Argumentation Frameworks with Recursive Attacks and Evidence-Based Supports

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Abstract. The purpose of this work is to study a generalisation of Dung's abstract argumentation frameworks that allows representing positive interactions (called *supports*). The notion of support studied here is based in the intuition that every argument must be supported by some chain of supports from some special arguments called *prima-facie*. The theory developed also allows the representation of both *recursive attacks* and *supports*, that is, a class of attacks or supports whose targets are other attacks or supports. We do this by developing a theory of argumentation where the classic role of *attacks* in defeating arguments is replaced by a subset of them, which is extension dependent and which, intuitively, represents a set of "valid attacks" with respect to the extension. Similarly, only the subset of "valid supports" is allowed to support other elements (arguments, attacks or supports). This theory displays a conservative generalisation of Dung's semantics (complete, preferred and stable) and also of their principles (conflict-freeness, acceptability and admissibility). When restricted to finite non-recursive frameworks, we are also able to prove a one-to-one correspondence with Evidence-Based Argumentation (EBA). When supports are ignored a one-to-one correspondence with Argumentation Frameworks with Recursive Attacks (AFRA) semantics is also established.

1 Introduction

Argumentation has become an essential paradigm for knowledge representation and, especially, for reasoning from contradictory information [2, 18] and for formalizing the exchange of arguments between agents in, *e.g.*, negotiation [3]. Formal abstract frameworks have greatly eased the modelling and study of argumentation. For instance, a Dung's argumentation framework (AF) [18] consists of a collection of arguments interacting through an attack relation, enabling to determine "acceptable" sets of arguments called *extensions*.

Two natural generalisations of Dung's argumentation frameworks consist in allowing positive interactions (usually expressed by a support relation) and allowing high-order attacks (that target other attacks or supports). These generalisations are not only for the "pleasure" to develop more complex concepts;

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they mainly allow the representation of richer argumentation problems. Here is an example in the legal field, borrowed from [4].

Example 1. The prosecutor says that the defendant has intention to kill the victim (argument b). A witness says that she saw the defendant throwing a sharp knife towards the victim (argument a). Argument a can be considered as a support for argument b. The lawyer argues back that the defendant was in a habit of throwing the knife at his wife's foot once drunk. This latter argument (argument c) is better considered attacking the support from a to b, than arguments a or b themselves. Now the prosecutor's argumentation seems no longer sufficient for proving the intention to kill. This example is represented as a recursive framework in Fig. 1. □

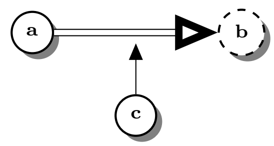


Fig. 1: An acyclic recursive framework where supports (resp. attacks) are represented by double (resp. simple) arrows ended with a white (resp. black) triangle. Circles with solid border represent prima-facie arguments while dashed border ones represent standard arguments.

Positive interaction between arguments has been first introduced in [20, 31]. In [13], the support relation is left general so that the bipolar framework keeps a high level of abstraction. The associated semantics are based on the combination of the attack relation with the support relation which results in new complex attack relations. However, there is no single interpretation of the support, and a number of researchers proposed specialized variants of the support relation (deductive support [7], necessary support [25, 26], evidential support [27, 28]). Each specialization can be associated with an appropriate modelling using an appropriate complex attack. These proposals have been developed quite independently, based on different intuitions and with different formalizations. [14] presents a comparative study in order to restate these proposals in a common setting, the bipolar argumentation framework (see also [15] for another survey).

We follow here an evidential understanding of the support relation [27] that allows to distinguish between two different kinds of arguments: *prima-facie* and *standard arguments*. *Prima-facie* arguments were already present in [31] as those that are justified whenever they are not defeated. On the other hand, *standard arguments* are not directly assumed to be justified and must inherit support from prima-facie arguments through a chain of supports. For instance, in Example 1, arguments a and c are considered as prima-facie arguments while b is regarded as a standard argument. Hence, while a and c can be accepted as in Dung's argumentation, b must inherit support from a: this holds if c is not accepted, but does not otherwise. Indeed, in the latter, the support from a to b is defeated by c.

Concerning frameworks with interactions between arguments and other interactions, a first version has been introduced in [21], then studied in [5] under the name of AFRA (Argumentation Framework with Recursive Attacks). This version describes abstract argumentation frameworks in which the interactions can be either attacks between arguments or attacks from an argument to another attack. In this case, as for the bipolar case, a translation of an AFRA into an equivalent AF can be defined by the addition of some new arguments and the attacks they produce or they receive. A generalization of AFRA has been proposed in [16] in order to take into account supports on arguments or on interactions. These frameworks are called ASAF (Attack-Support Argumentation Frameworks). As for an AFRA, a translation of an ASAF into an equivalent AF is proposed by the addition of arguments and attacks. More recently, alternative acceptability semantics have been defined in a direct way for argumentation frameworks with recursive attacks [10, 11].

In this paper, we are interested in a framework with high-order attacks and supports, with an evidential understanding of these supports. So, we apply the notion of prima-facie, not only to arguments, but also to interactions (attacks and supports). The intuition is that prima-facie elements (arguments, attack or supports) are elements that do not have to be supported. More precisely, we study a semantics for argumentation frameworks with recursive attacks *and* evidential supports, based on the following intuitive principles:

- **P1** The role played in Dung's argumentation frameworks by attacks in defeating arguments is now played by a subset of these attacks, which is extension dependent and represents the "valid attacks" with respect to that extension.
- **P2** The notion of acceptability for prima-facie (and supported) arguments (resp. attacks or supports) is as in recursive frameworks without supports.
- **P3** Non-prima-facie arguments (resp. attacks or supports) can only be "accepted" (resp. be "valid") if there is a chain of "valid supports" rooted in some prima-facie arguments. These "valid supports" are also extension dependent.
- **P4** It is a conservative generalisation of Dung's framework for the notions of conflict-free, admissible, complete, preferred, and stable extensions.

The paper is organized as follows: the necessary background is given in Section 2; new semantics for recursive and evidence-based frameworks are proposed in Section 3; a comparison with existing frameworks is given in sections 4 to 6; and we conclude in Section 7. Proofs of formal results can be found in [12].

2 Background

We next give preliminaries about the works the paper is based on. We first review some basic background about Dung's abstract argumentation frameworks [18], the recursive framework of [11] and Evidence-Based Argumentation (EBA) frameworks [27, 30].

2.1 Dung's Argumentation

Definition 1 (D-framework). *A* Dung's abstract argumentation framework (d-framework *for short) is a pair* $\mathbf{dAF} = \langle \mathbf{A}, \mathbf{R} \rangle$ *where* $\mathbf{A}$ *is a set of arguments and* $\mathbf{R} \subseteq \mathbf{A} \times \mathbf{A}$ *is a relation representing attacks over arguments.* □

Definition 2 (Defeated/acceptable argument). *Let* $\mathbf{dAF} = \langle \mathbf{A}, \mathbf{R} \rangle$ *be a d-framework and* $E \subseteq \mathbf{A}$*, an argument* $a \in \mathbf{A}$ *is said to be:*

1. defeated w.r.t. E *iff* $\exists b \in E$ *such that* $(b, a) \in \mathbf{R}$*, and*
2. acceptable w.r.t. E *iff for every argument* $b \in \mathbf{A}$ *with* $(b, a) \in \mathbf{R}$*, there is* $c \in E$ *such that* $(c, b) \in \mathbf{R}$*.* □

To obtain shorter definitions we will also use the following notations:

$$Def(E) \stackrel{\text{def}}{=} \{ \, a \in \mathbf{A} \mid \exists b \in E \text{ s.t. } (b, a) \in \mathbf{R} \, \}$$
$$Acc(E) \stackrel{\text{def}}{=} \{ \, a \in \mathbf{A} \mid \forall b \in \mathbf{A},\ (b, a) \in \mathbf{R} \text{ implies } b \in Def(E) \, \}$$

respectively denote the set of all defeated and acceptable arguments w.r.t. E.

