BLA: a new framework for group decision making

Florence Bannay

Romain Guillaume

IRIT, Université Paul Sabatier,
118 route de Narbonne, 31062 Toulouse, France

{bannay,guillaume}@irit.fr

Rapport IRIT
RR–2015–9–FR

Mai 2015

Contents

Abstract

In this paper we investigate the problem of admissibility of a candidate under incomplete distributed knowledge. We formalize this admissibility in terms of the goals that will be achieved by selecting this candidate. The knowledge about what feature can lead to achieve what goal, and the importance of each goal are stored in a structure called BLA (Bipolar Leveled set of Arguments). Once the BLA has been established the decision makers are only allowed to utter information about the features that characterize the candidates. The goals that would be achieved by selecting a given candidate will be derived directly from the BLA and the features uttered. This promotes a more transparent decision process since the criteria may not be discussed during the evaluation of actual candidates.

Keyword: Decision process, Incomplete Knowledge, Arguments.

1 Introduction

A standard way [28] to handle a decision problem is to define a utility function which enables the decision maker to evaluate the quality of each decision and select the one that have the best utility. This utility function should be designed in order to take into account the multi-criteria aspects of the problem. Moreover, it is often the case in practice that the decision maker does not want to select a best candidate but wants rather to ensure a choice of a convenient one, since maybe the best candidate is not good at all (if all the candidates are bad). This is why we choose to focus on deciding whether a candidate is admissible or not: this is well adapted when decision makers can accept more than one candidate or when the flow of candidates is continuous over time and they want to stop the selection process at the first admissible candidate. Our aim in this paper is to propose a new rational model of collaborative decision making (or deliberation) in the presence of incomplete distributed knowledge, through the definition of clear admissibility criteria by taking advantage of the notions of efficiency and simplicity that are central in industrial domain.

We formalize the decision problem about the admissibility of a candidate in terms of the goals that will be achieved by this selection, the goals are positive or negative and they can be ensured to be achieved or ensured to be failed. Moreover among the goals some are more important to achieve than others. The knowledge about what are the features of a candidate that can lead to achieve what goal, i.e., the link between features and goals, together with the strength of these links associated to the importance of each goal is stored in a structure called BLA (Bipolar Leveled set of Arguments) which is constructed *before* the deliberation. This structure was first introduced in [6] with a simplified view of arguments. In the following, arguments are defeasible reasons to believe that some goals are achieved or failed and the attacks between them come from the conflicts between their conclusion (i.e. the achievement of a goal). Moreover the importance level of the goals together with the certainty of their reasons induces the importance levels of the arguments. In AI literature some models have already been proposed based on a bipolar view of alternatives. Indeed, it is often the case that human people evaluate the possible alternatives considering positive and negative aspects separately [13]. Moreover, argumentation has already been proposed to govern decision making in a negotiation context (see for instance [4] and [27] for a survey).

Our paper offers two new main contributions:

- For the first time : a clear and well-founded semantics of a decision argument, together with attacks, validity and realized goal. All the notions are defined clearly and their meaning are rationally interconnected. In our formalism, the realization of a goal (which could be viewed as similar to the admissibility of an argument)

is not built on an abstract concepts but is related to the nature of attacks which is based on the content of arguments. This leads us to clearly define what is an acceptable decision (by means of admissibility thresholds). Besides as we will show in Section 2.1, Dung's view of argumentation [21] is not suitable for our purpose.
- We have formalize the idea of a consensual definition of the criteria and of the goals to reach *before* making the decision. The decision is then straightforwardly computed only from facts that are established, no agent can directly give her opinion about the achievement of a goal, she can only describe objectively what are the features that hold. This consensual definition is summarized by a BLA (see Section 2) and can be viewed as a kind of qualitative utility function (see Section 3) which is both very informative, compact and easy to read.

2 Bipolar Leveled set of Arguments (BLA)

A BLA defines the set of possible arguments which may be used in order to evaluate the admissibility of a given candidate. In the following we first explain what we mean by "argument".

2.1 Arguments

We consider a set $\mathscr{C}$ of candidates[1] about which some knowledge is available. In the following, an argument is a reason for believing that a given goal can be achieved by selecting a candidate c.

Hence an argument is not a logic-based argument like in [7] since there is no explicit deductive link between the reason and the goal. Those arguments could be viewed as enthymemes [9, 22] but with the precision that the support and the conclusion are of different nature, namely based respectively on beliefs and preferences. This relation between beliefs and preferences comes from the fact that in decision problem, arguments should encode a kind of utility notion. Moreover the argument is itself a more or less objective link between the reason and the goal realized: this gives more freedom to the decision makers for building the arguments, hence we do not impose a complex definition based on a value-based transition system like in [8] (in which they impose a deductive link between conditions, actions and goals).

We neither deal with Dung-style abstract arguments for decision like in [3], since as we will see in Section 2.2, when an argument is attacked it is no more useful, hence Dung's defense notion is not appropriate for our purpose.

[1]Candidates are also called alternatives in the decision literature.

More precisely, we consider two languages $\mathscr{L}_F$ (a propositional language based on a vocabulary $\mathcal{V}_F$) representing information about some features that are believed to hold for a candidate and $\mathscr{L}_G$ (another propositional language based on a distinct vocabulary $\mathcal{V}_G$) representing information about the achievement of some goals when a candidate is selected. In the propositional languages used here, the logical connectors "or", "and", "not" are denoted respectively by $\vee$, $\wedge$, and $\neg$. A *literal* is a propositional symbol x or its negation $\neg x$, the set of literals of $\mathscr{L}_G$ are denoted by LIT_G. Classical inference, logical equivalence and contradiction are denoted respectively by $\vdash$, $\equiv$, $\bot$.

In the following we denote by K a set of formulas representing features that are known to hold: hence $K \subseteq \mathscr{L}_F$ is the available knowledge. Using the inference operator $\vdash$, the fact that a formula $\varphi \in \mathscr{L}_F$ holds can be written $K \vdash \varphi$.

