Group Decision Making in a Bipolar Leveled Framework*

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Abstract

We study the use of a bipolar decision structure called BLF (bipolar leveled framework) in the context of collective decision making where the vote consists in giving factual information about a candidate which the group should accept or reject. A BLF defines the set of possible decision principles that may be used in order to evaluate the admissibility of a given candidate. A decision principle is a rule that relates some observations about the candidate to a given goal that the selection of this candidate may achieve or miss. The decision principles are ordered accordingly to the importance of the goal they support. Oppositions to decision principles are also described in the BLF under the form of observations that contradict the realization of the decision principles. We show how the use of a common BLF may reduce the impact of manipulation strategies in the context of group decision making.
Keyword: Group decision making, Qualitative decision theory, Arguments.

1 Introduction

A standard way [16] to handle a decision problem is to define a utility function which enables the decision maker to evaluate the quality of each decision and select the one that have the best utility. This utility function should be designed in order to take into account the multi-criteria aspects of the problem. The classical approaches for handling collective decision problems under uncertainty are based on (i) the identification of a decision making theory under uncertainty that captures the decision makers' behaviour with respect to uncertainty and (ii) the specification of a collective utility function (CUF) as it may be used when the problem is not pervaded with uncertainty [5].

In classical decision problems, all candidates are available to the decision maker simultaneously and he must choose one preferred candidate among them. Our approach deals with a different case of decision like the "secretary problem" (see e.g. [15]) where an administrator wants to hire the best secretary. In this case, only one candidate is evaluated at a time and a decision about each particular applicant is to be made immediately after the interview. Once rejected, an applicant cannot be recalled. Hence the decision maker should decide whether to make a final choice or continue searching for better candidates. In this kind of problem, the aim is not to compute the best candidate among all the possible candidates, but rather to select a candidate that is convenient wrt some criterias. We capture the notion of convenience by introducing "admissibility statuses" where the admissibility of a candidate is given in terms of the goals that would be achieved by accepting her, the goals are positive or negative and some are more important to achieve than others.

Recently, a framework based on default rules have been proposed by [12] for decision under uncertainty. Its aim was to propose a new rational model for decision making in the presence of incomplete knowledge, through the definition of clear admissibility criteria by taking advantage of the notions of efficiency and simplicity that are central in industrial domain. In particular the authors were inspired from the protocols promoted in business practices guidelines like the "Collaborative Planning, Forecasting and Replenishment" model [14]. This protocol aims at coordinating the supply chain from strategic to operational

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decision: at the end, the agents should select a production plan (the candidate) that will be convenient for all the members of the supply chain. In this context the knowledge is incomplete and distributed since all the consequences of a production policy (i.e. the selection of a given production plan) are not known by all the particular participants of the supply chain and agents usually do not want to make all their knowledge public. This paper has defined a new representation framework for decision making, called Bipolar Leveled Framework (BLF), which was first introduced in [3]. The BLF is a bipolar structure that enables the decision maker to *visualize the attributes and goals that are involved in the decision problem, together with their inhibitors and their importance levels*. Informally, a BLF may be viewed as a kind of qualitative utility function with some extra features: 1) the defeasible links between attributes and goals are made explicit into what is called "decision principles", 2) an opposition to a decision principle, called "inhibitor", is represented by an arc directed towards it, 3) the importance levels of decision principles are represented by the height of their position in the structure. The BLF can be established either by one person or by a group of people in order to define a decision making framework which will be instantiated when candidates will be evaluated.

In this paper we propose to use a BLF in the collective decision problem to accept or reject a candidate. We propose to organize the decision by letting agents vote about the features that hold for the candidate but agents are never allowed to vote about the goals that would be achieved by selecting the candidate. Hence we separate the decision into three phases, the phase where the criteria associated to a good decision are defined (the BLF construction which is out of the scope of this paper), the phase where the candidates are evaluated by the voters, and the final decision to accept or reject the candidate (which is an automatic phase using the BLF with the precise features concerning the current candidate). In the literature the specification of a CUF is an aggregation of the agent preferences hence this is somewhat mixing the three phases. Moreover, in order to show that this rich and visual framework is well founded we show how the use of a common BLF may reduce the impact of manipulation strategies in the context of group decision making. The term manipulation is used in a weak sense, since the results presented are not of a game-theoretic nature, in particular, they do not admit deviating behavior. However since agents have the right to omit some information we consider that this behavior is a kind of manipulation.

2 Decision making with a BLF

2.1 BLF: a structure encoding decision criteria

We consider a set $\mathscr{C}$ of candidates[1] about which some information is available and two languages $\mathscr{L}_F$ (a propositional language based on a vocabulary $\mathcal{V}_F$) representing information about some features that are believed to hold for a candidate and $\mathscr{L}_G$ (another propositional language based on a distinct vocabulary $\mathcal{V}_G$) representing information about the achievement of some goals when a candidate is selected. In the propositional languages used here, the logical connectors "or", "and", "not" are denoted respectively by $\vee$, $\wedge$, and $\neg$. A *literal* is a propositional symbol x or its negation $\neg x$, the set of literals of $\mathscr{L}_G$ are denoted by LIT_G. Classical inference, logical equivalence and contradiction are denoted respectively by $\models$, $\equiv$, $\perp$. The reason why we propose two distinct languages is to clearly differentiate beliefs (coming from observations) from desires (goals to be achieved when selecting a candidate). In the following we denote by K a set of formulas representing features that are believed to hold: hence $K \subseteq \mathscr{L}_F$ is the available information. Using the inference operator $\models$, the fact that a formula $\varphi \in \mathscr{L}_F$ holds[2] in K is written $K \models \varphi$.

