that correspond to the set of transitions denoted $\mathcal{T}_e$ in Definition 19.

Assignment of Mr. X about the prosecutor Mr. X thinks that the addition of an argument is as plausible as its removal. So he uses an assignment respecting $\mathcal{T}_e$ but with no other refinement.

Mr. X's knowledge about the prosecutor's argumentation graph is given by Figure 6.

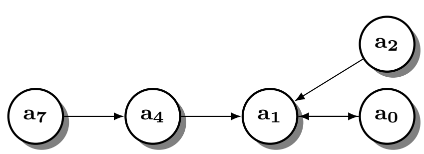


Figure 6: *System of the prosecutor $\mathcal{G}_{Pro}$.*

Thanks to Mr. X's assignment, we are able to represent the more plausible graphs that can be reached after the prosecutor intervention. Table 1 shows these graphs ranked by plausibility.

$(\ominus, a_0, \varnothing)(\mathcal{G}_D)$	$(\ominus, a_1, \varnothing)(\mathcal{G}_D)$
$(\ominus, a_4, \varnothing)(\mathcal{G}_D)$	$(\oplus, a_2, \{(a_2, a_1)\})(\mathcal{G}_D)$
$(\oplus, a_7, \{(a_7, a_4)\})(\mathcal{G}_D)$	
$(\ominus, a_2, \varnothing)(\mathcal{G}_D)$	$(\ominus, a_7, \varnothing)(\mathcal{G}_D)$
$(\oplus, a_4, \{(a_4, a_1)\})(\mathcal{G}_D)$	$(\oplus, a_5, \{(a_5, a_4)\})(\mathcal{G}_D)$
$(\oplus, \{a_2, a_7\}, \{(a_2, a_1), (a_7, a_4)\})(\mathcal{G}_D)$	$(\oplus, \{a_5, a_7\}, \{(a_5, a_4), (a_7, a_4)\})(\mathcal{G}_D)$
$\ldots$	

Table 1: *The most plausible reachable graphs after the prosecutor intervention are in the first layer, the least plausible ones (that are also the graphs to which the transitions are not authorized) are in the second layer.*

Goal of the prosecutor and update The prosecutor wants to make accepted the argument representing the guilt of the defendant; her goal is $\exists p, G(p) \wedge (\{a_1\} \subseteq p)$. Hence, Mr. X may want to know what would be the state of the debate if the prosecutor utters something, and Mr. X perfectly knows that whatever the prosecutor may say, this would be said in a way to achieve her goal. It means that Mr. X wants to compute the set of graphs that satisfy the prosecutor's goal and that are accessible from $\mathcal{G}_D$ by a most plausible elementary change:

$$[\Phi_{\mathbf{U}}(\mathcal{G}_D) \diamondsuit_{\mathcal{T}_e} (\exists p, G(p) \wedge (\{a_1\} \subseteq p))]$$

If we compute the extensions of the most preferred graphs of Table 1 we can find two transitions that achieve the goal: $(\ominus, a_4, \varnothing)$ and $(\oplus, a_7, \{(a_7, a_4)\})$.

In order to avoid the computation of the extensions of each graph, Mr. X may just want to check if the prosecutor would have had some interest to add a particular argument, for instance a_7. Hence, Mr. X wants to compute the set of graphs containing a_7 and the attack (a_7, a_4) that are accessible from $\mathcal{G}_D$ by a most plausible change:

$$[\Phi_{\mathbf{U}}(\mathcal{G}_D) \diamondsuit_{\mathcal{T}_e} (on(\{a_7\}) \wedge (\{a_7\} \rhd \{a_4\}))]$$

Using update property $\mathbf{A}_{C2}$ with $z = \{a_7\}$, $\mathcal{R}_z = \{(a_7, a_4)\}$, $x = \{a_1\}$ and $p = \{a_0, a_4\}$, and knowing that the following properties hold:

- $\Phi_{\mathbf{U}}(\mathcal{G}_D) \models G(p) \wedge singl(\{a_1\}) \wedge \neg(\{a_1\} \subseteq p)$,
- $(\mathcal{G}_D, \langle \oplus, a_7, \{(a_7, a_4)\} \rangle(\mathcal{G}_D)) \in \mathcal{T}_e$ and

- $\Phi_{\mathbf{U}}(\langle \oplus, a_7, \{(a_7, a_4)\}\rangle(\mathcal{G}_D)) \models singl(\{a_7\}) \wedge \neg(\exists t(on(t) \wedge (t \rhd \{a_7\}))) \wedge (\{a_7\} \rhd\!\!\rightsquigarrow\!\!\rhd \{a_1\}))$,

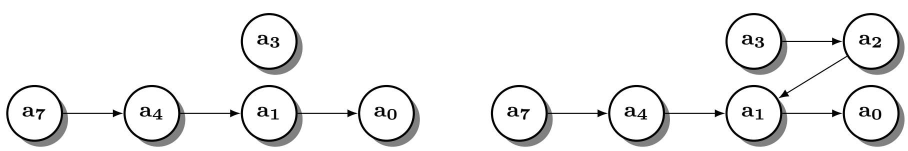
we obtain:

$$\exists p', \Phi_{\mathbf{U}}(\mathcal{G}_D) \diamondsuit_{\mathcal{T}_e} (on(\{a_7\}) \wedge (\{a_7\} \rhd \{a_4\})) \models G(p') \wedge (\{a_1\} \subseteq p')$$

Then, it is not necessary to compute the grounded extension of the graph with the addition of a_7 since, thanks to Property $\mathbf{A}_{C2}$, we are sure that a_1 will be in the extension after the change.

6.2 Lawyer turn

After quite a heated duel between the two opponents that gave rise to several new arguments, it is again the turn of the lawyer. Mr. X is sure that Arguments a_3 and a_7 have been enunciated, and consequently he added them to his representation of the current state of the debate, but he is not so sure that Argument a_2 has really been expressed clearly. As a result, he hesitates between the two argumentation graphs presented in Figure 7 for representing the current state of the debate.



(a) *Argumentation graph $\mathcal{G}_{D1}$.* **(b)** *Argumentation graph $\mathcal{G}_{D2}$.*

Figure 7: *Two possible cases for the debate according to Mr. X.*

Formula representing the debate The doubt of Mr. X about the presence of a_2 can be represented in YALLA$_{\mathbf{U}}$ by:

$$\begin{array}{rcl}
\varphi & = & on(\{a_0, a_1, a_3, a_4, a_7\}) \wedge \\
 & & \neg(on(\{a_5\})) \wedge \neg(on(\{a_6\})) \wedge \\
 & & (\{a_1\} \rhd \{a_0\}) \wedge (\{a_4\} \rhd \{a_1\}) \wedge (\{a_7\} \rhd \{a_4\}) \wedge \\
 & & \neg(\{a_0\} \rhd \{a_1\}) \wedge \neg(\{a_5\} \rhd \{a_4\}) \wedge \neg(\{a_6\} \rhd \{a_1\}) \wedge \\
 & & \left(\begin{array}{c} (\neg(on(\{a_2\})) \wedge \neg(\{a_2\} \rhd \{a_1\}) \wedge \neg(\{a_3\} \rhd \{a_2\})) \\ \vee \\ (on(\{a_2\}) \wedge (\{a_2\} \rhd \{a_1\}) \wedge (\{a_3\} \rhd \{a_2\})) \end{array} \right)
\end{array}$$

Note that $\varphi \equiv (\Phi_{\mathbf{U}}(\mathcal{G}_{D1}) \vee \Phi_{\mathbf{U}}(\mathcal{G}_{D2}))$.

Assignment of Mr. X about the lawyer Mr. X thinks that the lawyer prefers addition over removal and elementary changes over non elementary changes; hence, for the lawyer he uses the following assignment:

$\forall \mathcal{G} \in \Gamma_{\mathbf{U}}, \forall \mathcal{G}_i \in \Gamma_{\mathbf{U}}, \forall \mathcal{G}_k \in \Gamma_{\mathbf{U}} \setminus \mathcal{G}$,

- if $(\mathcal{G}, \mathcal{G}_i) \in \mathcal{T}_+$ and $(\mathcal{G}, \mathcal{G}_k) \in \mathcal{T}_-$, then $\mathcal{G}_i \prec_{\mathcal{G}}^{Law} \mathcal{G}_k$
- else, if $(\mathcal{G}, \mathcal{G}_i) \in \mathcal{T}_e$ and $(\mathcal{G}, \mathcal{G}_k) \notin \mathcal{T}_e$, then $\mathcal{G}_i \prec_{\mathcal{G}}^{Law} \mathcal{G}_k$.

Moreover Mr. X thinks that the system presented in Figure 8 is the lawyer's system.[30]

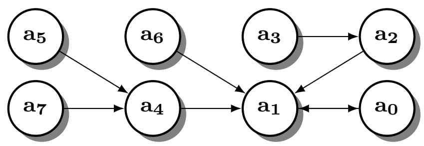


Figure 8: *System of the lawyer $\mathcal{G}_{Law}$.*

According to Mr. X's assignment for the lawyer, the most plausible graphs reachable after the lawyer intervention can be represented in Table 2.

$(\oplus, a_2, \{(a_2, a_1)\})(\mathcal{G}_{D1})$	$(\oplus, a_2, \{(a_2, a_1), (a_3, a_2)\})(\mathcal{G}_{D1})$
$(\oplus, a_5, \{(a_5, a_4)\})(\mathcal{G})$	$(\oplus, a_6, \{(a_6, a_1)\})(\mathcal{G})$
$(\ominus, a_0, \varnothing)(\mathcal{G})$	$(\ominus, a_1, \varnothing)(\mathcal{G})$
$(\ominus, a_3, \varnothing)(\mathcal{G})$	$(\ominus, a_4, \varnothing)(\mathcal{G})$
$(\ominus, a_7, \varnothing)(\mathcal{G})$	
$(\oplus, a_2, \{(a_2, a_1)\})(\mathcal{G}_{D2})$	$(\oplus, a_2, \{(a_2, a_1), (a_3, a_2)\})(\mathcal{G}_{D2})$
$(\ominus, a_2, \varnothing)(\mathcal{G})$	$(\ominus, a_5, \varnothing)(\mathcal{G})$
$(\oplus, a_4, \{(a_4, a_1)\})(\mathcal{G})$	$(\oplus, \{a_2, a_6\}, \{(a_2, a_1), (a_6, a_1)\})(\mathcal{G})$
$(\ominus, \{a_3, a_4\}, \varnothing)(\mathcal{G})$	$\cdots$

Table 2: *Most plausible graphs reachable after the lawyer intervention, in descending order (the first layer gathers the most preferred graphs, the second layer concerns less preferred graphs and the third one concerns graphs that are not accessible by an authorized transition). Note that $\mathcal{G} \in \{\mathcal{G}_{D1}, \mathcal{G}_{D2}\}$.*

[30]Here, at this moment, the lawyer graph is the universe. This means that the lawyer knows all the arguments of this case.

