Change in abstract bipolar argumentation systems

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Abstract

An argumentation system can undergo changes (addition or removal of arguments, addition or removal of interactions), particularly in multiagent systems. In this paper, we are interested in the change concerning abstract bipolar argumentation systems, *i.e.* argumentation systems using two kinds of interaction: attacks and supports. We propose some characterizations of this change that use and extend previous results defined in the case of Dung abstract argumentation systems.

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1 Introduction

The main feature of argumentation is the ability to deal with incomplete and / or contradictory information, especially for reasoning [Dung, 1995; Amgoud and Cayrol, 2002]. Moreover, argumentation can be used to formalize dialogues between several agents by modeling the exchange of arguments in, *e.g.*, negotiation between agents [Amgoud *et al.*, 2000]. An argumentation system (AS for short) consists of a collection of arguments interacting with each other through a relation reflecting conflicts between them, called *attack*. The issue of argumentation is then to determine "acceptable" sets of arguments (*i.e.*, sets able to defend themselves collectively while avoiding internal attacks), called *"extensions"*, and thus to reach a coherent conclusion. Another form of analysis of an AS is the study of the particular status of each argument based on its membership to the extensions. Formal frameworks have greatly eased the modeling and study of AS. In particular, the framework of [Dung, 1995] allows for abstracting from the "concrete" meaning of the arguments and relies only on binary interactions that may exist between them. This approach enables the user to focus on other aspects of argumentation, including its dynamic side. Indeed, in the course of a discussion or due to the acquisition of new pieces of information, an AS can undergo changes such as the addition of a new argument or the removal of an argument considered as illegal. This is particularly important for dialogs in a multiagent system since it is unrealistic to consider that the argumentation system reflecting the dialog can be statically defined. Thus, it is interesting to study these changes, to characterize them by giving properties describing a change operation and to provide conditions under which these properties hold. This has been done in several papers, especially [Bisquert *et al.*, 2013], for Dung AS with only attacks.

In this paper, we are interested in the extension of this work to bipolar AS (BAS for short), *i.e.* AS augmented with a second kind of interaction, the support relation. This relation represents a positive interaction between arguments and has been first introduced by [Karacapilidis and Papadias, 2001; Verheij, 2003]. In [Cayrol and Lagasquie-Schiex, 2005], the support relation is left general so that the resulting bipolar framework keeps a high level of abstraction. However there is no single interpretation of the support, and a number of researchers proposed specialized variants of the support relation: deductive support [Boella *et al.*, 2012], necessary support [Nouioua and Risch, 2010; Nouioua and Risch, 2011], evidential support [Oren and Norman, 2008; Oren *et al.*, 2010]. Each specialization can be associated with an appropriate modelling using appropriate complex attacks. These proposals have been developed quite independently, based on different intuitions and with different formalizations. [Cayrol and Lagasquie-Schiex, 2013] presents a comparative study in order to restate these proposals in a common setting, the *bipolar argumentation framework*. The idea is to keep the original arguments, to add complex attacks defined by the combination of the original attacks and the supports, and to modify the classical notions of acceptability. An important contribution of [Cayrol and Lagasquie-Schiex, 2013] is to highlight a kind of duality between the deductive and the necessary interpretations of support, which results in a duality in the modelling by complex attacks. Handling support is a growing concern: [Polberg and Oren, 2014] gives a translation between necessary supports and evidential supports; [Prakken, 2014] proposes a justification of the necessary support using the notion of subarguments; [Nouioua, 2013] studies an extension of the necessary support; [Gabbay, 2013] gives a logical study of bipolar systems; [Cohen *et al.*, 2014] proposes a general framework for taking into account recursive attacks and supports. However, there is no work concerning the study of the dynamics of a bipolar AS while it is an essential issue for modelling the actions of the participants to a multiagent system:

Ex. 1 *Journalists during an editorial board discuss about the publication of an information I:*

Journalist J_1 (Argument a): *I is important, we must publish it;*

Journalist J_2 (Argument b): *I is about a person X, it is forbidden to publish without the agreement of the concerned person and X disagrees with the publication;*

Journalist J_1 (Argument c): *X is a public person (she is the Prime Minister); in this case, her agreement is not mandatory;*

Journalist J_2 (Argument d): *However, I have heard about X's resignation;*

Journalist J_3 (Argument e): *I now understand why CNN has announced yesterday the postponement of the Council of Ministers;*

Journalist J_4 (Argument f): *However, yesterday was April Fools' Day; so CNN news announced yesterday are not reliable.*

This example illustrates a typical situation between agents that exchange arguments in order to take a decision (here, publish or not publish information I). In this dialog, one can see arguments (here, informal arguments corresponding to pieces of dialog), attacks (for instance Argument b attacks Argument a), supports (between Argument d and Argument e); and the dynamics of argumentation is illustrated by the dynamics of the dialog: at each step of the dialog, the global argumentation system evolves (here, by the addition of an argument and an interaction).

In this paper, we define the update of BAS and characterize it in a special case: a BAS reduced to an AS that is changed by the introduction of a new argument that interacts with another argument using supports. Such an update is realized using a combination of the works of both domains (bipolar argumentation and dynamics of argumentation).

Some background is given in Section 2 for AS and BAS, and in Section 3 for change operations. Then Section 4 proposes a change operation concerning a BAS. Characterizations of this new change operation are presented in Section 5. Finally, Section 6 concludes and suggests perspectives of our work. The proofs of our results are given in Appendix A.

2 Abstract bipolar argumentation system

The bipolar argumentation framework extends Dung's argumentation framework.

2.1 Abstract argumentation system

Dung's abstract framework consists of a set of arguments and only one type of interaction between these arguments, these interactions representing attacks.

Def. 1 (Dung AS) *A Dung argumentation system (AS, for short) is a pair $\langle \mathbf{A}, \mathbf{R} \rangle$ where $\mathbf{A}$ is a finite and non-empty set of arguments and $\mathbf{R}$ is a binary relation over $\mathbf{A}$ (a subset of $\mathbf{A} \times \mathbf{A}$), called the* attack relation.

An argumentation system can be represented by a directed graph denoted by $\mathcal{G}$, called the *interaction graph*, in which nodes represent arguments and edges are defined by the attack relation: $\forall a, b \in \mathbf{A}$, $a\mathbf{R}b$ is represented by $a \not\to b$.

Def. 2 (Admissibility) *Given* $\mathsf{AS} = \langle \mathbf{A}, \mathbf{R} \rangle$ *and* $S \subseteq \mathbf{A}$,

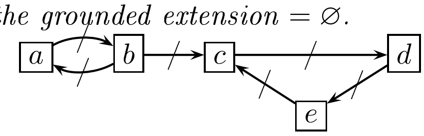
- *S is* conflict-free *in* AS *if and only if (iff for short) there are no arguments $a, b \in S$, such that (s.t. for short) $a\mathbf{R}b$.*

- $a \in \mathbf{A}$ *is* acceptable *in* AS *with respect to (wrt for short)* S *iff* $\forall b \in \mathbf{A}$ *s.t.* $b\mathbf{R}a$, $\exists c \in S$ *s.t.* $c\mathbf{R}b$. $\mathcal{F}$ *denotes the characteristic function of* AS *defined by* $\forall S \subseteq \mathbf{A}$, $\mathcal{F}(S) = \{x \text{ s.t. } x \text{ is acceptable in } \mathsf{AS} \text{ wrt } S\}$.
- S *is* admissible *in* AS *iff* S *is conflict-free and each argument in* S *is acceptable in* AS *wrt* S.

Standard semantics introduced by Dung (preferred, stable, grounded) enable to characterize admissible sets of arguments that satisfy a form of optimality (see [Baroni *et al.*, 2011] for a survey of semantics in abstract argumentation systems).

Def. 3 (Extensions) *Given* $\mathsf{AS} = \langle \mathbf{A}, \mathbf{R} \rangle$ *and* $S \subseteq \mathbf{A}$,
- S *is* a preferred extension *of* AS *iff it is a maximal (wrt* $\subseteq$*) admissible set in* AS.
- S *is* a stable extension *of* AS *iff it is conflict-free and for each* $a \notin S$, *there is* $b \in S$ *s.t.* $b\mathbf{R}a$.
- S *is* the grounded extension *of* AS *iff it is the least fixpoint of* $\mathcal{F}$.

Ex. 2 *Let* AS *be defined by* $\mathbf{A} = \{a, b, c, d, e\}$ *and* $\mathbf{R} = \{(a, b), (b, a), (b, c), (c, d), (d, e), (e, c)\}$ *and represented by the following graph. There are two preferred extensions (*$\{a\}$ *and* $\{b, d\}$*), one stable extension (*$\{b, d\}$*) and the grounded extension* $= \varnothing$.



The status of an argument is determined by its membership to the extensions of the selected semantics: *e.g.*, an argument is "skeptically accepted" (resp. "credulously") if it belongs to all the extensions (resp. at least to one extension) and "rejected" if it does not belong to any extension.