Remark 1 *If K gathers the pieces of knowledge of one or several agents, we assume that they agree on how to evaluate that a feature holds for a candidate c. Hence we assume that the common knowledge K* is consistent. *Note that this assumption may seem unrealistic but it is done in practical applications like in the Supply Chain Management context (see references in [6]).*

As it is summarized in Chesñevar et al.'s survey of logical models of arguments [14], arguments can be viewed as a chain of defeasible reasons enabling to reach a conclusion. Hence we propose to define an argument for selecting a candidate as a defeasible rule of the form, "if the formula φ is believed to hold for a candidate then a priori the goal g is achieved by selecting this candidate", i.e., when φ holds it is more possible that g is achieved than failed. More formally:

Definition 1 *A* basic argument *a is a pair (φ, g) where $\varphi \in \mathscr{L}_F$ and $g \in LIT_G$. Let $reas(a)$ and $concl(a)$ denote respectively the reason φ and the conclusion g of the argument a.*

In the following, this pair (φ, g) is interpreted as a default rule $\varphi \rightsquigarrow g$ (i.e., it could be encoded in a possibilistic setting [15] by: $\Pi(\varphi \wedge g) \geq \Pi(\varphi \wedge \neg g)$) meaning that in general when φ holds for the candidate c the goal g is achieved by selecting c, it is a general rule that can have exceptions.

In order to understand how arguments will be used, we are going to define their validity (see Section 3.1 for a more precise definition), informally, let (φ, g) be an argument, then knowing that φ does not hold for a given candidate c *invalidates* this argument for this candidate c, hence the argument (φ, g) can not be present for candidate c. Similarly, knowing that φ holds for a given candidate enables a decision maker to consider this argument.

Example 1 *If the candidates are people applying for a job then the argument (CV good readability, ability to well present herself) could be understood as "if the candidate has a CV easy to read then a priori the goal to have a person able to well present herself is achieved".*

Some arguments are more important than other either because their goal is more important or because they are more credible or both. Moreover depending on the fact that the achievement of this goal is either wished, then $pol(x) = \oplus$, or dreaded, denoted $pol(x) = \ominus$, we associate an argument with a polarity as follows:

Definition 2 *A basic argument $a = (\varphi, g)$ is associated with a* level *denoted $l(a)$, and a* polarity *denoted $pol(a)$ which comes from the goal polarity defined by a function $pol : \mathcal{V}_G \to \{\oplus, \ominus\}$ such that: $pol(\varphi, x) = pol(x)$ and $pol(\varphi, \neg x) = -pol(x)$ where x denotes a propositional symbol of $\mathcal{V}_G$ and $-$ is used to switch the polarity (i.e., $-\oplus = \ominus$ and $-\ominus = \oplus$).*

The level of an argument is supposed to be given by the decision makers (see Remark 2). Note that the precise value of the level of an argument is not meaningful, **only the rank ordering of the levels** is taken into account.

However, it would be possible to evaluate rationnally this level, since it depends on the importance of the goal and on the plausibility of the reasons that support it. In order to compute this level, we could use a function $f(m, d)$ that aggregates a belief measure m and an importance degree of a goal d. Indeed, on the one hand, we can associate each propositional symbol x of $\mathcal{V}_F$ with a *measure*, $m(x) \in]0, 1]$ where 1 is the highest level and if $m(x) > m(x')$ then x have more chance to be true than x' or x implies a better knowledge[2] than x', from this elementary measure we can induce a measure on formulas according to the theory associated to this measure (possibility theory, probability theory, belief function theory...). On the other hand, some goals are more important to achieve than others, each propositional symbol x of $\mathcal{V}_G$ is associated with a *degree*, $d(x) \in]0, 1]$ where 1 is the most important level and if $d(x) > d(x')$ then x is more important than x'.

Remark 2 *In the context of a BLA definition, the function $f(m, d)$ is not necessarily required but the group of decision makers could fix directly the level of the arguments (what is our satisfaction level about the achievement of the goal g when we know that the formulae φ holds). Indeed it is difficult to formalize how a human being integrates a belief measure with a preference degree (it may depend of the*

[2]Some formula may imply a better knowledge about a feature for instance feature f_1 can be considered to hold when the temperature belongs to the interval $[0, 100]$ and feature f_2 is true if the temperature is in $[50, 100]$, in that case f_2 demands a higher level of knowledge.

nature of the goal, the optimism or the pessimism of the decision maker..etc). In a formal approach, f measure should correspond to a risk evaluation, it can be based on any optimistic/pessimistic qualitative expectation measure (see [19] min/max, Sugeno Integral, ...) or even based on a quantitative expectation measure: namely the expected value.

Example 2 *Let us consider a recruitment problem. The recruitment is done according to some goals: e.g. ap: we do not want someone with an anti-social personality, polarity=$\ominus$, ej:we want to hire an efficient person for the job, polarity=$\oplus$, et: we want to find a person easy to train,*

Here are two examples of arguments associated to the vacant position a positive one (eb, ej) (a good educational background allows to infer that the candidate will be efficient for the job) and a negative one $(jhop \wedge \wedge lpe, \neg et)$ (a job hopper with a long experience will not be easy to train).

2.2 Conflicts between arguments

If two arguments of the same level of importance have goals that are logically inconsistent (i.e., their goals are opposite) then they are conflicting.

Definition 3 (conflicts) *Let $\mathcal{A}$ be a set of arguments conform to Definition 1 and let $a, b \in \mathcal{A}$, a and b are* conflicting *iff $concl(a) \wedge concl(b) \vdash \bot$ and $reas(a) \wedge reas(b) \nvdash \bot$ and $l(a) = l(b)$.*

In order to decide which is the argument that applies among two conflicting ones, the BLA is equipped with directed links called "attacks" between every pair of conflicting arguments. The link is directed according to the priority that is given to the applicability of the arguments when their features hold simultaneously. So for each pair of arguments a, b in the BLA that are in conflict (i.e that have an opposite conclusion about the achievement of a given goal) the argument whose conclusion holds when both reasons are present attacks the other argument, there are exactly two possible cases:
- either only one attack between them from a to b (resp. b to a), i.e., a *attacks* b, also denoted $(a, b) \in \mathcal{R}$, meaning that the agents have agreed that when $K \vdash reas(a) \wedge reas(b)$ the goal $concl(a)$ is achieved (resp. the goal $concl(b)$ is achieved)
- or two symmetric attacks: a attacks b and b attacks a (i.e., $(a, b), (b, a) \in \mathcal{R}$) meaning that the agents have agreed that when $K \vdash reas(a) \wedge reas(b)$ we don't know whether the goal expressed by $concl(a)$ or the goal expressed by $concl(b)$ is achieved.

Note that the latter is a case where the arguments destroy each other. Note also that the case where there is no attack between two conflicting arguments is not allowed.