Goal of the lawyer and update Mr. X knows that the lawyer wants to make accepted the argument representing his innocence; her goal is $\exists p, G(p) \wedge (\{a_0\} \subseteq p)$ and Mr. X would like to compute what would happen after his lawyer's intervention (without knowing exactly what she is going to say but knowing perfectly that his lawyer is brillant and would do anything to prove his innocence):

$$[\varphi \diamondsuit_{\mathcal{T}_e} (\exists p, G(p) \wedge (\{a_0\} \subseteq p))]$$

Note that since φ represents two argumentation graphs, we can use Postulate **E8**; we obtain:

$$[\Phi_{\mathbf{U}}(\mathcal{G}_{D1}) \diamondsuit_{\mathcal{T}_e} (\exists p, G(p) \wedge (\{a_0\} \subseteq p))] = \left\{ \begin{array}{l} (\oplus, a_2, \{(a_2, a_1)\})(\mathcal{G}_{D1}), \\ (\oplus, a_6, \{(a_6, a_1)\})(\mathcal{G}_{D1}) \end{array} \right\}$$

$$[\Phi_{\mathbf{U}}(\mathcal{G}_{D2}) \diamondsuit_{\mathcal{T}_e} (\exists p, G(p) \wedge (\{a_0\} \subseteq p))] = \left\{ (\oplus, a_6, \{(a_6, a_1)\})(\mathcal{G}_{D2}) \right\}$$

Hence,

$$[\varphi \diamondsuit_{\mathcal{T}_e} (\exists p, G(p) \wedge (\{a_0\} \subseteq p))] = \left\{ \begin{array}{l} (\oplus, a_2, \{(a_2, a_1)\})(\mathcal{G}_{D1}), \\ (\oplus, a_6, \{(a_6, a_1)\})(\mathcal{G}_{D1}), \\ (\oplus, a_6, \{(a_6, a_1)\})(\mathcal{G}_{D2}) \end{array} \right\}$$

Figure 9 shows the result of the update of φ by $(\exists p, G(p) \wedge (\{a_0\} \subseteq p))$ in terms of graphs.

Moreover, as done during the turn of the prosecutor, Mr. X may again avoid the computation of the extensions and can just check if the addition of a_6 is relevant for his lawyer's goal.

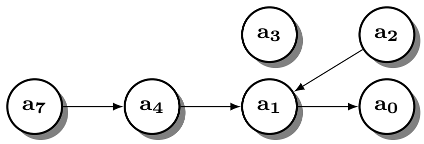
Using Property $\mathbf{A}_{C2}$ with $z = \{a_6\}$, $\mathcal{R}_z = \{(a_6, a_1)\}$, $x = \{a_0\}$ and $p = \{a_1, a_3, a_7\}$, we obtain:

$$\exists p', \varphi \diamondsuit_{\mathcal{T}_e} (on(a_6) \wedge (a_6 \rhd a_1)) \models G(p') \wedge (a_0 \subseteq p')$$

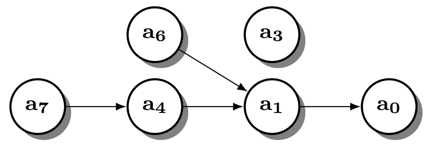
Here again, it is not necessary to compute the extension of the graph with the addition of a_6 since we know that a_0 will be in the extension after the change, thanks to $\mathbf{A}_{C2}$.

7 Related work

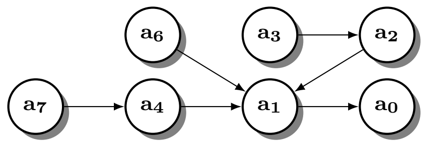
In this section, we review related works that either address the enforcement issue, or address change in argumentation within a logical setting.



(a) *Argumentation graph* $(\oplus, a_2, \{(a_2, a_1)\})(\mathcal{G}_{D1})$.



(b) *Argumentation graph* $(\oplus, a_6, \{(a_6, a_1)\})(\mathcal{G}_{D1})$.



(c) *Argumentation graph* $(\oplus, a_6, \{(a_6, a_1)\})(\mathcal{G}_{D2})$.

Figure 9: *The three possible graphs obtained after the update.*

Enforcement/update The main references about the particular goal of *enforcement* are [8] and [7], where change operations are restricted to the addition of new arguments and new attacks. More precisely, [7] introduces three types of changes called "expansions":

- the *normal expansion* adds new arguments and new attacks concerning at least one of the new arguments,
- the *weak expansion* refines the normal expansion by the addition of new arguments not attacking any old argument and
- the *strong expansion* refines the normal expansion by the addition of new arguments not being attacked by any old argument.

The authors showed that it is not the case in general that any desired set of arguments is enforceable using a particular expansion and that in some cases, several enforcements are possible, some of them requiring more effort than others. Baumann [7] has specified necessary and sufficient conditions, called "characteristics" under which enforcements are possible.

More precisely, a characteristic of a set of arguments is defined wrt a semantics and a set of possible expansions, as the minimal number of modifications (in terms of differences between attacks) that are needed in order to enforce this set of arguments. This number equals 0 when each argument of the desired set is already accepted. It equals infinity if no enforcement is possible. [7] provides means to compute a characteristic w.r.t. a given type of expansion and a given semantics. The characteristic of a set of arguments can be viewed as an evaluation of the change required to accept this set of arguments. Thanks to Theorem 3, we can translate the fact that the characteristic of a set of arguments E w.r.t. an argumentation graph $\mathcal{G}$ under the grounded semantics (for instance) is c by the following expression:

$$\mathcal{G}' \in [\Phi_{\mathbf{U}}(\mathcal{G}) \quad \diamondsuit^{att} \quad (E \subseteq p) \wedge G(p)] \text{ implies that } dist_{att}(\mathcal{G}, \mathcal{G}') = c$$

where $\diamondsuit^{att}$ is an update operator based on a preference ordering that minimizes $dist_{att}$ with $dist_{att}$ being the number of attacks that are modified (see Section 5.4), $\Phi_{\mathbf{U}}(\mathcal{G})$ is a formula describing the graph $\mathcal{G}$ and $G(p)$ means that the set p is the grounded extension (see Section 4). The previous expression means that in order to enforce E to be in the grounded extension, an update operation will lead to an argumentation graph that has c attacks that are changed wrt the initial argumentation graph. These characteristics are related to our characterizations since the latter are necessary or sufficient conditions for obtaining a given change property. However our characterizations are expressed in a more general setting in which the authorized transitions are not limited to Baumann's expansions and the goals are not restricted to enforcement.

Moreover, among the different kinds of changes proposed by Baumann, the normal expansion, *i.e.*, adding an argument with the attacks that concern it, can be easily encoded by a particular update operator ${\diamondsuit_{\mathcal{T}_B}}^{att}$: $\mathscr{L} \times \mathscr{L} \to \mathscr{L}$ with $\mathcal{T}_B = \{((\mathcal{A}_1, \mathcal{R}_1), (\mathcal{A}_2, \mathcal{R}_2)) \mid \mathcal{A}_1 \subset \mathcal{A}_2, \mathcal{R}_1 \subseteq \mathcal{R}_2, \mathcal{R}_2 \setminus \mathcal{R}_1 \cap (\mathcal{A}_1 \times \mathcal{A}_1) = \varnothing\}$

Let us notice that in Baumann's framework, the initial argumentation graph should be completely known, *i.e.*, the formula that describes it should correspond to only one graph. And the formula concerning the goal of enforcement should describe a set of arguments that should be accepted (under a given semantics) after the change, such as $G(p)$ for instance under the grounded semantics.[31]

The idea of enforcement can be found in the approach of Moguillansky et al. [44] through the concepts of *activation* and *de-activation* of arguments. The idea is to make arguments rejected in order to *activate* (make accepted) a given argument. Here the authors consider that they are facing a revision problem but neither representation theorems nor characterization postulates are given; the process of change is considered from the argumentation

[31]Note that since Baumann's enforcement is defined on one graph and not on a set of graphs, it is also a kind of belief revision since revision and update collapse when the initial world is completely known (this kind of belief revision won't be a pure AGM revision but rather a revision under transition constraints).

standpoint while keeping usual principles of belief revision, namely minimal change and success. The change machinery first involves the determination of *attacking lines* (sequences of arguments, where each argument attacks its predecessor) that prevent a given argument from being accepted, then the addition of new arguments so that the attacking lines are no more effective. The choice of the argument to attack in a given attacking line is based on the minimal change principle: the set of arguments of the line whose status is modified should be as small as possible.

Booth et al. [21] propose to represent the belief state of an agent by an argumentation graph and a constraint that encodes the outcome the agent believes the graph should have. This belief state can be changed in two ways: either by "strengthening" the constraint (more precisely, it amounts to add a restriction to the logical formula that describes the constraint), or by adding new arguments and/or attacks. However, it may happen that a change makes the new constraint and the outcome inconsistent. In that case, one solution proposed by [21] is to come up with an expansion of the argumentation graph that restores coherence. This issue of restoring coherence is related to the enforcement issue. Coste-Marquis et al. [26] define new enforcement operators for which enforcement can be achieved by adding arguments and attacks and also by questioning some attacks. They propose a boolean encoding for the new enforcement problems, which enables to formalize them as optimization problems.

Other works address dynamics in argumentation graphs using logical languages.

Logical languages [21] uses a logical labeling language for representing the constraints and the outcome of a graph. Formulae in this language are statements about the acceptance status of arguments. These formulae are interpreted on labelings, functions that assign to each argument of a graph a label in, out or undecided. In contrast, formulae in YALLA are interpreted on sets of argumentation graphs and may express the criteria underlying the traditional extension-based semantics as well as a wide range of properties that hold for some argumentation graphs. For instance, YALLA enables us to express in a same formula that a given set of arguments should be accepted under the complete semantics but not accepted under the preferred semantics. YALLA is a very expressive language capable of describing abstract argumentation, structural semantics (*i.e.* semantics based on the attack relation) and also change in argumentation systems.

Doutre et al. [29] encode argumentation frameworks and their dynamics in DL-PA, the Dynamic Logic of Propositional Assignments. More precisely, to every input formula representing a goal to be enforced is associated a DL-PA program implementing the corresponding update. Enforcement operators are defined, which map an argumentation graph and a goal to a set of argumentation graphs. Three postulates are given, namely a success postulate, a minimal change postulate and a postulate of syntax independence. Note that, as in Baumann's approach [7], an enforcement operator applies to only one argumentation graph, which corresponds in our work to a formula with only one model.

Moreover, our formalism handles a richer set of change operations which are captured through the notion of update with transition constraints. Another difference lies in the additional properties that we have established on the basis of the characterizations proposed by Bisquert et al. in [14] that are specific for update of argumentation graphs.

Coste et al. [24] consider revision of an argumentation graph as minimal change of the argument status. More precisely, given an argumentation graph and a given semantics, a revision formula expresses how the status of some arguments has to be changed. Then the revision process derives argumentation graphs that satisfy the revision formula, while having extensions as close as possible to the extensions of the input graph. Revision formulae are expressed in a logical propositional language, where the propositional variables are the arguments. A typical example of goal that can be encoded is "Argument a must be accepted and Arguments b and c must be both accepted or both refused".
Revision operators are defined in a two-step process. The first step selects from sets of arguments that satisfy the revision formula those as close as possible to the extensions of the input graph, called candidates. The second step generates the argumentation graphs whose extensions coincide with the candidates, this is done by building argumentation graphs with the same arguments as the initial graph and with attacks that are chosen in order to coincide with the candidates. Note that the addition of arguments is not allowed in the revision process. Moreover, the logical language cannot encode complex goals, nor the argumentation semantics.

Coste et al. [25] propose a translation-based approach for revising argumentation systems which is closer to our approach. They propose to encode an argumentation framework into two kinds of logical constraints: those concerning the structure of the argumentation framework and those concerning the acceptance of arguments wrt a given semantics. This logical encoding allows the use of classical revision operators to perform a rational minimal change. Let us first note that the choice of revising instead of updating is not justified by these authors. Another difference with our approach concerns the logical encoding. Coste et al. [25] use a propositional language allowing the representation of attacks between arguments (the attack between arguments x and y is represented by the variable $att(x, y)$) and of skeptical acceptance (the variable $acc(x)$ represents the skeptical acceptance of Argument x). Each semantics is encoded by a logical formula which expresses for each argument the condition under which it is skeptically accepted under this semantics. In contrast, YALLA is a first-order language with more expressivity. YALLA enables to encode sets of arguments, set-theoretic properties and attacks between sets of arguments. YALLA enables to reason about extensions, and not only about argument acceptance: for each semantics, there is a formula in YALLA expressing that a subset of arguments is an extension under this semantics.