Definition 3 (Semantics). *Given a d-framework* $\mathbf{dAF} = \langle \mathbf{A}, \mathbf{R} \rangle$*, a set of arguments* $E \subseteq \mathbf{A}$ *is said to be:*

1. conflict-free *iff* $E \cap Def(E) = \varnothing$*,*
2. admissible *iff it is conflict-free and* $E \subseteq Acc(E)$*,*
3. complete *iff it is conflict-free and* $E = Acc(E)$*,*
4. preferred *iff it is* $\subseteq$*-maximal admissible,*
5. stable *iff it is conflict-free and* $E \cup Def(E) = \mathbf{A}$*.* □

Theorem 1 (From [18]). *Given a d-framework* $\mathbf{dAF} = \langle \mathbf{A}, \mathbf{R} \rangle$*, the following assertions hold:*

1. *every complete set is also admissible,*
2. *every preferred set is also complete, and*
3. *every stable set is also preferred.* □

Example 2. Consider the d-framework corresponding to Fig.2. The argument a

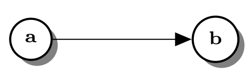


Fig. 2: A d-framework

is accepted w.r.t. any set E because there is no argument $x \in \mathbf{A}$ such that $(x, a) \in \mathbf{R}$. Furthermore, b is defeated and non-acceptable w.r.t. the set $\{a\}$. Then, it is easy to check that $\{a\}$ is stable (and, thus, conflict-free, admissible, complete and preferred). The empty set $\varnothing$ is admissible, but not complete; and the set $\{b\}$ is conflict-free, but not admissible.

2.2 Recursive Argumentation

Let us here recall the necessary background from [11], where high-order attacks are called "recursive".

Definition 4 (RAF). *A* recursive argumentation framework (RAF) *is a tuple* $\langle \mathbf{A}, \mathbf{K}, s, t\rangle$ *where* $\mathbf{A}$ *is a set of arguments,* $\mathbf{K}$ *is a set disjunct from* $\mathbf{A}$*, representing attack names, s is a function from* $\mathbf{K}$ *to* $\mathbf{A}$*, mapping each interaction to its source, t is a function from* $\mathbf{K}$ *to* $(\mathbf{A} \cup \mathbf{K})$ *mapping each interaction to its target.*

Acceptability semantics are defined by replacing the notion of extension (set of arguments) by a pair of a set of arguments and a set of attacks, called a "structure". The intuition is the fact that two arguments may be conflicting depends on the validity of the attack between them. So it would not be sound to give a definition of a set of arguments being conflict-free, independently of a set of attacks. More generally, the classic role of attacks in defeating arguments is played by a subset of attacks, which is extension dependent, and represents the valid attacks with respect to the extension.

Definition 5 (Structure). *A structure on* $\langle \mathbf{A}, \mathbf{K}, s, t\rangle$ *is a pair* $U = (S, \Gamma)$ *such that* $S \subseteq \mathbf{A}$ *and* $\Gamma \subseteq \mathbf{K}$*.* □

Intuitively, S represents the set of arguments that are accepted w.r.t. the structure U while Γ represents the set of attacks that are valid w.r.t. U.

Definition 6 (Defeat/Inhibition/Acceptability). *Given* $U = (S, \Gamma)$ *a structure on* $\langle \mathbf{A}, \mathbf{K}, s, t\rangle$*. Let* $a \in \mathbf{A}$ *and* $\alpha \in \mathbf{K}$*.*

1. *a is defeated wrt (S, Γ) iff $\exists \beta \in \Gamma$ such that $s(\beta) \in S$ and $t(\beta) = a$,*
2. *α is inhibited wrt (S, Γ) iff $\exists \beta \in \Gamma$ such that $s(\beta) \in S$ and $t(\beta) = \alpha$.*

$Def(U)$ (resp. $Inh(U)$) will denote the set of arguments (resp. attacks) that are defeated (resp. inhibited) wrt the structure U.

3. *a is acceptable wrt U iff $\forall \beta \in \mathbf{K}$ such that $t(\beta) = a$, either $\beta \in Inh(U)$ or $s(\beta) \in Def(U)$.*
4. *α is acceptable wrt U iff $\forall \beta \in \mathbf{K}$ such that $t(\beta) = \alpha$, either $\beta \in Inh(U)$ or $s(\beta) \in Def(U)$.*

$Acc(U)$ will denote the set of all acceptable arguments and attacks wrt U. □

Then, semantics are defined as follows:

Definition 7 (Semantics). *A structure* $U = (S, \Gamma)$ *on* $\langle \mathbf{A}, \mathbf{K}, s, t\rangle$ *is:*

1. *conflict-free iff $S \cap Def(U) = \varnothing$ and $\Gamma \cap Inh(U) = \varnothing$;*
2. *admissible iff it is conflict-free and $\forall x \in (S \cup \Gamma)$, x is acceptable wrt U;*
3. *complete iff it is conflict-free and $Acc(U) = S \cup \Gamma$;*
4. *stable iff it is conflict-free and satisfies :*
   (a) $\forall a \in \mathbf{A} \setminus S$, $a \in Def(U)$ and

(b) $\forall \alpha \in \mathbf{K} \setminus \Gamma$, $\alpha \in Inh(U)$;

5. *preferred iff it is a $\subseteq$-maximal*[1] *admissible structure.* □

It has been proved in [11] that every complete structure is admissible, every preferred structure is also complete and every stable structure is also preferred.

2.3 Evidence-Based Argumentation

We recall the formal definition of EBA frameworks. We follow here the definitions from [30] which correct some technical flaws from [27].

Definition 8 (Evidence-Based Argumentation framework). *An Evidence-Based Argumentation framework (EBAF) is a tuple* $\mathbf{EBAF} = \langle \mathbf{A}, \mathbf{R}_a, \mathbf{R}_e \rangle$ *where* $\mathbf{A}$ *represents a set of arguments,* $\mathbf{R}_a \subseteq (2^{\mathbf{A}} \setminus \varnothing) \times \mathbf{A}$ *is an attack relation and* $\mathbf{R}_e \subseteq (2^{\mathbf{A}} \setminus \varnothing) \times \mathbf{A}$ *is a support relation. A special argument* $\eta \in \mathbf{A}$ *is distinguished satisfying that there is no* $(B, \eta) \in \mathbf{R}_a \cup \mathbf{R}_e$ *for any set* B *nor there is* $(B, a) \in \mathbf{R}_a$ *with* $\eta \in B$*. We say that* $\mathbf{EBAF}$ *is* (in)finite *iff* $\mathbf{A}$ *is (in)finite.* □

The special argument η serves as a representation of the prima-facie arguments. Note that the attack relation is not a binary relation. Instead, there can be an attack from a *set of arguments* to another argument, something which is not the case in d-frameworks.

