# Structure-based semantics of argumentation frameworks with higher-order attacks and supports

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**Abstract** In this paper, we propose a generalisation of Dung's abstract argumentation framework that allows representing higher-order attacks and supports, that is attacks or supports whose targets are other attacks or supports. We follow the necessary interpretation of the support, based on the intuition that the acceptance of an argument requires the acceptance of each supporter. We propose semantics accounting for acceptability of arguments and validity of interactions, where the standard notion of extension is replaced by a triple of a set of arguments, a set of attacks and a set of supports. Our framework is a conservative generalisation of Argumentation Frameworks with Necessities (AFN). When supports are ignored, Argumentation Frameworks with Recursive Attacks are recovered.

**Keywords.** Abstract argumentation, bipolar argumentation, higher-order interactions

## 1. Introduction

Abstract argumentation frameworks have greatly eased the modelling and study of argumentation. Whereas Dung's framework [12] only accounts for an attack relation between arguments, two natural generalisations have been developed in order to allow positive interactions (usually expressed by a support relation) and higher-order interactions (attacks or supports that target other attacks or supports). Here is an example in the legal field, borrowed from [1], that illustrates both generalisations (this example corresponds to a dynamic process of exchange of pieces of information, each one being considered as an "argument").

**Ex. 1** *The prosecutor says that the defendant has intention to kill the victim (argument b). A witness says that she saw the defendant throwing a sharp knife towards the victim (argument a). Argument a can be considered as a* support *for argument b. The lawyer argues back that the defendant was in a habit of throwing the knife at his wife's foot once drunk. This latter argument (argument c) is better considered* attacking the support *from a to b, than arguments a or b themselves. Now the prosecutor's argumentation seems no longer sufficient for proving the intention to kill.*

Different interpretations for the notion of support were proposed: deductive support [3], evidential support [16], necessary support [15], that are compared in [8,9]. Recent works have focused on the necessary interpretation, for instance in Argumentation Frameworks with Necessities (AFN) [14], and in [10,11,4]. In [17], correspondences are provided between a

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framework with evidential support and an AFN. In evidential argumentation standard arguments need to be supported by special (called prima-facie) arguments in order to be considered as acceptable. So arguments need to be able to trace back to prima-facie arguments. With the necessary interpretation of support as in AFN, arguments need to be able to trace back to arguments that require no support in order to be considered as acceptable.
It is worth to note that [8,17,14] do not allow the representation of higher-order interactions. In contrast, higher-order interactions (attacks as well as supports) have been considered in [10,11,4], with different ways for defining acceptability semantics: a translation into a standard Dung's AF [10], meta-argumentation techniques [4], a direct characterization of extension-based acceptability semantics [11].
Very recently, a new framework called Recursive Evidence-Based Argumentation Framework (REBAF) has been proposed that accounts for higher-order attacks and higher-order evidential supports [6]. This framework handles both acceptability of arguments and validity of interactions (attacks or supports).

In this paper, our purpose is to propose a Recursive Argumentation Framework with Necessities (RAFN) with semantics accounting for acceptability of arguments and validity of interactions, in the case of higher-order attacks and higher-order necessary supports. Moreover, we are interested in a conservative generalisation of AFN. Taking advantage of the correspondences that have been established between evidential and necessary support in [17], our methodology and definitions draw on the REBAF of [6].
The paper is organized as follows: Section 2 gives some background about necessary support and about the REBAF; the definition and semantics for the RAFN are proposed in Section 3; in Section 4 we prove a one-to-one correspondence with AFN in the case of first-order interactions, and we give a comparison with recent work about recursive attacks and supports [11]; and we conclude in Section 5. Proofs are given in [7].

## 2. Background

We next review some basic background about the works the paper is based on: an abstract argumentation framework handling first-order necessary supports (AFN), and a recent approach dealing with higher-order attacks and evidential supports (REBAF).

***First-order necessary support (AFN).*** Binary necessary support was initially introduced in [15], then discussed in [10,11,4] in a more general context (particularly with higher-order interactions). Let $a$ and $b$ be two arguments, "$a$ necessarily supports $b$" means that the acceptance of $a$ is necessary to get the acceptance of $b$, or equivalently that the acceptance of $b$ implies the acceptance of $a$. Necessary support has been extended to express the fact that a given argument requires at least one element among a set of arguments. In [14], an Argumentation Framework with Necessities (AFN) is defined as follows:

**Def. 1 (AFN [14])** *An* Argumentation Framework with Necessities (AFN) *is a tuple* $\langle \mathbf{A}, \mathbf{R}, \mathbf{N} \rangle$ *, where* $\mathbf{A}$ *is a finite and non-empty set of arguments,* $\mathbf{R} \subseteq \mathbf{A} \times \mathbf{A}$ *represents the* attack relation *and* $\mathbf{N} \subseteq (2^{\mathbf{A}} \setminus \varnothing) \times \mathbf{A}$ *represents the* necessity relation.