Indeed, the direction of an attack between two arguments depends on extra-information about these arguments. For instance, if we consider that the belief measure is a possibility measure (i.e. $m()$ stands for $\Pi()$) and that the function $f(\Pi, d)$ is the optimistic expectation: $min(\Pi(\varphi), d(g))$ with the two arguments $a_1 = (\varphi_1, g)$ and $a_2 = (\varphi_2, \neg g)$ s.t. $\Pi(\varphi_1) = 1$, $\Pi(\varphi_2) = 0.8$ and $d(g) = 0.5$ we have $l(a_1) = l(a_2) = 0.5$. However, the conclusion of a_1 is more credible hence choosing to define an attack from a_1 to a_2 is a justified choice.

Example 3 *Now suppose that we know that when a candidate is unmotivated, even if she has a good professional experience or a good educational background, the goal "efficient for the job" will not be achieved by hiring this candidate, it means that $(u, \neg ej)$ attacks (eb, ej) and (pe, ej). Moreover a job hopper that has a long professional experience is generally not easy to train but if this experience was not in the specialty of the job, it means that this person has a good adaptability hence she will be easy to train. So the more specific argument attacks the other one: $(jhop \wedge \neg spe \wedge lpe, et)$ attacks $(jhop \wedge lpe, \neg et)$.*

We are now in position to define the structure called BLA that will be used to make a common decision. We assume that the participants of the decision are able to build together their own collective BLA respecting the previous definitions. Once they have agreed on a common BLA, they can refer to it each time they want in order to make a decision about a candidate (see Section 3).

Definition 4 (BLA) *A BLA is a quadruple $\langle \mathcal{A}, l, pol, \mathcal{R} \rangle$ where $\mathcal{A}$ is a set of arguments conformed to Definition 1, l is a function giving the levels of the arguments, pol is a function giving the polarity of the arguments and $\mathcal{R}$ is an attack relation between arguments conform to Definition 3.*

The four elements of the BLA are supposed to be available before the decision and to result from a common deliberation of all the agents implied in the future decisions.

Example 4 *Figure 1 illustrates the BLA corresponding to a recruitment example where the features are: cbs (CV bad spelling), cgr (CV good readability), cps (CV poorly structured), eb (educational background), gp (good personality), i (introverted candidate), $jhop$ (job hopper), lpe (long professional experience), spe (professional experience in the specialty of the job) and u (unmotivated candidate). The goals are: ap (anti-social personality), ej: (efficient for the job), ph (a person able to present herself), et (easy to train), st (a stable person).*

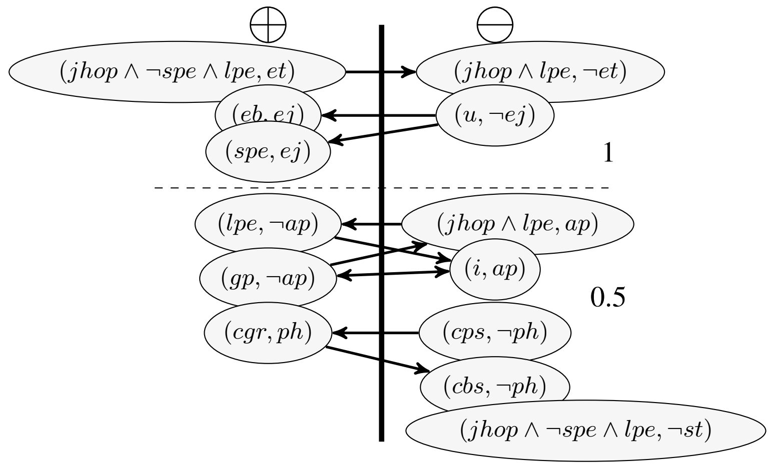

Figure 1: Recruitment BLA

Remark 3 *The attack relation $\mathcal{R}$ is not transitive, since if $a_1 \mathcal{R} a_2$ and $a_2 \mathcal{R} a_3$ then a_1 and a_3 have the same conclusion, hence they are not conflicting.*

3 Selection of a candidate using a BLA

The BLA defines all possible information required to make a decision. Unfortunately, in uncertain context, only little information maybe available. In this section we propose a method for analyzing the acceptability of a candidate wrt a BLA. First, we present the available information and the notion of instantiated BLA, called valid BLA. Then, we define thresholds for acceptability and study their relations with classical decision rules of qualitative bipolar decision making.

3.1 Valid BLA

Given a candidate $c \in \mathscr{C}$, and a knowledge base K_c, $K_c \subseteq \mathscr{L}_F$, representing the knowledge of a decision maker about the candidate c, and given a formula φ describing a configuration of features ($\varphi \in \mathscr{L}_F$), the decision maker can have three kinds of knowledge about c: φ holds for candidate c (denoted $K_c \vdash \varphi$), or not (denoted $K_c \vdash (\neg\varphi)$ or the feature φ is unknown for c (denoted $K_c \nvdash \varphi$ and $K_c \nvdash \neg\varphi$). When there is no ambiguity about the candidate c, K_c will be denoted K.

Definition 5 (Valid argument according to K) *The set $\mathcal{A}_K = \{(\varphi, g) \in \mathcal{A}$, s.t. $K \vdash \varphi\}$ denotes the set of arguments in $\mathcal{A}$ whose reason φ holds in K.*

Definition 6 (Valid BLA according to K) *A valid BLA is a graph represented by a quadruple $\langle \mathcal{A}_K, l, pol, \mathcal{R}_K \rangle$ where $\mathcal{A}_K$ is the set of valid arguments according to K and $\mathcal{R}_K$ is the restriction of $\mathcal{R}$ on $\mathcal{A}_K$.*

When there is no ambiguity about the knowledge available, $\mathcal{A}_K$ will be denoted $\mathcal{A}$. and $\mathcal{R}_K$ abbreviated to $\mathcal{R}$.

Example 5 *Let us consider that we know that a candidate has the features eb, lpe and $jhop$ (see Example 4), the Valid BLA is represented in Figure 2.*

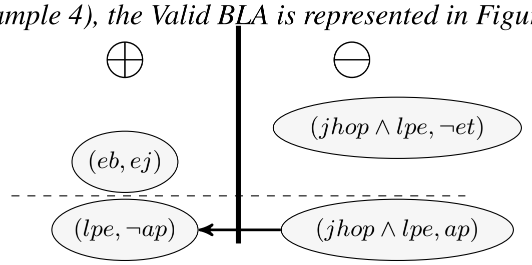

Figure 2: A Valid BLA

In the following we suppose that the candidate c and the available knowledge K_c are fixed. A goal is realized if there is a valid argument in its favor that is not attacked by any valid argument:

Definition 7 (realized goal) *Let g be a literal in LIT_G, the goal g is realized wrt a valid BLA $\langle \mathcal{A}_K, l, pol, \mathcal{R}_K \rangle$ iff $\exists a \in \mathcal{A}_K$ such that $concl(a) \equiv g$ and $\nexists b \in \mathcal{A}_K$ such that $b \mathcal{R} a$. The set of realized goals is denoted $\mathtt{R}$.*

Example 6 *According to the valid BLA of Example 5, the goal ej is realized since there is only one argument, namely (eb, ej), concerning this goal in the Valid BLA. Similarly, the goal $\neg et$ is realized, hence et is failed. The goal ap is also realized since the argument $(jhop \wedge lpe, ap)$ attacks the argument $(lpe, \neg ap)$. The goal ph is not realized since there is no argument concerning this goal that is valid (i.e., whose reasons are known) in our context.*

3.2 Admissibility threshold

From the status of a goal, we can define an admissibility status for a candidate $c \in \mathscr{C}$ given a Valid BLA $\langle \mathcal{A}, l, pol, \mathcal{R} \rangle$ for this candidate. But we have first to define the λ-section of a set of goals.

Definition 8 *Given a Valid BLA $\langle \mathcal{A}, l, pol, \mathcal{R} \rangle$, the λ-section of a set of goals $G \subset \mathscr{L}_G$ is the goals of G which can be justified by an argument with level λ.*

$$G_\lambda = \{g \in G, l(\varphi, g) = \lambda\}$$

Let us denote by $\mathtt{R}_\lambda$, the λ-section of realized goals, i.e., $\mathtt{R}_\lambda = \mathtt{R} \cap G_\lambda$.

Moreover the set of positive, respectively negative, goals is abbreviated $\overline{\oplus}$, respectively $\overline{\ominus}$; i.e., $\overline{\oplus} = \{g \in \mathscr{L}_G, pol(g) = \oplus\}$ and $\overline{\ominus} = \{g \in \mathscr{L}_G, pol(g) = \ominus\}$. Let $\mathtt{R}^\oplus = \mathtt{R} \cap \overline{\oplus}$ and $\mathtt{R}^\ominus = \mathtt{R} \cap \overline{\ominus}$. We denote $\mathtt{R}^\oplus_\lambda$, resp. $\mathtt{R}^\ominus_\lambda$ the λ-section of positive, resp. negative realized goals. We are going to focus on the most important level of a realized goal in a Valid BLA, named e, so we will study the e-section of realized goals:

Definition 9 (admissibility status) *Given a candidate $c \in \mathscr{C}$ and given a Valid BLA $\langle \mathcal{A}, l, pol, \mathcal{R} \rangle$ for this candidate, let $e = \max_{g \in \mathtt{R}} l(\varphi, g)$ with $e = 0$ if $\mathtt{R} = \varnothing$. The status of c is:*
- Necessary admissible *(N_{ad}) if* $\mathtt{R}^\oplus_e \neq \varnothing$ *and* $\mathtt{R}^\ominus_e = \varnothing$
- Possibly admissible *(Π_{ad}) if* $\mathtt{R}^\oplus_e \neq \varnothing$
- Indifferent *(Id) if* $\mathtt{R} = \varnothing$
- Possibly inadmissible *($\Pi_{\neg ad}$) if* $\mathtt{R}^\ominus_e \neq \varnothing$
- Necessary inadmissible *($N_{\neg ad}$) if* $\mathtt{R}^\ominus_e \neq \varnothing$ *and* $\mathtt{R}^\oplus_e = \varnothing$
- Controversial *(Ct) if* $\mathtt{R}^\oplus_e \neq \varnothing$ *and* $\mathtt{R}^\ominus_e \neq \varnothing$

In other words, a *necessary admissible* candidate has positive arguments with goals of maximum importance that are realized (i.e., unattacked) and all the negative goals of the same importance are failed. A *possibly admissible* candidate has at least one unattacked positive argument of maximum importance. An *indifferent candidate* has no unattacked arguments (positive or negative), while a *controversial candidate* is both supported and criticized by unattacked arguments of maximum importance. A candidate is *possibly inadmissible* if there is a negative unattacked argument of maximum importance concerning her and *necessary inadmissible* if there is a negative unattacked argument of maximum importance concerning her and all positive goals appearing in arguments concerning her at this level are failed.

Remark 4 *If the group of decision makers wants to encode the fact that having several goals of little importance that are satisfied is better than having only one important goal satisfied, they can create one or several new arguments with a new goal and with a conjunction of the reasons that were inside the less important arguments.*

Example 7 *The candidate described by the arguments given in the valid BLA of Example 5 is controversial, since at the most important level we have both a realized positive goal, namely ej and a realized negative goal: $\neg et$.*

The above definition is related to possibility theory [20],where necessary (resp. possibly) admissible could be understood as it is certain (resp. possible) that the candidate is admissible. The indifference case is linked to a lack of unattacked (positive or negative) arguments concerning a candidate, thus an impossibility to decide. However it is not related to a standard definition of possibilistic ignorance about the admissibility of a candidate, which rather corresponds to a controversial candidate that is both possibly admissible and possibly inadmissible.

With these admissibility statuses, we can propose 3 thresholds of admissibility (from the strongest to the weakest) as shown in Figure 3.

- Threshold 1: $c \in N_{ad}$, under this threshold, the candidate is admissible with no doubt, they are unattacked arguments about the candidate and those arguments are all positive.
- Since there are two ways to have doubts about a candidate, namely when she is indifferent (Id) or controversial (Ct), the second threshold is divided into two parts:
  - threshold 2a: $c \in N_{ad} \cup Id$ (i.e., $c \in \mathscr{C} \setminus \Pi_{\neg ad}$), under this threshold, we place candidates under the first threshold together with those for which no unattacked argument concerning them is available (neither positive nor negative),
  - threshold 2b: $c \in N_{ad} \cup Ct$ (i.e., $c \in \Pi_{ad}$), the candidates under this threshold are candidates under threshold 1 together with the candidates that are concerned by negative unattacked argument provided that they are also concerned at least by one positive unattacked argument.
- threshold 3: $c \in Id \cup Ct \cup N_{ad}$ (i.e, $c \in \Pi_{ad} \cup Id = \mathscr{C} \setminus N_{\neg ad}$) it gathers the union of the sets 1, 2a and 2b.

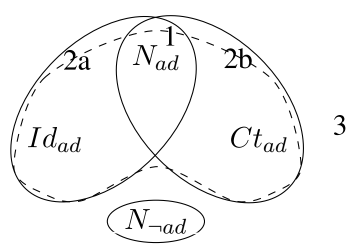

Figure 3: Admissibility thresholds.