Definition 9 (Evidential Support). *An argument* $a \in \mathbf{A}$ *is* e-supported *by a set* $B \subseteq \mathbf{A}$ *iff the two following conditions hold:*

1. $a = \eta$, *or*
2. *there is a non-empty* $C \subseteq B$ *s.t.* $(C, a) \in \mathbf{R}_e$ *and every* $c \in C$ *is e-supported by* $B \setminus \{a\}$. □

B is said to be a minimal e-support for a iff there is no $C \subset B$ such that a is e-supported by C. □

Note that η is e-supported by any set $B \subseteq \mathbf{A}$.

Definition 10 (Evidence-Supported Attack). *A pair* (B, a) *is said to be an* evidence-supported attack (e-attack) *iff* (i) *there is* $(C, a) \in \mathbf{R}_a$ *with* $C \subseteq B$ *and* (ii) *all elements in* C *are e-supported by* B*.* (B, a) *is said to be a minimal e-attack if there is no e-attack* (C, a) *with* $C \subset B$*.* □

We will say that B e-supports a or that (B, a) is an e-support when a is e-supported by B and that B e-attacks a when (B, a) is an e-attack.

Definition 11 (Acceptability). *Given some framework* $\mathbf{EBAF} = \langle \mathbf{A}, \mathbf{R}_a, \mathbf{R}_e \rangle$*, an argument* $a \in \mathbf{A}$ *is said to be* acceptable *w.r.t. a set* $E \subseteq \mathbf{A}$ *iff the following two conditions are satisfied:*

1. *a is e-supported by E, and*

[1] Where $U = (S, \Gamma) \subseteq U' = (S', \Gamma')$ iff $(S \cup \Gamma) \subseteq (S' \cup \Gamma')$.

2. *for every minimal e-attack (B, a), it holds that E e-attacks some $b \in B$.* □

Definition 12 (Semantics). *A set of arguments $E \subseteq \mathbf{A}$ is said to be*

1. self-supporting *iff all arguments $a \in E$ are e-supported by E,*
2. conflict-free *iff, for every $a \in E$, there is no $B \subseteq E$ such that $(B, a) \in \mathbf{R}_a$,*
3. admissible *iff it is conflict-free and all arguments $a \in E$ are acceptable w.r.t. E,*
4. complete *iff it is admissible and all acceptable arguments w.r.t. E are in E,*
5. preferred *iff it is a $\subseteq$-maximal admissible set,*
6. stable *iff it is self-supporting, conflict-free and any argument $a \notin E$ which is e-supported by $\mathbf{A}$ satisfies that E e-attacks either a or every minimal e-support B of a.* □

3 Recursive Evidence-Based Argumentation

In this section, we extend the semantics proposed for recursive attacks in [11] with the purpose of handling evidence-based supports.

3.1 Recursive Evidence-Based Argumentation Frameworks

Definition 13 (Recursive Evidence-Based Argumentation Framework). *An* (evidence-based recursive) argumentation framework $\mathbf{AF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{S}, \mathbf{s}, \mathbf{t}, \mathbf{P} \rangle$ *is a sextuple where $\mathbf{A}$, $\mathbf{K}$ and $\mathbf{S}$ are three (possible infinite) pairwise disjunct sets respectively representing arguments, attacks and supports names, and where $\mathbf{P} \subseteq \mathbf{A} \cup \mathbf{K} \cup \mathbf{S}$ is a set representing the* prima-facie *elements that do not need to be supported. Functions $\mathbf{s} : (\mathbf{K} \cup \mathbf{S}) \longrightarrow 2^{\mathbf{A}}$ and $\mathbf{t} : (\mathbf{K} \cup \mathbf{S}) \longrightarrow (\mathbf{A} \cup \mathbf{K} \cup \mathbf{S})$ respectively map each attack and support to its source and its target.* □

As in EBAFs, the source of attacks and supports is a set of arguments. It is obvious that any attack (a, b) in a d-framework can be represented by assigning to it some name α that satisfies $\mathbf{s}(\alpha) = \{a\}$ and $\mathbf{t}(\alpha) = b$. It is also worth mentioning that, from an evidential point of view, every argument and attack of a d-framework is prima-facie. That is, given some $\mathbf{dAF} = \langle \mathbf{A}, \mathbf{R} \rangle$, we can build a corresponding recursive framework $\mathbf{AF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{S}, \mathbf{s}, \mathbf{t}, \mathbf{P} \rangle$ where $\mathbf{K}$ is a set of names of the same cardinality of $\mathbf{R}$, where $\mathbf{S} = \varnothing$ is the empty set of supports, $\mathbf{s}$ and $\mathbf{t}$ map each attack name to its corresponding source and target, and the set of prima-facie elements $\mathbf{P} = \mathbf{A} \cup \mathbf{K}$ includes all arguments and attacks.

Example 3. In particular, the d-framework associated with Figure 2 corresponds to the $\mathbf{AF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{S}, \mathbf{s}, \mathbf{t}, \mathbf{P} \rangle$ with $\mathbf{A} = \{a, b\}$, $\mathbf{K} = \{\alpha\}$, $\mathbf{s}(\alpha) = \{a\}$, $\mathbf{t}(\alpha) = b$ and $\mathbf{P} = \{a, b, \alpha\}$. □

Note also that, different from EBAFs, the set $\mathbf{P}$ may contain several prima-facie elements (arguments, attacks and supports). This is not a substantial difference, but allows that any graph representing a d-framework has the same semantics when interpreted in our framework. For instance, Figure 3 depicts

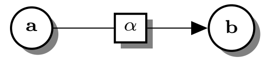


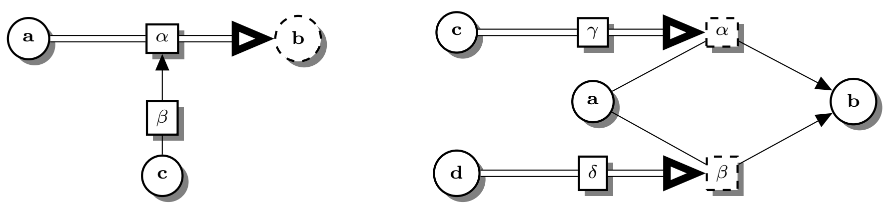
Fig. 3: An AF with named attack.

the framework of Figure 2 making explicit the attack name. Note that we use squares in the middle of the arrows to represent attack and support names. As with arguments, a solid border denotes prima-facie elements while a dashed border denotes standard elements. By following this notation every graph within Dung's theory preserves the same semantics, something which is in accordance with principle **P4**. Note also that, in contrast with EBAFs, we do not assume any constraint on the prima-facie elements, they can be attacked or supported (though supporting prima-facie elements do not make any semantical difference from not doing so).