The BLF is a structure that contains two kinds of information: decision principles and inhibitors. A decision principle can be viewed as a defeasible reason enabling to reach a conclusion about the achievement of a goal. More precisely, a decision principle is a pair (φ, g), it represents the default rule meaning that "if the formula φ is believed to hold for a candidate then the goal g is a priori believed to be achieved by selecting this candidate":

Definition 1 (decision principle (DP)) *A* decision principle p *is a pair* $(\varphi, g) \in \mathscr{L}_F \times LIT_G$*, where* φ *is the reason denoted* $reas(p)$ *and* g *the conclusion of* p *denoted* $concl(p)$*.* $\mathcal{P}$ *denotes the set of decision principles.*

[1] Candidates are also called alternatives in the literature.

[2] The agent's knowledge K being considered to be certain, we write "φ holds" instead of "φ is believed to hold".

We illustrate the BLF notions on a toy example concerning a recruitment problem.

Example 1 *If the candidates are people applying for a job then the decision principle (CV good readability, ability to well present herself) could be understood as "if the candidate has a CV easy to read then a priori the goal to have a person able to well present herself is achieved".*

Depending on whether the achievement of its goal is wished or dreaded, a decision principle may have either a positive or a negative polarity. Moreover some decision principles are more important than others because their goal is more important. The decision principles are totally ordered accordingly.

Definition 2 (polarity and importance) *A function $pol : \mathcal{V}_G \to \{\oplus, \ominus\}$ gives the polarity of a goal $g \in \mathcal{V}_G$, this function is extended to goal literals by $pol(\neg g) = -pol(g)$ with $-\oplus = \ominus$ and $-\ominus = \oplus$. Decision principles are polarized accordingly: $pol(\varphi, g) = pol(g)$. The set of positive and negative goals are abbreviated $\overline{\oplus}$ and $\overline{\ominus}$ respectively: $\overline{\oplus} = \{g \in LIT_G : pol(g) = \oplus\}$ and $\overline{\ominus} = \{g \in LIT_G : pol(g) = \ominus\}$.*

LIT_G is totally ordered by the relation $\preceq$ ("less or equally important than"). Decision principles are ordered accordingly: $(\varphi, g) \preceq (\psi, g')$ iff $g \preceq g'$.

The polarities and the relative importances of the goals in $\mathcal{V}_G$ are supposed to be given by the decision maker. In the following example, the decision maker (our agent) may want to avoid to select an anti-social person (hence ap is a negative goal), while selecting a candidate who is efficient for the job is a positive goal, moreover he may give more importance to the efficiency for the job than to the ability to present oneself. In this example we propose to affect the same importance to the negative goal to have a person not efficient for the job $\neg ej$ and to the positive goal to have an easy to train person et, since when a candidate has those features it is difficult to say whether the positive outweights the negative or the reverse.

Example 2 *Let us consider a recruitment problem. The recruitment is done according to the following goals, listed with their abbreviation and polarity:*

goal	*meaning*	*polarity*
ap	*we do not want someone with an anti-social personality*	$\ominus$
ej	*we want to hire an efficient person for the job*	$\oplus$
ph	*we want to find a person able to present herself*	$\oplus$
et	*we want to find a person easy to train*	$\oplus$
st	*we want to hire a stable person*	$\oplus$

Hence in this example $LIT_G = \{ap, \neg ap, ej, \neg ej, ph, \neg ph, et, \neg et, st, \neg st\}$. The levels of the goals of LIT_G are s.t.[3] *$et \simeq \neg ej \succ \neg et \succ ap \succ ph \simeq \neg ph \succ \neg st \succ \neg ap \simeq ej \simeq st$. These levels of importance translate e.g. that finding an efficient and easy to train person is strictly more important than achieving any of the three other goals.*

The set of features $\mathcal{V}_F$ describing a candidate is summarized in the following tables:

Feature	*Meaning*
cbs	*CV bad spelling*
eb	*educational background*
i	*introverted candidate*
lpe	*long professional experience*
u	*unmotivated candidate*

Feature	*Meaning*
cgr	*CV good readability*
gp	*good personality*
$jhop$	*job hopper*
spe	*professional experience in the specialty of the job*

A decision principle (φ, g) is a defeasible piece of information because sometimes there may exist some reason φ' to believe that it does not apply in the situation, this reason is called an *inhibitor*.

The fact that φ' inhibits a decision principle (φ, g) is interpreted as follows: "when the decision maker only knows $\varphi \wedge \varphi'$ then he is no longer certain that g is achieved". In that case, the inhibition is represented with an arc towards the decision principle. The decision principles and their inhibitors are supposed to be given by the decision maker. An interpretation in terms of possibility theory is described in [12].

We are now in position to define the structure BLF.

[3] The equivalence relation associated to $\preceq$ is denoted $\simeq$ ($x \simeq y \Leftrightarrow x \preceq y$ and $y \preceq x$) and the strict order is denoted $\prec$ ($x \prec y \Leftrightarrow x \preceq y$ and not $y \preceq x$) hence $x \simeq y$ represents the fact that the goals x and y are equally important, $x \succ y$ that x is strictly more important than y.

Definition 3 (BLF) *Given a set of goals $\mathcal{V}_G$, a BLF is a quadruplet $(\mathcal{P}, \mathcal{R}, pol, \preceq)$ where $\mathcal{P}$ is a set of decision principles ordered accordingly to their goals by $\preceq$ and with a polarity built on pol as defined in Definition 2, $\mathcal{R} \subseteq (\mathscr{L}_F \times \mathcal{P})$ is an inhibition relation.*

The four elements of the BLF are supposed to be available prior to the decision and to be settled for future decisions as if it was a kind of utility function. A graphical representation of a BLF is given below, it is a tripartite graph represented in three columns, the DPs with a positive level are situated on the right column, the inhibitors are in the middle, and the DPs with a negative polarity are situated on the left. The more important (positive and negative) DPs are in the higher part of the graph, equally important DPs are drawn at the same horizontal level. By convention the highest positive level is at the top right of the figure and the lowest negative level is at the top left. The height of the inhibitors is not significant only their existence is used.