Diller et al. [28] study revision operators that produce a single argumentation framework as output, still using a minimal change principle on the extensions. This differs from [25] in which a set of argumentation frameworks was obtained. More precisely, [28] proposes two revision operators according to the representation of the new information which can be either 1) a propositional formula expressing the desired change in the extensions of the original system, or 2) a new argumentation system. Thanks to our first-order language YALLA the two kinds of revision that are proposed can be captured in our formalism. Moreover since there is no uncertainty about the initial system, revision and update collapse hence our framework is a generalization of [28] in which it is imposed that the resulting formula should only have one model.

Some other works provide a logical analysis of argumentation.

Villata et al. [54] propose a logical formalism for representing (and reasoning about) the extensions of traditional semantics. This work follows work of Besnard and Doutre [10]. The arguments of an argumentation graph are denoted by symbols of the language enabling the user to write formulae whose models are sets of arguments. However, the purpose of this work is to characterize the extensions, in particular, it is not possible to write formulae that relate the structure of an argumentation graph and its extensions.

A language with a similar expressivity has been proposed by Coste-Marquis et al. [27], with another purpose. The idea was to generalize Dung's formal framework [31] by taking into account additional constraints (expressed in a logical form) about the admissible sets of arguments.

A logical language was also proposed by Wooldridge et al. [58] in which it is possible to express acceptability, conflicts and defense between sets of arguments. However, this formalism is devoted to specific kinds of arguments namely logical arguments (*i.e.*, pairs (support, conclusion) where the support is a set of formulae that infers logically the formula constituting the conclusion).

Those works are related to our YALLA proposal since these languages enable also to describe and reason about argumentation graphs; however none of them enables the user to express structural properties of an abstract argumentation graph together with its semantic properties (which is the main purpose of YALLA).

8 Conclusion

This article proposes a complete framework for handling the dynamics of an abstract argumentation graph. Our first aim was to define a frame in which it is possible to represent several knowledge bases under the form of several argumentation graphs. More precisely, in our formalism, it is possible to express (and study) how an agent who has her own argumentation graph can interact on a target argumentation graph (that may represent a state of knowledge at a given stage of a debate). The two argumentation graphs are defined inside a referential argumentation graph called the universe (which constitutes a kind of "common language").

An important issue about knowledge dynamics is the establishment of a set of axioms that characterize rational changes. Hence it was important to situate our framework in the field of belief change theory. We first generalized classical update postulates in order to take into account a set of authorized transitions. Since the update approach is based on classical propositional logic, we have defined a logical language (called YALLA) for representing argumentation graphs. Then we have shown that the change operations on argumentation graphs are update operations. This discovery is very important since many works about enforcement (*e.g.* [44, 25, 28]) are only considering revision which we think inappropriate for reasoning in a public debate context. Since we are facing an update process we have been able to show that it guarantees the existence of a preference relation on transitions between argumentation systems. Hence it justifies the idea that it is rational to compute minimal change operations, or optimal plan. Moreover due to previous works about dynamics in argumentation we have been in position to provide a set of new properties that are specific of argumentation update. We have illustrated our framework on an example that takes place in a judicial context with a prosecutor and a lawyer.

Another interest of our formal study is the definition of the language YALLA. YALLA is a first order logical language able to express complex information about an argumentation system, not only its structure but also the relations between sets of arguments: *e.g.* attack/defense between sets of arguments, inclusion, union of sets. This permits to express the principles underlying the usual semantics (*e.g.* conflict-freeness, admissibility, complement-attack, maximality), and so the extensions can be derived from logical formulae. In the future YALLA will allow us to capture any definition of an acceptable set of arguments (*e.g.* semi-stable semantics ...). Several similar logical languages that express the relation between the attack relation and the extensions have been proposed in the recent years (see Section 7), but as far as we know, none of them has the same expressivity than YALLA. Moreover providing a logic-based tool that characterizes properties of evolving argumentation systems may enable us to use standard logic-based solvers (for instance, automatic theorem provers or SMT-solvers) in order to find new argumentation-based update properties.

In conclusion, the main result of this paper is a unified view of dynamics in argumentation with the use of a new logical language and the discovery of a set of properties specific for argumentation dynamics.

Many directions are opened and would be interesting to follow, we list four possible trails below.

- In our definition of the operations that are executable on an argumentation graph, we restrict to operations that really achieve a change, *i.e.* it is not possible to add an argument that is already present or to suppress an argument that is not there. This restriction is natural but can lead to ignore some possible changes when the initial graph is not completely known, since an operation could be excluded if it is not executable in one of the possible initial situations. An idea would be to relax the executability constraint in order to allow for adding arguments already present or removing non existing arguments (in that case the operation will have no effect).

- Concerning non elementary operations, it would be interesting to realize a comparative analysis of the effect of performing a set of elementary operations in parallel (or simultaneously) wrt performing them in sequence.

- The planning aspect of change in argumentation system is a growing area in the literature. From a strategical point of view, it would be interesting to study if there is a way for one of the agents to reach her goal on the target system whatever the future actions of the other agents are. This kind of “long term” objective is important in many debates where the agent does not necessarily want to “win” at the next step but she wants to win at the end of the debate.

- A very appealing perspective is to study how a private argumentation graph evolves in presence of a public argumentation graph (in which the operations can be viewed as public announcements [53]). This is clearly related to belief revision and could also be situated wrt approaches about fusion of argumentation graphs.

A Proofs

Proposition 1. ***U1*** *is implied by* ***E3****,* ***E5*** *and* ***E8****.*

Proof of Proposition 1. $\forall\varphi,\alpha \in \mathscr{L}$, if $\exists\omega' \in [\varphi\diamondsuit_{\mathcal{T}}\alpha]$ then, due to **E3**, $(\varphi,\alpha) \models \mathcal{T}$, *i.e.*, $[\varphi] \neq \varnothing$ and $\forall\omega \in [\varphi], \exists\omega_1 \in [\alpha]$ such that $(\omega,\omega') \in \mathcal{T}$. Hence using **E8**, $[\varphi\diamondsuit_{\mathcal{T}}\alpha] = \cup_{\omega\in[\varphi]}[\Phi(\omega)\diamondsuit_{\mathcal{T}}\alpha]$. Hence there is a $\omega \in [\varphi]$ such that $\omega' \in [\Phi(\omega)\diamondsuit_{\mathcal{T}}\alpha]$. Due to **E5**, $(\Phi(\omega)\diamondsuit_{\mathcal{T}}\alpha)\wedge\Phi(\omega') \models \Phi(\omega)\diamondsuit_{\mathcal{T}}(\alpha\wedge\Phi(\omega'))$. Hence $\omega' \in [\Phi(\omega)\diamondsuit_{\mathcal{T}}(\alpha \wedge \Phi(\omega'))]$. Due to **E3**, $(\Phi(\omega),(\alpha \wedge \Phi(\omega'))) \models \mathcal{T}$ so $[\alpha \wedge \Phi(\omega')] \neq \varnothing$. Hence $\omega' \in [\alpha]$. Hence, **U1** holds. □

Proposition 2. ***E3****,* ***U4****,* ***E5****,* ***E8****,* ***U9*** *constitute a minimal set: no postulate can be derived from the others.*

Proof of Proposition 2. Let us recall the set of postulates: $\forall\varphi,\psi,\alpha,\beta \in \mathscr{L}$,

E3: $[\varphi\diamondsuit_{\mathcal{T}}\alpha] \neq \varnothing$ if and only if $(\varphi,\alpha) \models \mathcal{T}$

U4: $[\varphi] = [\psi]$ and $[\alpha] = [\beta] \implies [\varphi \diamond \alpha] = [\psi \diamond \beta]$

E5: If $card([\varphi]) = 1$ then $(\varphi\diamondsuit_{\mathcal{T}}\alpha) \wedge \beta \models \varphi\diamondsuit_{\mathcal{T}}(\alpha \wedge \beta)$

E8: if $([\varphi] \neq \varnothing$ and $[\varphi\diamondsuit_{\mathcal{T}}\alpha] = \varnothing)$ or $([\psi] \neq \varnothing$ and $[\psi\diamondsuit_{\mathcal{T}}\alpha] = \varnothing)$ then $[(\varphi \vee \psi)\diamondsuit_{\mathcal{T}}\alpha] = \varnothing$ else $[(\varphi \vee \psi)\diamondsuit_{\mathcal{T}}\alpha] = [(\varphi\diamondsuit_{\mathcal{T}}\alpha) \vee (\psi\diamondsuit_{\mathcal{T}}\alpha)]$

U9: If $card([\varphi]) = 1$ then if $[(\Phi(\omega)\diamondsuit_{\mathcal{T}}\alpha) \wedge \beta] \neq \varnothing$ then $[(\varphi\diamondsuit_{\mathcal{T}}\alpha) \wedge \beta] \neq \varnothing \implies \varphi\diamondsuit_{\mathcal{T}}(\alpha \wedge \beta) \models (\varphi\diamondsuit_{\mathcal{T}}\alpha) \wedge \beta$

In order to prove this result we have to provide for each postulate an operator satisfying the other postulates but not this one.

Independency of E3: Let $\diamondsuit_{\mathcal{T}}$ be such that $\forall \varphi, \alpha \in \mathscr{L}$, if $(\varphi, \alpha) \not\models \mathcal{T}$ then $[\varphi \diamondsuit_{\mathcal{T}} \alpha] = [\alpha]$ else $[\varphi \diamondsuit_{\mathcal{T}} \alpha] = \varnothing$.

- **E3** does not hold.
- **U4** holds: since we define $\diamondsuit_{\mathcal{T}}$ only from the models of φ and α.
- **E5** holds: since if $(\Phi(\omega), \alpha) \models \mathcal{T}$ then $[\Phi(\omega) \diamondsuit_{\mathcal{T}} \alpha] = \varnothing$, hence **U5** holds, else $[\Phi(\omega) \diamondsuit_{\mathcal{T}} \alpha] = [\alpha]$ and $(\Phi(\omega), \alpha) \not\models \mathcal{T}$ which implies $(\Phi(\omega), \alpha \wedge \beta) \not\models \mathcal{T}$, hence $[\Phi(\omega) \diamondsuit_{\mathcal{T}} (\alpha \wedge \beta)] = [\alpha \wedge \beta]$ hence **E5** holds.
- **E8** holds: If $\varphi \neq \varnothing$ and $[\varphi \diamondsuit_{\mathcal{T}} \alpha] = \varnothing$ then $(\varphi, \alpha) \models \mathcal{T}$, which implies $(\varphi \vee \psi, \alpha) \models \mathcal{T}$, hence $[(\varphi \vee \psi) \diamondsuit_{\mathcal{T}} \alpha] = \varnothing$ (**E8** holds). Else $(\varphi, \alpha) \not\models \mathcal{T}$ and $(\psi, \alpha) \not\models \mathcal{T}$, thus $(\varphi \vee \psi, \alpha) \not\models \mathcal{T}$ which means $[\varphi \vee \psi \diamondsuit_{\mathcal{T}} \alpha] = [\alpha] = [\varphi \diamondsuit_{\mathcal{T}} \alpha] = [\psi \diamondsuit_{\mathcal{T}} \alpha]$.
- **U9** holds: If $[(\varphi \diamondsuit_{\mathcal{T}} \alpha) \wedge \beta] \neq \varnothing$ then it means that $[(\varphi \diamondsuit_{\mathcal{T}} \alpha) \neq \varnothing$ hence $(\varphi, \alpha) \not\models \mathcal{T}$, *i.e.*, $[(\varphi \diamondsuit_{\mathcal{T}} \alpha) = [\alpha]$, so $[(\varphi \diamondsuit_{\mathcal{T}} \alpha) \wedge \beta] = [\alpha \wedge \beta]$. Moreover, $(\varphi, \alpha) \not\models \mathcal{T}$ implies $(\varphi, \alpha \wedge \beta) \not\models \mathcal{T}$. This means that $[\varphi \diamondsuit_{\mathcal{T}} (\alpha \wedge \beta)] = [\alpha \wedge \beta]$.