Example 4. As an illustration of frameworks with recursive attacks and supports, consider the argumentation frameworks $\mathbf{AF}_1 = \langle \mathbf{A}_1, \mathbf{K}_1, \mathbf{S}_1, \mathbf{s}_1, \mathbf{t}_1, \mathbf{P}_1 \rangle$ and $\mathbf{AF}_2 = \langle \mathbf{A}_2, \mathbf{K}_2, \mathbf{S}_2, \mathbf{s}_2, \mathbf{t}_2, \mathbf{P}_2 \rangle$ where $\mathbf{A}_1 = \{a, b, c\}$, $\mathbf{K}_1 = \{\beta\}$, $\mathbf{S}_1 = \{\alpha\}$, $\mathbf{A}_2 = \{a, b, c, d\}$, $\mathbf{K}_2 = \{\alpha, \beta\}$, $\mathbf{S}_2 = \{\gamma, \delta\}$, functions $\mathbf{s}_1$, $\mathbf{t}_1$, $\mathbf{s}_2$ and $\mathbf{t}_2$ satisfy

$$\mathbf{s}_1(\alpha) = \{a\} \qquad \mathbf{t}_1(\alpha) = b$$
$$\mathbf{s}_1(\beta) = \{c\} \qquad \mathbf{t}_1(\beta) = \alpha$$
$$\mathbf{s}_2(\alpha) = \{a\} \qquad \mathbf{t}_2(\alpha) = b \qquad \mathbf{s}_2(\gamma) = \{c\} \qquad \mathbf{t}_2(\gamma) = \alpha$$
$$\mathbf{s}_2(\beta) = \{a\} \qquad \mathbf{t}_2(\beta) = b \qquad \mathbf{s}_2(\delta) = \{d\} \qquad \mathbf{t}_2(\delta) = \beta$$

and $\mathbf{P}_1 = \{a, c, \alpha, \beta\}$, and $\mathbf{P}_2 = \{a, b, c, d, \gamma, \delta\}$. These two frameworks can be respectively depicted as the graphs in Figures 4a and 4b. It is worth to note



(a) The graph of Fig.1 with attack and support names

(b) A recursive framework representing attacks in different contexts

Fig. 4: Recursive frameworks with prima-facie elements

that Figure 4a is just the result of naming attacks and supports in Figure 1. On the other hand, Figure 4b represents a framework with two attacks between a and b that hold in different contexts: α and β are two standard attacks that are respectively supported by different prima-facie arguments, c and d respectively, that represent those different contexts. □

Example 5. Consider the following four arguments:[2]

(a) "The Bible says that God is all good",
(b) "God is all good",
(c) "The Bible was written by human beings",
(d) "Human beings are not infallible".

Argument (a) may be considered as a support α for argument (b), while (c) and (d) taken together may be considered as an attack β to the support α. Indeed, arguments (c) and (d), alone or together, contradict neither (a) nor (b). Moreover, (c) alone (resp. (d) alone) does not attack α. We must take (c) and (d) together in order to attack α. This example can be formalised as $\mathbf{AF}_3 = \langle \mathbf{A}_3, \mathbf{K}_3, \mathbf{S}_3, \mathbf{s}_3, \mathbf{t}_3, \mathbf{P}_3 \rangle$ where $\mathbf{A}_3 = \{a, b, c, d\}$, $\mathbf{K}_3 = \{\beta\}$, $\mathbf{S}_3 = \{\alpha\}$, and $\mathbf{P}_3 = \mathbf{A}_3 \backslash \{b\} = \{a, c, d\}$.

$$\mathbf{s}_3(\alpha) = \{a\} \qquad \mathbf{t}_3(\alpha) = b$$
$$\mathbf{s}_3(\beta) = \{c, d\} \qquad \mathbf{t}_3(\beta) = \alpha$$

□

It is worth to mention that the reason to use explicit names for attacks and supports in Definition 13 instead of just relations is twofold. First, this allows the existence of several attacks or supports between the same elements that can be used to represent different contexts as illustrated in Example 4. The second

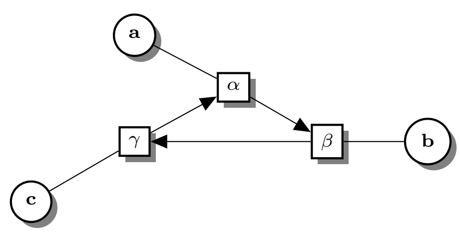


Fig. 5: A cyclic recursive framework

reason is due to the possible existence of cycles of attacks or supports, which has no trivial finite representation as a relation: for instance, attack α in Figure 5 would correspond to the infinite object $(\{a\}, (\{b\}, (\{c\}, (\{a\}, \dots))))$.

3.2 Semantics of Recursive Evidence-Based Argumentation Frameworks

We generalize next the notion of structure introduced in [11], which will allow us to characterise which arguments are regarded as "acceptable," and which

[2] This example is a slight variation of the one discussed in [24]. Having at our disposal supports allows us to explicitly represent the implicit support in "The Bible says that God is all good, so God is all good" which was there expressed as a single argument.

attacks and supports are regarded as "valid," with respect to some argumentation framework. The notion of structure is analogous to the notion of set of arguments and it will be the basis of defining the corresponding argumentation semantics for recursive frameworks.

Definition 14 (Structure). *A triple* $\mathfrak{A} = \langle E, \Gamma, \Delta \rangle$ *is said to be a* structure *of some* $\mathbf{AF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{S}, \mathbf{s}, \mathbf{t}, \mathbf{P} \rangle$ *iff it satisfies:* $E \subseteq \mathbf{A}$, $\Gamma \subseteq \mathbf{K}$ *and* $\Delta \subseteq \mathbf{S}$. □

Intuitively, the set E represents the set of "acceptable" arguments w.r.t. the structure $\mathfrak{A}$, while Γ and Δ respectively represent the set of "valid attacks" and "valid supports" w.r.t. $\mathfrak{A}$. Any attack[3] $\alpha \in \overline{\Gamma}$ is understood as non-valid and, in this sense, it cannot defeat the element that it is targeting. Similarly, any support $\beta \in \overline{\Delta}$ is understood as non-valid and it cannot support the element that it is targeting.

For the rest of this section we assume that all definitions and results are relative to some given framework $\mathbf{AF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{S}, \mathbf{s}, \mathbf{t}, \mathbf{P} \rangle$. We extend now the definition of defeated arguments (Definition 2) using the set Γ instead of the attack relation $\mathbf{R}$: given a structure of the form $\mathfrak{A} = \langle E, \Gamma, \Delta \rangle$, we define:

$$Def_X(\mathfrak{A}) \stackrel{\text{def}}{=} \{ x \in X \mid \exists \alpha \in \Gamma,\ \mathbf{s}(\alpha) \subseteq E \text{ and } \mathbf{t}(\alpha) = x \} \tag{1}$$

with $X \in \{\mathbf{A}, \mathbf{K}, \mathbf{S}\}$. In other words, an element x is defeated w.r.t. $\mathfrak{A}$ iff there is a "valid attack" w.r.t. $\mathfrak{A}$ that targets x and whose source is "acceptable" w.r.t. $\mathfrak{A}$. It is interesting to observe that we may define the *attack relation* associated with some structure $\mathfrak{A} = \langle E, \Gamma, \Delta \rangle$ as follows:

$$\mathbf{R}_{\mathfrak{A}} \stackrel{\text{def}}{=} \{ (\mathbf{s}(\alpha), \mathbf{t}(\alpha)) \mid \alpha \in \Gamma \} \tag{2}$$

and that, using this relation, we can rewrite (1) as:

$$Def_X(\mathfrak{A}) \stackrel{\text{def}}{=} \{ x \in X \mid \exists B \subseteq E \text{ s.t. } (B, x) \in \mathbf{R}_{\mathfrak{A}} \} \tag{3}$$

Now, it is easy to see that our definition for $Def_{\mathbf{A}}(\mathfrak{A})$ can be obtained from Dung's definition of defeat (Definition 2) just by replacing the attack relation $\mathbf{R}$ by the attack relation $\mathbf{R}_{\mathfrak{A}}$ associated with the structure $\mathfrak{A}$ and $\exists b \in E$ by $\exists B \subseteq E$, or in other words, by replacing the set of all attacks in the argumentation framework by the set of the "valid attacks" w.r.t. the structure $\mathfrak{A}$, as stated in **P1**; and allowing the source of attacks to be, not just arguments, but sets of them.