Some interesting properties have been identified:

Prop. 1 *[Dung, 1995]*
1. *There is at least one preferred extension, always a unique grounded extension, while there may be zero, one or several stable extensions.*
2. *Each admissible set is included in a preferred extension.*
3. *Each stable extension is a preferred extension, the converse is false.*
4. *The grounded extension is included in each preferred extension.*
5. *Each argument which is not attacked belongs to the grounded extension (hence to each preferred and to each stable extension).*
6. *If* $\mathbf{R}$ *is finite, then the grounded extension can be computed by iteratively applying the function* $\mathcal{F}$ *from the empty set.*
7. *If* $\mathbf{A}$ *is non empty, then a stable extension is always non empty.*

Prop. 2 *[Dunne and Bench-Capon, 2001; Dunne and Bench-Capon, 2002]*
1. *If* $\mathcal{G}$ *contains no cycle, then* $\langle \mathbf{A}, \mathbf{R} \rangle$ *has a unique preferred extension, which is also the grounded extension and the unique stable extension.*
2. *If* $\{\}$ *is the unique preferred extension of* $\langle \mathbf{A}, \mathbf{R} \rangle$*, then* $\mathcal{G}$ *contains an odd-length cycle.*
3. *If* $\langle \mathbf{A}, \mathbf{R} \rangle$ *has no stable extension, then* $\mathcal{G}$ *contains an odd-length cycle.*
4. *If* $\mathcal{G}$ *contains no odd-length cycle, then preferred and stable extensions coincide.*
5. *If* $\mathcal{G}$ *contains no even-length cycle, then* $\langle \mathbf{A}, \mathbf{R} \rangle$ *has a unique preferred extension.*

2.2 Abstract bipolar argumentation system

The abstract bipolar argumentation framework presented in [Cayrol and Lagasquie-Schiex, 2010] extends Dung's framework in order to take into account both negative interactions expressed by the attack relation and positive interactions expressed by a support relation (see [Amgoud *et al.*, 2008] for a more general survey about bipolarity in argumentation).

Def. 4 (BAS) *A bipolar argumentation system (BAS, for short) is a tuple* $\langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$ *where* $\mathbf{A}$ *is a finite and non-empty set of arguments,* $\mathbf{R}_{\text{att}}$ *is a binary relation over* $\mathbf{A}$ *called the* attack relation *and* $\mathbf{R}_{\text{sup}}$ *is a binary relation over* $\mathbf{A}$ *called the* support relation.

A BAS can still be represented by a directed graph $\mathcal{G}_b$ called the *bipolar interaction graph*, with two kinds of edges. Let a and $b \in \mathbf{A}$, $a\mathbf{R}_{\text{att}}b$ (resp. $a\mathbf{R}_{\text{sup}}b$) means that a attacks b (resp. a supports b) and it is represented by $a \not\to b$ (resp. by $a \to b$).

Among the different variants defined for interpreting a support between arguments, [Boella *et al.*, 2012] proposed the notion of deductive support. This notion is intended to enforce the following constraint: If $b\mathbf{R}_{\text{sup}}c$ then the acceptance of b implies the acceptance of c, and as a consequence the non-acceptance of c implies the non-acceptance of b. The support used in Example 1 can be considered as a deductive one (If X has resigned then the Council of Ministers must be postponed):

Ex.1 (cont'd) *The bipolar argumentation system corresponding to the editorial board can be represented by:*

$$\boxed{f} \not\to \boxed{e} \leftarrow \boxed{d} \not\to \boxed{c} \not\to \boxed{b} \not\to \boxed{a}$$

Then, in order to compute semantics of a BAS, one of the main proposals is to translate the BAS into an AS expressing the new attacks due to the presence of supports (this kind of "flattening" is studied for instance in [Gabbay, 2013]). In the case of deductive support, two kinds of attack can be added. The first one, called mediated attack, corresponds to the case when $b\mathbf{R}_{\text{sup}}c$ and $a\mathbf{R}_{\text{att}}c$: the acceptance of a implies the non-acceptance of c and so the non-acceptance of b:

Def. 5 (Mediated attack) *[Boella* et al.*, 2012] Let* $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$*. There is a* mediated attack *from a to b iff there is a sequence* $a_1\mathbf{R}_{\text{sup}} \ldots \mathbf{R}_{\text{sup}}a_{n-1}$*, and* $a_n\mathbf{R}_{\text{att}}a_{n-1}$*,* $n \geq 3$*, with* $a_1 = b$*,* $a_n = a$*.* $\mathbf{M}^{\mathbf{R}_{\text{sup}}}_{\mathbf{R}_{\text{att}}}$ *denotes the set of mediated attacks generated by* $\mathbf{R}_{\text{sup}}$ *on* $\mathbf{R}_{\text{att}}$*.*

Moreover, the deductive interpretation of support justifies the introduction of another attack (called supported attack in [Cayrol and Lagasquie-Schiex, 2010]): if $a\mathbf{R}_{\text{sup}}c$ and $c\mathbf{R}_{\text{att}}b$, the acceptance of a implies the acceptance of c and the acceptance of c implies the non-acceptance of b; so, the acceptance of a implies the non-acceptance of b.

Def. 6 (Supported attack) *[Cayrol and Lagasquie-Schiex, 2010] Let* $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$*. There is a* supported attack *from a to b iff there is a sequence* $a_1\mathbf{R}_{\text{sup}} \ldots \mathbf{R}_{\text{sup}}a_{n-1}\mathbf{R}_{\text{att}}a_n$*,* $n \geq 3$*, with* $a_1 = a$*,* $a_n = b$*.* $\mathbf{S}^{\mathbf{R}_{\text{sup}}}_{\mathbf{R}_{\text{att}}}$ *denotes the set of supported attacks generated by* $\mathbf{R}_{\text{sup}}$ *on* $\mathbf{R}_{\text{att}}$*.*

So, the deductive interpretation of support produces new kinds of attack, from a to b, in the following cases:

Supported attacks:	Mediated attacks:
$\boxed{a} \to \ldots \to \boxed{c} \not\to \boxed{b}$	$\boxed{b} \to \ldots \to \boxed{c}$, $\boxed{a} \not\to \boxed{c}$

By iterating the construction, `d-attacks` can be defined:

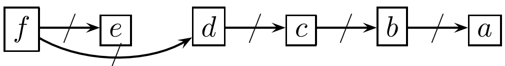
Def. 7 (`d-attacks`) *[Cayrol and Lagasquie-Schiex, 2013] Let* $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$ *with* $\mathbf{R}_{\text{sup}}$ *being a set of deductive supports. There exists a* `d-attack` *from a to b iff*

- *either* $a\mathbf{R}_{\text{att}}b$*, or* $a\mathbf{S}^{\mathbf{R}_{\text{sup}}}_{\mathbf{R}_{\text{att}}}b$*, or* $a\mathbf{M}^{\mathbf{R}_{\text{sup}}}_{\mathbf{R}_{\text{att}}}b$ *(***Basic case***),*
- *or there exists an argument c s.t. there is a sequence of supports from a to c and c* `d-attacks` *b (***Case 1***),*

- *or there exists an argument c s.t. a* `d-attacks` *c and there is a sequence of supports from b to c (***Case 2***).*

$\mathbf{D}^{\mathbf{R}_{\text{sup}}}_{\mathbf{R}_{\text{att}}}$ *denoted the set of* `d-attacks` *generated by $\mathbf{R}_{\text{sup}}$ on $\mathbf{R}_{\text{att}}$. $\langle \mathbf{A}, \mathbf{D}^{\mathbf{R}_{\text{sup}}}_{\mathbf{R}_{\text{att}}} \rangle$ is called the* deductive associated Dung AS of BAS *and denoted by $\mathsf{AS}^{\mathsf{BAS}}$.*

Ex.1 (cont'd) *The deductive associated Dung AS can be represented by (a mediated attack appears from f to d):*



Then, in this system, using for instance the preferred semantics, one can conclude to the acceptability of a (so the information I will be published).

Note that if $\mathbf{R}_{\text{sup}}$ is reduced to a singleton (a, b), Case 1 and Case 2 of Definition 7 do not apply. In this case, the attack (a, c) is added in $\mathsf{AS}^{\mathsf{BAS}}$ iff $(b, c) \in \mathbf{R}_{\text{att}}$ (this is a supported attack) and the attack (c, a) is added in $\mathsf{AS}^{\mathsf{BAS}}$ iff $(c, b) \in \mathbf{R}_{\text{att}}$ (this is a mediated attack).

Turning BAS into $\mathsf{AS}^{\mathsf{BAS}}$ enables to consider the semantics defined by Dung. Moreover, the first step leading to add new attacks, it falls within works about dynamics of AS.