Independency of U4: Given a formula $\alpha \in \mathscr{L}$ and a world $\omega \in \Omega$, let us denote by $r_\omega(\alpha)$ the number of the first literal in α that holds in ω. Let $\diamondsuit_{\mathcal{T}}$ be such that $\forall \omega \in \Omega$, $[\Phi(\omega) \diamondsuit_{\mathcal{T}} \alpha] = \{\omega_1 \in [\alpha]$ such that $(\omega, \omega_1) \in \mathcal{T}$ and for any other world $\omega_2 \in [\alpha]$ such that $(\omega, \omega_2) \in \mathcal{T}$, $r_{\omega_2}(\alpha) \geq r_{\omega_1}(\alpha)\}$. And $[\varphi \diamondsuit_{\mathcal{T}} \alpha] = \varnothing$ if $\exists \omega \in [\varphi]$ such that $[\Phi(\omega) \diamondsuit_{\mathcal{T}} \alpha] = \varnothing$ otherwise$[\varphi \diamondsuit_{\mathcal{T}} \alpha] = \cup_{\omega \in [\varphi]} [\Phi(\omega) \diamondsuit_{\mathcal{T}} \alpha]$.

- **E3** holds.
- **U4** does not hold: let us consider $\mathscr{L}$ based on $\mathcal{A} = \{a, b\}$ and three worlds $\omega, \omega_1, \omega_2$ such that $\Phi(\omega_1) = on(a) \wedge \neg on(b)$ and $\Phi(\omega_2) = \neg on(a) \wedge on(b)$, let us suppose that $\mathcal{T}$ contains at least the transitions (ω, ω_1) and (ω, ω_2). If we set $\alpha = \Phi(\omega_1) \vee \Phi(\omega_2)$ and $\beta = \Phi(\omega_2) \vee \Phi(\omega_1)$, then it leads to $[\Phi(\omega) \diamondsuit_{\mathcal{T}} \alpha] = \{\omega_1\}$ and $[\Phi(\omega) \diamondsuit_{\mathcal{T}} \beta] = \{\omega_2\}$.
- **E5** and **U9** hold since the ordering of the literals is not changed in the conjunction.
- **E8** holds by definition.

Independency of E5: Let $\diamondsuit_{\mathcal{T}}$ be such that $\forall \varphi, \alpha \in \mathscr{L}$, $\exists \omega_0 \in \Omega$ such that if $(\varphi, \alpha) \models \mathcal{T}$ then $[\varphi \diamondsuit_{\mathcal{T}} \alpha] = \{\omega_0\}$ else $[\varphi \diamondsuit_{\mathcal{T}} \alpha] = \varnothing$.

E3 holds.

U4 holds.

E5 does not hold: since $\forall \omega_1 \in \Omega$ such that $\omega_1 \neq \omega_0$ and $\forall \omega \in \Omega$ such that $(\omega, \omega_1) \in \mathcal{T}$, we have $[\Phi(\omega) \diamondsuit_{\mathcal{T}} \Phi(\omega_1)] = \{\omega_0\}$, hence $[(\Phi(\omega) \diamondsuit_{\mathcal{T}} \Phi(\omega_1)) \wedge \Phi(\omega_0)] = \{\omega_0\}$, while $[\Phi(\omega) \diamondsuit_{\mathcal{T}} (\Phi(\omega_1) \wedge \Phi(\omega_0))] = \varnothing$ (since $[\Phi(\omega_1) \wedge \Phi(\omega_0)] = \varnothing$, which implies that $(\Phi(\omega), \Phi(\omega_1) \wedge \Phi(\omega)) \not\models \mathcal{T}$).

E8 holds: If $\varphi \neq \varnothing$ and $[\varphi \diamondsuit_{\mathcal{T}} \alpha] = \varnothing$ then $(\varphi, \alpha) \not\models \mathcal{T}$, which implies $(\varphi \vee \psi, \alpha) \not\models \mathcal{T}$, hence $[(\varphi \vee \psi) \diamondsuit_{\mathcal{T}} \alpha] = \varnothing$ (**E8** holds). Else $(\varphi, \alpha) \models \mathcal{T}$ and $(\psi, \alpha) \models \mathcal{T}$, thus $(\varphi \vee \psi, \alpha) \models \mathcal{T}$ which means $[\varphi \vee \psi \diamondsuit_{\mathcal{T}} \alpha] = \{\omega_0\} = [\varphi \diamondsuit_{\mathcal{T}} \alpha] = [\psi \diamondsuit_{\mathcal{T}} \alpha]$.

U9 holds: If $[(\varphi \diamondsuit_{\mathcal{T}} \alpha) \wedge \beta] \neq \varnothing$ then it means that $[\varphi \diamondsuit_{\mathcal{T}} \alpha] \neq \varnothing$ hence $(\varphi, \alpha) \models \mathcal{T}$, *i.e.*, $[\varphi \diamondsuit_{\mathcal{T}} \alpha] = \{\omega_0\}$. Hence $\omega_0 \in [\beta]$, this means that $(\varphi, \alpha \wedge \beta) \models \mathcal{T}$, hence $[\varphi \diamondsuit_{\mathcal{T}} (\alpha \wedge \beta)] = \{\omega_0\}$.

Independency of E8: Let $\diamondsuit_{\mathcal{T}}$ be such that $\forall \varphi, \alpha \in \mathscr{L}$, $\exists \omega_1, \omega_2 \in \Omega$, such that $\omega_1 \neq \omega_2$ and if $(\varphi, \alpha) \models \mathcal{T}$ and $Card([\varphi]) = 1$ then $[\varphi \diamondsuit_{\mathcal{T}} \alpha] = \{\omega_1\}$, if $(\varphi, \alpha) \models \mathcal{T}$ and $Card([\varphi]) \neq 1$, $[\varphi \diamondsuit_{\mathcal{T}} \alpha] = \{\omega_2\}$, otherwise $[\varphi \diamondsuit_{\mathcal{T}} \alpha] = \varnothing$.

E3 holds.

U4 holds.

E5 holds: if $\omega_1 \in [(\Phi(\omega) \diamondsuit_{\mathcal{T}} \alpha) \wedge \beta]$, then $[\Phi(\omega) \diamondsuit_{\mathcal{T}} \alpha] = \{\omega_1\}$ hence $\omega_1 \in [\beta]$ and $(\Phi(\omega), \alpha) \models \mathcal{T}$. Thus, $(\Phi(\omega), \alpha \wedge \beta) \models \mathcal{T}$ hence $[\Phi(\omega) \diamondsuit_{\mathcal{T}} (\alpha \wedge \beta)] = \{\omega_1\}$. Hence the result.

E8 does not hold: the first part of **E8** may hold but when $Card([\varphi]) = Card([\psi]) = 1$ and $\varphi \diamondsuit_{\mathcal{T}} \alpha \neq \varnothing$ and $\psi \diamondsuit_{\mathcal{T}} \alpha \neq \varnothing$ then we have $[(\varphi \vee \psi) \diamondsuit_{\mathcal{T}} \alpha] = \{\omega_2\}$ while $[\varphi \diamondsuit_{\mathcal{T}} \alpha] = [\psi \diamondsuit_{\mathcal{T}} \alpha] = \{\omega_1\}$. Hence **E8** does not hold.

U9 holds: Same method as the one for **U5**.

Independency of U9: Let $\diamondsuit_{\mathcal{T}}$ be such that $\forall \varphi, \alpha \in \mathscr{L}$, $\exists \omega_0 \in \Omega$, if $(\varphi, \alpha) \models \mathcal{T}$ then (if $[\alpha] \neq \Omega$ then $[\varphi \diamondsuit_{\mathcal{T}} \alpha] = [\alpha]$ else $[\varphi \diamondsuit_{\mathcal{T}} \alpha] = \{\omega_0\}$) otherwise $[\varphi \diamondsuit_{\mathcal{T}} \alpha] = \varnothing$.

E3 holds

U4 holds.

E5 holds: if $\omega_1 \in [(\Phi(\omega)\diamondsuit_{\mathcal{T}}\alpha)\wedge\beta]$ then if $[\alpha] \neq \Omega$ then $\omega_1 \in ([\alpha]\cap[\beta])$. Hence, $[\alpha \wedge \beta] \neq \varnothing$ and $[\alpha \wedge \beta] \neq \Omega$, it means that $[\Phi(\omega)\diamondsuit_{\mathcal{T}}(\alpha \wedge \beta)] = [\alpha\wedge\beta]$, hence **E5** holds. Now, if $[\alpha] = \Omega$ then $[(\Phi(\omega)\diamondsuit_{\mathcal{T}}\alpha)\wedge \beta] = \{\omega_0\} \cap [\beta]$, this means that $\omega_1 = \omega_0$ and $\omega_0 \in [\beta]$, hence $\omega_0 \in [\alpha\wedge\beta]$. Now, either $[\alpha\wedge\beta] = \Omega$ and $[\Phi(\omega)\diamondsuit_{\mathcal{T}}(\alpha\wedge\beta)] = \{\omega_0\}$, or $\alpha \wedge \beta \neq \Omega$ and $[\Phi(\omega)\diamondsuit_{\mathcal{T}}(\alpha \wedge \beta)] = [\alpha \wedge \beta]$. In both cases, **E5** holds.

E8 holds: the definition of $\diamondsuit_{\mathcal{T}}$ does not depend on the first parameter (except for detecting failure) hence **E8** holds (same kind of proof as for **E5** independency).

U9 does not hold: For instance let us consider $\omega_1 \in \Omega$ such that $\omega_1 \neq \omega_0$ and $(\varphi, \Phi(\omega_1)) \models \mathcal{T}$, we have $[(\varphi\diamondsuit_{\mathcal{T}}\top) \wedge (\Phi(\omega_0) \vee \Phi(\omega_1))] \neq \varnothing$ while $[\varphi\diamondsuit_{\mathcal{T}}(\top\wedge(\Phi(\omega_0)\vee\Phi(\omega_1))] = \{\omega_0, \omega_1\} \not\subseteq [(\varphi\diamondsuit_{\mathcal{T}}\top)\wedge(\Phi(\omega_0)\vee \Phi(\omega_1)] = \{\omega_0\}$.

□

Theorem 2. *Given a set $\mathcal{T} \subseteq \Omega \times \Omega$ of authorized transitions, there is an operator $\diamondsuit_{\mathcal{T}} : \mathscr{L} \times \mathscr{L} \to \mathscr{L}$ satisfying **E3**, **U4**, **E5**, **E8**, **U9** if and only if there is an assignment respecting $\mathcal{T}$ such that $\forall\omega \in \Omega$, $\forall\varphi, \alpha \in \mathscr{L}$,*

(1) $[\varphi \diamondsuit_{\mathcal{T}} \alpha] = \varnothing$ *if* $\exists\omega \in [\varphi]$ *such that* $[\Phi(\omega) \diamondsuit_{\mathcal{T}} \alpha] = \varnothing$

(2) $[\varphi \diamondsuit_{\mathcal{T}} \alpha] = \displaystyle\bigcup_{\omega\in[\varphi]} [\Phi(\omega) \diamondsuit_{\mathcal{T}} \alpha]$ *otherwise*

(3) $$[\Phi(\omega)\diamondsuit_{\mathcal{T}}\alpha] = \left\{\omega_1 \in \Omega \;\middle|\; \begin{array}{l} \omega_1 \in [\alpha] \textit{ and} \\ (\omega, \omega_1) \in \mathcal{T} \textit{ and} \\ (\forall\omega_2 \in [\alpha] \textit{ such that } (\omega, \omega_2) \in \mathcal{T}, \omega_1 \preceq_\omega \omega_2) \end{array}\right\}$$

Proof of Theorem 2. The proof is similar to the one done by Katsuno and Mendelzon except that it does not rely on postulate **U2**, and uses the restricted versions **E3**, **E5** and **E8**. Moreover it takes into account the set $\mathcal{T}$. Note that this proof uses Proposition 4 which is defined later (but only relies on **E3**). This proof is carried out in two steps.