For $E \subseteq \mathbf{A}$, $E\mathbf{N}b$ reads "$E$ is a necessary support for $b$", which means that if no argument of $E$ is accepted then $b$ cannot be accepted, or equivalently that the acceptance of $b$ requires the acceptance of *at least one element* of $E$. Moreover, in AFN semantics, acyclicity of the support relation is required among accepted arguments. Intuitively, in a given extension, support for each argument is provided by at least one of its necessary arguments and there is no risk of a deadlock due to necessity cycles. These requirements have been formalized in [14] and can be reformulated as follows:

**Def. 2 (Semantics in AFN)** *Given* $AFN = \langle \mathbf{A}, \mathbf{R}, \mathbf{N} \rangle$ *and* $T \subseteq \mathbf{A}$.

- *$T$ is* support-closed *iff for each $a \in T$, if $E$***N***$a$, then $E \cap T \neq \varnothing$. Assume that $T$ is support-closed. $a \in T$ is* support-cycle-free in *$T$ iff $\forall E \subseteq$* **A** *such that $E$***N***$a$, there is $b \in E \cap T$ such that $b$ is support-cycle-free in $T \setminus \{a\}$. $T$ is* coherent *iff $T$ is support-closed and every $a \in T$ is support-cycle-free in $T$.*
- *$a \in$* **A** *is* deactivated by *$T$ iff $\forall C \subseteq$* **A** *coherent subset containing $a$, $T$***R***$C$ (i.e. there is $x \in T$ and $c \in C$ such that $x$***R***$c$). $a \in$* **A** *is* acceptable w.r.t. *$T$ iff (i) $T \cup \{a\}$ is coherent and (ii) $\forall b \in$* **A** *such that $b$***R***$a$, $b$ is deactivated by $T$.*
- *$T$ is* admissible *iff $T$ is conflict-free, coherent, and every $a$ in $T$ is acceptable w.r.t. $T$. $T$ is a* complete *extension iff $T$ is admissible and $\forall a \in$* **A**, *if $a$ is acceptable w.r.t. $T$, then $a \in T$. $T$ is a* preferred *extension iff $T$ is a $\subseteq$-maximal complete extension. $T$ is a* stable *extension iff $T$ is complete and $\forall a \in$* **A**, *$a \in$* **A** *$\setminus T$ iff $a$ is deactivated by $T$. $T$ is a* grounded *extension iff $T$ is a $\subseteq$-minimal complete extension.*

**Ex. 2** *Consider the framework representing an attack from $a$ to $b$ and no necessary support. The unique extension under complete, preferred, stable and grounded semantics is $\{a\}$. Indeed, the AFN framework is a conservative generalisation of Dung's framework.*

**Ex. 3** *Consider the framework representing a necessary support from $\{a\}$ to $b$ and no attack. $\{\}$ and $\{a\}$ are admissible sets. However, due to the necessary support, an admissible set containing $b$ must also contain $a$. So, $\{b\}$ is not admissible, and the unique complete extension is $\{a,b\}$.*

**Ex. 4** *Consider the framework representing a cycle of necessary supports between $a$ and $b$, and no attack. This cycle is represented by $\{a\}$***N***$b$ and $\{b\}$***N***$a$. There is no non-empty admissible set. Indeed, there is no way to trace back with a chain of supports from $a$ (resp. $b$) to arguments that require no support.*

***Recursive Evidence-Based Argumentation Frameworks (REBAF).*** Recently introduced in [6], the REBAF allows representing higher-order attacks and higher-order supports. It is a generalisation of the Evidence-Based Argumentation Framework (EBAF) [17]. In these frameworks, the "evidential" understanding of the support relation allows to distinguish between two different kinds of arguments: *prima-facie* and *standard arguments*. *Prima-facie* arguments are justified whenever they are not defeated. On the other hand, *standard arguments* are not assumed to be justified and must inherit support from prima-facie arguments through a chain of supports. In the REBAF, the semantics handle both acceptability of arguments and validity of interactions (attacks or supports), and account for the fact that acceptability of arguments may depend on the validity of interactions and vice-versa. As a consequence, the standard notion of extension is replaced by a triple of a set of arguments, a set of attacks and a set of supports, called "structure". We briefly recall the main definitions.