3 Dynamics in argumentation systems

When studying argumentation dynamics, an important issue is to save computation, that is to reuse as far as possible previous computations carried out in the original argumentation system. This issue has been extensively discussed in [Bisquert *et al.*, 2013] with the following methodology: A typology of change operations has been proposed and the impact of each change operation on the computation of the extensions has been studied. So, the work of [Bisquert *et al.*, 2013] is particularly suitable for our purpose and easily adaptable.[1] In this paper, following Example 1, we use the change operations corresponding to either the addition of an argument and the interactions (only attacks) involving it, or the addition of some interactions:

Def. 8 (Addition in an AS) *Let $\mathsf{AS} = \langle \mathbf{A}, \mathbf{R} \rangle$. Two change operations are considered:*

1. *Let z be an argument and $\mathcal{I}_z$ be a set of interactions s.t. $\mathcal{I}_z \subseteq (\mathbf{A} \times \{z\}) \cup (\{z\} \times \mathbf{A})$.* Adding *$z$ and $\mathcal{I}_z$ is a change operation, denoted by $\oplus^z_{\mathcal{I}_z}$, providing a new system s.t.: $\oplus^z_{\mathcal{I}_z} \langle \mathbf{A}, \mathbf{R} \rangle = \langle \mathbf{A} \cup \{z\}, \mathbf{R} \cup \mathcal{I}_z \rangle$.*
2. *Let $\mathcal{I}$ be a set of interactions s.t. $\mathcal{I} \subseteq (\mathbf{A} \times \mathbf{A})$ and $\mathcal{I} \cap \mathbf{R} = \varnothing$.* Adding *$\mathcal{I}$ is a change operation, denoted by $\oplus_{\mathcal{I}}$, providing a new system s.t.: $\oplus_{\mathcal{I}} \langle \mathbf{A}, \mathbf{R} \rangle = \langle \mathbf{A}, \mathbf{R} \cup \mathcal{I} \rangle$.*

The system resulting of a change, denoted by $\mathsf{AS}' = \langle \mathbf{A}', \mathbf{R}' \rangle$, will be represented by the graph $\mathcal{G}'$.

In each case, given a semantics, the set of extensions of AS (resp. AS') is denoted by $\mathbf{E}$ (resp. $\mathbf{E}'$), with $\mathcal{E}_1, \ldots, \mathcal{E}_n$ (resp. $\mathcal{E}'_1, \ldots, \mathcal{E}'_n$) standing for the extensions. We consider the same semantics before and after the change.

The impact of a change operation has been studied in [Bisquert *et al.*, 2013] through the notion of *change property* that can be seen as a set of pairs $(\mathcal{G}, \mathcal{G}')$, where $\mathcal{G}$ and $\mathcal{G}'$ are argumentation graphs. Here we just recall some of these properties.

[1] Other works could be considered for addressing the issue of incremental computation in a dynamic context. [Baroni *et al.*, 2014] for instance presents a more general approach dealing with modularity in abstract argumentation, based on the partition of an argumentation framework in interacting subframeworks. However, the application to our purpose is not straightforward and requires further investigation.

Properties about the set of extensions Change properties express structural modifications of an AS that are caused by a change operation. For that purpose, a partition based on three possible cases of evolution of the set of extensions, has been defined in [Bisquert *et al.*, 2013]:

- the *extensive* case, in which the number of extensions increases,
- the *restrictive* case, in which the number of extensions decreases,
- the *constant* case, in which the number of extensions remains the same.

For each case, numerous sub-cases are proposed and denoted by a letter (e for the *extensive* case, r for the *restrictive* case and c for the *constant* case) subscripted by the expression $\gamma - \gamma'$, where γ (resp. γ') describes the set of extensions before (resp. after) the change. Thus γ and γ' can be:

- $\varnothing$: the set of extensions is empty,
- $1e$: the set of extensions is reduced to one empty extension,
- $1ne$: the set of extensions is reduced to one non-empty extension,
- k (resp. j): the set of extensions contains k (resp. j) extensions s.t. $1 < k$ (resp. $1 < j < k$: note that the symbol j is used only if the symbol k belongs also to the expression $\gamma - \gamma'$).

For instance, the notation $e_{\varnothing-1ne}$ means that the change increases the number of extensions (so it is an *extensive* case), with no initial extension ($\varnothing$) and one non-empty final extension ($1ne$).

Nevertheless, some special sub-cases of the *constant* case are denoted by another method since they are based on notions distinct from the emptiness or the number of the extensions; for these sub-cases, the subscript is replaced by a qualifier. For instance, the *c-conservative* case describes the case where the extensions remain unchanged after the change.

Here is the formal definition of these changes. First, we study the case in which a change increases (resp. decreases) the number of extensions, called *extensive* (resp. *restrictive*) change.

Def. 9 (Extensive and Restrictive changes) *The change from $\mathcal{G}$ to $\mathcal{G}'$ is* extensive *(resp.* restrictive*) iff $|\mathbf{E}| < |\mathbf{E}'|$ (resp. $|\mathbf{E}| > |\mathbf{E}'|$).*[2]
The sub-cases of extensive *changes from $\mathcal{G}$ to $\mathcal{G}'$ are:*
1. $e_{\varnothing-1ne}$ *iff* $|\mathbf{E}| = 0$ *and* $|\mathbf{E}'| = 1$, *with* $\mathcal{E}' \neq \varnothing$.
2. $e_{\varnothing-k}$ *iff* $|\mathbf{E}| < |\mathbf{E}'|$, $|\mathbf{E}| = 0$ *and* $|\mathbf{E}'| > 1$.
3. e_{1e-k} *iff* $|\mathbf{E}| < |\mathbf{E}'|$ *and* $|\mathbf{E}| = 1$, *with* $\mathcal{E} = \varnothing$.
4. e_{1ne-k} *iff* $|\mathbf{E}| < |\mathbf{E}'|$ *and* $|\mathbf{E}| = 1$, *with* $\mathcal{E} \neq \varnothing$.
5. e_{j-k} *iff* $1 < |\mathbf{E}| < |\mathbf{E}'|$.

The sub-cases of restrictive *changes from $\mathcal{G}$ to $\mathcal{G}'$ are:*
1. $r_{1ne-\varnothing}$ *iff* $|\mathbf{E}| = 1$, *with* $\mathcal{E} \neq \varnothing$, *and* $|\mathbf{E}'| = 0$.
2. $r_{k-\varnothing}$ *iff* $|\mathbf{E}| > |\mathbf{E}'|$, $|\mathbf{E}| > 1$ *and* $|\mathbf{E}'| = 0$.
3. r_{k-1e} *iff* $|\mathbf{E}| > |\mathbf{E}'|$ *and* $|\mathbf{E}'| = 1$, *with* $\mathcal{E}' = \varnothing$.
4. r_{k-1ne} *iff* $|\mathbf{E}| > |\mathbf{E}'|$ *and* $|\mathbf{E}'| = 1$, *with* $\mathcal{E}' \neq \varnothing$.
5. r_{k-j} *iff* $1 < |\mathbf{E}'| < |\mathbf{E}|$.

The *constant* change corresponds to the case where the number of extensions remains unchanged while inclusion relations may exist between extensions of $\mathcal{G}$ and extensions of $\mathcal{G}'$.

Def. 10 (Constant change) *The change from $\mathcal{G}$ to $\mathcal{G}'$ is* constant *iff $|\mathbf{E}| = |\mathbf{E}'|$. The sub-cases of* constant *changes from $\mathcal{G}$ to $\mathcal{G}'$ are:*
1. **c-conservative** *iff* $\mathbf{E} = \mathbf{E}'$.
2. c_{1e-1ne} *iff* $\mathbf{E} = \{\{\}\}$ *and* $\mathbf{E}' = \{\mathcal{E}'\}$, *with* $\mathcal{E}' \neq \varnothing$.
3. c_{1ne-1e} *iff* $\mathbf{E} = \{\mathcal{E}\}$, *with* $\mathcal{E} \neq \varnothing$ *and* $\mathbf{E}' = \{\{\}\}$.

[2]Let S be a set, $|S|$ denotes the cardinality of S.

4. **c-expansive** *iff* $\mathbf{E} \neq \varnothing$ *and* $|\mathbf{E}| = |\mathbf{E}'|$ *and* $\forall \mathcal{E}_i \in \mathbf{E}, \exists \mathcal{E}'_j \in \mathbf{E}', \varnothing \neq \mathcal{E}_i \subset \mathcal{E}'_j$ *and* $\forall \mathcal{E}'_j \in \mathbf{E}', \exists \mathcal{E}_i \in \mathbf{E}, \varnothing \neq \mathcal{E}_i \subset \mathcal{E}'_j$.
5. **c-narrowing** *iff* $\mathbf{E} \neq \varnothing$ *and* $|\mathbf{E}| = |\mathbf{E}'|$ *and* $\forall \mathcal{E}_i \in \mathbf{E}, \exists \mathcal{E}'_j \in \mathbf{E}', \varnothing \neq \mathcal{E}'_j \subset \mathcal{E}_i$ *and* $\forall \mathcal{E}'_j \in \mathbf{E}', \exists \mathcal{E}_i \in \mathbf{E}, \varnothing \neq \mathcal{E}'_j \subset \mathcal{E}_i$.
6. **c-altering** *iff* $|\mathbf{E}| = |\mathbf{E}'|$ *and it is neither* c-conservative, *nor* c_{1e-1ne}, *nor* c_{1ne-1e}, *nor* c-expansive, *nor* c-narrowing.