Step 1: Let $\diamondsuit_{\mathcal{T}} : \mathscr{L} \times \mathscr{L} \to \mathscr{L}$ be an operator satisfying **E3**, **U4**, **E5**, **E8**, **U9**. For any $\omega \in \Omega$, let us define $\preceq_\omega$ such that $\omega_1 \preceq_\omega \omega_2$ if and only if $\omega_1 \in [\Phi(\omega)\diamondsuit_{\mathcal{T}}(\Phi(\omega_1) \vee \Phi(\omega_2))]$ or $[\Phi(\omega)\diamondsuit_{\mathcal{T}}(\Phi(\omega_1) \vee \Phi(\omega_2))] = \varnothing$.

We can show that $\preceq_\omega$ is a complete preorder. In the following we abbreviate $\Phi(\omega)\diamondsuit_{\mathcal{T}}(\Phi(\omega_x) \vee \Phi(\omega_y)$ by $\diamondsuit(x, y)$ and $\Phi(\omega)\diamondsuit_{\mathcal{T}}(\Phi(\omega_1) \vee \Phi(\omega_2) \vee \Phi(\omega_3))$ by $\diamondsuit(1, 2, 3)$

- Transitivity: Let us consider $\omega, \omega_1, \omega_2, \omega_3 \in \Omega$, such that $\omega_1 \preceq_\omega \omega_2$ and $\omega_2 \preceq_\omega \omega_3$, there are four cases:
  - $\omega_1 \in \diamondsuit(1,2)$ and $\omega_2 \in \diamondsuit(2,3)$: due to Proposition 4, we get $\diamondsuit(1,2,3) \neq \varnothing$. Hence, due to **U1**, $[\diamondsuit(1,2,3)] \subseteq \{\omega_1, \omega_2, \omega_3\}$.
    * if $[\diamondsuit(1,2,3) \wedge (\Phi(\omega_1) \vee \Phi(\omega_2))] = \varnothing$ then $[\diamondsuit(1,2,3)] = \{\omega_3\}$, which means that $[\diamondsuit(1,2,3) \wedge (\Phi(\omega_2) \vee \Phi(\omega_3))] \neq \varnothing$, hence using both **U9** and **E5**, $\diamondsuit(1,2,3) \wedge (\Phi(\omega_2) \vee \Phi(\omega_3)) = \diamondsuit(2,3)$ this is in contradiction with the hypothesis that $\omega_2 \in \diamondsuit(2,3)$.
    * if $[\diamondsuit(1,2,3) \wedge (\Phi(\omega_1) \vee \Phi(\omega_2))] \neq \varnothing$ then using **E5** and **U9**, $\diamondsuit(1,2) = \diamondsuit(1,2,3) \wedge [(\Phi(\omega_1) \vee \Phi(\omega_2))]$. Hence, $\omega_1 \in \diamondsuit(1,2,3)$. Due to **E5** and **U9** again, $\diamondsuit(1,3) = \diamondsuit(1,2,3) \wedge [(\Phi(\omega_1) \vee \Phi(\omega_3))]$, hence $\omega_1 \in \diamondsuit(1,3)$.
  - $\diamondsuit(1,2) = \varnothing$ and $\omega_2 \in \diamondsuit(2,3)$: due to the converse of Proposition 4, we get $[\Phi(\omega)\diamondsuit_{\mathcal{T}}\Phi(\omega_2)] = \varnothing$. Now, due to **E5**, $\diamondsuit(2,3) \wedge \Phi(\omega_2) \models \Phi(\omega)\diamondsuit_{\mathcal{T}}\Phi(\omega_2)$ hence $\omega_2 \notin \diamondsuit(2,3)$, contradiction: so this case never occurs.
  - $\omega_1 \in \diamondsuit(1,2)$ and $\diamondsuit(2,3) = \varnothing$, using Proposition 4, we get that $[\diamondsuit(1,2,3)] \neq \varnothing$, from **U1**, $[\diamondsuit(1,2,3)] \subseteq \{\omega 1, \omega 2, \omega 3\}$. Using **E5** and **U9**, as previously we get that $\omega_2 \notin \diamondsuit(1,2,3)$ and $\omega_3 \notin \diamondsuit(1,2,3)$. Hence, $[\diamondsuit(1,2,3) \wedge (\Phi(\omega_1) \vee \Phi(\omega_2))]$ should contain ω_1. Using **E5** we get $\omega_1 \in \diamondsuit(1,3)$.
  - $\diamondsuit(1,2) = \varnothing$ and $\diamondsuit(2,3) = \varnothing$, using Proposition 4, we get that $[\Phi(\omega)\diamondsuit_{\mathcal{T}}\Phi(\omega_1)] = \varnothing$ and $[\Phi(\omega)\diamondsuit_{\mathcal{T}}\Phi(\omega_3)] = \varnothing$, using **E5** we get that $\diamondsuit(1,3) = \varnothing$.
- Reflexivity: $\forall \omega, \omega_1 \in \Omega$, using **U1**, $\Phi(\omega)\diamondsuit_{\mathcal{T}}\Phi(\omega_1) \models \Phi(\omega_1)$, hence either $\omega_1 \in [\Phi(\omega)\diamondsuit_{\mathcal{T}}\Phi(\omega_1)]$ or $[\Phi(\omega)\diamondsuit_{\mathcal{T}}\Phi(\omega_1)] = \varnothing$. Hence $\omega_1 \preceq_\omega \omega_1$.
- Completeness: $\forall \omega, \omega_1, \omega_2 \in \Omega$, if $\omega_1 \not\preceq \omega_2$ then $[\Phi(\omega)\diamondsuit_{\mathcal{T}}(\Phi(\omega_1) \vee \Phi(\omega_2))] \neq \varnothing$ and $\omega_1 \notin [\Phi(\omega)\diamondsuit_{\mathcal{T}}(\Phi(\omega_1) \vee \Phi(\omega_2))]$, due to **U1** it means that $\omega_2 \in [\Phi(\omega)\diamondsuit_{\mathcal{T}}(\Phi(\omega_1) \vee \Phi(\omega_2))]$. Hence $\omega_2 \preceq_\omega \omega_1$.
- $\preceq_\omega$ respects $\mathcal{T}$: if $(\omega, \omega_1) \in \mathcal{T}$ and $(\omega, \omega_2) \notin \mathcal{T}$ and $\omega_2 \preceq_\omega \omega_1$ it means that either $[\Phi(\omega)\diamondsuit_{\mathcal{T}}(\Phi(\omega_1) \vee \Phi(\omega_2))] = \varnothing$ or $\omega_2 \in [\Phi(\omega)\diamondsuit_{\mathcal{T}}(\Phi(\omega_1) \vee \Phi(\omega_2))]$.
  Due to **E3**, since $(\omega, \omega_1) \in \mathcal{T}$, $[\Phi(\omega)\diamondsuit_{\mathcal{T}}\Phi(\omega_1)] \neq \varnothing$.
  Due to **U1**, $\omega_1 \in [\Phi(\omega)\diamondsuit_{\mathcal{T}}\Phi(\omega_1)]$. Hence using **E5** and **U9**, $[\Phi(\omega)\diamondsuit_{\mathcal{T}}(\Phi(\omega_1) \vee \Phi(\omega_2)) \wedge \Phi(\omega_1)] = [\Phi(\omega)\diamondsuit_{\mathcal{T}}(\Phi(\omega_1)] = \{\omega 1\}$. Contradiction, hence $\omega_1 \prec_\omega \omega_2$.

Let us show now that $\forall \omega \in \Omega, \forall \alpha \in \mathscr{L}, [\Phi(\omega)\Diamond_{\mathcal{T}}\alpha] = \{\omega_1 \in [\alpha]$ such that $(\omega, \omega_1) \in \mathcal{T}$ and $\forall \omega_2 \in [\alpha]$, such that $(\omega, \omega_2) \in \mathcal{T}$, $\omega_1 \preceq_\omega \omega_2\}$.

- If $\omega_1 \in [\Phi(\omega)\Diamond_{\mathcal{T}}\alpha]$, due to **U1**, $\omega_1 \in [\alpha]$. If $\exists \omega_2 \in [\alpha]$ such that $(\omega, \omega_2) \in \mathcal{T}$ and $\omega_2 \prec_\omega \omega_1$, it means that $\omega_1 \notin [\Phi(\omega)\Diamond_{\mathcal{T}}(\Phi(\omega_1) \vee \Phi(\omega_2))]$.
  Due to **E5**, $(\Phi(\omega)\Diamond_{\mathcal{T}}\alpha) \wedge (\Phi(\omega_1) \vee \Phi(\omega_2)) \models \Phi(\omega)\Diamond_{\mathcal{T}}(\Phi(\omega_1) \vee \Phi(\omega_2))$ which implies $\omega_1 \in [\Phi(\omega)\Diamond_{\mathcal{T}}(\Phi(\omega_1) \vee \Phi(\omega_2))]$, hence $\omega_1 \preceq_\omega \omega_2$. Moreover, due to **E5**, $[(\Phi(\omega)\Diamond_{\mathcal{T}}\alpha) \wedge \Phi(\omega_1)] \subseteq [\Phi(\omega)\Diamond_{\mathcal{T}}\Phi(\omega_1)]$, hence $\omega_1 \in [\Phi(\omega)\Diamond_{\mathcal{T}}\Phi(\omega_1)]$, hence $(\omega, \omega_1) \in \mathcal{T}$.
  Thus $[\Phi(\omega)\Diamond_{\mathcal{T}}\alpha] \subseteq \{\omega_1 \in [\alpha], (\omega, \omega_1) \in \mathcal{T}$ and $\forall \omega_2 \in [\alpha]$ such that $(\omega, \omega_2) \in \mathcal{T}, \omega_1 \preceq_\omega \omega_2\}$.
- Conversely, let $\omega_1 \in [\alpha]$ such that $(\omega, \omega_1) \in \mathcal{T}$ and $\forall \omega_2 \in [\alpha]$ such that $(\omega, \omega_2) \in \mathcal{T}, \omega_1 \preceq_\omega \omega_2$. Let us first consider $\omega_2 \in [\alpha]$ such that $(\omega, \omega_2) \notin \mathcal{T}$, Due to **E5**: $[(\Phi(\omega)\Diamond_{\mathcal{T}}(\Phi(\omega_1) \vee \Phi(\omega_2))) \wedge \Phi(\omega_2)] \subseteq [\Phi(\omega)\Diamond_{\mathcal{T}}\Phi(\omega_2)]$ and, since $(\omega, \omega_2) \notin \mathcal{T}$, due to **E3**, $[\Phi(\omega)\Diamond_{\mathcal{T}}\Phi(\omega_2)] = \varnothing$. Moreover since $(\omega, \omega 1) \in \mathcal{T}$, we get using **E3** that $[\Phi(\omega)\Diamond_{\mathcal{T}}(\Phi(\omega_1) \vee \Phi(\omega_2)] \neq \varnothing$. Due to **U1**, $[\Phi(\omega) \diamond (\Phi(\omega_1) \vee \Phi(\omega_2))] \subseteq \{\omega_1, \omega_2\}$. Hence, $[\Phi(\omega)\Diamond_{\mathcal{T}}(\Phi(\omega_1) \vee \Phi(\omega_2))] = \{\omega_1\}$ this result, say (a), holds for any $\omega_2 \in [\alpha]$ such that $(\omega, \omega_2) \notin \mathcal{T}$.

  Furthermore, due to our hypothesis about ω_1, $\forall \omega_2 \in [\alpha]$ such that $(\omega, \omega_2) \in \mathcal{T}$, if $[\Phi(\omega)\Diamond_{\mathcal{T}}(\Phi(\omega_1) \vee \Phi(\omega_2))] \neq \varnothing$ then $\omega_1 \in [\Phi(\omega)\Diamond_{\mathcal{T}} (\Phi(\omega_1) \vee \Phi(\omega_2))]$. Let us call (b) this result.

  Let us denote, without loss of generality, $\{\omega_1, \ldots, \omega_n\} = [\alpha]$ the set of worlds in which α holds (this set is finite since $\mathcal{A}$ is finite and R is fixed).