By $Def(\mathfrak{A}) \stackrel{\text{def}}{=} Def_{\mathbf{A}}(\mathfrak{A}) \cup Def_{\mathbf{K}}(\mathfrak{A}) \cup Def_{\mathbf{S}}(\mathfrak{A})$, we will denote the set of all defeated arguments. By $\overline{Def_X(\mathfrak{A})} \stackrel{\text{def}}{=} X \backslash Def_X(\mathfrak{A})$ with $X \in \{\mathbf{A}, \mathbf{K}, \mathbf{S}\}$, we denote the non-defeated arguments (resp. attacks, supports) w.r.t. $\mathfrak{A}$. Furthermore, by $\overline{Def(\mathfrak{A})} \stackrel{\text{def}}{=} (\mathbf{A} \cup \mathbf{K} \cup \mathbf{S}) \backslash Def(\mathfrak{A})$, we denote the set of all non-defeated elements.

Example 4 (cont'd) Consider the framework corresponding to Figure 4a, and the structure $\mathfrak{A} = \langle E, \Gamma, \Delta \rangle$ with $E = \{a, c\}$, $\Gamma = \{\beta\}$ and $\Delta = \varnothing$. Then, we have that $Def(\mathfrak{A}) = \{\alpha\}$. □

[3] By $\overline{\Gamma} \stackrel{\text{def}}{=} \mathbf{K} \backslash \Gamma$ we denote the set complement of Γ w.r.t. $\mathbf{K}$. Similarly, by $\overline{\Delta} \stackrel{\text{def}}{=} \mathbf{S} \backslash \Delta$ we denote the set complement of Δ w.r.t. $\mathbf{S}$.

Let us now introduce the notion of *supported elements* w.r.t. a structure. Intuitively, it should be noted that the prima-facie elements (arguments, attacks, supports) of a given framework are supported for any structure. Then, a standard element is supported if there exists a chain of supported supports, leading to it, which is rooted in prima-facie arguments. Formally, given some framework $\mathbf{AF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{S}, \mathbf{s}, \mathbf{t}, \mathbf{P} \rangle$ and some structure $\mathfrak{A} = \langle E, \Gamma, \Delta \rangle$, the set of supported elements $Sup(\mathfrak{A})$ is recursively defined as follows[4]:

$$Sup(\mathfrak{A}) \stackrel{\text{def}}{=} \mathbf{P} \cup \{\, \mathbf{t}(\alpha) \mid \exists \alpha \in \Delta \cap Sup(\mathfrak{A}')\ , \mathbf{s}(\alpha) \subseteq E \cap Sup(\mathfrak{A}')\ \} \tag{4}$$

with[5] $\mathfrak{A}' = \mathfrak{A} \backslash \{\mathbf{t}(\alpha)\}$. By $Sup_X(\mathfrak{A}) \stackrel{\text{def}}{=} Sup(\mathfrak{A}) \cap X$ with $X \in \{\mathbf{A}, \mathbf{K}, \mathbf{S}\}$, we respectively denote the set of all supported arguments, attacks and supports.

Example 4 (cont'd) Consider the framework corresponding to Figure 4a, and the structure $\mathfrak{A} = \langle E, \Gamma, \Delta \rangle$ with $E = \{a, b, c\}$, $\Gamma = \varnothing$ and $\Delta = \{\alpha\}$. Let us prove that $b \in Sup(\mathfrak{A})$. Note that $b = \mathbf{t}(\alpha)$ with $\alpha \in \Delta$. So we have to prove that α and $a \in \mathbf{s}(\alpha) = \{a\}$ both belong to $Sup(\mathfrak{A} \backslash \{b\})$. That is true since α and a both belong to $\mathbf{P}$.

Example 6. As a further example, consider the framework corresponding to the graph depicted in Figure 6 and let $\mathfrak{A} = \langle E, \Gamma, \Delta \rangle$ be a structure with

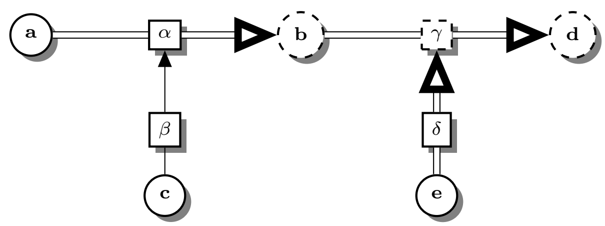


Fig. 6: A recursive framework with prima-facie elements

$E = \{a, b, c, d, e\}$, $\Gamma = \varnothing$ and $\Delta = \{\alpha, \gamma, \delta\}$. Then, we have that $Sup(\mathfrak{A}) = \{a, b, c, d, e, \alpha, \beta, \gamma, \delta\}$. Note that a, c, e, α, β and δ are supported because they are prima-facie elements. It is also easy to see that b is supported as in the previous example and that γ is supported through δ by e. So, b and γ both belong to $Sup(\mathfrak{A} \backslash \{d\})$. Hence, d is also supported. □

Now, drawing on the notion of supported elements w.r.t. a given structure $\mathfrak{A}$, we are able to define the *supportable* elements w.r.t. $\mathfrak{A}$. Intuitively, an element is considered as being still supportable as long as there exists some non-defeated support with all its source elements non-defeated and regarded, in its turn, as

[4] Note that $E = \varnothing$ and $\Delta = \varnothing$ act as base cases, because $E = \varnothing$ (resp. $\Delta = \varnothing$) implies $Sup(\mathfrak{A}) = \mathbf{P}$.

[5] By abuse of notation, we write $\mathfrak{A} \backslash T$ instead of $\langle E \backslash T, \Gamma \backslash T, \Delta \backslash T \rangle$ with $T \subseteq (\mathbf{A} \cup \mathbf{K} \cup \mathbf{S})$.

supportable. Formally, an element x is supportable w.r.t. $\mathfrak{A}$ iff x is supported w.r.t. $\mathfrak{A}' = \langle \overline{Def_{\mathbf{A}}(\mathfrak{A})}, \mathbf{K}, \overline{Def_{\mathbf{S}}(\mathfrak{A})} \rangle$. Elements that are defeated or that are unsupportable cannot be accepted. In this sense, by $UnAcc(\mathfrak{A}) \stackrel{\text{def}}{=} Def(\mathfrak{A}) \cup \overline{Sup(\mathfrak{A}')}$ we denote the *unacceptable* elements w.r.t. $\mathfrak{A}$. Moreover, we say that an attack $\alpha \in \mathbf{K}$ is *unactivable*[6] iff either it is unacceptable or some element in its source is unacceptable, that is,

$$UnAct(\mathfrak{A}) \stackrel{\text{def}}{=} \{ \alpha \in \mathbf{K} \mid \alpha \in UnAcc(\mathfrak{A}) \text{ or } \mathbf{s}(\alpha) \cap UnAcc(\mathfrak{A}) \neq \varnothing \}$$

Definition 15 (Acceptability). *An element* $x \in \mathbf{A} \cup \mathbf{K} \cup \mathbf{S}$ *is said to be* acceptable *w.r.t. a structure* $\mathfrak{A}$ *iff (i)* $x \in Sup(\mathfrak{A})$ *and (ii) every attack* $\alpha \in \mathbf{K}$ *with* $\mathbf{t}(\alpha) = x$ *is unactivable, that is,* $\alpha \in UnAct(\mathfrak{A})$. □

By $Acc(\mathfrak{A})$, we denote the set containing all arguments, attacks and supports that are acceptable with respect to $\mathfrak{A}$.