Now, let us study the relation between the admissibility thresholds and classical rules of bipolar decision. The rules of bipolar decision problem introduced in [11] are Pareto Dominance, Bipolar Possibility relation and Bipolar leximin, they are based on the order of magnitude of a set of goals:

Definition 10 *Given a Valid BLA $\langle \mathcal{A}, l, pol, \mathcal{R} \rangle$, the order of magnitude of a set of goals $G \subset \mathscr{L}_G$ is:*

$$OM(G) = \max_{g \in G} l(\varphi, g) \quad \text{and} \quad OM(\varnothing) = 0$$

Definition 11 (decision rules [11]) *Given two candidates c and c' with their associated realized goals $\mathtt{R}$ and $\mathtt{R}'$, Pareto (denoted $\succeq_{Pareto}$), Bipolar Possibility (denoted $\succeq_{BiPoss}$), Bipolar Leximin (denoted $\succeq_{BiLexi}$) dominance relations are defined by the following rules:*

- $c \succeq_{Pareto} c'$ *iff* $OM(\mathtt{R}^{\oplus}) \geq OM(\mathtt{R}'^{\oplus})$ *and* $OM(\mathtt{R}^{\ominus}) \leq OM(\mathtt{R}'^{\ominus})$
- $c \succeq_{BiPoss} c'$ *iff* $OM(\mathtt{R}^{\oplus} \cup \mathtt{R}'^{\ominus}) \geq OM(\mathtt{R}^{\ominus} \cup \mathtt{R}'^{\oplus})$
- $c \succeq_{BiLexi} c'$ *iff* $|\mathtt{R}^{\oplus}_{\delta}| \geq |\mathtt{R}'^{\oplus}_{\delta}|$ *and* $|\mathtt{R}^{\ominus}_{\delta}| \leq |\mathtt{R}'^{\ominus}_{\delta}|$
  where $\delta = Argmax_{\lambda}\{|\mathtt{R}^{\oplus}_{\lambda}| \neq |\mathtt{R}'^{\oplus}_{\lambda}|$ *or* $|\mathtt{R}^{\ominus}_{\lambda}| \neq |\mathtt{R}'^{\ominus}_{\lambda}|\}$

where $\succeq_r$ stands for "is r-preferred to".

The following proposition shows that controversial candidates and indifferent candidates cannot be discriminated wrt $Pareto$, $BiLexi$ and $Biposs$ selection rules.

Proposition 1 $\forall c \in Ct$ *and* $\forall c' \in Id$,

- $c \not\succeq\not\preceq_r c'$, $\forall r \in \{Pareto,\ BiLexi\}$
- $c \sim_{BiPoss} c'$

where $c \not\succeq\not\preceq_r c'$ stands for incomparability (i.e., neither $c \succeq c'$ nor $c' \succeq c$) and $c \sim c'$ stands for equivalence (i.e., $c \succeq c'$ and $c' \succeq c$).

Proof: $\forall c \in Ct$, we have $\mathtt{R}^{\oplus}_e \neq \varnothing$ and $\mathtt{R}^{\ominus}_e \neq \varnothing$, with $e = \max_{g \in \mathtt{R}} l(g)$, it means that there are some realized goals in $\oplus$ and $\ominus$ at the highest level e where some goals are realized, hence OM$(\mathtt{R}^{\oplus})$ = OM$(\mathtt{R}^{\ominus}) = e$. $\forall c' \in Id$, we have $\mathtt{R}' = \varnothing$ hence OM$(\mathtt{R}'^{\oplus})$ = OM$(\mathtt{R}'^{\ominus}) = 0$. Consequently, we have: OM$(\mathtt{R}^{\oplus}) >$ OM$(\mathtt{R}'^{\oplus})$ and OM$(\mathtt{R}^{\ominus}) >$ OM$(\mathtt{R}'^{\ominus})$ so $c \not\succeq\not\preceq_{Pareto} c'$.
Moreover, $e = Argmax_{\lambda}\{\ |\mathtt{R}^{\oplus}_{\lambda}| \neq |\mathtt{R}'^{\oplus}_{\lambda}|$ or $|\mathtt{R}^{\ominus}_{\lambda}| \neq |\mathtt{R}'^{\ominus}_{\lambda}|\}$ but $|\mathtt{R}^{\oplus}_e| > |\mathtt{R}'^{\oplus}_e|$ holds, while $|\mathtt{R}^{\ominus}_e| \leq |\mathtt{R}'^{\ominus}_e|$ does not hold (since $|\mathtt{R}^{\ominus}_e| \neq 0$ hence $|\mathtt{R}^{\ominus}_e| > |\mathtt{R}'^{\ominus}_e| = 0$) so $c \not\succeq\not\preceq_{BiLexi} c'$.
Lastly, OM$(\mathtt{R}^{\oplus} \cup \mathtt{R}'^{\ominus})$ = OM$(\mathtt{R}^{\oplus})$ and OM$(\mathtt{R}^{\ominus} \cup \mathtt{R}'^{\oplus})$ = OM$(\mathtt{R}^{\ominus})$ and OM$(\mathtt{R}^{\oplus})$ = OM$(\mathtt{R}^{\ominus})$ hence the result. □

The following proposition shows that necessary admissible candidates are preferred or incomparable to controversial candidates wrt $Pareto$ and $BiLexi$ selection rules, while they are preferred or equivalent wrt $Biposs$ selection rule.

Proposition 2 *$\forall c \in N_{ad}$ and $\forall c' \in Ct$,*

- *$c \succ_r c'$ or $c \ \hat{\approx}_r \ c'$, $\forall r \in \{Pareto, BiLexi\}$*
- *$c \succeq_{BiPoss} c'$*

Proof: Let $p = \text{OM}(\mathtt{R}^{\oplus})$, $p' = \text{OM}(\mathtt{R}'^{\oplus})$, $m = \text{OM}(\mathtt{R}^{\ominus})$ and $m' = \text{OM}(\mathtt{R}'^{\ominus})$, with this notations the dominance rules can be written as follows[3]:

$$c \succeq_{Pareto} c' \text{ iff } p' \leq p \text{ and } m \leq m'$$

$$c \succeq_{Biposs} c' \text{ iff } \max(p, m') \geq \max(m, p')$$

$$c \succeq_{BiLexi} c' \text{ iff } |\mathtt{R}^{\oplus}_{\delta}| \geq |\mathtt{R}'^{\oplus}_{\delta}| \text{ and } |\mathtt{R}^{\ominus}_{\delta}| \leq |\mathtt{R}'^{\ominus}_{\delta}| \text{ with } \delta = \max(p, m, p', m')$$