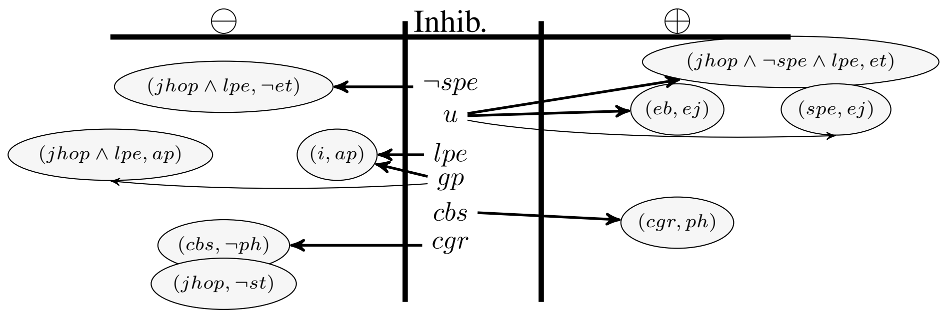

Figure 1: Recruitment BLF

Example 2 (continued): *Figure 1 illustrates the BLF corresponding to Example 2. In this example, we can see that it is different to have a decision principle with the negation of a goal like $\neg et$ (not easy to train) in $(jhop \wedge lpe, \neg et)$ (a job hopper with a long experience is not easy to train) from having an inhibition u towards a DP with the goal et, here $(jhop \wedge \neg spe \wedge lpe, et)$ (a job hopper non specialist with a long experience is generally easy to train except when he is unmotivated). Note that the utility of $\neg et$ is considered, it has a disutility (at a lower level than the utility of et), but sometimes the utility of the opposite goal is not considered as it is done for $\neg ap$ here (which is translated by an attribution of the lowest utility).*

In the following, the BLF $(\mathcal{P}, \mathcal{R}, pol, \preceq)$ is set and we show how it can be used for analyzing the acceptability of a candidate. First, we present the available information and the notion of instantiated BLF, called valid-BLF.

Given a candidate $c \in \mathscr{C}$, we consider that the knowledge of the decision maker about c has been gathered in a knowledge base K_c with $K_c \subseteq \mathscr{L}_F$. Given a formula φ describing a configuration of features ($\varphi \in \mathscr{L}_F$), the decision maker can have three kinds of knowledge about c: φ holds for candidate c (i.e., $K_c \models \varphi$), or not ($K_c \models \neg\varphi$) or the feature φ is unknown for c ($K_c \not\models \varphi$ and $K_c \not\models \neg\varphi$). When there is no ambiguity about the candidate c, K_c is denoted K.

Definition 4 (K-valid-BLF) *Given a* consistent *knowledge base K, a K-valid-BLF is a quadruplet ($\mathcal{P}_K$, $\mathcal{R}_K$, pol, $\preceq$) where*

- $\mathcal{P}_K = \{(\varphi, g) \in \mathcal{P}$, *s.t.* $K \models \varphi\}$ *is the set of DPs in $\mathcal{P}$ whose reason φ holds in K, called* valid-DP*s.*
- $\mathcal{R}_K = \{(\varphi, p) \in \mathcal{R}$, *s.t.* $K \models \varphi$ *and* $p \in \mathcal{P}_K\}$ *is the set of* valid inhibitions *according to K.*

When there is no ambiguity, we simply use "valid-BLF" instead of "K-valid-BLF". The validity of a DP only depends on the fact whether the features that constitute its reason φ hold or not, it does not depend on its goal g since the link between the reasons and the goal is given in the BLF (hence it is no longer questionable).

Example 2 (continued): *The agent has information about a candidate c_0: she is a job hopper $jhop$, with a long personal experience, lpe, this experience is not in the specialty of the job $\neg spe$ and the candidate is unmotivated u but she has a CV with a good readability cgr. Figure 2 is the valid-BLF representing the knowledge about c_0.*

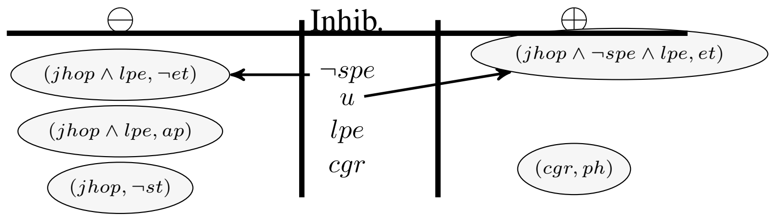

Figure 2: Valid BLF with $K = \{jhop, lpe, \neg spe, u, cgr\}$

Now in the valid-BLF the principles that are not inhibited are the ones that are going to be trusted. A goal in $\mathcal{V}_G$ is said to be *realized* if there is a valid-DP that is not inhibited by any valid-inhibitor.

Definition 5 (realized goal) *Let g be a goal in LIT_G, g is* realized *w.r.t. a K-valid-BLF $B = (\mathcal{P}_K, \mathcal{R}_K,$ $pol, \preceq)$ iff $\exists(\varphi, g) \in \mathcal{P}_K$ and $\nexists(\varphi', (\varphi, g)) \in \mathcal{R}_K$.*
The set of realized goals w.r.t. B is denoted $\mathtt{R}(B)$, the positive and negative realized goals are denoted by $\mathtt{R}(B)^{\oplus} = \mathtt{R}(B) \cap \overline{\oplus}$ and $\mathtt{R}(B)^{\ominus} = \mathtt{R}(B) \cap \overline{\ominus}$ respectively.

When there is no ambiguity about the BLF B, $\mathtt{R}(B)$ is simply denoted by $\mathtt{R}$.

Example 2 (continued): *The goals $\neg et$ and et are not realized for candidate c_0 since the inhibitions $\neg spe$ and u are valid. Two negative goals, ap and $\neg st$, and one positive goal, ph, are realized.*

In Section 2.2 we explain how to use a BLF in order to make a decision. The decision consists in saying whether or not a candidate is admissible based on the goals that are realized in its corresponding valid-BLF.