  Due to **E3** and **U1**, since $(\omega, \omega_1) \in \mathcal{T}$ and $\omega_1 \in [\alpha]$, we get $[(\Phi(\omega)\Diamond_{\mathcal{T}}\ \alpha)] \neq \varnothing$ and $[(\Phi(\omega)\Diamond_{\mathcal{T}}\alpha)] \subseteq [\alpha] = \{\omega_1, \ldots, \omega_n\}$. Hence, for some $k \in [1, n]$, it holds that $[(\Phi(\omega)\Diamond_{\mathcal{T}}\alpha) \wedge (\Phi(\omega_1) \vee \Phi(\omega_k))] \neq \varnothing$. Let us notice that $\forall i \in [2, n]$, $[\alpha \wedge (\Phi(\omega_1) \vee \Phi(\omega_i)] = [(\Phi(\omega_1) \vee \Phi(\omega_i)]$. Due to **U4**, we deduce that $[\Phi(\omega)\Diamond_{\mathcal{T}}(\alpha \wedge (\Phi(\omega_1) \vee \Phi(\omega_i))] = [\Phi(\omega)\Diamond_{\mathcal{T}}(\Phi(\omega_1) \vee \Phi(\omega_i))]$

  By definition of k, we can apply both **E5** and **U9**, we get $[\Phi(\omega)\Diamond_{\mathcal{T}} (\Phi(\omega_1) \vee \Phi(\omega_k))] = [(\Phi(\omega)\Diamond_{\mathcal{T}}\alpha) \wedge (\Phi(\omega_1) \vee \Phi(\omega_k))] \neq \varnothing$.

  If $(\omega, \omega_k) \notin \mathcal{T}$ then due to (a) we get $[((\Phi(\omega)\Diamond_{\mathcal{T}}\alpha) \wedge (\Phi(\omega_1) \vee \Phi(\omega_k))] = \{\omega_1\}$. If $(\omega, \omega_k) \in \mathcal{T}$ then due to (b) we get $\omega_1 \in [((\Phi(\omega)\Diamond_{\mathcal{T}}\alpha) \wedge (\Phi(\omega_1) \vee \Phi(\omega_k))]$. In both cases, $\omega_1 \in [\Phi(\omega)\Diamond_{\mathcal{T}}\alpha]$.

Now, if $[\varphi] = \varnothing$ then $\cup_{\omega\in[\varphi]}[\Phi(\omega)\diamondsuit_{\mathcal{T}}\alpha] = \varnothing$. Else let us denote $\omega_1, \ldots \omega_n$ the worlds in which φ holds: $[\varphi] = [\Phi(\omega_1) \vee \ldots \Phi(\omega_n)]$, using **E8**, if $\exists\omega \in [\varphi]$ such that $[\Phi(\omega)\diamondsuit_{\mathcal{T}}\alpha] = \varnothing$ then $[\varphi\diamondsuit_{\mathcal{T}}\alpha] = \varnothing$ else we get that $[\varphi\diamondsuit_{\mathcal{T}}\alpha] = \bigcup_{\omega\in[\varphi]} [\Phi(\omega)\diamondsuit_{\mathcal{T}}\alpha]$. Hence, the result.

Step 2: Let $\preceq_\omega$ be a complete preorder on $\Omega \times \Omega$ respecting $\mathcal{T}$, let $\diamondsuit_{\mathcal{T}}$ be defined by $\forall\omega \in \Omega$, $[\Phi(\omega)\diamondsuit_{\mathcal{T}}\alpha] = \{\omega_1 \in [\alpha]$ such that $(\omega, \omega_1) \in \mathcal{T}$ and $\forall\omega_2 \in [\alpha]$ such that $(\omega, \omega_2) \in \mathcal{T}$, $\omega_1 \preceq_\omega \omega_2\}$ and $[\varphi\diamondsuit_{\mathcal{T}}\alpha] = \varnothing$ if $\exists\omega \in [\varphi]$ such that $[\Phi(\omega)\diamondsuit_{\mathcal{T}}\alpha] = \varnothing$. Otherwise $[\varphi\diamondsuit_{\mathcal{T}}\alpha] = \bigcup_{\omega\in[\varphi]} [\Phi(\omega)\diamondsuit_{\mathcal{T}}\alpha]$.

Let us show that $\diamondsuit_{\mathcal{T}}$ satisfies **E3**, **U4**, **E5**, **E8** and **U9**:

E3:

($\Rightarrow$:) $[\varphi\diamondsuit_{\mathcal{T}}\alpha] \neq \varnothing$ if and only if $[\varphi] \neq \varnothing$ and $\forall\omega \in [\varphi]$, $[\Phi(\omega)\diamondsuit_{\mathcal{T}}\alpha] \neq \varnothing$. This implies that $(\varphi, \alpha) \models \mathcal{T}$ since it is equivalent to $[\varphi] \neq \varnothing$ and $\forall\omega \in [\varphi]$, $\exists\omega' \in [\alpha]$ such that $(\omega, \omega') \in \mathcal{T}$.

($\Leftarrow$:) if $(\varphi, \alpha) \models \mathcal{T}$ then $[\varphi] \neq \varnothing$ and $\forall\omega \in [\varphi]$, $\exists\omega' \in [\alpha]$ such that $(\omega, \omega') \in \mathcal{T}$. Due to the completeness of $\preceq_\omega$ it means that $\exists\omega_1 \in [\alpha]$ such that $(\omega, \omega_1) \in \mathcal{T}$ and $\forall\omega' \in [\alpha], \omega_1 \preceq_\omega \omega'$. By definition it means that $[\varphi\diamondsuit_{\mathcal{T}}\alpha] \neq \varnothing$.

U4: since the definition of $\diamondsuit_{\mathcal{T}}$ is only based on the models of φ and α, it is easy to check that **U4** holds.

E5: if $[(\Phi(\omega)\diamondsuit_{\mathcal{T}}\alpha) \wedge \beta] \neq \varnothing$ then $\exists\omega_a \in [\Phi(\omega)\diamondsuit_{\mathcal{T}}\alpha] \cap [\beta]$. Thus, $\omega_a \in [\beta]$ and $\omega_a \in [\alpha]$ and $(\omega, \omega_a) \in \mathcal{T}$ and $\forall\omega_1 \in [\alpha]$ such that $(\omega, \omega_1) \in \mathcal{T}$, $\omega_a \preceq_\omega \omega_1$. Hence $\forall\omega_2 \in [\alpha \wedge \beta]$ such that $(\omega, \omega_2) \in \mathcal{T}$, ω_2 being in $[\alpha]$, it means that $\omega_a \preceq_\omega \omega_2$. Hence, $\omega_a \in [(\Phi(\omega)\diamondsuit_{\mathcal{T}}(\alpha \wedge \beta)]$.

E8: by definition.

U9: if $[(\Phi(\omega)\diamondsuit_{\mathcal{T}}\alpha) \wedge \beta] \neq \varnothing$ then let $\omega_b \in [\Phi(\omega)\diamondsuit_{\mathcal{T}}(\alpha \wedge \beta)]$ then $\omega_b \in [\alpha \wedge \beta]$ and $(\omega, \omega_b) \in \mathcal{T}$ and $\forall\omega_1 \in [\alpha \wedge \beta]$ such that $(\omega, \omega_1) \in \mathcal{T}$, $\omega_b \preceq_\omega \omega_1$. Now let $\omega_a \in [\Phi(\omega)\diamondsuit_{\mathcal{T}}\alpha) \wedge \beta]$, it means that $\omega_a \in [\alpha \wedge \beta]$ and $(\omega, \omega_a) \in \mathcal{T}$, hence $\omega_b \preceq_\omega \omega_a$. Moreover, by definition of ω_a, $\forall\omega_1 \in [\alpha]$ such that $(\omega, \omega_1) \in \mathcal{T}$, $\omega_a \preceq_\omega \omega_1$. Thus, by transitivity, $\forall\omega_1 \in [\alpha]$ such that $(\omega, \omega_1) \in \mathcal{T}$, $\omega_b \preceq_\omega \omega_1$. Thus, $\omega_b \in [\Phi(\omega)\diamondsuit_{\mathcal{T}}\alpha) \wedge \beta]$.

□

Proposition 3. *If $\diamondsuit_{\mathcal{T}}$ satisfies **E3**, **U4**, **E5**, **E8** and **U9** then* $([\varphi] = \varnothing$ *or* $[\alpha] = \varnothing) \implies [\varphi \diamondsuit_{\mathcal{T}} \alpha] = \varnothing$.

Proof of Proposition 3. The proof is straightforward from the equation relating $\diamondsuit_{\mathcal{T}}$ and $\preceq$ in Theorem 2. □

Proposition 4. *If $\diamondsuit_{\mathcal{T}}$ satisfies **E3** then* $([\varphi] \neq \varnothing$ *and* $[\varphi \diamondsuit_{\mathcal{T}} \alpha] \neq \varnothing) \implies (\forall \gamma, [\varphi \diamondsuit_{\mathcal{T}} (\alpha \vee \gamma)] \neq \varnothing)$.

Proof of Proposition 4. Let $\varphi, \alpha \in \mathscr{L}$ such that $[\varphi] \neq \varnothing$ and $[\varphi \diamondsuit_{\mathcal{T}} \alpha] \neq \varnothing$ and $\diamondsuit_{\mathcal{T}}$ satisfies E_3. It means that $(\varphi, \alpha) \models \mathcal{T}$, *i.e.*, $\forall \omega \in [\varphi]$, $\exists \omega_1 \in [\alpha]$, $(\omega, \omega_1) \in \mathcal{T}$. Now, $\forall \beta \in \mathscr{L}$, $[\alpha] \subseteq [\alpha \vee \beta]$, hence $\omega_1 \in [\alpha \vee \beta]$. This means that $(\varphi, \alpha \vee \beta) \models \mathcal{T}$. Using **E3**, we get $[\varphi \diamondsuit_{\mathcal{T}} (\alpha \vee \beta)] \neq \varnothing$. □

Proposition 5. *Given a* reflexive *relation $\mathcal{T} \subseteq \Omega \times \Omega$ of authorized transitions, there is an operator $\diamondsuit_{\mathcal{T}} : \mathscr{L} \times \mathscr{L} \to \mathscr{L}$ satisfying **E3**, **U4**, **E5**, **E8**, **U9** that satisfies **U2** if and only if there is a faithful assignment respecting $\mathcal{T}$ defined as in Theorem 2.*

Proof of Proposition 5. Let $\mathcal{T}$ be a reflexive relation on $\Omega \times \Omega$.

(a). Let $\diamondsuit_{\mathcal{T}} : \mathscr{L} \times \mathscr{L} \to \mathscr{L}$ be an operator satisfying **U2**, **E3**, **U4**, **E5**, **E8**, **U9**. For any $\omega \in \Omega$, let us define $\preceq_\omega$ such that $\omega_1 \preceq_\omega \omega_2$ if and only if $\omega_1 \in [\Phi(\omega) \diamondsuit_{\mathcal{T}} (\Phi(\omega_1) \vee \Phi(\omega_2))]$ or $[\Phi(\omega) \diamondsuit_{\mathcal{T}} (\Phi(\omega_1) \vee \Phi(\omega_2))] = \varnothing$. Due to Theorem 2, $\preceq_\omega$ is a complete preorder on Ω respecting $\mathcal{T}$. $\forall \omega_1 \in \Omega$, let us compute $[\Phi(\omega) \diamondsuit_{\mathcal{T}} (\Phi(\omega) \vee \Phi(\omega_1))]$. Due to **U2**, since $\Phi(\omega) \models \Phi(\omega) \vee \Phi(\omega_1)$, we have $[\Phi(\omega) \diamondsuit_{\mathcal{T}} (\Phi(\omega) \vee \Phi(\omega_1))] = [\Phi(\omega)]$. Hence, $\omega \prec_\omega \omega_1$. It means that the assignment is faithful.