Moreover, $c \in N_{ad}$ implies $0 \leq m < p$ (since $\mathtt{R}^{\oplus}_p \neq \varnothing$ and $\mathtt{R}^{\ominus}_p = \varnothing$ and $p = \max_{g \in \mathtt{R}} l(g)$) and $c' \in Ct$ implies $p' = m' \neq 0$ (since $\mathtt{R}'^{\oplus}_{p'} \neq \varnothing$ and $\mathtt{R}'^{\ominus}_{p'} \neq \varnothing$ and $p' = \max_{g \in \mathtt{R}'} l(g)$). Hence, we have three cases:

- $0 < (p' = m') \leq m < p$: it holds that $c \ \hat{\approx}_{Pareto} \ c'$, $c \succ_{Biposs} c'$. For BiLexi, we have $\delta = p$ and $|\mathtt{R}^{\oplus}_p| \neq 0$ while $|\mathtt{R}'^{\oplus}_p| = |\mathtt{R}^{\ominus}_p| = |\mathtt{R}'^{\ominus}_p| = 0$ hence $c \succ_{Bilexi} c'$.
- $0 \leq m < (p' = m') \leq p$: it holds that $c \succ_{Pareto} c'$, $c \succeq_{Biposs} c'$. For BiLexi, we have $\delta = p$ and $|\mathtt{R}^{\oplus}_p| \neq 0$, it may happen that $|\mathtt{R}'^{\oplus}_p|$ and $|\mathtt{R}'^{\ominus}_p|$ are not zero, in any case $|\mathtt{R}^{\ominus}_p| = 0$ hence $c \succ_{Bilexi} c'$ or $c \ \hat{\approx}_{BiLexi} \ c'$ (when $|\mathtt{R}'^{\oplus}_p| \geq \mathtt{R}^{\oplus}_p|$)
- $0 \leq m < p \leq (p' = m')$: it holds that $c \succ_{Pareto} c'$ or $c \ \hat{\approx}_{Pareto} \ c'$ (when $p' > p$). We have $c \sim_{BiPoss} c'$. Concerning BiLexi, $\delta = p'$ and we have the same case than before i.e., $c \succ_{Bilexi} c'$ or $c \ \hat{\approx}_{BiLexi} \ c'$.

□

The next proposition establishes that a necessary admissible candidate is always preferred by BiPoss or BiLexi to an indifferent one, but they can be incomparable with Pareto.

Proposition 3 *$\forall c \in N_{ad}$ and $\forall c' \in Id$, $c \succ_{Pareto} c'$ or $c \ \hat{\approx}_{Pareto} \ c'$, and $c \succ_r c'$, $\forall r \in \{ BiPoss, BiLexi\}$*

[3]For BiLexi, we can set δ to $\max(p, m, p', m')$ since at least m is different from the others.

Proof: $c \in N_{ad}$ implies $\mathtt{R}_e^{\oplus} \neq \varnothing$ and $\mathtt{R}_e^{\oplus} = \varnothing$, but it may be the case that $OM(\mathtt{R}^{\ominus}) \neq 0$. Moreover, $c' \in Id$ implies that $\mathtt{R}'^{\oplus} = \mathtt{R}'^{\ominus} = \varnothing$ i.e., OM($\mathtt{R}'^{\oplus}$) = OM($\mathtt{R}'^{\ominus}$) $= 0$. This means that either $c \succ_{Pareto} c'$ or $c \;\char"2241_{Pareto}\; c'$, and that $c \succ_{Biposs} c'$ and that $e = Argmax_{\lambda}\{\, |\mathtt{R}_{\lambda}^{\oplus}| \neq |\mathtt{R}'^{\oplus}_{\lambda}|$ or $|\mathtt{R}_{\lambda}^{\ominus}| \neq |\mathtt{R}'^{\ominus}_{\lambda}|\}$ hence $c \succ_{BiLexi} c'$. □

Proposition 4 shows that inadmissible candidates (belonging to $N_{\neg ad}$) are always less preferred than others.

Proposition 4 $\forall c' \in N_{\neg ad}$

- $\forall c \in N_{ad} \cup Id$,
  - $c \succ_{Pareto} c'$ *or* $c \not\gtrless_{Pareto} c'$,
  - *and* $c \succ_r c'$, $\forall r \in \{\, BiPoss, BiLexi\}$
- $\forall c \in Ct$,
  - $c \succ_r c'$ *or* $c \not\gtrless_r c'$, $\forall r \in \{Pareto, BiLexi\}$
  - $c \succeq_{BiPoss} c'$

Proof: Let us use the same notations than for the proof of Proposition 2, $c' \in N_{\neg ad}$ implies $0 \leq p' < m'$

- $c \in N_{ad}$ implies $0 \leq m < p$. Hence $\max(p, m') > \max(p', m)$ so $c \succ_{Biposs} c'$. Concerning BiLexi $Argmax_{\lambda}\{\, |\mathtt{R}_{\lambda}^{\oplus}| \neq |\mathtt{R}'^{\oplus}_{\lambda}|$ or $|\mathtt{R}_{\lambda}^{\ominus}| \neq |\mathtt{R}'^{\ominus}_{\lambda}|\} = \delta = \max(p, m, p', m')$ hence δ is either equal to p or m' or to both of them, in any cases $|\mathtt{R}_{\delta}^{\ominus}| = |\mathtt{R}'^{\oplus}_{\delta}| = 0$ and at least one among $|\mathtt{R}_{\delta}^{\oplus}|$ and $|\mathtt{R}'^{\ominus}_{\delta}|$ is non zero, thus $c \succ_{BiLexi} c'$.

  Considering the Pareto rule, there are two cases where $c \not\gtrless_{Pareto} c'$ namely $0 \leq m < p < p' < m'$ and $0 \leq p' < m' < m < p$ in all other cases $c \succ_{Pareto} c'$
- $c \in Id$ the proof is similar to the one used for Proposition 3.
- $c \in Ct$ the proof is similar to the one used for Proposition 2.

□

From these propositions we can establish (Theorem 1) the rationality of the admissibility thresholds $\{1, 2a, 2b, 3\}$ for the rules $Pareto$, $Biposs$ and $BiLexi$ since an inadmissible candidate can not be preferred (with respect to these rules) to an admissible candidate.