**Def. 3 (Recursive EBAF and structure)** *A* Recursive Evidence-Based Argumentation Framework (REBAF) *is a sextuple $\langle \mathbf{A}, \mathbf{R}, \mathbf{S}, s, t, \mathbf{P} \rangle$, where* **A**, **R** *and* **S** *are three pairwise disjunct sets respectively representing arguments, attacks and supports names, and where $\mathbf{P} \subseteq \mathbf{A} \cup \mathbf{R} \cup \mathbf{S}$ is a set representing the* prima-facie *elements that do not need to be supported.*[3] *Functions $s : (\mathbf{R} \cup \mathbf{S}) \longrightarrow 2^{\mathbf{A}}$ and $t : (\mathbf{R} \cup \mathbf{S}) \longrightarrow (\mathbf{A} \cup \mathbf{R} \cup \mathbf{S})$ respectively map each attack and support to its source and its target.*
*A* structure *of $\langle \mathbf{A}, \mathbf{R}, \mathbf{S}, s, t, \mathbf{P} \rangle$ is a triple $U = (T, \Gamma, \Delta)$ with $T \subseteq \mathbf{A}$, $\Gamma \subseteq \mathbf{R}$ and $\Delta \subseteq \mathbf{S}$.*

The notion of structure allows characterizing which arguments are regarded as "acceptable" and which attacks and supports are regarded as "valid" with respect to a given framework. It is the basis of defining the semantics for recursive frameworks. Intuitively, the set $T$ rep-

[3] Note that the set **P** may contain several prima-facie elements (arguments, attacks and supports) without any constraint (they can be attacked or supported).

resents the set of "acceptable" arguments w.r.t. the structure $U$, while $\Gamma$ and $\Delta$ respectively represent the set of "valid attacks" and "valid supports" w.r.t. $U$. For the rest of this section, we consider a REBAF $\langle \mathbf{A}, \mathbf{R}, \mathbf{S}, s, t, \mathbf{P} \rangle$ and a structure $U$ of this REBAF. An element $x$ (argument, attack or support) is defeated w.r.t. $U$ iff there is a "valid attack" w.r.t. $U$ that targets $x$ and whose source is "acceptable" w.r.t. $U$. As for the notion of *supported elements* w.r.t. a structure, the prima-facie elements of a REBAF are supported w.r.t. any structure. Then, a standard element is supported if there exists a chain of supported supports, leading to it, which is rooted in prima-facie arguments. Formally, the set of defeated (resp. supported) elements is defined as follows:

**Def. 4 ([6])** *$Def_X(U) = \{x \in X / \exists \alpha \in \Gamma, s(\alpha) \subseteq T$ and $t(\alpha) = x\}$ with $X \in \{\mathbf{A}, \mathbf{R}, \mathbf{S}\}$. Let $U_{-x}$ denote $(T \setminus \{x\}, \Gamma \setminus \{x\}, \Delta \setminus \{x\})$. $Supp(U) = \mathbf{P} \cup \{t(\alpha) / \exists \alpha \in (\Delta \cap Supp(U_{-t(\alpha)}))$ with $s(\alpha) \subseteq (T \cap Supp(U_{-t(\alpha)})))\}$.*

Drawing on the notions of defeated elements and supported elements, the *supportable* elements can be defined. An element is supportable if there exists some non-defeated support with all its source elements non-defeated and regarded as supportable. Formally, an element $x$ is supportable w.r.t. $U$ iff $x$ is supported w.r.t. $U' = (\overline{Def_{\mathbf{A}}(U)}, \mathbf{R}, \overline{Def_{\mathbf{S}}(U)})$.[4] Elements that are defeated or unsupportable cannot be accepted. $UnAcc(U) = Def(U) \cup \overline{Supp(U')}$ denotes the set of *unacceptable* elements w.r.t. $U$. Moreover, an attack $\alpha \in \mathbf{R}$ is *unactivable*[5] iff either it is unacceptable or some element in its source is unacceptable. $UnAct(U) = \{\alpha \in \mathbf{R} / \alpha \in UnAcc(U)$ or $s(\alpha) \cap UnAcc(U) \neq \varnothing\}$. Finally, an element is acceptable w.r.t. $U$ iff it is supported w.r.t. $U$ and, in addition, every attack against it is unactivable w.r.t. $U$, because either some argument in its source or itself has been regarded as unacceptable w.r.t. $U$. $Acc(U)$ denotes the set of all elements that are acceptable w.r.t. $U$. Semantics are defined as follows:

**Def. 5 (Semantics in REBAF [6])** *Let $U$ be the structure $(T, \Gamma, \Delta)$. $U$ is* self-supporting *iff $(T \cup \Gamma \cup \Delta) \subseteq Supp(U)$. $U$ is* conflict-free *iff $T \cap Def_{\mathbf{A}}(U) = \varnothing$, $\Gamma \cap Def_{\mathbf{R}}(U) = \varnothing$ and $\Delta \cap Def_{\mathbf{S}}(U) = \varnothing$. $U$ is* admissible *iff it is conflict-free and $(T \cup \Gamma \cup \Delta) \subseteq Acc(U)$. $U$ is* complete *iff it is conflict-free and $(T \cup \Gamma \cup \Delta) = Acc(U)$. $U$ is* preferred *iff it is a $\subseteq$-maximal*[6] *admissible structure. $U$ is* stable *iff $(T \cup \Gamma \cup \Delta) = \overline{UnAcc(U)}$.*

## 3. Handling higher-order necessary supports

Our purpose is to propose a framework that allows representing higher-order attacks and higher-order necessary supports, using similar definitions as those at work in the REBAF.

**Def. 6 (Recursive AFN)** *A* Recursive Argumentation Framework with Necessities (RAFN) *is a tuple $\langle \mathbf{A}, \mathbf{R}, \mathbf{N}, s, t \rangle$, where $\mathbf{A}$, $\mathbf{R}$ and $\mathbf{N}$ are three pairwise disjunct sets respectively representing arguments, attacks and supports names, $s$ is a function from $\mathbf{R} \cup \mathbf{N}$ to $(2^{\mathbf{A}} \setminus \varnothing)$ mapping each interaction to its source,*[7] *and $t$ is a function from $\mathbf{R} \cup \mathbf{N}$ to $(\mathbf{A} \cup \mathbf{R} \cup \mathbf{N})$ mapping each interaction to its target. It is assumed that $\forall \alpha \in \mathbf{R}, s(\alpha)$ is a singleton.*
*A* structure *of $\langle \mathbf{A}, \mathbf{R}, \mathbf{N}, s, t \rangle$ is a triple $U = (T, \Gamma, \Delta)$ with $T \subseteq \mathbf{A}$, $\Gamma \subseteq \mathbf{R}$ and $\Delta \subseteq \mathbf{N}$.*

Let us consider a RAFN $\langle \mathbf{A}, \mathbf{R}, \mathbf{N}, s, t \rangle$ and a structure $U$ of this RAFN. We keep the definition for an element being defeated recalled in Section 2 (which can be simplified as $\forall \alpha \in \mathbf{R}, s(\alpha)$ is a singleton). In contrast, a difference appears with the notion of *supported*

[4]Let $U$ be a structure, $X \in \{\mathbf{A}, \mathbf{R}, \mathbf{S}\}$ and $f_X(U)$ a subset of $X$. $\overline{f_X(U)}$ denotes the set $X \setminus f_X(U)$. Moreover, $f(U)$ is short for $f_{\mathbf{A}}(U) \cup f_{\mathbf{R}}(U) \cup f_{\mathbf{S}}(U)$. And as usual, $\overline{f(U)}$ denotes $\mathbf{A} \cup \mathbf{R} \cup \mathbf{S} \setminus f(U)$.

[5] Intuitively, such an attack cannot be "activated" in order to defeat the element that it is targeting.

[6]For any pair of structures $U = (T, \Gamma, \Delta)$ and $U = (T', \Gamma', \Delta')$, $U \subseteq U'$ means that $(T \cup \Gamma \cup \Delta) \subseteq (T' \cup \Gamma' \cup \Delta')$.

[7]In contrast with ASAF (see [11]), the source of a support in a RAFN is a *set* of arguments.