Def.10.1, 10.2, 10.3 and 10.6 are fairly straightforward. Def.10.4 states that a *c-expansive* change is a change where all the extensions of $\mathcal{G}$, which are initially not empty, are increased by some arguments. A *c-narrowing* change, according to Def.10.5, is a change where all the extensions of $\mathcal{G}$ are reduced by some arguments without becoming empty.

Ex.1 (cont'd) *In this example, all the agents always propose constant changes, since they want to take a decision without ambiguity.*

Properties about the acceptability of a set of arguments A change can also have an impact on the acceptability of sets of arguments. For instance, in a dialog, it would be interesting to know whether the addition (or the removal) of an argument modifies the acceptability of the arguments previously accepted. We say "monotony from $\mathcal{G}$ to $\mathcal{G}'$" when every argument accepted *before* the change is still accepted *after* the change, *i.e.*, no accepted argument is lost and there is a (not necessarily strict) *expansion* of acceptability.[3]

Def. 11 (Simple expansive monotony) *The change from* $\mathcal{G}$ *to* $\mathcal{G}'$ *satisfies the property of* simple expansive monotony *iff* $\forall \mathcal{E}_i \in \mathbf{E}, \exists \mathcal{E}'_j \in \mathbf{E}',\ \mathcal{E}_i \subseteq \mathcal{E}'_j$.

Note that [Bisquert *et al.*, 2013] describes many other properties such as, for instance, a property of "enforcement" that would be interesting for J_1 in Example 1 in order to obtain the acceptability of Argument a.

4 Definition of a change operation taking into account support

First of all, it should be noted that turning $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$ into its deductive associated Dung system $\mathsf{AS}^{\mathsf{BAS}}$ corresponds to the *update of a specific system*, $\mathsf{AS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}} \rangle$, the reduction of BAS to its direct attacks (see Figure 1). The next step is to allow for updating a BAS. So Def.8 is generalized:

$$\begin{tikzcd}
\boxed{\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle} \arrow[r, "\text{reduction of BAS}", "\text{to its direct attacks}"'] \arrow[d, "\text{translation (Def. 7)}"'] & \boxed{\mathsf{AS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}} \rangle} \arrow[dl, "\text{change by addition of attacks (Def. 8.2)}"] \\
\boxed{\mathsf{AS}^{\mathsf{BAS}} = \langle \mathbf{A}, \mathbf{D}^{\mathbf{R}_{\text{sup}}}_{\mathbf{R}_{\text{att}}} \rangle} &
\end{tikzcd}$$

Figure 1: The translation of BAS into $\mathsf{AS}^{\mathsf{BAS}}$ is an update

Def. 12 (Addition in a BAS) *Let* $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$. *Two change operations are considered:*

[3] A second case, referred as "monotony from $\mathcal{G}'$ to $\mathcal{G}$", has been described in [Bisquert *et al.*, 2013]. It is not used in this paper.

1. *Let z be an argument, $\mathcal{I}a_z$ be a set of attacks concerning z and $\mathcal{I}s_z$ be a set of supports concerning z ($\mathcal{I}s_z \cup \mathcal{I}a_z$ is denoted by $\mathcal{I}_z$). We assume that $\mathcal{I}_z \subseteq (\mathbf{A} \times \{z\}) \cup (\{z\} \times \mathbf{A})$.*
   Adding *z and $\mathcal{I}_z$ is a change operation, denoted by $\oplus^z_{(\mathcal{I}a,\mathcal{I}s)}$, providing a new bipolar system s.t.: $\oplus^z_{(\mathcal{I}a,\mathcal{I}s)}\langle \mathbf{A}, \mathbf{R}_{\mathrm{att}}, \mathbf{R}_{\mathrm{sup}}\rangle = \langle \mathbf{A} \cup \{z\}, \mathbf{R}_{\mathrm{att}} \cup \mathcal{I}a_z, \mathbf{R}_{\mathrm{sup}} \cup \mathcal{I}s_z\rangle$.*
2. *Let $\mathcal{I}a$ be a set of attacks and $\mathcal{I}s$ be a set of supports ($\mathcal{I}s \cup \mathcal{I}a$ is denoted by $\mathcal{I}$). We assume that $\mathcal{I} \subseteq (\mathbf{A} \times \mathbf{A})$ and $\mathcal{I} \cap (\mathbf{R}_{\mathrm{att}} \cup \mathbf{R}_{\mathrm{sup}}) = \varnothing$.*
   Adding *$\mathcal{I}$ is a change operation, denoted by $\oplus_{(\mathcal{I}a,\mathcal{I}s)}$, providing a new bipolar system s.t.: $\oplus_{(\mathcal{I}a,\mathcal{I}s)}\langle \mathbf{A}, \mathbf{R}_{\mathrm{att}}, \mathbf{R}_{\mathrm{sup}}\rangle = \langle \mathbf{A}, \mathbf{R}_{\mathrm{att}} \cup \mathcal{I}a, \mathbf{R}_{\mathrm{sup}} \cup \mathcal{I}s\rangle$.*

The system resulting of a change is denoted by $\mathsf{BAS}' = \langle \mathbf{A}', \mathbf{R}_{\mathrm{att}}', \mathbf{R}_{\mathrm{sup}}'\rangle$ and its deductive associated Dung AS is denoted by $\mathsf{AS}^{\mathsf{BAS}'}$.

Due to lack of place, **in this paper, we only study the case corresponding to Definition 12.1**. As we consider deductive support and from Definitions 12 and 7, the following consequence obviously holds:

Conseq. 1 *Let $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\mathrm{att}}, \mathbf{R}_{\mathrm{sup}}\rangle$. Let $\oplus^z_{(\mathcal{I}a,\mathcal{I}s)}$ be a change operation on BAS producing BAS'. $\mathsf{AS}^{\mathsf{BAS}'} = \langle \mathbf{A} \cup \{z\}, \mathbf{D}^{\mathbf{R}_{\mathrm{sup}} \cup \mathcal{I}s_z}_{\mathbf{R}_{\mathrm{att}} \cup \mathcal{I}a_z}\rangle$.*

Due to the above result, it seems natural to study the update of BAS by comparing $\mathsf{AS}^{\mathsf{BAS}}$ and $\mathsf{AS}^{\mathsf{BAS}'}$. However, it is not always possible to identify a *unique change* on $\mathsf{AS}^{\mathsf{BAS}}$, *as defined in Definition 8*, that produces $\mathsf{AS}^{\mathsf{BAS}'}$. Indeed, the addition of an argument with interactions in BAS can induce the addition in $\mathbf{D}^{\mathbf{R}_{\mathrm{sup}}}_{\mathbf{R}_{\mathrm{att}}}$ of new attacks between arguments of $\mathbf{A}$ as shown by the following example:

Ex. 3 *Let $\mathsf{BAS} = \langle \{a, b\}, \varnothing, \varnothing\rangle$, let us apply on BAS the change $\oplus^z_{(\mathcal{I}a,\mathcal{I}s)}$ with $\mathcal{I}a_z = \{(a, z)\}$ and $\mathcal{I}s_z = \{(b, z)\}$; in this case, following Def. 12.1 and 7, $\mathsf{AS}^{\mathsf{BAS}'}$ contains the new attack (a, b) that does not concern z.*

Another example shows that this problem also exists even if $\mathcal{I}a_z = \varnothing$:

Ex. 4 *Consider $\mathsf{BAS} = \langle \{a, b, c\}, \{(c, a)\}, \varnothing\rangle$, and apply on BAS the change $\oplus^z_{(\mathcal{I}a,\mathcal{I}s)}$ with $\mathcal{I}a_z = \varnothing$ and $\mathcal{I}s_z = \{(b, z), (z, c)\}$; in this case, following Def. 12.1 and 7, $\mathsf{AS}^{\mathsf{BAS}'}$ contains the new attack (b, a) that does not concern z.*

So, if we add an argument z with at least one support in BAS, the change of $\mathsf{AS}^{\mathsf{BAS}}$ into $\mathsf{AS}^{\mathsf{BAS}'}$ cannot always be expressed using either Def. 8.1 (since attacks are added that do not concern z), or Def. 8.2 (since the argument z is added). The links between the different systems are illustrated by Figure 2.