Theorem 1
- *$\forall c \in Ad$ with $Ad \in \{1, 2a, 2b, 3\}$ and $\forall c' \in \mathscr{C} \setminus Ad$, $c' \not\succ_r c$, $\forall r \in \{Pareto,\ BiPoss,\ BiLexi\}$*

- *For all c in threshold 1 and for all c' not in threshold 1 but in a threshold inside $\{2a, 2b, 3\}$, $c' \not\succ_r c$, $\forall r \in \{Pareto,\ BiPoss,\ BiLexi\}$.*

- *Threshold $2a$ and Threshold $2b$ are not distinguishable with $\{Pareto,\ BiPoss,\ BiLexi\}$.*

Proof:

- Due to Proposition **??**, $Ad = N_{ad} \cup Ct \cup Id$ and $\mathscr{C} \setminus Ad = N_{\neg ad}$. Now, $\forall c \in N_{ad} \cup Ct \cup Id$ and $\forall c' \in N_{\neg ad}$, Proposition 4 expresses that $c' \not\succ_r c$, $\forall r \in \{Pareto, BiPoss, BiLexi\}$.

- Proposition 2 and 3 imply that $\forall c \in N_{ad}, \forall c' \in Ct \cup Id$, $c' \not\succ_r c$, $\forall r \in \{Pareto, BiPoss, BiLexi\}$.

- Threshold $2a = N_{ad} \cup Id$ and $2b = N_{ad} \cup Ct$, Proposition 1 expresses that $\forall c \in Ct$ and $\forall c' \in Id$, $c \not\approx_r c'$ or $c \sim_r c'$ $\forall r \in \{Pareto, BiPoss, BiLexi\}$.

□

Note that inside the first threshold, $Pareto$, $Biposs$ and $BiLexi$ rules can help to refine the selection: i.e., among two necessary admissible candidates one may be preferred to the other w.r.t. one of those rules. This can be the case when the positive realized goals of a candidate are higher than the positive realized goals of another one provided that the negative realized goals of each candidate are of a lower level than the positive ones.

4 Related work

Our model is related to several domains, namely argumentative decision support technology, bipolar multi-criteria decision and argumentation-based decision.

The studies in decision support technology domain are well in accordance with our intuition, since a BLA is a visual representation of arguments. However our formalism differs on our definition of the arguments and their relations, since we have clearly restricted the expressive power thanks to a formal language with a clear semantics, this is not the case in those works that allow for natural language sentences that maybe related by several kinds of links (more or less easy to establish or validate). Moreover the fact that the knowledge is incomplete and distributed is well apprehended in this kind of approaches. A recent proposal by Ouerdane et al. [26] could also help us to build our BLA, indeed a BLA may correspond to their defeasible "cognitive artifact" (i.e., the formulation of the decision problem and the evaluation model).

In the domain of multi-criteria decision, [16] and [17] propose a qualitative bipolar approach in which a candidate is associated with two distinct sets of positive "arguments" (pros) and negative "arguments" (cons). In this model, arguments are not structured, they are only elements pertaining to a set (either the set of pros or the set of cons), the arguments are positive or negative wrt the decision goal without attack relation between arguments. A function π assesses the level of importance of each argument for the decision maker. The authors provide an axiomatic characterization of different natural decision rules in this setting. The papers [11] and [18] study the use of these rules in practice by human decision makers, and show that the bipolar lexicographic rule is largely favored by humans, these results are confirmed by [12]. These polar notions are also used by Tchangani et al. [30] who proposes two measures called "selectability" and "rejectability" that are built wrt the positive or negative effects that a set of features has on the global objective, these measures are used to compare the candidates. The decision rules introduced in the above approaches are based on a comparison of the possible candidates, in our framework, our aim is different since we want to know if a candidate is acceptable or not, whatever the other candidates are. Our acceptability thresholds are based on the notions of "realized" and "failed" positive or negative goals but the acceptability is not computed by comparison with other candidates. However, we show in this paper that our thresholds behave rationally with respect to these rules. A second difference in our framework is the fact that we handle incomplete distributed knowledge which is not the case in those settings. Moreover our approach is a generalization of the previous one since we deal with uncertainty: the knowledge maybe built on uncertain data as we have shown in Section 2.

In the same domain, Amgoud and Prade [2] propose a bipolar argumentation-based approach for decision and define some statuses close to our thresholds. However they do not relates these statuses to classical decision rules. [3] proposes to distinguish two types of arguments: the epistemic arguments and the practical arguments. The epistemic arguments are used to deal with inconsistent knowledge and the practical arguments are in favor or against a decision. Practical argumentation has been widely studied (see e.g. [32, 1]) since the seminal work of Raz [29] and the philosophical justification provided by Walton [31] and consists in answering the question "what is the right thing to do in a given situation". This question is clearly related to a decision problem and several works are using argumentative approaches to tackle it: for instance Bonet and Geffner [10] have a very similar view of what we call arguments, since they use defeasible rules in favor or against a given action. However, in this kind of approach there is no attack relation defined between two practical arguments. In our framework an argument can be viewed as a pair of (epistemic information, practical information) which defines the practical conclusion that should be fired under given epistemic information. More precisely, in this current proposal, we are not interested in dealing with inconsistency problem (this has been broadly developed in the argumentation field see for instance [24, 23] with the "acceptability" notion defined at the argument level) but rather on the practical part of the problem. Our need to have arguments for making a decision on the ground of factual reasons could be related to the idea to use arguments for reasoning about actions and values of Fox and Parsons [25] however in their approach arguments can be viewed as an encoding of expected utilities and outcomes of actions, this is different from our definition where arguments encode what goal will be achieved if some facts are true. Amgoud and Vesic [5] have studied the use of Dung's [21] argumentation framework for decision and compare the results with the results that can be obtained by the decision rules of Dubois and Fargier's framework [17]. More precisely, they propose to attach a Dung-style argumentation framework to a set of candidates, then the candidates are evaluated and compared on the basis of the arguments pros and/or the arguments cons. They conclude that [17] performs better than its argumentative counterpart. As shown in Section 2, our approach is not a Dung-style argumentative approach, since for instance the defense notion does not make sense in our setting, this explains why our approach is in agreement with [17] (with a more refined expressive power).

5 Conclusion

This paper presents a new framework for decision making under incomplete and distributed knowledge. The decision that will be taken will comply with the priorities among goals and the contradictions among arguments that is summarized in a structure called BLA that is given before start. The decision will depend on the instance of the BLA for a given candidate, i.e., it will depend on the features that the candidate possesses that are present on arguments of the BLA. This framework can be used by one human agent in order to decide whether a candidate is admissible but it can also be used by a group of human agents who are going to give the features that concern the candidate, by a simultaneous vote. This vote will instantiate the BLA and will automatically lead to an admissibility decision. In this paper we have shown that our definitions comply with classical rules of multi-criteria decision. A first benefit of the BLA is its visual aspect allowing to be easy to read and to create, a second benefit is that it provides a neutral process to compute a group decision.