2.2 Admissibility statuses of candidates

The admissibility status of a candidate c is computed from a BLF and a knowledge base K_c describing what is known about c, its corresponding K_c-valid-BLF should be denoted $\langle \mathcal{P}_{K_c}, \mathcal{R}_{K_c}, pol, \preceq \rangle$. However, when there is no ambiguity about the knowledge available, $\mathcal{P}_{K_c}$ is denoted $\mathcal{P}$ and $\mathcal{R}_{K_c}$ is abbreviated $\mathcal{R}$.

We first define the notion of order of magnitude which requires to define the levels of a set of goals. We attribute levels to sets of goals starting from the least important ones that are assigned a level 1 and stepping by one each time the importance grows.

Definition 6 (levels and order of magnitude) *Given a set of goals $G \subseteq LIT_G$ and the relation $\preceq$ on LIT_G, the levels of G are defined by induction:*

- $G_1 = \{g \in G : \nexists g' \in G \text{ s.t. } g' \prec g\}$
- $G_{i+1} = \{g \in G : \nexists g' \in G \setminus (\bigcup_{k=1}^{i} G_k) \text{ s.t. } g' \prec g\}$

The order of magnitude of G is: $OM(G) = \max_{g \in G}\{\lambda : g \in G_\lambda\}$ with $OM(\varnothing) = 0$.

Definition 7 (admissibility status) *Given a candidate $c \in \mathscr{C}$, given the knowledge K_c about c and given a K_c-valid-BLF $\langle \mathcal{P}, \mathcal{R}, pol, \preceq \rangle$. The status of c is:*
- necessarily admissible *if* $\mathtt{R}_M^{\oplus} \neq \varnothing$ *and* $\mathtt{R}_M^{\ominus} = \varnothing$
- possibly admissible *if* $\mathtt{R}_M^{\oplus} \neq \varnothing$
- indifferent *if* $\mathtt{R} = \varnothing$
- possibly inadmissible *if* $\mathtt{R}_M^{\ominus} \neq \varnothing$
- necessarily inadmissible *if* $\mathtt{R}_M^{\ominus} \neq \varnothing$ *and* $\mathtt{R}_M^{\oplus} = \varnothing$

- controversial *if* $\mathtt{R}_M^{\oplus} \neq \varnothing$ *and* $\mathtt{R}_M^{\ominus} \neq \varnothing$
where $M = OM(\mathtt{R})$ *(hence* $\mathtt{R}_M$ *is the set of most important realized goals).*

We respectively denote by N_{ad}*,* Π_{ad}*,* Id*,* $\Pi_{\neg ad}$*,* $N_{\neg ad}$ *and* Ct *the set of necessarily admissible, possibly admissible, indifferent, possibly inadmissible, necessarily inadmissible and controversial candidates.*

In other words, a *necessarily admissible* candidate is supported by positive principles with goals of maximum importance that are realized (*i.e.,* uninhibited) and all the negative goals of the same importance do not hold. A *possibly admissible* candidate has at least one uninhibited positive principle of maximum importance in its favor. An *indifferent candidate*[4] is not concerned by any uninhibited principle (nor positive nor negative), while a *controversial candidate* is both supported and criticized by uninhibited DPs of maximum importance We define three sets of admissibility:

- $S1 = N_{ad}$, in this set, the candidates are admissible with no doubt, there are uninhibited principles about the candidates which are all positive.
- Since there are two ways to have doubts about a candidate, namely when she is indifferent (Id) or controversial (Ct), we define two weaker sets:
  - $S2a = N_{ad} \cup Id$ (*i.e.,* $S2a = \mathscr{C} \setminus \Pi_{\neg ad}$). In this set, we place candidates of S1 together with those for which no uninhibited principle is available (neither positive nor negative),
  - $S2b = N_{ad} \cup Ct$ (*i.e.,* $S2b = \Pi_{ad}$). It gathers S1 together with the candidates that are concerned by negative uninhibited principles provided that they are also concerned at least by one positive uninhibited principle.
- S3 $= Id \cup Ct \cup N_{ad}$ (*i.e.*, $\Pi_{ad} \cup Id = \mathscr{C} \setminus N_{\neg ad}$). It contains also S1.

Note that in this paper we focus on the question whether to accept or not a given candidate. Regarding the question about the selection of one candidate among a set of candidates, different rules [7] have been proposed for comparing candidates in a qualitative bipolar decision framework and have been adapted to the BLF setting [12].

Example 2 (continued): *The status of the candidate* c_0 *is* necessarily inadmissible *since only ap, ph and* $\neg st$ *are realized in her valid BLF, but* $ap \succ ph$ *and* $pol(ap) = \ominus$*. Hence* c_0 *will not be accepted since the most important realized goal for her is negative.*

3 Vote between several agents under a common BLF

The BLF demonstrates the full extent of its usefulness in the case where knowledge is distributed over several agents who have personal preferences but who want to collaborate in order to make a good decision for the group. In this paper, we focus on the case where the agents vote in order to accept or reject one candidate. The vote action consists in revealing that some feature holds or not (wrt to the agent knowledge).

In order to evaluate the BLF with regard to manipulation, we are going to consider that each agent has some private preferences about the candidates i.e. she may want to support one candidate or not. However, we assume that the common knowledge is consistent and that agent can not lie, hence, two agents cannot utter contradictory facts.

More formally, we consider a set $\mathcal{V}$ of agents (voters), each agent $v \in \mathcal{V}$ divides the set of candidates $\mathscr{C}$ into two subsets $\mathscr{C}_v^+$ and $\mathscr{C}_v^-$. $c \in \mathscr{C}_v^+$ means that the agent v is in favor of the candidate c and $c \in \mathscr{C}_v^-$ means that the agent v is against accepting c. In this section we propose several vote strategies and study the relation between the strategies and the admissible thresholds. Each agent has her own private knowledge, K_v, and we suppose that the set of all the available knowledge K ($K = \bigcup_{v \in \mathcal{V}} K_v$) is consistent, however each agent does not know the private knowledge of the other agents.