The difficulties pointed by Examples 3 and 4 suggest to consider two particular cases. The first one concerns a BAS with only one support *from* z to a, z being unattacked. In this case, Definition 7 obviously implies that z has in $\mathsf{AS}^{\mathsf{BAS}}$ exactly the same role as a in AS:

Prop. 3 *Let $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\mathrm{att}}, \mathbf{R}_{\mathrm{sup}}\rangle$ with $\mathbf{R}_{\mathrm{sup}} = \{(z, a)\}$ and z is not attacked in BAS. The following properties hold:*
- *if a is unattacked in BAS then z is unattacked in $\mathsf{AS}^{\mathsf{BAS}}$ (no direct attack, no direct or inductive supported or mediated attack on z);*
- *if a is attacked by b in BAS then z is attacked by b in $\mathsf{AS}^{\mathsf{BAS}}$ (this is a mediated attack on z);*
- *if a attacks b in BAS then z attacks b in $\mathsf{AS}^{\mathsf{BAS}}$ (this is a supported attack).*
- *if a is defended by c against b in BAS then z is defended by c against b in $\mathsf{AS}^{\mathsf{BAS}}$ (the defence of a direct attack on a can be used for the defence of the mediated attack on z).*

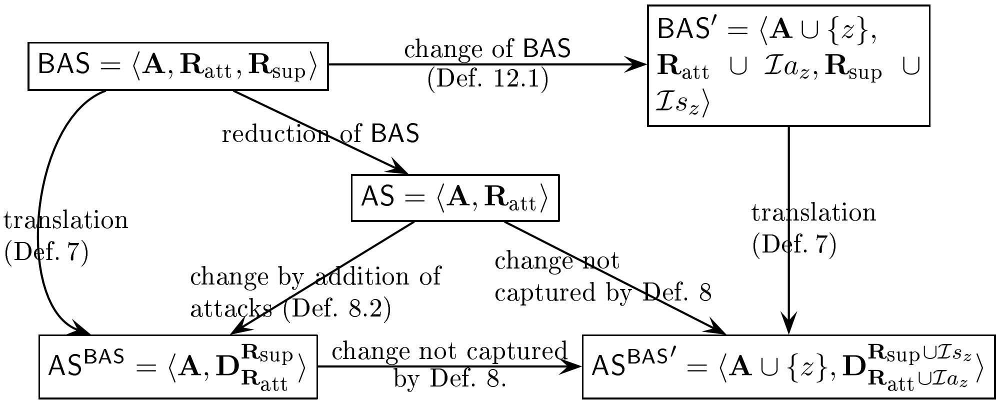


Figure 2: Links between the different systems

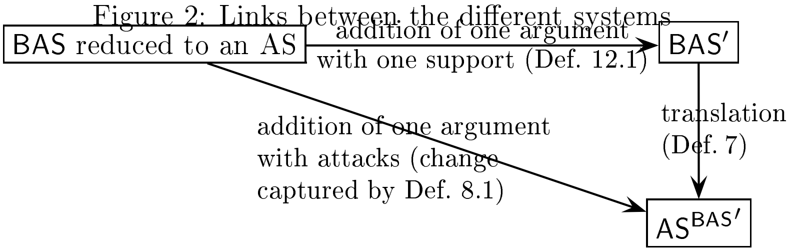


Figure 3: Links between systems if there is no support in BAS

- *if c is defended by b against a in* BAS *then c is defended by b against z in* $\mathsf{AS}^{\mathsf{BAS}}$ *(a mediated attack can be used as a defence against a supported attack).*

A second particular case concerns a BAS with only one support *on* an unattacked argument. In this case, Definition 7 obviously implies that the set of attacks remains unchanged:

Prop. 4 *Let* $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$ *with* $\mathbf{R}_{\text{sup}} = \{(a, z)\}$ *and z unattacked by* BAS*. Then* $\mathbf{D}^{\mathbf{R}_{\text{sup}}}_{\mathbf{R}_{\text{att}}} = \mathbf{R}_{\text{att}}$.

Moreover, in these particular cases, following Definition 12.1, Propositions 3 and 4, the addition of one argument involved in only one support in BAS cannot add attacks between arguments of $\mathbf{A}$ and preserves acceptability:

Prop. 5 *Let* $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$ *s.t.* $\mathbf{R}_{\text{sup}} = \varnothing$.[4] *Let* $\oplus^{z}_{(\mathcal{I}a, \mathcal{I}s)}$ *be a change operation defined on* BAS *with* $\mathcal{I}a_z = \varnothing$, $|\mathcal{I}s_z| = 1$ *and producing* BAS'.

- $\forall x, y \in \mathbf{A}$, *s.t. y does not attack x in* BAS *then there is no attack from y to x in* $\mathsf{AS}^{\mathsf{BAS}'}$.
- $\forall y \in \mathbf{A}$, *if y is unattacked in* BAS *then it remains unattacked in* $\mathsf{AS}^{\mathsf{BAS}'}$.
- *Consider $\mathcal{F}$ (resp. $\mathcal{F}'$) the characteristic function of* AS *(resp.* $\mathsf{AS}^{\mathsf{BAS}'}$*).* $\forall S \subseteq \mathbf{A}$, $\mathcal{F}(S) \subseteq \mathcal{F}'(S)$.

Thus, considering a BAS reduced to an AS (*i.e. without any support*), if we add only one argument with one support, the links between the different systems are given by Figure 3.

So we are able to characterize the addition of a support by an addition of attacks. In the next section, we study this simplified change operation.

5 Characterizing the addition of an argument and a support

In Section 5.1 (resp. Section 5.2), we give some results about the characterization of the addition of a supported (resp. supporting) argument in a BAS.

[4]In this case, BAS is reduced to an AS. So BAS, its reduction AS and $\mathsf{AS}^{\mathsf{BAS}}$ collapse.

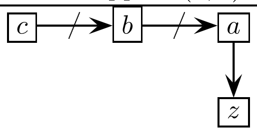
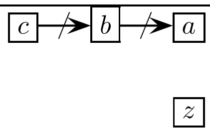
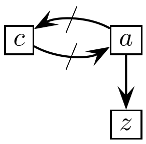
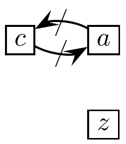
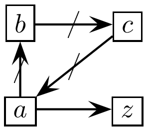
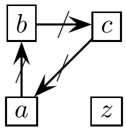
<table>
<tr><td rowspan="2">BAS (reduced to an AS) updated with z and the support (a, z)</td><td rowspan="2">$\mathsf{AS}^{\mathsf{BAS}'}$</td><td colspan="2">Extensions</td></tr>
<tr><td>before change</td><td>after change</td></tr>
<tr><td rowspan="2"></td><td rowspan="2"></td><td>$\{a, c\}$ is the grounded, preferred and stable extension</td><td>$\{a, c, z\}$ is the grounded, preferred and stable extension</td></tr>
<tr><td colspan="2">The change is c-expansive</td></tr>
<tr><td rowspan="2"></td><td rowspan="2"></td><td>$\varnothing$ is the grounded extension; $\{a\}$ and $\{c\}$ are the preferred and stable extensions</td><td>$\{z\}$ is the grounded extension; $\{a, z\}$ and $\{c, z\}$ are the preferred and stable extensions</td></tr>
<tr><td colspan="2">The change is c-expansive(preferred, stable) or c_{1e-1ne}(grounded)</td></tr>
<tr><td rowspan="2"></td><td rowspan="2"></td><td>$\varnothing$ is the grounded and preferred extensions; there is no stable extension</td><td>$\{z\}$ is the grounded and preferred extensions; there is no stable extension</td></tr>
<tr><td colspan="2">The change is c-expansive(preferred), or c_{1e-1ne}(grounded), or c-conservative(stable)</td></tr>
</table>

Table 1: Addition of a supported argument in an AS

5.1 Case of an added supported argument

In this case, as a direct application of Proposition 4, we prove that the update of a BAS without supports has a deductive associated Dung AS that corresponds to the addition of an argument without interaction into the initial BAS.

Prop. 6 *Let* $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\mathrm{att}}, \mathbf{R}_{\mathrm{sup}} \rangle$ *s.t.* $\mathbf{R}_{\mathrm{sup}} = \varnothing$*. Let* $\oplus^{z}_{(\mathcal{I}a, \mathcal{I}s)}$ *be a change operation defined on* BAS *with* $\mathcal{I}a_z = \varnothing$ *and* $\mathcal{I}s_z = \{(a, z)\}$ *and producing* BAS'*.* $\mathsf{AS}^{\mathsf{BAS}'} = \oplus^{z}_{\varnothing} \langle \mathbf{A}, \mathbf{R}_{\mathrm{att}} \rangle$*.*

Due to Proposition 6, Definitions 8.1 and 12.1, we have:

Prop. 7 *Let* $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\mathrm{att}}, \mathbf{R}_{\mathrm{sup}} \rangle$ *s.t.* $\mathbf{R}_{\mathrm{sup}} = \varnothing$*. Let* $\oplus^{z}_{(\mathcal{I}a, \mathcal{I}s)}$ *be a change operation defined on* BAS *with* $\mathcal{I}a_z = \varnothing$ *and* $\mathcal{I}s_z = \{(a, z)\}$ *and producing* BAS'*.*

- *Let s be a semantics belonging to {grounded, preferred, stable}.* $\mathcal{E}$ *is an extension of* AS *under s iff* $\mathcal{E}' = \mathcal{E} \cup \{z\}$ *is an extension of* $\mathsf{AS}^{\mathsf{BAS}'}$ *under s.*
- *There is no stable extension in* AS *iff there is no stable extension in* $\mathsf{AS}^{\mathsf{BAS}'}$*.*

And an obvious consequence of Proposition 7 is:

Conseq. 2 *The change* $\oplus^{z}_{(\varnothing, \{(a,z)\})}$ *is only either* c-expansive*, or* c_{1e-1ne}*, or* c-conservative*. In the last case, the only possibility is* $\mathbf{E} = \mathbf{E}' = \varnothing$*.*

Some examples of this change are given in Table 1.