*elements*: elements (arguments, attacks, supports) which receive no necessary support do not require any support, so they are supported w.r.t. any structure. That corresponds to the set $\mathbf{P}$ in Def. 7 below. Moreover, in an AFN, for $E \subseteq \mathbf{A}$, $E\mathbf{N}x$ means that the acceptance of $x$ requires the acceptance of *at least one* element of $E$. Then, an element $x$ is supported w.r.t. $U$ if *for each* support $\alpha$ (which can be regarded as supported), the source of $\alpha$ contains *at least one* argument of $U$ that can be regarded as supported. Formally, we have:

**Def. 7** *Given a structure* $U = (T, \Gamma, \Delta)$

- $Def_X(U) = \{x \in X / \exists \alpha \in \Gamma, s(\alpha) \in T \text{ and } t(\alpha) = x\}$ *with* $X \in \{\mathbf{A}, \mathbf{R}, \mathbf{N}\}$.
- *Let* $\mathbf{P} = \{x \in \mathbf{A} \cup \mathbf{R} \cup \mathbf{N} /$ *there is no* $\alpha \in \mathbf{N}$ *with* $t(\alpha) = x\}$. $Supp(U) = \mathbf{P} \cup \{x / \forall \alpha \in \Delta$ *such that* $t(\alpha) = x$, *if* $\alpha \in Supp(U_{-x})$ *then* $s(\alpha) \cap (T \cap Supp(U_{-x})) \neq \varnothing\}$.
- $U$ *is* self-supporting *iff* $(T \cup \Gamma \cup \Delta) \subseteq Supp(U)$.

Pursuing the analogy with REBAF, an element of a RAFN is considered as being still supportable as long as *for each* non-defeated support, *there exists at least one* argument in it source, which is non-defeated and regarded as supportable. Formally, an element $x$ is supportable w.r.t. $U$ iff $x$ is supported w.r.t. $U' = (\overline{Def_{\mathbf{A}}(U)}, \mathbf{R}, \overline{Def_{\mathbf{N}}(U)})$. Drawing on these new notions of supported (resp. unsupportable) element, we keep the definitions used in a REBAF for unacceptable elements and unactivable attacks. Namely, elements that are defeated or that are unsupportable are said to be *unacceptable* (they cannot be accepted). Then an attack $\alpha \in \mathbf{R}$ is *unactivable* (such an attack cannot be "activated" in order to defeat the element that it is targeting) iff either it is unacceptable or its source is unacceptable. Finally we keep the definition for acceptability used in a REBAF.

**Def. 8** *Given a structure* $U = (T, \Gamma, \Delta)$, *let* $U' = (\overline{Def_{\mathbf{A}}(U)}, \mathbf{R}, \overline{Def_{\mathbf{N}}(U)})$.

- $UnSupp(U) = \overline{Supp(U')}$.
- $UnAcc(U) = Def(U) \cup UnSupp(U)$ *denotes the set of* unacceptable *elements w.r.t.* $U$.
- $UnAct(U) = \{\alpha \in \mathbf{R} / \alpha \in UnAcc(U) \text{ or } s(\alpha) \subseteq UnAcc(U)\}$ *denotes the set of* unactivable *attacks w.r.t.* $U$.
- $x \in \mathbf{A} \cup \mathbf{R} \cup \mathbf{N}$ *is acceptable w.r.t.* $U$ *iff* $x \in Supp(U)$ *and for each* $\alpha \in \mathbf{R}$ *with* $t(\alpha) = x$, $\alpha \in UnAct(U)$. $Acc(U)$ *denotes the set of all elements that are acceptable w.r.t.* $U$.

**Ex. 5** *Consider the framework* $RAFN = \langle \{a, x, y, z, t\}, \{\beta\}, \{\alpha_1, \alpha_2, \alpha_3\}, s, t \rangle$ *with* $s(\alpha_1) = \{a\}$, $s(\alpha_2) = \{z\}$, $s(\alpha_3) = \{t\}$, $s(\beta) = \{y\}$, $t(\alpha_1) = x, t(\alpha_2) = y$, $t(\alpha_3) = y$, $t(\beta) = x$.
*Let* $U = (\{a\}, \{\beta\}, \{\alpha_1, \alpha_2, \alpha_3\})$. $U' = (\mathbf{A}, \mathbf{R}, \mathbf{N})$. $x \in Supp(U)$. *However,* $x \notin Acc(U)$ *as it is the target of the attack* $\beta$, $s(\beta) = \{y\}$ *and* $y \notin UnAcc(U)$. *Indeed* $y$ *is not attacked and* $y \in Supp(U')$ *since* $\alpha_2$ *and* $\alpha_3$ *belong to* $\mathbf{P}$ *and* $z$ *and* $t$ *do not belong to* $Def_{\mathbf{A}}(U)$.