(b). $\forall \omega \in \Omega$, let $\preceq_\omega$ be a complete preorder on $\Omega \times \Omega$ respecting $\mathcal{T}$, and such that $\forall \omega_1, \omega \prec_\omega \omega_1$. Let $\diamondsuit_{\mathcal{T}}$ be defined by $\forall \omega \in \Omega$, $[\Phi(\omega) \diamondsuit_{\mathcal{T}} \alpha] = \{\omega_1 \in [\alpha]$ such that $(\omega, \omega_1) \in \mathcal{T}$ and $\forall \omega_2 \in [\alpha], \omega_1 \preceq_\omega \omega_2\}$ and $[\varphi \diamondsuit_{\mathcal{T}} \alpha] = \varnothing$ if $\exists \omega \in [\varphi]$ such that $[\Phi(\omega) \diamondsuit_{\mathcal{T}} \alpha] = \varnothing$. Otherwise $[\varphi \diamondsuit_{\mathcal{T}} \alpha] = \bigcup_{\omega \in [\varphi]} [\Phi(\omega) \diamondsuit_{\mathcal{T}} \alpha]$. Due to Theorem 2, $\diamondsuit_{\mathcal{T}}$ satisfies **U1**, **E3**, **U4**, **E5**, **E8** and **U9**. Let us check if **U2** holds: let $\varphi, \alpha \in \mathscr{L}$, such that $\varphi \models \alpha$.

  (a) If $[\varphi] = \varnothing$ then due to Proposition 3 $[\varphi \diamondsuit_{\mathcal{T}} \alpha] = \varnothing$ (hence **U2** holds).

  (b) If $[\varphi] \neq \varnothing$ then let $\omega \in [\varphi]$, $[\Phi(\omega) \diamondsuit_{\mathcal{T}} \alpha] = \{\omega_1 \in [\alpha]$ such that $(\omega, \omega_1) \in \mathcal{T}$ and $\forall \omega_2 \in [\alpha]$ such that $(\omega, \omega_2) \in \mathcal{T}$, $\omega_1 \preceq_\omega \omega_2\}$.

Since $\varphi \models \alpha$, it meant that $\omega \models \alpha$. Moreover since $\mathcal{T}$ is reflexive then $(\omega, \omega) \in \mathcal{T}$, lastly, due to faithfulness, $\forall \omega_1$, $\omega \prec_\omega \omega_1$. Hence $\omega \in [\Phi(\omega) \diamondsuit_\mathcal{T} \alpha]$ and $\forall \omega_1 \neq \omega$, $\omega_1 \notin [\Phi(\omega) \diamondsuit_\mathcal{T} \alpha]$. It means that $[\Phi(\omega) \diamondsuit_\mathcal{T} \alpha] = \{\omega\} = [\Phi(\omega)]$. This is true for any $\omega \in [\varphi]$, hence $\bigcup\limits_{\omega \in [\varphi]} [\Phi(\omega) \diamondsuit_\mathcal{T} \alpha] = \bigcup\limits_{\omega \in [\varphi]} [\Phi(\omega)] = [\varphi]$ (hence **U2** holds).

$\square$

Proposition 6. *If $\mathcal{T} = \Omega \times \Omega$ then $\diamondsuit_\mathcal{T}$ satisfies **U2, E3, U4, E5, E8, U9** if and only if $\diamondsuit_\mathcal{T}$ satisfies **U1, U2, U3, U4, U5, U8** and **U9**.*

Proof of Proposition 6. Let $\mathcal{T} = \Omega \times \Omega$.

(a). Let $\diamondsuit_\mathcal{T}$ be an operator satisfying **U2**, **E3**, **U4**, **E5**, **E8**, **U9**, let us show that it satisfies **U1**, **U3**, **U5** and **U8**:

U1: This is due to Proposition 1.

U3: If $[\varphi] \neq \varnothing$ then due to **E3**, $[\varphi \diamondsuit_\mathcal{T} \alpha] \neq \varnothing$ if and only if $(\varphi, \alpha) \in \mathcal{T}$. Hence, $\mathcal{T}$ being equal to $\Omega \times \Omega$, if $[\alpha] \neq \varnothing$ then $(\varphi, \alpha) \in \mathcal{T}$ thus $[\varphi \diamondsuit_\mathcal{T} \alpha] \neq \varnothing$.

U5: This is due to **U8** and **E5**.

U8: If $[\varphi \diamondsuit_\mathcal{T} \alpha] = \varnothing$ or $[\psi \diamondsuit_\mathcal{T} \alpha] = \varnothing$ then, due to **E8**, we have $[(\varphi \vee \psi) \diamondsuit_\mathcal{T} \alpha] = \varnothing$, hence **U8** holds. If $[\varphi \diamondsuit_\mathcal{T} \alpha] \neq \varnothing$ and $[\psi \diamondsuit_\mathcal{T} \alpha] \neq \varnothing$ then, due to **E8**, **U8** holds.

(b). Let $\diamondsuit_\mathcal{T}$ be an operator satisfying **U1**, **U2**, **U3**, **U4**, **U5**, **U8** and **U9** let us show that it satisfies **E3**, **E5** and **E8**:

E3: When $\mathcal{T} = \Omega \times \Omega$, $(\varphi, \alpha) \in \mathcal{T} \Leftrightarrow [\alpha] \neq \varnothing$, hence the result.

E5: It is a particular case of **U5**.

E8: If $([\varphi] \neq \varnothing$ and $[\varphi \diamondsuit_\mathcal{T} \alpha] = \varnothing)$ then due to **U3** it means that $[\alpha] = \varnothing$, hence, due to **U1**, $[(\varphi \vee \psi) \diamondsuit_\mathcal{T} \alpha = \varnothing$. (**E8** holds).

Now, if $[\varphi] = \varnothing$ and $[\alpha] \neq \varnothing$ then, due to **U3**, $[\varphi \diamondsuit_\mathcal{T} \alpha] \neq \varnothing$, idem for ψ. It means that the other case is when $[\varphi \diamondsuit_\mathcal{T} \alpha] \neq \varnothing$ and $[\psi \diamondsuit_\mathcal{T} \alpha] \neq \varnothing$ in that case **U8** applies hence **E8** holds.

$\square$

Proposition 7. *$(\mathcal{A}, \mathcal{R})$ is the unique model of $\Phi_\mathbf{U}(\mathcal{A}, \mathcal{R})$.*

Proof of Proposition 7. Let $\mathcal{G} = (\mathcal{A}, \mathcal{R}_{\mathcal{A}})$. Let $\mathcal{G}_1 \in \Omega_{\mathcal{A}}$, $\mathcal{G}_1 = (\mathcal{A}_1, \mathcal{R}_{\mathcal{A}_1})$, such that $\mathcal{G}_1 \in [\Phi_{\mathbf{U}}(\mathcal{G})]$. Using Definitions 14, 13 and the axioms proposed in Section 4.2.2, $\forall x \in \mathcal{A}$, $x \in \mathcal{G}$ if and only if $on(x)$ is true in $\mathcal{G}_1$. Hence, $\mathcal{G}_1$ has the same vertices as $\mathcal{G}$, *i.e.* $\mathcal{A}_1 = \mathcal{A}$. So $\mathcal{R}_{\mathcal{A}_1} = \mathcal{R}_A$ and $\mathcal{G}_1 = \mathcal{G}$. $\square$

Proposition 8. *Let $\mathcal{A}_{\mathbf{U}}$ be a set of arguments, and $(\mathcal{A}, \mathcal{R})$ an argumentation graph such that $\mathcal{A} \subseteq \mathcal{A}_{\mathbf{U}}$ and $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$. Let t, t_1, t_2, t_3 be terms of $YALLA_{\mathbf{U}}$. We have:*

- *t is conflict-free in $(\mathcal{A}, \mathcal{R})$ if and only if $(\mathcal{A}, \mathcal{R}) \models on(t) \wedge (\neg(t \rhd t))$. The latter formula is denoted by $F(t)$.*
- *t_1 defends each element of t_2 in $(\mathcal{A}, \mathcal{R})$ if and only if $(\mathcal{A}, \mathcal{R}) \models (\forall t_3\ ((singl(t_3) \wedge (t_3 \rhd t_2)) \implies (t_1 \rhd t_3)))$, which is denoted by $(\mathcal{A}, \mathcal{R}) \models t_1 \ \rhd\!\!\!\rhd\, t_2$.*
- *If $(\mathcal{A}, \mathcal{R}) \models singl(t_2)$, then t_1 indirectly defends the unique element of t_2 in $(\mathcal{A}, \mathcal{R})$ (which is denoted by $(\mathcal{A}, \mathcal{R}) \models t_1 \rhd\!\!\leadsto\!\!\rhd t_2$) if and only if $(\mathcal{A}, \mathcal{R}) \models (t_1 \ \rhd\!\!\!\rhd\, t_2) \vee (\exists y\ ((t_1 \ \rhd\!\!\!\rhd\, y) \wedge (y \rhd\!\!\leadsto\!\!\rhd t_2)))$.*
- *t is admissible in $(\mathcal{A}, \mathcal{R})$ if and only if $(\mathcal{A}, \mathcal{R}) \models (F(t) \wedge (t \ \rhd\!\!\!\rhd\, t))$, which is denoted by $(\mathcal{A}, \mathcal{R}) \models A(t)$.*
- *t is a complete extension of $(\mathcal{A}, \mathcal{R})$ if and only if $(\mathcal{A}, \mathcal{R}) \models (A(t) \wedge \forall t_2\ ((singl(t2) \wedge (t \ \rhd\!\!\!\rhd\, t_2)) \implies (t_2 \subseteq t)))$, which is denoted by $(\mathcal{A}, \mathcal{R}) \models C(t)$.*
- *t is the grounded extension of $(\mathcal{A}, \mathcal{R})$ if and only if $(\mathcal{A}, \mathcal{R}) \models (C(t) \wedge \forall t_2\ (C(t_2) \implies (t \subseteq t_2)))$, which is denoted by $(\mathcal{A}, \mathcal{R}) \models G(t)$.*
- *t is a stable extension of $(\mathcal{A}, \mathcal{R})$ if and only if $(\mathcal{A}, \mathcal{R}) \models (F(t) \wedge \forall t_2\ ((singl(t_2) \wedge \neg(t_2 \subseteq t)) \implies (t \rhd t_2)))$, which is denoted by $(\mathcal{A}, \mathcal{R}) \models S(t)$.*
- *t is a preferred extension of $(\mathcal{A}, \mathcal{R})$ if and only if $(\mathcal{A}, \mathcal{R}) \models (A(t) \wedge \forall t_2\ (((t_2 \neq t) \wedge (t \subseteq t_2)) \implies \neg A(t_2)))$, which is denoted by $(\mathcal{A}, \mathcal{R}) \models P(t)$.*

Proof of Proposition 8. It follows easily from Definition 13 and from the definitions of the semantics. $\square$

Theorem 3. *Given $\mathcal{T} \subseteq \Gamma_{\mathbf{U}} \times \Gamma_{\mathbf{U}}$ a set of authorized transitions, there exists an operator $\Diamond_{\mathcal{T}}$ satisfying **E3**, **U4**, **E5**, **E8**, **U9** if and only if there is an assignment respecting $\mathcal{T}$ such that $\forall \mathcal{G} \in \Gamma_{\mathbf{U}}, \forall \varphi, \alpha \in YALLA_{\mathbf{U}}$,* *(1)* $[\varphi \diamondsuit_{\mathcal{T}} \alpha] = \varnothing$ *if* $\exists \mathcal{G} \in [\varphi]$ *such that* $[\Phi_{\mathbf{U}}(\mathcal{G}) \diamondsuit_{\mathcal{T}} \alpha] = \varnothing$

(2) $[\varphi \diamondsuit_{\mathcal{T}} \alpha] = \bigcup_{\mathcal{G} \in [\varphi]} [\Phi_{\mathbf{U}}(\mathcal{G}) \diamondsuit_{\mathcal{T}} \alpha]$ *otherwise*