5.2 Case of an added supporting argument

In this case, the existence of cycles is preserved as shown by:

Prop. 8 *Let* $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$ *s.t.* $\mathbf{R}_{\text{sup}} = \varnothing$. *Let* $\oplus^{z}_{(\mathcal{I}a,\mathcal{I}s)}$ *be a change operation defined on* BAS *with* $\mathcal{I}a_z = \varnothing$ *and* $\mathcal{I}s_z = \{(z,a)\}$ *and producing* BAS'.

- *If* a *belongs to a cycle of attacks in* BAS *then* z *belongs to a new cycle of attacks in* $\mathsf{AS}^{\mathsf{BAS}'}$ *and the length of both cycles is the same.*
- *If* a *does not belong to a cycle of attacks in* BAS *then there is no cycle of attacks in* $\mathsf{AS}^{\mathsf{BAS}'}$ *involving* z.

This result is proven using Definitions 5 to 7 and by *reductio ad absurdum* for the second item.

Following Definition 7 and Proposition 3, we can characterize the impact of this change for stable semantics:

Prop. 9 *Let* $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$ *s.t.* $\mathbf{R}_{\text{sup}} = \varnothing$. *Let* $\oplus^{z}_{(\mathcal{I}a,\mathcal{I}s)}$ *be a change operation defined on* BAS *with* $\mathcal{I}a_z = \varnothing$ *and* $\mathcal{I}s_z = \{(z,a)\}$ *and producing* BAS'. *Let* $\mathcal{E}$ *be a stable extension of* AS*:*

- *if* $a \notin \mathcal{E}$ *then* $\mathcal{E}$ *is a stable extension of* $\mathsf{AS}^{\mathsf{BAS}'}$*;*
- *if* $a \in \mathcal{E}$ *then* $\mathcal{E} \cup \{z\}$ *is a stable extension of* $\mathsf{AS}^{\mathsf{BAS}'}$.

And more generally, the *simple expansive monotony* of the change operation can be proven:

Prop. 10 *Let* $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$ *s.t.* $\mathbf{R}_{\text{sup}} = \varnothing$. *Let* s *be a semantics belonging to* $\{$*grounded, preferred, stable*$\}$. *Let* $\oplus^{z}_{(\mathcal{I}a,\mathcal{I}s)}$ *be a change operation defined on* BAS *with* $\mathcal{I}a_z = \varnothing$ *and* $\mathcal{I}s_z = \{(z,a)\}$ *and producing* BAS'.

$\forall \mathcal{E}$ *extension of* AS *under* s, $\exists \mathcal{E}'$ *an extension of* $\mathsf{AS}^{\mathsf{BAS}'}$ *under* s *s.t.* $\mathcal{E} \subseteq \mathcal{E}'$.

This result is proven using Definition 3, Propositions 3 and 5, by induction on the characteristic function for the grounded semantics, showing that $\mathcal{E}$ is admissible in $\mathsf{AS}^{\mathsf{BAS}'}$ for the preferred semantics and following Proposition 9 for the stable semantics.

An obvious consequence of the two previous results is:

Conseq. 3 *The change* $\oplus^{z}_{(\varnothing,\{(z,a)\})}$ *cannot be* restrictive*, nor* c-narrowing*, nor* c-altering*, nor* c_{1ne-1e}.

Some examples of this change are given in Table 2.

6 Conclusion and future works

This paper presents preliminary work about change for abstract bipolar argumentation systems, *i.e.* where there exist two kinds of interaction, attacks and supports. The central idea is to take advantage of two kinds of previous works, works about dynamics in argumentation systems (AS) and works about bipolar argumentation systems (BAS). Indeed, it has been shown that a BAS can be turned into a standard Dung's AS by adding appropriate attacks. Our main contribution is to show how the addition of one argument together with one support involving it (and without any attack) impacts the extensions of the resulting system. In this particular case, we have clearly identified the attacks that must be added and we have obtained specific properties which enable to characterize this change. These characterizations refine and complete the results presented in [Bisquert *et al.*, 2013] that cannot be used directly for characterizing the impact of these new attacks (the conditions used in [Bisquert *et al.*, 2013] are too strong with regard to our case and thus they cannot be satisfied here). Our work is of particular interest in a multiagent context if we do not want to recompute the extensions when a agent gives a new argument that supports (or is supported by) an already existing argument.

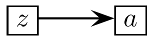
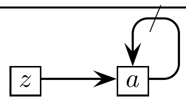
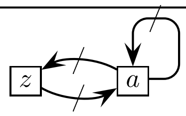
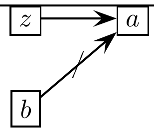
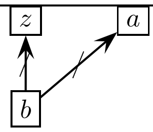
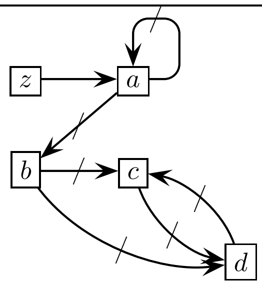
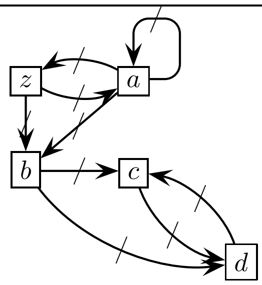
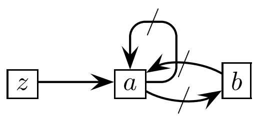
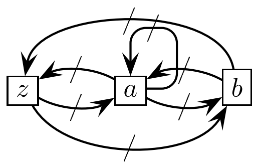
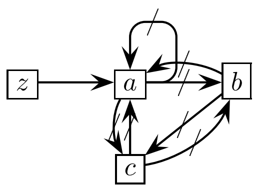
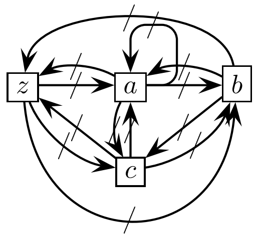
<table>
<tr><th rowspan="2">BAS (reduced to an AS) updated with z and the support (z, a)</th><th rowspan="2">$\mathsf{AS}^{\mathsf{BAS}'}$</th><th colspan="2">Extensions</th></tr>
<tr><th>before change</th><th>after change</th></tr>
<tr><td rowspan="2"></td><td rowspan="2"></td><td>$\{a\}$ is the grounded, preferred and stable extension</td><td>$\{a, z\}$ is the grounded, preferred and stable extension</td></tr>
<tr><td colspan="2">The change is c-expansive</td></tr>
<tr><td rowspan="2"></td><td rowspan="2"></td><td>$\varnothing$ is the grounded and preferred extension; there is no stable extension</td><td>$\{z\}$ is the grounded, preferred and stable extension</td></tr>
<tr><td colspan="2">The change is c_{1e-1ne} (grounded, preferred) or $e_{\varnothing-1ne}$ (stable)</td></tr>
<tr><td rowspan="2"></td><td rowspan="2"></td><td>$\{b\}$ is the grounded, preferred and stable extension</td><td>$\{b\}$ is the grounded, preferred and stable extension</td></tr>
<tr><td colspan="2">The change is c-conservative</td></tr>
<tr><td rowspan="2"></td><td rowspan="2"></td><td>$\varnothing$ is the grounded and preferred extension; there is no stable extension</td><td>$\varnothing$ is the grounded extension; $\{z, c\}$ and $\{z, d\}$ are the preferred and stable extensions</td></tr>
<tr><td colspan="2">The change is c-conservative (grounded) or e_{1e-k} (preferred), or $e_{\varnothing-k}$ (stable)</td></tr>
<tr><td rowspan="2"></td><td rowspan="2"></td><td>$\varnothing$ is the grounded extension; $\{b\}$ is the preferred and stable extension</td><td>$\varnothing$ is the grounded extension; $\{b\}$ and $\{z\}$ are the preferred and stable extensions</td></tr>
<tr><td colspan="2">The change is c-conservative (grounded) or e_{1ne-k} (preferred, stable)</td></tr>
<tr><td rowspan="2"></td><td rowspan="2"></td><td>$\varnothing$ is the grounded extension; $\{b\}$ and $\{c\}$ are the preferred and stable extensions</td><td>$\varnothing$ is the grounded extension; $\{b\}$, $\{c\}$ and $\{z\}$ are the preferred and stable extensions</td></tr>
<tr><td colspan="2">The change is c-conservative (grounded) or e_{j-k} (preferred, stable)</td></tr>
</table>

Table 2: Addition of a supporting argument in an AS

Although our results are given for very simple cases (addition of one argument and one support), we think that they can be generalized considering that the addition of a set of arguments with interactions can be viewed as a sequence of "simple" additions. Nevertheless, in order to achieve this generalization, there are two issues to be solved: (1) characterize the addition of an argument with attacks (as was done for AS; results given in [Bisquert *et al.*, 2013] will be useful) and (2) study the addition of interactions (this operation has been defined in [Bisquert *et al.*, 2013] for AS and in our paper for BAS but not completely studied). This last study could also give a way for computing directly the $\mathsf{AS}^{\mathsf{BAS}}$ of a BAS. It will be the subject of future works.