**Ex. 6** *Consider the RAFN obtained by adding an attack* $\gamma$ *from* $a$ *to* $z$ *in the RAFN of Ex. 5 and the new structure* $U = (\{a\}, \{\beta, \gamma\}, \{\alpha_1, \alpha_2, \alpha_3\})$. *With this new structure, we have* $z \in Def_{\mathbf{A}}(U)$. *So* $y \notin Supp(U')$ *and therefore* $x$ *becomes acceptable w.r.t.* $U$.

The Fundamental Lemma cannot be generalized, since the function *Supp* is not monotonic: Let us consider Ex. 3 modified as follows: $RAFN = \langle \{a, b, c\}, \varnothing, \{\alpha, \delta\}, s, t \rangle$ with $s(\alpha) = \{a\}$, $s(\delta) = \{c\}$, $t(\alpha) = b$, $t(\delta) = \alpha$. Let $U = (\{b\}, \varnothing, \{\alpha, \delta\})$. $b \in Supp(U)$ since $c \notin T$ and so $\alpha$ is not supported. However, $b \notin Supp(U \cup \{c\})$ since $a \notin T$.

As a consequence, semantics are defined as follows:[8]

**Def. 9 (Semantics in RAFN)** *Let* $U$ *be the structure* $(T, \Gamma, \Delta)$. $U$ *is* conflict-free *iff* $T \cap Def_{\mathbf{A}}(U) = \varnothing$, $\Gamma \cap Def_{\mathbf{R}}(U) = \varnothing$ *and* $\Delta \cap Def_{\mathbf{N}}(U) = \varnothing$. $U$ *is* admissible *iff it is conflict-*

[8] As there is no Fundamental Lemma, preferred and stable extensions are assumed to be complete sets.

*free and* $(T\cup\Gamma\cup\Delta)\subseteq Acc(U)$.[9] *U is* complete *iff it is conflict-free and* $(T\cup\Gamma\cup\Delta) = Acc(U)$. *U is* preferred *iff it is a* $\subseteq$*-maximal complete structure. U is* stable *iff it is complete and* $\overline{(T\cup\Gamma\cup\Delta)} = UnAcc(U)$. *U is* grounded *iff it is a* $\subseteq$*-minimal complete structure.*

**Ex. 7** *Consider the framework* $RAFN = \langle\{a,b,c,d,e\},\{\alpha_3\},\ \{\alpha_1,\alpha_2\},s,t\rangle$ *with* $s(\alpha_1) = \{b,c\}$, $s(\alpha_2) = \{d\}$, $s(\alpha_3) = \{e\}$, $t(\alpha_1) = t(\alpha_2) = a$, $t(\alpha_3) = b$. *We have* $\mathbf{P} = \{b,c,d,e,\alpha_1,\alpha_2,\alpha_3\}$. *Let us study different structures:*

- $U_1 = (\{a,b,d,e\},\varnothing,\{\alpha_1,\alpha_2\})$. *$U_1$ is conflict-free (as $\alpha_3\notin\Gamma_1$) and self-supporting. As $b,d,e,\alpha_1,\alpha_2$ belong to* $\mathbf{P}$, *we just have to prove that $a\in Supp(U_1)$. Due to Def. 7, we have to consider $\alpha_1$ and $\alpha_2$, the supports in $\Delta_1$ that target a. As both of them belong to* $\mathbf{P}$, *we have to consider their source. $s(\alpha_2) = \{d\}\subseteq \mathbf{P}\cap T_1$, and $s(\alpha_1)$ contains b that is an element of $T_1\cap\mathbf{P}$. So $a\in Supp(U_1)$. However, $b\notin Acc(U_1)$. Indeed $\alpha_3\notin Unact(U_1)$ as $\alpha_3$ and $s(\alpha_3)$ both belong to* $\mathbf{P}$ *and to $\overline{Def(U_1)}$. So $U_1$ is not admissible.*
- $U_2 = (\{a,c,d,e\},\varnothing,\{\alpha_1,\alpha_2\})$. *$U_2$ is conflict-free. It is also self-supporting (it can be proved as for $U_1$ replacing b by c) and no element of $U_2$ is attacked. So $U_2$ is admissible.*
- $U_3 = (\{a,c,d,e\},\{\alpha_3\},\{\alpha_1,\alpha_2\})$ *is the unique preferred structure. Note that $U_3$ follows the intuition behind Def. 7, that at least one element in the source of the support $\alpha_1$ has to be accepted (here c) in order to accept the target (here a).*

**Ex. 4 (cont'd)** *Consider the RAFN corresponding to the AFN ($\alpha_1$ and $\alpha_2$ being the names of the supports). $U = (\varnothing,\varnothing,\{\alpha_1,\alpha_2\})$ is the unique stable structure. So differently from Dung's approach, it can be the case that an element is not in the stable structure even if it is not defeated by it (it is left out because it is unsupported by the structure).*

## 4. Related works

First, we consider the particular case of RAFN without support, then we compare our framework with AFN and ASAF.