It is worth to note that, intuitively, an element is acceptable iff it is supported and, in addition, every attack against it can be considered as unactivable because either some argument in its source or itself has been regarded as unacceptable.

Example 7. Consider the argumentation framework of Figure 7, and the structure $\mathfrak{A} = \langle \{a, b, c, e\}, \{\alpha, \kappa, \gamma\}, \varnothing \rangle$. We have that c is acceptable w.r.t. $\mathfrak{A}$. Note that there are two attacks against c: β is defeated through α by a, while γ is unactivable because d is unsupportable since δ is defeated by κ. □

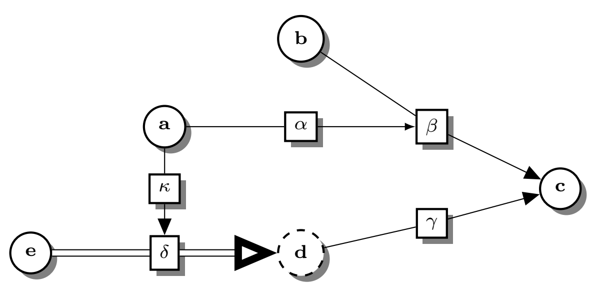


Fig. 7: Argumentation framework corresponding to Example 7.

We also define the following order relations that will help us defining preferred structures: for any pair of structures $\mathfrak{A} = \langle E, \Gamma, \Delta \rangle$ and $\mathfrak{A}' = \langle E', \Gamma', \Delta' \rangle$, we write $\mathfrak{A} \sqsubseteq \mathfrak{A}'$ iff $(E \cup \Gamma \cup \Delta) \subseteq (E' \cup \Gamma' \cup \Delta')$. As usual, we say that a structure $\mathfrak{A}$ is $\sqsubseteq$-maximal iff every $\mathfrak{A}'$ that satisfies $\mathfrak{A} \sqsubseteq \mathfrak{A}'$ also satisfies $\mathfrak{A}' \sqsubseteq \mathfrak{A}$.

Definition 16. *A structure* $\mathfrak{A} = \langle E, \Gamma, \Delta \rangle$ *is said to be:*

1. self-supporting *iff* $(E \cup \Gamma \cup \Delta) \subseteq Sup(\mathfrak{A})$,

[6] Intuitively, such an attack cannot be "activated" in order to defeat the element that it is targeting.

2. conflict-free *iff* $X \cap Def_Y(\mathfrak{A}) = \varnothing$ *for any* $(X, Y) \in \{(E, \mathbf{A}), (\varGamma, \mathbf{K}), (\varDelta, \mathbf{S})\}$,
3. admissible *iff it is conflict-free and* $E \cup \varGamma \cup \varDelta \subseteq Acc(\mathfrak{A})$,
4. complete *iff it is conflict-free and* $Acc(\mathfrak{A}) = E \cup \varGamma \cup \varDelta$,
5. preferred *iff it is a* $\sqsubseteq$*-maximal admissible structure,*
6. stable[7] *iff* $(E \cup \varGamma \cup \varDelta) = \overline{UnAcc(\mathfrak{A})}$.

□

Example 4 (cont'd) The framework of Figure 4a has a unique complete, preferred and stable structure $\mathfrak{A} = \langle \{a, c\}, \{\beta\}, \varnothing \rangle$. Note that α cannot be accepted because it is defeated by c through β, while b cannot be accepted because, now, it lacks support.

Example 6 (cont'd) The framework of Figure 6 has also a unique complete, preferred and stable structure $\mathfrak{A} = \langle \{a, c, e\}, \{\beta\}, \{\gamma, \delta\} \rangle$. As above, α cannot be accepted because it is defeated by c through β which implies that b and d cannot be accepted because of lack of support. γ is acceptable because it is supported through δ by e and not attacked. □

Example 7 (cont'd) $\mathfrak{A} = \langle \{a, b, c, e\}, \{\alpha, \kappa, \gamma\}, \varnothing \rangle$ is the unique complete, preferred and stable structure w.r.t. the framework of Figure 7. □

We show now that, as in Dung's argumentation theory, there is also a kind of Fundamental Lemma for argumentation frameworks with recursive attacks and evidence-based supports. Intuitively, this lemma says that elements of an admissible structure continue to be acceptable when the structure is "reasonably" extended, that is extended with an acceptable element.

Lemma 1 (Fundamental Lemma). *Let* $\mathfrak{A} = \langle E, \varGamma, \varDelta \rangle$ *be an admissible structure and* $x, y \in Acc(\mathfrak{A})$ *be any pair of acceptable elements. Then,*[8] *(i)* $\mathfrak{A}' = \mathfrak{A} \cup \{x\}$ *is an admissible structure, and (ii)* $y \in Acc(\mathfrak{A}')$. □

Moreover, admissible structures form a complete partial order with preferred structures as maximal elements:

Proposition 1. *The set of all admissible structures forms a complete partial order with respect to* $\sqsubseteq$*. Furthermore, for every admissible structure* $\mathfrak{A}$*, there exists a preferred one* $\mathfrak{A}'$ *such that* $\mathfrak{A} \sqsubseteq \mathfrak{A}'$. □

The following result shows that the usual relation between extensions also holds for structures.

Theorem 2. *The following assertions hold:*

1. *every admissible structure is also self-supporting,*
2. *every complete structure is also admissible,*
3. *every preferred structure is also complete, and*
4. *every stable structure is also preferred.* □

[7] Note also this already implies conflict-freeness.

[8] By abuse of notation, we write $\mathfrak{A} \cup T$ instead of $\langle E \cup (T \cap \mathbf{A}), \varGamma \cup (T \cap \mathbf{K}), \varDelta \cup (T \cap \mathbf{S}) \rangle$ with $T \subseteq (\mathbf{A} \cup \mathbf{K} \cup \mathbf{S})$.

Example 8. As a further example, consider the framework corresponding to Figure 8. This framework has a unique complete and preferred structure $\mathfrak{A} =$

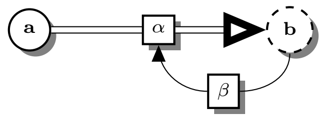


Fig. 8: A cyclic recursive framework

$\langle\{a\},\{\beta\},\varnothing\rangle$, but no stable one. Note that α and b are neither acceptable nor unacceptable w.r.t. $\mathfrak{A}$: α is not unacceptable because it is supportable (it is prima-facie) and it is not defeated (b is not in the structure) and it is not acceptable because it is attacked by b, which is still not unacceptable. Similarly, b is not unacceptable because it is still supportable through α, but it is not supported (and, thus not acceptable) because α is not in the structure. □