Let $Voted_v^s \subseteq \mathscr{L}_F$ be the set of voted formulas under strategy s by the agent v i.e. $f \in Voted_v^s$ if under strategy s the agent v gives the information that f holds to the group (the subscript v will be forgotten when

[4] Note that the indifference definition uses $\mathtt{R}$ and not $\mathtt{R}_M$.

there is no ambiguity about the agent). In this paper we define two strategies, the optimistic strategy (Definition 8) and the pessimistic strategy (Definition 9). In the vote procedure, we impose that the agents are only allowed to use literals extracted from the BLF: more formally, let $LIT_{\mathcal{F}} = \bigcup_{a\in\mathcal{P},x\in reas(a)}\{x, \neg x\}$, $\forall s, \forall v \in V, Voted^s_v \subseteq LIT_{\mathcal{F}}$. In other words, the BLF defines the vocabulary that can be used for voting.

3.1 Optimistic and pessimistic strategies

In the following definition we slightly abuse notations by using $f \in \varphi$ with f being a literal, it means that the variable on which f is built appears in φ, i.e., $\forall f \in LIT_F$, $f \in \varphi$ is a shortcut for ((f is a variable and $f \in \varphi$) or ($f = \neg x$ and x is a variable s.t. $x \in \varphi$)). We call optimistic strategy Def.8 (or Confident strategy) a strategy in which as soon as something is in a positive decision principle or in an inhibitor of a negative decision principle the agent will utter it without looking at the possible negative effect of this utterance. More formally:

Definition 8 (Optimistic strategy) *Let $v \in \mathcal{V}$ be a agent and $c \in \mathscr{C}$ a candidate such that $c \in \mathscr{C}^+_v$, let $K_{v,c}$ the facts that are known by v about c. Let us consider $(\mathcal{P}, \mathcal{R}, pod, \preceq)$ a BLF with $(\mathcal{P}_c, \mathcal{R}_c, pod, \preceq)$ the associated $K_{v,c}$-valid BLF. The* optimistic strategy *is: $\forall f \in LIT_{\mathcal{F}}$,*

$$f \in Voted^o_v \textit{ iff } \begin{cases} K_{v,c} & \vdash f \\ \textit{and} & (\quad \exists p \in \mathcal{P}^{\oplus}_c \textit{ s.t. } f \in reas(p) \textit{ and } f \wedge reas(p) \nvdash \bot \\ & \textit{or} \quad \exists(\varphi, p) \in \mathcal{R}^{\ominus}_c \textit{ s.t. } f \in \varphi \textit{ and } f \wedge \varphi \nvdash \bot) \end{cases}$$

where
$\mathcal{P}^{\oplus}_c = \{(\varphi, g) \in \mathcal{P}_c$ s.t. $g \in \oplus\}$ is the set of DPs in the $K_{v,c}$-valid BLF that have a positive goal and $\mathcal{R}^{\ominus}_c = \{(\varphi', (\varphi, g)) \in \mathcal{R}_c$ s.t. $g \in \ominus\}$ denotes the set of inhibition that inhibit a DP with a negative goal in the $K_{v,c}$-valid BLF.

In other words the optimistic strategy consists in giving *all*[5] the literals that are known to hold and that appear either in a positive DP or in a inhibitor of a negative DP as long as they are not inconsistent with the reason of this positive DP, nor inconsistent with this inhibitor. Hence by using an optimistic strategy the agent will give a lot of information, hoping that it will help to validate at least one positive DP or to invalidate at least one negative DP.

Example 2 (continued): *Let us suppose that the agent wants to accept the candidate c_0. If she uses the optimistic strategy then she will omit to say u because it does not appear in a positive DP nor in a Inhibitor against a negative DP. The valid BLF obtained by the vote of this agent is described in Figure 3 with a cross on the omitted literal. In this case the candidate becomes necessarily admissible.*

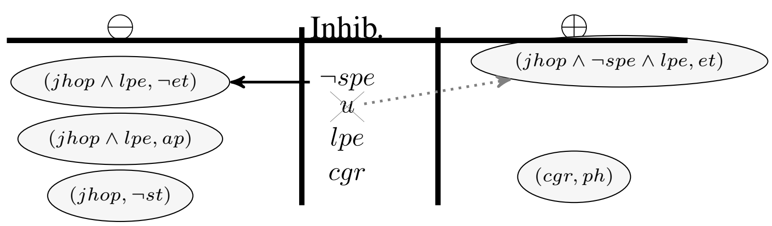

Figure 3: Optimistic BLF with $K = \{jhop, lpe, \neg spe, u, cgr\}$

Note that with the optimistic strategy, the agent would tell everything she knows as soon as there is a possibility that other agents complete her vote in a favorable way.

Example 2 (continued): *The agent utters $jhop$ even if there is a negative unihibited DP $(jhop \wedge lpe, ap)$. Unfortunately, in our case, we know that lpe holds, hence by saying $jhop$ it opens the way to validate the negative DP, moreover the agent herself has uttered lpe (since she is a naive optimistic, a more skilled strategy would reason more globally on the whole set of voted formulas, which would require a more complex computation). Besides since the most important DP is inhibited by u, the agent takes the risk that another agent utters u, in that case the candidate c_0 would be necessarily rejected.*

[5] Due to the use of "iff" in Definition 8.

As we have seen, by using the optimistic strategy the agent may validate some negative DP. She can also inhibit other positive DPs when the literal is present both in a reason for a positive DP and in an inhibitor. The pessimistic approach is more cautious and check (naively) if a voted literal cannot create collateral damages.