Moreover, our work concerns only a special variant of support, the deductive one. Using the duality between necessary and deductive supports, our results can be easily translated for necessary support. However, it remains to adapt them to the case of a generalized support (a support from a set of arguments to an argument as proposed by [Nouioua, 2013]).

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A Proofs

Conseq.1: Let $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\mathrm{att}}, \mathbf{R}_{\mathrm{sup}} \rangle$. Let $\oplus^{z}_{(\mathcal{I}a,\mathcal{I}s)}$ be a change operation on BAS producing BAS'. $\mathsf{AS}^{\mathsf{BAS}'} = \langle \mathbf{A} \cup \{z\}, \mathbf{D}^{\mathbf{R}_{\mathrm{sup}} \cup \mathcal{I}s_z}_{\mathbf{R}_{\mathrm{att}} \cup \mathcal{I}a_z} \rangle$. □

Proof of Conseq.1: By Definition 12.1, $\mathsf{BAS}' = \langle \mathbf{A} \cup \{z\}, \mathbf{R}_{\text{att}} \cup \mathcal{I}a_z, \mathbf{R}_{\text{sup}} \cup \mathcal{I}s_z \rangle$. Then, following Definition 7, $\mathsf{AS}^{\mathsf{BAS}'} = \langle \mathbf{A} \cup \{z\}, \mathbf{D}^{\mathbf{R}_{\text{sup}} \cup \mathcal{I}s_z}_{\mathbf{R}_{\text{att}} \cup \mathcal{I}a_z} \rangle$. □

Prop.5: Let $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$ s.t. $\mathbf{R}_{\text{sup}} = \varnothing$. Let $\oplus^z_{(\mathcal{I}a, \mathcal{I}s)}$ be a change operation defined on BAS with $\mathcal{I}a_z = \varnothing$, $|\mathcal{I}s_z| = 1$ and producing BAS'.

- For all $x, y \in \mathbf{A}$, s.t. y does not attack x in BAS then there is no attack from y to x in $\mathsf{AS}^{\mathsf{BAS}'}$.
- For all $y \in \mathbf{A}$, if y is unattacked in BAS then it remains unattacked in $\mathsf{AS}^{\mathsf{BAS}'}$.
- Consider $\mathcal{F}$ (resp. $\mathcal{F}'$) the characteristic function of AS (resp. $\mathsf{AS}^{\mathsf{BAS}'}$). $\forall S \subseteq \mathbf{A}$, $\mathcal{F}(S) \subseteq \mathcal{F}'(S)$.

□

Proof of Prop.5:

- The first item is proven using Definition 5 to Definition 7: we know that all the attacks in $\mathbf{D}^{\mathbf{R}_{\text{sup}} \cup \mathcal{I}s_z}_{\mathbf{R}_{\text{att}} \cup \mathcal{I}a_z}$ are produced using $\mathbf{R}_{\text{att}}$ and $\mathbf{R}_{\text{sup}} \cup \mathcal{I}s_z$ (either directly, or inductively by building the supported or mediated attacks); and we assume that $\mathbf{R}_{\text{sup}} = \varnothing$ and $\mathcal{I}s_z$ is reduced to one support (either (z, a) or (a, z)), so the only support concerns z that is not in $\mathbf{A}$; so, following Definition 12.1, Proposition 3 and Proposition 4, the set of attacks between arguments of $\mathbf{A}$ remain unchanged in $\mathsf{AS}^{\mathsf{BAS}'}$.
- The second item is trivially deduced from the first one.
- For the third item, consider $\mathcal{F}$ (resp. $\mathcal{F}'$) the characteristic function of AS (resp. $\mathsf{AS}^{\mathsf{BAS}'}$). Let $x \in \mathcal{F}(S)$ s.t. x is attacked in $\mathsf{AS}^{\mathsf{BAS}'}$. Either x is attacked in $\mathsf{AS}^{\mathsf{BAS}'}$ by only arguments of $\mathbf{A}$ and then following the previous items, x is defended by S in $\mathsf{AS}^{\mathsf{BAS}'}$; or x is also attacked in $\mathsf{AS}^{\mathsf{BAS}'}$ by z and then x was also attacked by a in AS (following Definition 7) and defended by S in AS and in $\mathsf{AS}^{\mathsf{BAS}'}$ (following Proposition 3).

□

Prop.6: Let $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$ s.t. $\mathbf{R}_{\text{sup}} = \varnothing$. Let $\oplus^z_{(\mathcal{I}a, \mathcal{I}s)}$ be a change operation defined on BAS with $\mathcal{I}a_z = \varnothing$ and $\mathcal{I}s_z = \{(a, z)\}$ and producing BAS'. $\mathsf{AS}^{\mathsf{BAS}'} = \oplus^z_{\varnothing} \langle \mathbf{A}, \mathbf{R}_{\text{att}} \rangle$.
□

Proof of Prop.6: By Definition 12.1, $\mathsf{BAS}' = \langle \mathbf{A} \cup \{z\}, \mathbf{R}_{\text{att}}, \{(a, z)\} \rangle$. In this case, following Proposition 4, the set of `d-attacks` exactly corresponds to $\mathbf{R}_{\text{att}}$. Then $\mathsf{AS}^{\mathsf{BAS}'} = \langle \mathbf{A} \cup \{z\}, \mathbf{R}_{\text{att}} \rangle$ and trivally corresponds to $\oplus^z_{\varnothing} \langle \mathbf{A}, \mathbf{R}_{\text{att}} \rangle$ (see Definition 8.1). □

Prop.7: Let $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$ s.t. $\mathbf{R}_{\text{sup}} = \varnothing$. Let $\oplus^z_{(\mathcal{I}a, \mathcal{I}s)}$ be a change operation defined on BAS with $\mathcal{I}a_z = \varnothing$ and $\mathcal{I}s_z = \{(a, z)\}$ and producing BAS'.

- Let s be a semantics belonging to {grounded, preferred, stable}. $\mathcal{E}$ is an extension of AS under s iff $\mathcal{E}' = \mathcal{E} \cup \{z\}$ is an extension of $\mathsf{AS}^{\mathsf{BAS}'}$ under s.
- There is no stable extension in AS iff there is no stable extension in $\mathsf{AS}^{\mathsf{BAS}'}$.

□

Proof of Prop.7:

- Following Proposition 6, $\mathsf{AS}^{\mathsf{BAS}'} = \oplus^z_{\varnothing}\langle \mathbf{A}, \mathbf{R}_{\text{att}}\rangle$. So, following Definition 8.1, $\mathsf{AS}^{\mathsf{BAS}'} = \langle \mathbf{A} \cup \{z\}, \mathbf{R}_{\text{att}}\rangle$. Since z is involved in no attack, z must be added to any (grounded, preferred, stable) extension of $\mathsf{BAS} = \mathsf{AS}$ and no other argument is affected.
- It follows directly from the previous item by contraposition. Note that this point makes sense only for stable semantics. Note also that $\mathbf{A}$ is not empty since there exists at least the argument $a \in \mathbf{A}$ that supports z.

$\square$

Prop.8: Let $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}}\rangle$ s.t. $\mathbf{R}_{\text{sup}} = \varnothing$. Let $\oplus^z_{(\mathcal{I}a,\mathcal{I}s)}$ be a change operation defined on BAS with $\mathcal{I}a_z = \varnothing$ and $\mathcal{I}s_z = \{(z, a)\}$ and producing BAS'.

- If a belongs to a cycle of attacks in BAS then z belongs to a new cycle of attacks in $\mathsf{AS}^{\mathsf{BAS}'}$ and the length of both cycles is the same.
- If a does not belong to a cycle of attacks in BAS then there is no cycle of attacks in $\mathsf{AS}^{\mathsf{BAS}'}$ involving z.