4 Relation with Recursive Argumentation Frameworks

As mentioned in Section 3, our framework is a conservative generalisation of the Recursive Argumentation Framework (RAF) defined in [11] with the addition of supports and joint attacks. RAF's attacks are similar to Dung's attacks with the only difference that they may target, not only arguments, but also other attacks. Hence, translating RAF's (or Dung's) attacks into joint attacks is trivial: every attack with source a is replaced by an attack with the singleton set $\{a\}$ as its source. On the other hand, like Dung's frameworks, RAFs do not encompass the notion of support. From an evidential point of view it is as every argument or attack was externally supported, or in other words, as attacks and arguments were prima-facie. In this sense, every $\mathbf{RAF} = \langle\mathbf{A},\mathbf{K},\mathbf{s},\mathbf{t}\rangle$ can be translated into a corresponding recursive evidence-based argumentation framework of the form $\mathbf{AF} = \langle\mathbf{A},\mathbf{K},\mathbf{S},\mathbf{s}',\mathbf{t},\mathbf{P}\rangle$ with $\mathbf{S} = \varnothing$ (no supports), where every element is considered as prima-facie, that is $\mathbf{P} = \mathbf{A}\cup\mathbf{K}$, and where $\mathbf{s}'$ satisfies $\mathbf{s}'(\alpha) = \{\mathbf{s}(\alpha)\}$ for every attack $\alpha \in \mathbf{K}$. It is easy to check that a structure $\langle E,\Gamma\rangle$ is conflict-free (resp. admissible, complete, preferred, stable) w.r.t. some $\mathbf{RAF}$ iff $\langle E,\Gamma,\varnothing\rangle$ is conflict-free (resp. admissible, complete, preferred, stable) w.r.t. its corresponding $\mathbf{AF}$. Furthermore, there is a one-to-one correspondence between complete, preferred and stable structures in RAF's and their corresponding Dung's extensions, so this correspondence is also carried over to our argumentation frameworks with evidence-based support. In [11], it also has been shown that there is a one-to-one correspondence between RAF and AFRA [5], which is also carried over to our frameworks (when we restrict ourselves to frameworks without supports). Note that AFRA has been extended with supports in [16, 17] and called Attack-Support Argumentation Framework (ASAF). However, ASAF supports are understood as necessary conditions for their targets instead. This is quite different from the evidential understanding followed here as shown by the following example.

Example 9. According to ASAF, the set $\{a, b, \alpha, \beta\}$ is a complete, preferred and stable w.r.t. the framework of Figure 9. On the other hand, in our framework,

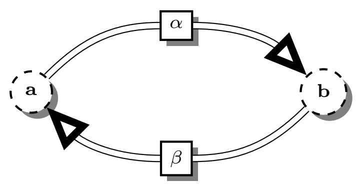


Fig. 9: A framework with a cycle of supports

$\langle\{a, b\}, \varnothing, \{\alpha, \beta\}\rangle$ is not admissible (and, thus, not complete, preferred nor stable) because neither a nor b are supported by a chain rooted in some prima-facie argument. □

5 Relation with Dung's Argumentation Frameworks

It is also worth to mention that the one-to-one correspondence between RAF (or either AFRA or ASAF) and Dung's frameworks is not directly applicable to conflict-free or admissible sets as illustrated by the following example:

Example 2 (cont'd) Consider the argumentation framework corresponding to Figure 3. According to Dung's theory, this framework has three conflict-free sets, namely $\varnothing$, $\{a\}$ and $\{b\}$, which respectively correspond to the structures: $\langle\varnothing, \{\alpha\}, \varnothing\rangle$, $\langle\{a\}, \{\alpha\}, \varnothing\rangle$ and $\langle\{b\}, \{\alpha\}, \varnothing\rangle$. On the other hand, $\langle\{a, b\}, \varnothing, \varnothing\rangle$ is a conflict-free structure because the attack α is not considered valid. Similarly, $\{a, b\}$ is a conflict-free set according to AFRA or ASAF. □

The difference between Dung's argumentation frameworks and these three semantics for recursive attacks, illustrated by the above example, can be explained by the fact that, in Dung's theory, every attack is considered as "valid" in the sense that it may affect its target. In [11], it has been shown that a one-to-one correspondence with Dung's theory, for conflict-free and admissible sets, can be recovered by adding a kind of reinstatement principle on attacks, which forces all attacks that cannot be defeated to be "valid". The following extends the definition of d-structure from [11] to the case of supports by strengthening the notion of structure according to the above intuition:

Definition 17 (D-structure). *Given some framework* $\mathbf{AF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{S}, \mathbf{s}, \mathbf{t}, \mathbf{P} \rangle$*, a structure* $\mathfrak{A} = \langle E, \Gamma, \Delta \rangle$ *is said to be a* d-structure *iff it satisfies* $(Acc(\mathfrak{A}) \cap \mathbf{K}) \subseteq \Gamma$

and $(Acc(\mathfrak{A}) \cap \mathbf{S}) \subseteq \Delta$*. Then, a* conflict-free *(resp.* admissible, complete, preferred or stable*)* d-structure *is a conflict-free (resp. admissible, complete, preferred, stable) structure which is also a d-structure.* □

As a direct consequence of Definition 16 and Theorem 2, we have:

Observation 1. *Every complete (resp. preferred or stable) structure is also a d-structure.* □

It is easy to check that a structure $\langle E, \Gamma \rangle$ is a d-structure w.r.t. some **RAF** (as defined in [11]) iff $\langle E, \Gamma, \varnothing \rangle$ is a d-structure w.r.t. its corresponding **AF**. Hence, the following result is an immediate consequence of Theorem 4 in [11]:

Theorem 3. *Let* $\mathbf{AF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{S}, \mathbf{s}, \mathbf{t}, \mathbf{P} \rangle$ *be some non-recursive framework with* $\mathbf{S} = \varnothing$*,* $\mathbf{P} = \mathbf{A} \cup \mathbf{K}$*, and that satisfies* $|\mathbf{s}(\alpha)| = 1$ *and* $\mathbf{t}(\alpha) \in \mathbf{A}$*, for all* $\alpha \in \mathbf{K}$*. Then, a d-structure* $\mathfrak{A} = \langle E, \mathbf{K}, \varnothing \rangle$ *is conflict-free (resp. admissible, complete, preferred or stable) w.r.t.* **AF** *(Definition 17) iff it is conflict-free (resp. admissible, complete, preferred or stable) w.r.t.* $\mathbf{dAF} = \langle \mathbf{A}, \mathbf{R}_{\mathbf{AF}} \rangle$ *(Definition 3) with the relation* $\mathbf{R}_{\mathbf{AF}} \stackrel{\text{def}}{=} \{\, (a, \mathbf{t}(\alpha)) \mid \alpha \in \mathbf{K} \text{ and } \mathbf{s}(\alpha) = \{a\} \,\}$*.* □

Theorem 3 formalises how any d-framework can be represented as an **AF**: in particular, in these frameworks, all elements are prima-facie $\mathbf{P} = \mathbf{A} \cup \mathbf{K}$ (so supports are not needed $\mathbf{S} = \varnothing$). Furthermore, an attack only targets arguments, $\mathbf{t}(\alpha) \in \mathbf{A}$ for all $\alpha \in \mathbf{K}$, and the source is a single argument, represented by the restriction to singleton sets $|\mathbf{s}(\alpha)| = 1$.

6 Relation with Evidence-Based Argumentation Frameworks

As mentioned in the introduction, (non-recursive) EBAFs were first introduced in [27]. When we are restricted to non-recursive frameworks, the major difference between EBAFs and our frameworks comes from the way in which the notion of acceptability is defined. In both cases, every acceptable argument must also be supported but while, in EBAFs, acceptability relies on what is called *evidence-supported attack* (*e-attack* for short), in our theory, it relies on the idea that arguments are *unacceptable* if they cannot be supported or are defeated. Intuitively, an e-attack is a pair (B, a) where B groups together the arguments necessary to attack a and all the arguments necessary to support all those arguments. Then, acceptability is defined requiring defence against e-attacks instead of standard attacks. In this sense, an EBAF can be understood as a (possibly exponential in size) Dung's framework in which arguments are self-supporting sets and attacks are the e-attacks [28].