***RAFN without support.*** In that case we get exactly the definitions of the Recursive Argumentation Framework (RAF) of [5]. Besides, [5] provided correspondences between RAF-structures and AFRA-extensions of [2]. The RAFN without support also corresponds to the REBAF without support (in the particular case of binary attacks). Moreover, a RAFN with only first-order attacks and without support is a RAF with only first-order attacks. That case has been proved to be a conservative generalisation of Dung's framework in [5].

***Relation with AFN.*** We show that the RAFN is a conservative generalisation of the AFN. Given an AFN, we give a translation into a RAFN, and prove a one-to-one correspondence between complete (resp. preferred, stable, grounded) extensions of the AFN and complete (resp. preferred, stable, grounded) structures of the corresponding RAFN.

**Def. 10** *Given* $AFN = \langle\mathbf{A},\mathbf{R},\mathbf{N}\rangle$, *the corresponding RAFN is* $\langle\mathbf{A},\mathbf{R}',\mathbf{N}',s',t'\rangle$, *where* $\mathbf{R}'$ *and* $\mathbf{N}'$ *are two disjunct sets with the same cardinality as* $\mathbf{R}$ *and* $\mathbf{N}$ *respectively, and $s'$ and $t'$ map each interaction to their corresponding source and target, that is:*

- *for* $(a,b)\in\mathbf{R}$, *and $\alpha$ the associated name in* $\mathbf{R}'$, *we have* $s'(\alpha) = \{a\}$ *and* $t'(\alpha) = b$.
- *for* $(X,b)\in\mathbf{N}$, *and $\beta$ the associated name in* $\mathbf{N}'$, *we have* $s'(\beta) = X$ *and* $t'(\beta) = b$.

Following Def. 7, $\mathbf{P}' = \{x\in\mathbf{A}/$ there is no $\alpha\in\mathbf{N}'$ with $t'(\alpha) = x\}\cup\mathbf{R}'\cup\mathbf{N}'$.
Note that in an AFN, each attack (resp. support) can be considered as "valid", as it is neither attacked nor supported. Hence, in the corresponding RAFN, such an interaction must be acceptable w.r.t. any structure. Accordingly, given a set $T\subseteq\mathbf{A}$, by $U_T = (T,\mathbf{R}',\mathbf{N}')$ we denote its corresponding structure. Then it can be proved:

[9] It follows that an admissible structure is also self-supporting.

**Prop. 1** *Let $T \subseteq \mathbf{A}$.*

1. *Let $a \in T$. If $T$ is support-closed in AFN, then $a \in Supp(U_T)$ iff $a$ is support-cycle-free in $T$. Moreover $U_T$ is self-supporting in RAFN iff $T$ is coherent in AFN.*
2. *Let $a \in \mathbf{A}$. If $a$ is acceptable w.r.t. $T$ in AFN, then $a$ is acceptable w.r.t. $U_T$ in RAFN. If $T$ is self-supporting and $a$ is acceptable w.r.t. $U_T$ in RAFN, then $a$ is acceptable w.r.t. $T$ in AFN.*

**Prop. 2** *Let $T \subseteq \mathbf{A}$. $T$ is an admissible (resp. complete, preferred, stable, grounded) extension of AFN iff $U_T$ is an admissible (resp. complete, preferred, stable, grounded) structure of the corresponding RAFN.*

***Relation with ASAF.*** We next compare the RAFN semantics with ASAF semantics [11]. We consider particular cases of RAFN, as ASAF excludes cycles of necessary supports, and assumes that interactions are binary ones (the source of an attack or a support is a unique argument). The common idea is that the extensions may not only include arguments but also attacks and supports. However, several differences can be outlined. First, in ASAF, attacks and supports are combined to obtain extended (direct or indirect) defeats. Conflict-freeness for a set of elements (arguments, attacks, supports) is defined w.r.t. these extended defeats. So the conflict-freeness requirement takes support into account. In contrast, in RAFN, the notions of support and attack are dealt with separately (see Def. 7). As for acceptability, in ASAF, an element is acceptable w.r.t. a set of elements whenever it can be defended against each defeat. So, in the particular case when there is no attack, each argument would be acceptable w.r.t. any set. In contrast, Def. 8 explicitly requires a support.