$\square$

Proof of Prop.8: Consider $z \notin \mathbf{A}$ and $a \in \mathbf{A}$ s.t. z supports a. Let $\mathsf{AS}^{\mathsf{BAS}'}$ be the deductive associated Dung AS of BAS'

- If a belongs to a cycle of attacks in BAS then $\exists n \geq 1$ s.t. $a = a_1\mathbf{R}_{\text{att}}a_2\mathbf{R}_{\text{att}}\ldots\mathbf{R}_{\text{att}}a_n\mathbf{R}_{\text{att}}a_1 = a$; so considering Definition 5 and Definition 6, there exist a supported attack (z, a_2) and a mediated attack (a_n, z) in $\mathsf{AS}^{\mathsf{BAS}'}$; moreover since attacks in BAS are also attacks in BAS' and remain in $\mathsf{AS}^{\mathsf{BAS}'}$ (see Definition 7), z belongs to the cycle of attacks $(z, a_2, \ldots, a_n, z)$ in $\mathsf{AS}^{\mathsf{BAS}'}$; moreover the length of this cycle in $\mathsf{AS}^{\mathsf{BAS}'}$ is equals to the length of the cycle containing a in BAS.
- Proof by *reductio ad absurdum*: if z belongs to a cycle of attacks $(z, a_1, \ldots, a_n, z)$ in $\mathsf{AS}^{\mathsf{BAS}'}$, then considering than $z \notin \mathbf{A}$ and the fact that $\mathbf{D}^{\mathbf{R}_{\text{sup}}\cup\mathcal{I}s_z}_{\mathbf{R}_{\text{att}}\cup\mathcal{I}a_z}$ is built with $\mathbf{R}_{\text{sup}} = \mathcal{I}a_z = \varnothing$, we can deduce that the attacks (z, a_1) and (a_n, z) are new attacks generated by the support (z, a), whereas (due to Proposition 6) the other attacks in the cycle belong to $\mathbf{R}_{\text{att}}$; moreover the attack (z, a_1) can appear only if there exists x s.t. z supports x and $x\mathbf{R}_{\text{att}}a_1$; similarly the attack (a_n, z) can appear only if there exists y s.t. z supports y and $a_n\mathbf{R}_{\text{att}}y$; knowing that there is only one support added to BAS, $x = y = a$; so there exists in BAS a sequence $a\mathbf{R}_{\text{att}}a_1\mathbf{R}_{\text{att}}\ldots\mathbf{R}_{\text{att}}a_n\mathbf{R}_{\text{att}}a$; this means that a belongs to a cycle of attacks in BAS and that is in contradiction with the assumption.

$\square$

Conseq.2: The change $\oplus^z_{(\varnothing,\{(a,z)\})}$ is only either *c-expansive*, or c_{1e-1ne}, or *c-conservative*. In the last case, the only possibility is $\mathbf{E} = \mathbf{E}' = \varnothing$. $\square$

Proof of Conseq.2: Following Proposition 7 and the definitions of change properties, if there exists at least one extension before the change, it is obvious that the change is *c-expansive* or c_{1e-1ne} (since at each extension $\mathcal{E}$ of BAS corresponds an extension of BAS' that stricly contains $\mathcal{E}$). And, following Proposition 7, if there is no extension before the change (this is possible only with stable semantics) then there is also no extension after the change (*c-conservative* change). $\square$

Prop.9: Let $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}}\rangle$ s.t. $\mathbf{R}_{\text{sup}} = \varnothing$. Let $\oplus^z_{(\mathcal{I}a,\mathcal{I}s)}$ be a change operation defined on BAS with $\mathcal{I}a_z = \varnothing$ and $\mathcal{I}s_z = \{(z, a)\}$ and producing BAS'. Let $\mathcal{E}$ be a stable extension of AS:

- if $a \notin \mathcal{E}$ then $\mathcal{E}$ is a stable extension of $\mathsf{AS}^{\mathsf{BAS}'}$;

- if $a \in \mathcal{E}$ then $\mathcal{E} \cup \{z\}$ is a stable extension of $\mathsf{AS}^{\mathsf{BAS}'}$.

$\square$

Proof of Prop.9: Let $\mathcal{E}$ be a stable extension. $\mathcal{E}$ stable in AS means that $\mathcal{E}$ is conflictfree in AS and $\mathcal{E}$ attacks $\mathbf{A} \setminus \mathcal{E}$.

- Consider the case when $a \notin \mathcal{E}$. As $\mathcal{E}$ is conflictfree in AS, due to Proposition 5, $\mathcal{E}$ remains conflictfree in $\mathsf{AS}^{\mathsf{BAS}'}$. Then a is attacked by an argument x of $\mathcal{E}$. Following Proposition 3, z is also attacked by x in $\mathsf{AS}^{\mathsf{BAS}'}$ and so $\mathcal{E}$ attacks $\mathbf{A} \cup \{z\} \setminus \mathcal{E}$; that implies that $\mathcal{E}$ is a stable extension of $\mathsf{AS}^{\mathsf{BAS}'}$.
- Consider the case when $a \in \mathcal{E}$. $\mathcal{E} \cup \{z\}$ attacks $\mathbf{A} \setminus (\mathcal{E} \cup \{z\})$. We show by *reductio ad absurdum* that $\mathcal{E} \cup \{z\}$ is conflictfree; we assume that there is an argument $x \in \mathcal{E}$ such that either x attacks z, or z attacks x; in the first case, following Definition 7, there exists in AS an attack from x to a, so $\mathcal{E}$ is not conflictfree; and in the second case, once again following Definition 7, there exists in AS an attack from a to x, so $\mathcal{E}$ is not conflictfree; in each case, there is a contradiction. Thus $\mathcal{E} \cup \{z\}$ is conflictfree in $\mathsf{AS}^{\mathsf{BAS}'}$ and it is a stable extension of $\mathsf{AS}^{\mathsf{BAS}'}$.

$\square$

Prop.10: Let $\mathsf{BAS} = \langle \mathbf{A}, \mathbf{R}_{\text{att}}, \mathbf{R}_{\text{sup}} \rangle$ s.t. $\mathbf{R}_{\text{sup}} = \varnothing$. Let s be a semantics belonging to {grounded, preferred, stable}. Let $\oplus^{z}_{(\mathcal{I}a, \mathcal{I}s)}$ be a change operation defined on BAS with $\mathcal{I}a_z = \varnothing$ and $\mathcal{I}s_z = \{(z, a)\}$ and producing BAS'.

$\forall \mathcal{E}$ extension of AS under s, $\exists \mathcal{E}'$ an extension of $\mathsf{AS}^{\mathsf{BAS}'}$ under s s.t. $\mathcal{E} \subseteq \mathcal{E}'$. $\square$

Proof of Prop.10:

- **Grounded semantics** $\mathcal{F}$ (resp. $\mathcal{F}'$) denotes the characteristic function of AS (resp. $\mathsf{AS}^{\mathsf{BAS}'}$). Let prove by induction on $i \geq 1$ that $\forall i \geq 1$, $\mathcal{F}^i(\varnothing) \subseteq \mathcal{F}'^i(\varnothing)$.
  The case $i = 1$ is trivial, following Proposition 5.
  Assume that $\mathcal{F}^i(\varnothing) \subseteq \mathcal{F}'^i(\varnothing)$. Take $S = \mathcal{F}^i(\varnothing)$. From Proposition 5, $\mathcal{F}(S) \subseteq \mathcal{F}'(S)$. So $\mathcal{F}^{i+1}(\varnothing) \subseteq \mathcal{F}'(\mathcal{F}^i(\varnothing))$. As $\mathcal{F}'$ is monotonic and using the inductive assumption, we have $\mathcal{F}'(\mathcal{F}^i(\varnothing)) \subseteq \mathcal{F}'(\mathcal{F}'^i(\varnothing)) = \mathcal{F}'^{i+1}(\varnothing)$.
  So $\forall i \geq 1$, $\mathcal{F}^i(\varnothing) \subseteq \mathcal{F}'^i(\varnothing)$. Hence, $\mathcal{E} \subseteq \mathcal{E}'$.
- **Preferred semantics** It is sufficient to show that each preferred extension $\mathcal{E}$ of AS is admissible in $\mathsf{AS}^{\mathsf{BAS}'}$. Let $\mathcal{E}$ be a preferred extension of AS. $\mathcal{E}$ is conflictfree in AS and so it is also conflictfree in $\mathsf{AS}^{\mathsf{BAS}'}$ (*cf* Proposition 5). Assume that $y \in \mathcal{E}$ is attacked by x in $\mathsf{AS}^{\mathsf{BAS}'}$. Two cases are possible: either $x \in \mathbf{A}$ or $x = z$.
  If $x \in \mathbf{A}$, the attack (x, y) is already in AS and since $\mathcal{E}$ is admissible in AS there exists $e \in \mathcal{E}$ s.t. e attacks x in AS. So e defends y in $\mathsf{AS}^{\mathsf{BAS}'}$.
  If $x = z$, then the attack (z, y) in $\mathsf{AS}^{\mathsf{BAS}'}$ is generated using the attack (a, y) in AS and the support (z, a). Since $\mathcal{E}$ is admissible, there exists $e \in \mathcal{E}$ s.t. e attacks a in AS and, following Proposition 3, e defends y against $x = z$ in $\mathsf{AS}^{\mathsf{BAS}'}$.
  In each case, $\mathcal{E}$ defends y against x in $\mathsf{AS}^{\mathsf{BAS}'}$. Thus $\mathcal{E}$ is admissible in $\mathsf{AS}^{\mathsf{BAS}'}$ and so included in a preferred extension of $\mathsf{AS}^{\mathsf{BAS}'}$.
- **Stable semantics** Trivially follows Proposition 9

$\square$

Conseq.3: The change $\oplus^{z}_{(\varnothing, \{(z,a)\})}$ cannot be *restrictive*, nor *c-narrowing*, nor *c-altering*, nor c_{1ne-1e}. $\square$

Proof of Conseq.3: It is obvious following Proposition 10 since each extension of BAS is always included in an extension of BAS' (so the change cannot be *c-narrowing*, nor *c-altering*). Moreover, the number of extensions cannot be decreased (so the change cannot be *restrictive*) and an empty extension cannot be appeared (so the change cannot be c_{1ne-1e}). $\square$