References

[1] L. Amgoud, C. Devred, and M.-C. Lagasquie-Schiex. A constrained argumentation system for practical reasoning. In *Argumentation in M.Agent Systems*, pages 37–56. Springer, 2009.

[2] L. Amgoud and H. Prade. Comparing decisions on the basis of a bipolar typology of arguments. In G. Della Riccia, D. Dubois, R. Kruse, and H. J. Lenz, editors, *Preferences and Similarities*, pages 249–264. Springer, 2008.

[3] L. Amgoud and H. Prade. Using Arguments for Making and Explaining Decisions. *Artificial Intelligence*, 173:413–436, 2009.

[4] L. Amgoud and S. Vesic. A formal analysis of the role of argumentation in negotiation dialogues. *Journal of Logic and Computation*, 22:957–978, 2012.

[5] L. Amgoud and S. Vesic. On the use of argumentation for multiple decision making. In *Int. Conf. on Information Processing and Management of Uncertainty in Knowledge-based Systems (IPMU), Italy*, pages 480–489. Springer Berlin - Heidelberg, 2012.

[6] F. Bannay and R. Guillaume. Towards a transparent deliberation protocol inspired from supply chain collaborative planning. In *Int. Conf. on Information Processing and Management of Uncertainty in Knowledge-based Systems (IPMU)*, number 443 in CCIS, pages 335–344. Springer, 2014.

[7] P. Besnard and A. Hunter. A logic-based theory of deductive arguments. *Artificial Intelligence*, 128(1):203–235, 2001.

[8] E. Black and K. Atkinson. Choosing persuasive arguments for action. In *10th Int. Conf. on Autonomous Agents and Multi-Agent Systems*, 2011.

[9] E. Black and A. Hunter. Using enthymemes in an inquiry dialogue system. In *Proc of 7th Int. Conf. on Autonomous Agents and Multiagent Systems (AAMAS'08)*, pages 437–444, 2008.

[10] B. Bonet and H. Geffner. Arguing for decisions: A qualitative model of decision making. In *Proceedings of the Twelfth International Conference on Uncertainty in Artificial Intelligence*, pages 98–105. Morgan Kaufmann Publishers Inc., 1996.

[11] J. Bonnefon, D. Dubois, and H. Fargier. An overview of bipolar qualitative decision rules. In G. D. Riccia, D. Dubois, R. Kruse, and H.-J. Lenz, editors,

Preferences and Similarities, volume 504 of *CISM Courses and Lectures*, pages 47–73. Springer, 2008.

[12] J.-F. Bonnefon, D. Dubois, H. Fargier, and S. Leblois. Qualitative heuristics for balancing the pros and cons. *Theory and Decision*, 65:71–95, 2008.

[13] J. Cacioppo and G. Berntson. Relationship between attitudes and evaluative space: A critical review, with emphasis on the separability of positive and negative substrates. *Psychol. Bulletin*, 115(3):401, 1994.

[14] C. Chesñevar, A. Maguitman, and R. Loui. Logical models of argument. *ACM Comput. Surv.*, 32(4):337–383, 2000.

[15] J. L. D. Dubois and H. Prade. Possibilistic logic. *Handbook of Logic for Artificial Intelligence and Logic Programming*, 3:439–513, 1994.

[16] D. Dubois and H. Fargier. Qualitative decision making with bipolar information. *KR*, 6:175–186, 2006.

[17] D. Dubois and H. Fargier. Qualitative bipolar decision rules: Toward more expressive settings. In *Preferences and Decisions*, pages 139–158. Springer, 2010.

[18] D. Dubois, H. Fargier, and J.-F. Bonnefon. On the qualitative comparison of decisions having positive and negative features. *Journal of Artificial Intelligence Research*, 32:385–417, 2008.

[19] D. Dubois and H. Prade. Possibility theory as a basis for qualitative decision theory. In *IJCAI*, volume 95, pages 1924–1930, 1995.

[20] D. Dubois and H. Prade. Possibility theory: qualitative and quantitative aspects. In *Quantified Representation of Uncertainty and Imprecision*, pages 169–226. Kluwer Academic Publishers, 1998.

[21] P. M. Dung. On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. *Artificial Intelligence*, 77:321–357, 1995.

[22] F. Dupin de Saint Cyr. Handling enthymemes in time-limited persuasion dialogs. In S. Benferhat and J. Grant, editors, *International Conference on Scalable Uncertainty Management (SUM)*, number 6929 in LNAI, pages 149–162, Dayton, Ohio, 2011. Springer-Verlag.

[23] M. Elvang-Gøransson and A. Hunter. Argumentative logics: Reasoning with classically inconsistent information. *Data & Knowledge Engineering*, 16(2):125–145, 1995.

[24] M. Elvang-Gøransson, P. Krause, and J. Fox. Acceptability of arguments as "logical uncertainty". In *Symbolic and Quantitative Approaches to Reasoning and Uncertainty*, pages 85–90. Springer, 1993.

[25] J. Fox and S. Parsons. On using arguments for reasoning about actions and values. In *Proceedings of the AAAI Spring Symposium on Qualitative Preferences in Deliberation and Practical Reasoning, Stanford*, pages 55–63, 1997.

[26] W. Ouerdane, Y. Dimopoulos, K. Liapis, and P. Moraitis. Towards automating decision aiding through argumentation. *J. of Multi-Criteria Decision Analysis*, 18(5-6):289–309, 2011.

[27] I. Rahwan, S. D. Ramchurn, N. R. Jennings, P. McBurney, S. Parsons, and L. Sonenberg. Argumentation-based negotiation. *Knowledge Engineering Review*, 18(4):343–375, 2003.

[28] H. Raiffa. *Decision Analysis: Introductory lectures on choices under uncertainty*. Addison-Wesley, Reading, Massachusetts, 1970.

[29] J. Raz. *Practical reasoning*. Oxford University Press, 1978.

[30] A. Tchangani, Y. Bouzarour-Amokrane, and F. Pérès. Evaluation model in decision analysis: Bipolar approach. *Informatica*, 23(3):461–485, 2012.

[31] D. Walton. *Argumentation schemes for presumptive reasoning*. (first edition 1996) Routledge, 2013.

[32] M. J. Wooldridge. *Reasoning about rational agents*. MIT press, 2000.