**Ex. 3 (cont'd)** *The corresponding RAFN of AFN is $\langle\{a,b\},\varnothing,\{\alpha\},s,t\rangle$ with $s(\alpha)=\{a\}$, $t(\alpha)=b$. With ASAF semantics, the set $\{b,\alpha\}$ is admissible, whereas the structure ($\{b\}$, $\varnothing$, $\{\alpha\}$) is not admissible in RAFN.*

Another difference was already pointed in [5], where correspondences have been provided between a RAF and an ASAF without support. Indeed, in an ASAF, an attack is not acceptable whenever its source is not acceptable (Prop. 2 in [11]).

**Ex. 6 (cont'd)** *With RAFN semantics, $\beta$ is not attacked and not supported so $\beta$ must belong to each complete structure. With ASAF semantics, if $\beta$ is acceptable w.r.t. a set E, then y must be also acceptable w.r.t. E. If E is a complete extension, E contains a, $\gamma$ and $\alpha_2$. As y is defeated by $\gamma$ given $\alpha_2$ it cannot be the case that y is acceptable w.r.t. E. So $\beta$ cannot belong to any complete extension.*

So, following the work of [5], we define the following mappings:

- Let $\langle\mathbf{A},\mathbf{R},\mathbf{N},s,t\rangle$ be a RAFN. Given a structure $U=(T,\Gamma,\Delta)$, by $E_U = T \cup \{\alpha \in \Gamma$ such that $s(\alpha) \subseteq T\} \cup \Delta$, we denote the corresponding ASAF extension.
- Let $\langle\mathbf{A},\mathbf{R},\mathbf{N}\rangle$ be an ASAF. Given $E \subseteq (\mathbf{A}\cup\Gamma\cup\mathbf{N})$ an ASAF extension, by $U_E = (T_E,\Gamma_E,\Delta_E)$, we denote the corresponding RAFN structure, where $T_E = \mathbf{A}\cap E$, $\Gamma_E = (\mathbf{R}\cap E)\cup\{\alpha \in (\mathbf{R}\cap Acc(U'_E))$ such that $s(\alpha)\notin E\}$ with $U'_E$ denoting the structure $(T_E,\mathbf{R}\cap E,\mathbf{N}\cap E)$ and $\Delta_E = (\mathbf{N}\cap E)\cup(\mathbf{N}\cap Acc(U'_E))$.

Our intuition is that, despite the differences between conflict-free and acceptability requirements, the above mappings will enable to achieve correspondences between ASAF and RAFN for the complete (and also grounded and preferred) semantics.

**Ex. 6 (cont'd)** *Consider the unique complete structure $U=(\{a,x,t\},\{\beta,\gamma\},\{\alpha_1,\alpha_2,\alpha_3\})$. The corresponding ASAF extension is $E_U=\{a,x,t,\gamma,\alpha_1,\alpha_2,\alpha_3\}$. It can be checked that it is an ASAF complete extension. Conversely, let $E=\{a,x,t,\gamma,\alpha_1,\alpha_2,\alpha_3\}$. We have $U'_E=(\{a,x,t\},\{\gamma\},\{\alpha_1,\alpha_2,\alpha_3\})$. Obviously, $\beta\in Acc(U'_E)$ as $\beta$ is neither attacked nor supported. So $\beta$ can be added to $\Gamma_E$ and $U_E=U$.*

## 5. Conclusion

We have proposed an abstract framework that deals with higher-order interactions, using two types of interaction: attacks and necessary supports. That framework generalises both abstract frameworks with necessities (AFN, see [15,14]) and recursive abstract frameworks (RAF, see [5]), and so is called RAFN. We have defined semantics accounting for acceptability of arguments and also validity of interactions. As a source of inspiration, we have used the approach presented in [6] that does a similar work for REBAF, another framework dealing with higher-order interactions using evidential supports in place of necessary ones. In the literature, there exist few works handling higher-order attacks and necessary supports, except the ASAF framework [10,11]. However, ASAF excludes cycles of support and is restricted to binary interactions. Our framework is a conservative generalisation of AFN and RAF, and we are able to outline the differences with ASAF semantics proposed in [11]. In this work, we have defined structure-based semantics in a similar way as done in [6] for evidential support. That paves the way for studying a more general framework capable of taking into account both necessary supports and evidential supports. We aim to address that issue as future work. We also plan to connect RAFN to Logic Programming, following existing works relating Dung's framework to logic programs and ASP (for instance [13]).

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