Definition 9 (Pessimistic strategy) *Let $v \in \mathcal{V}$ be a agent and $c \in \mathscr{C}$ a candidate such that $c \in \mathscr{C}_v^+$ and let $K_{v,c}$ the facts that are known by v about c. Let us consider $(\mathcal{P}, \mathcal{R}, pol, \preceq)$ a BLF with $(\mathcal{P}_c, \mathcal{R}_c, pol, \preceq)$ the associated $K_{v,c}$-valid BLF. The* pessimistic strategy *is: $\forall f \in LIT_{\mathcal{F}}$,*
$\forall f \in LIT_{\mathcal{F}}$,

$$f \in Voted_v^p \textit{ iff } \begin{cases} K_{v,c} & \vdash f \\ \textit{and} & (\exists p \in \mathcal{P}_c^{\oplus} \textit{ s.t. } f \in reas(p) \textit{ and } f \wedge reas(p) \nvdash \bot \\ & \textit{or } \exists(\varphi, p) \in \mathcal{R}_c^{\ominus} \textit{ s.t. } f \in \varphi \textit{ and } f \wedge \varphi \nvdash \bot) \\ \textit{and} & \nexists p \in \mathcal{P}_c^{\ominus} \textit{ such that } f \in reas(p) \textit{ and } f \wedge reas(p) \nvdash \bot \\ \textit{and} & \nexists(\varphi, p) \in \mathcal{R}_c^{\oplus} \textit{ s.t. } f \in \varphi \textit{ and } f \wedge \varphi \nvdash \bot) \end{cases}$$

where $\mathcal{P}_c^{\oplus}$ is defined as before and
$P_c^{\ominus} = \{(\varphi, g) \in \mathcal{P}_c \text{ s.t. } g \in \ominus\}$ is the set of DPs in the $K_{v,c}$-valid BLF that have a negative goal and $\mathcal{R}_c^{\ominus}$ is defined as before and
$\mathcal{R}_c^{\oplus} = \{(\varphi', (\varphi, g)) \in \mathcal{R}_c \text{ s.t. } g \in \oplus\}$ denotes the set of inhibition that inhibit a DP with a positive goal in the $K_{v,c}$-valid BLF.

In other words the pessimistic strategy consists in giving a piece of information f about a preferred candidate only if it may help to validate positive DP or to inhibit negative DP, but if this piece cannot be used against the candidate. Hence the agent with a pessimistic strategy will not utter a literal f if it could help to validate a negative DP or if f could help to validate an inhibitor of a positive argument.

Example 2 (continued): *If our agent uses the pessimistic strategy then she would omit to say u but would also omit $jhop$ and lpe. The valid BLF obtained by the vote of this agent is described in Figure 4 with the only valid DP encircled by solid lines. In this case the candidate becomes necessarily admissible.*

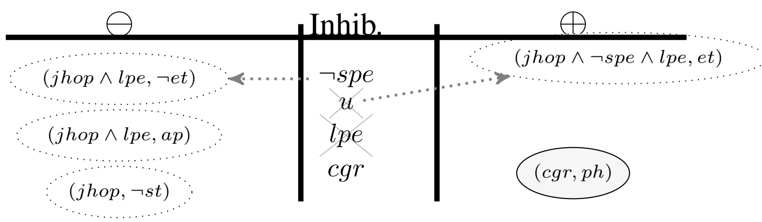

Figure 4: Pessimistic BLF with $K = \{jhop, lpe, \neg spe, u, cgr\}$

We can notice that the pessimistic strategy is hiding a lot of information. Note that we have shown the BLF obtained after the vote of only one agent, let us see in the next section what happens in presence of several agents. The previous strategies are naive in the sense that they focus on each literal independently while they could focus on achieving the goals. More complex strategy could be defined, namely we could propose to select a set of literals which can be uttered together in order to influence admissibility statuses.However, we are going to show that no strategy can guarantee to obtain the desired admissibility status.

3.2 Analysis of the strategies

In this section, we study the properties of the optimistic and pessimistic strategies. First, let us recall that global knowledge is assumed to be consistent. We also assume that the agents cannot lie (except by omission). This comes from two reasons: first, idealy agents are gathered in order to take the best decision for the group, hence they have a moral duty to do it honestly, second a lie may lead to an inconsistency hence could be discovered by the group and cause the agent to be ashame. Hence the only way to manipulate the vote is to omit to declare some pieces of information that they know.

Proposition 1 [6] $\forall v \in \mathcal{V}, \forall s \in \{o,p\} \quad K_v \vdash Voted_v^s$ *and* $Voted_v^s \nvdash \perp$

Proposition 1 states that under optimistic and pessimistic strategy what is voted should be deduced from the knowledge of the voters and it is consistent. The following proposition shows that the pessimistic strategy cannot reveal more information than the optimistic one.

Proposition 2 $\forall v \in \mathcal{V}, \quad Voted_v^p \subseteq Voted_v^o$

In the following, $Voted$ is the set of voted knowledge and $\mathtt{R}_K$ and $\mathtt{R}_{Voted}$ denotes the set of realized goals under total knowledge and voted knowledge respectively. We are now going to study the link between strategies and the admissibility thresholds. Note that if all the agents use the optimistic strategy then $Voted = \bigcup_{v \in \mathcal{V}} Voted_v^o$. The following propositions concern basic admissibility.

Proposition 3 *$\forall c \in \mathscr{C}$ s.t. $\forall v \in \mathcal{V}$, $c \in \mathscr{C}_v^+$, if all the agents use the optimistic strategy, then $\mathtt{R}_K^{\oplus} \subseteq \mathtt{R}_{Voted}^{\oplus}$.*

Note that the inclusion maybe strict since there maybe some positive goals which are realized by an optimistic strategy made by all the agents but which are not realized under total knowledge, since they can have inhibitors that would be omitted by the strategical agents. Hence there can be more positive realized goals by using a global optimistic strategy than by sharing all the knowledge.