Let us start by defining the non-recursive framework that corresponds to some EBAF with finite set of arguments.

Definition 18. *Given an* $\mathbf{EBAF} = \langle \mathbf{A}, \mathbf{R}_a, \mathbf{R}_e \rangle$*, by* $\mathbf{AF}_{\mathbf{EBAF}} = \langle \mathbf{A}, \mathbf{K}, \mathbf{S}, \mathbf{s}, \mathbf{t}, \mathbf{P} \rangle$ *we denote the argumentation framework where* $\mathbf{K}$ *and* $\mathbf{S}$ *are two (disjunct) sets with* *the same cardinality as $\mathbf{R}_a$ and $\mathbf{R}_e$, respectively; $\mathbf{P} = \mathbf{K} \cup \mathbf{S} \cup \{\eta\}$ and functions* **s** *and* **t** *map each attack and support name to their corresponding source and target,*[9] *that is, they satisfy:*

$$\begin{aligned} \mathbf{R}_a &= \{ \ (\mathbf{s}(\alpha), \mathbf{t}(\alpha)) \ \big| \ \alpha \in \mathbf{K} \ \} \\ \mathbf{R}_e &= \{ \ (\mathbf{s}(\beta), \mathbf{t}(\beta)) \ \big| \ \beta \in \mathbf{S} \ \ \} \end{aligned}$$

Given a set $E \subseteq \mathbf{A}$, by $\mathfrak{A}_E \stackrel{\text{def}}{=} \langle E, \mathbf{K}, \mathbf{S} \rangle$ we denote its corresponding structure. □

Observation 2. Since there are no attacks against other attacks or supports, *every d-structure w.r.t. some $\mathbf{AF}_{\mathbf{EBAF}}$ is of the form $\mathfrak{A}_E$ for some set of arguments $E \subseteq \mathbf{A}$.* □

In order to establish the existence of a one-to-one correspondence between finite EBAFs and non-recursive argumentation frameworks in our theory, let us define $\mathtt{struct}_{\mathbf{EBAF}}(\cdot)$ as the function mapping any set of arguments E into the structure $\mathfrak{A}_E = \langle E, \mathbf{K}, \mathbf{S} \rangle$.

Theorem 4. *Let* **EBAF** *be some finite EBA framework. Then, the function $\mathtt{struct}_{\mathbf{EBAF}}(\cdot)$ is a one-to-one correspondence between its self-supporting (resp. conflict-free, admissible, complete, preferred or stable) sets according to Definition 12 and the self-supporting (resp. conflict-free, admissible, complete, preferred or stable) d-structures of its corresponding framework $\mathbf{AF}_{\mathbf{EBAF}}$.* □

The above result holds for the finite case. That immediately rises the question whether this correspondence can be generalised to non-finite frameworks. The following example answers this question in a negative way.

Example 10. Let $\mathbf{EBAF} = \langle \mathbf{A}, \mathbf{R}_a, \mathbf{R}_e \rangle$ be some EBAF with a set of arguments $\mathbf{A} = \{\eta, a, b, c_1, c_2 \ldots\}$, a set of attacks $\mathbf{R}_a = \{(\{a\}, b)\}$ and a set of supports

$$\begin{aligned} \mathbf{R}_e = \quad & \{(\{\eta\}, b)\} \cup \{(\{\eta\}, c_1), \ (\{\eta\}, c_2), \ldots\} \\ & \cup \{(\{c_1, c_2, \ldots, \}, a), \ (\{c_2, \ldots\}, a), \ldots\} \end{aligned}$$

Let $E = \mathbf{A} \backslash \{a\}$ be a set of arguments. It is easy to see that every argument is supported according to Definition 9 and, thus, that a and all c_i are acceptable because there is no attack against them. This implies that b is not acceptable because it is attacked by a which is supported and not defeated and, thus, that E is not admissible. On the other hand, according to Definition 11, argument b is also acceptable w.r.t. E. Just note that, for every e-attack (C, b) against b, the set C must include a and infinitely many c_i's and thus, there is always some e-attack (C', b) against b with $C' = C \backslash \{c_i\}$ and $c_i \in C$. Hence, there is no minimal e-attack against b, which immediately implies that b is acceptable and that E is admissible. □

[9] In other words, for a given $(C, a) \in \mathbf{R}_a$, if α denotes the associated name in $\mathbf{K}$, we have $s(\alpha) = C$ and $t(\alpha) = a$.

It is worth to note that Example 10 can be also used to show that some usual results of abstract argumentation framework are not satisfied for non-finite EBAFs. In particular, the following example illustrates that neither the Fundamental Lemma nor the usual relations between semantics are satisfied:

Example 10 (cont'd) Note that a is acceptable w.r.t. the admissible set E, but $E \cup \{a\}$ is not conflict-free (and, thus, not admissible) because a attacks b. This is a counterexample to the Fundamental Lemma. Furthermore, this also implies that E is a preferred set, though it is not a complete one, so the usual relations among semantics are not satisfied. □

7 Conclusion

In this work we have extended Dung's abstract argumentation framework with recursive attacks and supports. One of the essential characteristics of this extension is that semantics are given with respect to the notion of "valid attacks and supports" which respectively play a role analogous to attacks in Dung's frameworks and supports in Evidence-Based Argumentation (EBA). The bases for this extension were first settled in [11], where semantics for frameworks with recursive attacks without supports were studied. The notions of "grounded attack/support" and "valid attack/support" have been introduced in [9] and encoded through a two-step translation into a meta-argumentation framework.[10] In the first step, a meta-argument is associated to an attack, and a support relation is added from the source of the attack to the meta-argument. In the second step, a support relation is encoded by the addition of a new meta-argument and new attacks. So [9] uses a method for flattening a recursive framework. As a consequence, extensions contain different kinds of argument. In contrast, we propose a theory where valid attacks remain explicit, and distinct from arguments, within the notion of structure.

It is worth mentioning that this extension is a conservative extension with respect to Dung's approach (when d-structures are considered) and that we have proved a one-to-one correspondence with finite EBA frameworks. We have also shown that non-finite EBA frameworks do not satisfy the Fundamental Lemma nor the usual relations among semantics. In this sense, our approach is an alternative semantics for non-finite frameworks with evidence-based supports that satisfies these properties. In addition, with restricted frameworks without supports, we inherit, from [11], a one-to-one correspondence with AFRA-extensions [5] in the case of the complete, preferred and stable semantics.

For a better understanding of the recursive frameworks, future work should include the study of other semantics (stage, semi-stable, grounded and ideal), extending our approach by taking into account other bipolar interactions [16, 32],

[10] Note that meta-argumentation frameworks have been often used for flattening complex argumentation frameworks (such as bipolar or recursive ones, see for instance [5, 16, 9]). More generally, meta-level argumentation is concerned with using arguments to reason over arguments (see for instance [22, 23]).

and enriching the translation proposed by [6, 8, 19, 29] from Dung's framework into propositional logic and ASP, in order to capture RAF. This is the best way for encoding these frameworks (either directly, or by a flattening process) in order to obtain efficient practical implementations that could be tested in the ICCMA competition (see[1]).

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