(3) $[\Phi_{\mathbf{U}}(\mathcal{G}) \diamondsuit_{\mathcal{T}} \alpha] = \left\{ \mathcal{G}_1 \in \Gamma_{\mathbf{U}} \;\middle|\; \begin{array}{l} \mathcal{G}_1 \in [\alpha] \textit{ and} \\ (\mathcal{G}, \mathcal{G}_1) \in \mathcal{T} \textit{ and} \\ (\forall \mathcal{G}_2 \in [\alpha] \textit{ such that } (\mathcal{G}, \mathcal{G}_2) \in \mathcal{T}, \mathcal{G}_1 \preceq_{\mathcal{G}} \mathcal{G}_2) \end{array} \right\}$

Proof of Theorem 3. It follows easily from Theorem 2 and from the parallel between graphs and sets of worlds. □

Proposition 9. *Given a set of authorized transitions* $\mathcal{T} \subseteq \Gamma_{\mathbf{U}} \times \Gamma_{\mathbf{U}}$ *such that*[32] $\mathcal{T}_e \subseteq \mathcal{T}$*, for any characterization* C *of the form:*

> *"If* $\mathcal{G} \models \gamma_1$ *and* $\exists z, \mathcal{R}_z$ *such that* $o = \langle \oplus, z, \mathcal{R}_z \rangle$ *is executable by an agent on* $\mathcal{G}$ *and* $o(\mathcal{G}) \models \alpha_z$ *then* $o(\mathcal{G}) \models \gamma_2$*",*

it holds that every assignment respecting $\mathcal{T}$ *and* $\mathcal{T}_e$ *allows us to define a generalized enforcement operator* $\diamondsuit_{\mathcal{T}}$ *in the same way as in Theorem 3 satisfying* ***E3****,* ***U4****,* ***E5****,* ***E8****,* ***U9*** *and the additional postulate*

$$\begin{array}{lll} \boldsymbol{A}_C\textit{:} & & \forall \mathcal{A} \subseteq \mathcal{A}_{\mathbf{U}}, \forall \mathcal{R}_{\mathcal{A}} \subseteq \mathcal{A} \times \mathcal{A}, \forall z \in \mathcal{A}_{\mathbf{U}} \setminus \mathcal{A}, \\ & & \forall \mathcal{R}_z \subseteq (\{z\} \times \mathcal{A}) \cup (\mathcal{A} \times \{z\}), \\ & \textit{let} & \mathcal{G} = (\mathcal{A}, \mathcal{R}_{\mathcal{A}}) \textit{ and } o = \langle \oplus, z, \mathcal{R}_z \rangle, \\ & \textit{if} & \Phi_{\mathbf{U}}(\mathcal{G}) \models \gamma_1 \textit{ and } (\mathcal{G}, o(\mathcal{G})) \in \mathcal{T}_e \textit{ and } \Phi_{\mathbf{U}}(o(\mathcal{G})) \models \alpha_z \\ & \textit{then} & \Phi_{\mathbf{U}}(\mathcal{G}) \diamondsuit_{\mathcal{T}} (on(z) \wedge \varphi_{\mathcal{R}_z}) \models \gamma_2. \end{array}$$

where $\varphi_{\mathcal{R}_z} = \bigwedge_{(x,y) \in \mathcal{R}_z} (x \rhd y) \wedge \bigwedge_{x \in \mathcal{G} | (x,z) \notin \mathcal{R}_z} \neg(x \rhd z) \wedge \bigwedge_{y \in \mathcal{G} | (z,y) \notin \mathcal{R}_z} \neg(z \rhd y)$ *is a formula that describes the attacks that are in* $\mathcal{R}_z$ *and that excludes any other attack concerning* z*.*

Proof of Proposition 9. Due to Theorem 3, every assignment respecting $\mathcal{T}$ allows us to define a generalized enforcement operator $\diamondsuit_{\mathcal{T}}$ satisfying **E3**, **U4**, **E5**, **E8** and **U9**.

Let us show that $\diamondsuit_{\mathcal{T}}$ satisfies $\mathbf{A}_C$. For any graph $\mathcal{G} = (\mathcal{A}, \mathcal{R}_{\mathcal{A}})$ and any argument $z \notin \mathcal{A}$ and any set of attacks $\mathcal{R}_z$ that are concerning z, let $o = \langle \oplus, z, \mathcal{R}_z \rangle$ and let us assume that (a1) $\Phi_{\mathbf{U}}(\mathcal{G}) \models \gamma_1$, (a2) $(\mathcal{G}, o(\mathcal{G})) \in \mathcal{T}_e$

[32] The elementary operations that are executable should be authorized, *i.e.* included in the set of authorized transitions.

and (a3) $\Phi_{\mathbf{U}}(o(\mathcal{G})) \models \alpha_z$. (a1) means that $\mathcal{G} \models \gamma_1$. Moreover, (a2) means that o is an elementary operation executable by an agent on $\mathcal{G}$. (a3) means that $o(\mathcal{G}) \models \alpha_z$. Hence we can apply the characterization C and we get that $o(\mathcal{G}) \models \gamma_2$.

In order to prove $\mathbf{A}_C$, we are going to show that $o(\mathcal{G})$ is the only model of $\Phi_{\mathbf{U}}(\mathcal{G}) \diamondsuit_{\mathcal{T}} (on(z) \wedge \varphi_{\mathcal{R}_z})$.

Using Theorem 3, we have $[\Phi_{\mathbf{U}}(\mathcal{G}) \diamondsuit_{\mathcal{T}} (on(z) \wedge \varphi_{\mathcal{R}_z})] = \{\mathcal{G}_1 \in [on(z) \wedge \varphi_{\mathcal{R}_z}] \mid (\mathcal{G}, \mathcal{G}_1) \in \mathcal{T}$ and $(\forall \mathcal{G}_2 \in [on(z) \wedge \varphi_{\mathcal{R}_z}]$ such that $(\mathcal{G}, \mathcal{G}_2) \in \mathcal{T}, \mathcal{G}_1 \preceq_{\mathcal{G}} \mathcal{G}_2)\}$.

Now, an operation from $\mathcal{G}$ (that does not contain z) to a graph in $[on(z) \wedge \varphi_{\mathcal{R}_z}]$ should at least add the argument z together with exactly $\mathcal{R}_z$ interactions (and no other interaction concerning z), hence the only elementary operation that could do that is $o = \langle \oplus, z, \mathcal{R}_z \rangle$. Moreover, since $\mathcal{T}_e \subseteq \mathcal{T}$ it means that $(\mathcal{G}, o(\mathcal{G})) \in \mathcal{T}$ (since o is an elementary change executable by an agent on $\mathcal{G}$). Now if there exists another operation o' from $\mathcal{G}$ to a graph in $[on(z) \wedge \varphi_{\mathcal{R}_z}]$ and such that $(\mathcal{G}, o'(\mathcal{G})) \in \mathcal{T}$ then this operation is not elementary (*i.e.* $o'(\mathcal{G})$ contains z with exactly its corresponding attacks but $o'(\mathcal{G})$ either contains some other arguments that were not in $\mathcal{G}$ or it is the case that some arguments that were in $\mathcal{G}$ are no longer in $o'(\mathcal{G})$). Hence $o(\mathcal{G}) \prec_{\mathcal{G}} o'(\mathcal{G})$. This is true for every operation o' distinct from o which means that $\{o(\mathcal{G})\} = [\Phi_{\mathbf{U}}(\mathcal{G}) \diamondsuit_{\mathcal{T}} (on(z) \wedge \varphi_{\mathcal{R}_z})]$. We have seen that $o(\mathcal{G}) \models \gamma_2$ hence $\Phi_{\mathbf{U}}(\mathcal{G}) \diamondsuit_{\mathcal{T}} (on(z) \wedge \varphi_{\mathcal{R}_z}) \models \gamma_2$. □

Proposition 10. *Given a set of authorized transitions $\mathcal{T} \subseteq \Gamma_{\mathbf{U}} \times \Gamma_{\mathbf{U}}$ such that $\mathcal{T}_e \subseteq \mathcal{T}$, for any characterization C of the form:*

> *"If $\mathcal{G} \models \gamma_1$ and $\exists z$ such that $o = \langle \ominus, z, \varnothing \rangle$ is executable by an agent on $\mathcal{G}$ and $\mathcal{G} \models \alpha_z$ then $o(\mathcal{G}) \models \gamma_2$",*

*it holds that every assignment respecting $\mathcal{T}$ and $\mathcal{T}_e$ allows us to define a generalized enforcement operator $\diamondsuit_{\mathcal{T}}$ in the same way as in Theorem 3 satisfying **E3**, **U4**, **E5**, **E8**, **U9** and the additional property*

$\boldsymbol{A}_C$:	$\forall \mathcal{A} \subseteq \mathcal{A}_{\mathbf{U}}, \forall \mathcal{R}_{\mathcal{A}} \subseteq \mathcal{A} \times \mathcal{A}, \forall z \in \mathcal{A}$,
let	$\mathcal{G} = (\mathcal{A}, \mathcal{R}_{\mathcal{A}})$ *and* $o = \langle \ominus, z, \varnothing \rangle$,
if	$\Phi_{\mathbf{U}}(\mathcal{G}) \models \gamma_1$ *and* $(\mathcal{G}, o(\mathcal{G})) \in \mathcal{T}_e$ *and* $\Phi_{\mathbf{U}}(\mathcal{G}) \models \alpha_z$
then	$\Phi_{\mathbf{U}}(\mathcal{G}) \diamondsuit_{\mathcal{T}} \neg on(z) \models \gamma_2$

Proof of Proposition 10. Due to Theorem 3, every assignment respecting $\mathcal{T}$ allows us to define a generalized enforcement operator $\diamondsuit_{\mathcal{T}}$ satisfying **E3**,

U4, **E5**, **E8** and **U9**. Now since $\mathcal{T}_e \subseteq \mathcal{T}$ it means that all elementary transitions are authorized. Therefore an assignment respecting $\mathcal{T}$ and preferring elementary changes will associate to any graph $\mathcal{G}$ a preference relation $\preceq_{\mathcal{G}}$ that will prefer to perform an elementary change if it exists. Thus the preferred (wrt to $\preceq_{\mathcal{G}}$) graphs satisfying the formula $\neg on(z)$ are the graphs that are obtained by an elementary change done on $\mathcal{G}$ if they exist. Moreover since $z \in \mathcal{A}$ this elementary change is necessarily the removal of z. There is only one elementary change o that removes z it is exactly $\langle \ominus, z, \varnothing \rangle$. This means that the only model of $[\Phi_{\mathbf{U}}(\mathcal{G}) \diamondsuit_{\mathcal{T}} \neg on(z)]$ is $\langle \ominus, z, \varnothing \rangle(\mathcal{G})$; then using the characterization we know that if $\mathcal{G} \models \gamma_1$ and if o is executable by an agent on $\mathcal{G}$ and $o(\mathcal{G}) \models \alpha_z$ then $o(\mathcal{G}) \models \gamma_2$. Hence if o is executable by an agent on $\mathcal{G}$ it means that $[\Phi_{\mathbf{U}}(\mathcal{G}) \diamondsuit_{\mathcal{T}} \neg on(z)] = \{o(\mathcal{G})\}$, thus if $\Phi_{\mathbf{U}}(\mathcal{G}) \models \gamma_1$ and $(\mathcal{G}, o(\mathcal{G})) \in T_e$ and $o(\mathcal{G}) \models \alpha_z$ then $\Phi_{\mathbf{U}}(\mathcal{G}) \diamondsuit_{\mathcal{T}} \neg on(z) \models \gamma_2$. □

Proposition 11. *$\diamondsuit^{on}$ satisfies all the characterization properties.*

Proof of Proposition 11. Since the assignment of $\preceq^{on}$ is preferring elementary changes then, due to Propositions 9 and 10, we obtain directly the result. □

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