Proposition 4 *$\forall c \in \mathscr{C}$ s.t. $\forall v \in \mathcal{V}$, $c \in \mathscr{C}_v^+$, if all the agents use the optimistic strategy then $\mathtt{R}_{Voted}^{\ominus} \subseteq \mathtt{R}_K^{\ominus}$.*

Similarly as before, the inclusion maybe strict since there maybe some negative DP that are not validated by the vote. Hence there can be less negative realized goals by using a global optimistic strategy than by sharing all the knowledge. From propositions 3 and 4, it follows that under the optimistic strategy used by all the agents, a candidate that is preferred by all the agents is not guaranteed to be accepted, this depends on how the complete knowledge is shared and on the BLA structure. More precisely, we have:

Theorem 1 *$\forall c \in \mathscr{C}$ s.t. $\forall v \in \mathcal{V}$, $c \in \mathscr{C}_v^+$, if all the agents use the optimistic strategy*
- $c \in N_{ad}$ under total knowledge then $c \in N_{ad}$ under voted knowledge
- $c \in Ct$ under total knowledge then $c \in N_{ad} \cup Ct$ under voted knowledge
- $c \in Id$ under total knowledge then $c \in N_{ad} \cup Id$ under voted knowledge
- $c \in N_{\neg ad}$ under total knowledge then c may have any status under voted knowledge ($c \in N_{ad} \cup Id \cup Ct \cup N_{\neg ad}$).

The last point of Theorem 1 shows that a unanimity of personal preferences may lead to accept a candidate which is not admissible for the group according to the BLF.

Example 3 *To illustrate Theorem 1, we propose an example with 4 agents with $c \in \mathscr{C}_v^+$ with $K_1 = \{eb, \neg spe\}$, $K_2 = \{lpe, i\}$, $K_3 = \{eb, cgr\}$ and $K_4 = \{i, u, cbs\}$. Using optimistic strategy $Voted = \{eb, \neg spe, lpe, cgr\}$. Hence, $\mathtt{R}_K^{\oplus} = \{ej, ph\}$ and $\mathtt{R}_K^{\ominus} = \varnothing$ so the candidate is N_{ad} while the candidate is Id under total knowledge (see Figure* **??***). Note that Agent 4's preferences are totally in contradiction with its valid-BLF. This contradiction allows Agent 4 to omit information in order to increase the accepted status of the candidate. One can see that if Agent 4 changes its personal preferences the candidate becomes Id.*

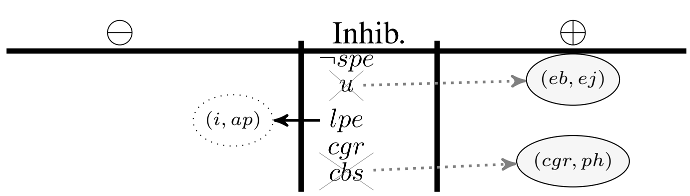

Fig.5 $K = \{eb, \neg spe\} \cup \{lpe, i\} \cup \{eb, cgr\} \cup \{i, u, cbs\}$, Voted=$\{eb, \neg spe, lpe, cgr\}$

Let us study the pessimistic strategy. Note that under this strategy, the realized goals may e pessimistic differ from what could have been obtained by having the whole information.

[6] The proofs are in Annex.

Proposition 5 *If all the agents use the pessimistic strategy then* $\mathtt{R}^{\ominus}_{Voted} = \varnothing$.

$\mathtt{R}^{\oplus}_{Voted}$ can be different from $\mathtt{R}^{\oplus}_{K}$ (i.e., we can have $\mathtt{R}^{\oplus}_{Voted} \nsubseteq \mathtt{R}^{\oplus}_{K}$ and $\mathtt{R}^{\oplus}_{K} \nsubseteq \mathtt{R}^{\oplus}_{Voted}$). Since we may have some positive DPs that can be uninhibited by voted DPs while they are inhibited under complete knowledge, and we may also have some positive DPs that contains facts that belong to inhibitors of positive DPs or that belong to negative DPs that will not be valid under voted knowledge while they would be validated by sharing the complete knowledge.

From Proposition 5 it follows that a candidate that is preferred by all agents is either necessary admissible or indifferent when the pessimistic strategy is used by everyone:

Corollary 1 $\forall c \in \mathscr{C}$, *if all the agents use pessimistic strategy and if* c *is s.t.* $\forall v \in \mathcal{V}$, $c \in \mathscr{C}_v^+$ *then* $c \in N_{ad} \cup Id$ *under voted knowledge.*

Theorem 1 and Corollary 1 show the impact of the unanimity of the agents' preferences on the satisfaction of the group. Optimistic strategy can increase the evaluation of a candidate but this may be impossible to do if there is no feature that holds (under total knowledge) that can help to produce a DP in favor of the candidate. Concerning pessimistic strategy, even if it is naive, the possibilities of manipulation of the results are limited, i.e., it cannot transform a candidate which is preferred by all the agents into a necessary admissible candidate, sometimes the best they can obtain will be Id (for instance when the agents possess only bad features for this candidate, then they will say nothing, hence the candidate will be considered as indifferent).

We are now in position to relate the choice of a strategy and the choice of an admissibility threshold. Indeed, from Theorem 1, it follows that given an admissibility threshold in the set $\{1, 2a, 2b, 3\}$, if a candidate would have been selected according to that threshold under total knowledge and if this candidate is preferred by all the agents then it would also have been selected by these agents playing all the optimistic strategy and using the same threshold:

Corollary 2 $\forall c \in Ad$ *under total knowledge with* $Ad \in \{1, 2a, 2b, 3\}$, *if* $\forall v \in \mathcal{V}$, v *uses the optimistic strategy and* $c \in \mathscr{C}_v^+$ *then* $c \in Ad$ *under voted knowledge.*

It follows from Corollary 1 that with the pessimistic strategy used by all the agents, the common utility modeled by a BLF is ignored in favor of a unanimity of personal preferences if the admissibility threshold is $2a$ (hence a good way to lower manipulation would be to rather choose the $2b$ threshold, forbidding the pessimistic strategies to act against the common utility).

Corollary 3 *If the group uses the threshold 2a* ($N_{ad} \cup Id$) *and if all agents are using the pessimistic strategy and if* c *is s.t.* $\forall v \in \mathcal{V}$, $c \in \mathscr{C}_v^+$ *then the candidate is accepted.*

In a symmetric way, if no one wants to accept the candidate and if the threshold is 2a, the candidate can be accepted in the Voted BLF.

The use of a BLF reflects that knowledge is power: since knowledge gives the power to advance facts for triggering or inhibits DPs, this is why we are going to use a framework in which all the agents have the same knowledge. More formally, $K = K_v, \forall v \in \mathcal{V}$. With equal knowledge the optimistic strategy is more efficient since if there are pros and cons it will be the consensual BLF that will make the decision.

Proposition 6 $\forall c \in \mathscr{C}$ *s.t.* $\exists v \in \mathcal{V}$, $c \in \mathscr{C}_v^+$ *and* $\exists v' \in \mathcal{V}$, $c \in \mathscr{C}_{v'}^-$, *if all the agents use the optimistic strategy and* $K = K_v, \forall v \in \mathcal{V}$ *then* $Voted = K$.

In the case where an agent is pro and another agent is con and the knowledge is the same for all agents, the vote amounts to share all the knowledge in a neutral way, hence the candidate will have the same admissibility as if there were no strategy.

Corollary 4 (Statuses under optimistic strategy 2) $\forall c \in \mathscr{C}$ *s.t.* $\exists v \in \mathcal{V}$, $c \in \mathscr{C}_v^+$ *and* $\exists v' \in \mathcal{V}$, $c \in \mathscr{C}_{v'}^-$, $\forall Ad \in \{1, 2a, 2b, 3\}$, *if* $c \in Ad$ *under total knowledge then* $c \in Ad$ *under voted knowledge.*

Let us consider now the pessimistic strategy under a shared equal knowledge.

Proposition 7 *$\forall c \in \mathscr{C}$ s.t. $\exists v \in \mathcal{V}$, $c \in \mathscr{C}_v^+$ and $\exists v' \in \mathcal{V}$, $c \in \mathscr{C}_v^-$, if all the agents use the pessimistic strategy and $K = K_v, \forall v \in \mathcal{V}$ then $\forall Ad \in \{1, 2a, 2b, 3\}$, if $c \in Ad$ under total knowledge then $c \in N_{ad} \cup Id \cup Ct \cup N_{\neg ad}$ under voted knowledge.*

From Corollary 4 and Proposition 7 it appears that the optimistic strategy is more rational and does not depend on the structure of the BLF, since with pessimistic strategy c can have any status under voted knowledge.

We are now going to compare the use of the optimistic or pessimistic strategy in the case where the knowledge is the same for all and some agents are pro others are con. For this, we consider a problem with two agents, the first one is pro the candidate and uses the optimistic strategy and the second one is con and uses the pessimistic strategy.

Proposition 8 *Let $\mathcal{V} = \{v_1, v_2\}$ with $K = K_{v1} = K_{v2}$, $\forall c \in \mathscr{C}$ s.t. $c \in \mathscr{C}_{v_1}^+$ and $c \in \mathscr{C}_{v_2}^-$ with v_1 (resp. v_2) using optimistic (resp. pessimistic) strategy, it holds that $\mathtt{R}_K^{\oplus} \subseteq \mathtt{R}_{Voted}^{\oplus}$ and $\mathtt{R}_{Voted}^{\ominus} \subseteq \mathtt{R}_K^{\ominus}$ and:*

- *$c \in N_{ad}$ under total knowledge then $c \in N_{ad}$ under voted knowledge*
- *$c \in Ct$ under total knowledge then $c \in N_{ad} \cup Ct$ under voted knowledge*
- *$c \in Id$ under total knowledge then $c \in N_{ad} \cup Id$ under voted knowledge*
- *$c \in N_{\neg ad}$ under total knowledge then c may have any status under voted knowledge ($c \in N_{ad} \cup Id \cup Ct \cup N_{\neg ad}$).*

Before closing this section about naive strategies, let us underline the fact that strategies are not well suited with the collaborative spirit of the BLF, since, by strategy, a agent can choose to hide some information that have been judged to be relevant for the realization of a common goal by all the group. Hence using a strategy in order to favor a candidate may be viewed as a betrayal with respect to the group welfare.

4 Discussion and related work

In AI literature the bipolar view has often been used, inspired by the fact that human usually evaluate the possible alternatives considering positive and negative aspects separately [9]. In the domain of multi-criteria decision, Dubois and Fargier [10] propose a qualitative bipolar approach in which a candidate is associated with two distinct sets of positive "arguments" (pros) and negative "arguments" (cons). This polarity is given wrt the decision goal. The arguments are abstract and there is no relation between them, but their level of importance is given. The papers [11, 8] study the use of decision rules in practice by human decision makers, and show that the bipolar lexicographic rule is largely favored by humans. Another way to deal with this kind of situation can be based on the maching between a post profile and each candidate. When uncertainty is taken into account, this matching can be expressed by criteria based on the possibility or necessity measure that the candidate matches the profile [13]. A drawback of these approaches is that it requires to dispose of a precise possibility distribution about candidates and it implies a commensurability between the preferences about the research profile and the uncertainty about the candidate. Moreover it is difficult to define the threshold over which the candidate could be accepted.

The notion of "argument" in favor or against a decision has also been developped in practical argumentation domain which has been widely studied (see e.g. [17, 1]). Practical argumentation aims at answering the question "what is the right thing to do in a given situation" which is clearly related to a decision problem. Several works are using argumentative approaches to tackle it: for instance [6] have a very similar view of what we call Principles, since they use defeasible rules in favor or against a given action, Amgoud and Prade [2] propose a bipolar argumentation-based approach distinguishing epistemic and practical arguments. Our decision principle can be viewed as a pair of (epistemic information, practical information) which defines the practical conclusion that should be fired under the epistemic information.

Another theoretical framework has been proposed for multi-criteria / multi-agent (non sequential) decision making under possibilistic uncertainty [4, 5]. Our approach differs from classical approaches of group decision under uncertainty on the following points: (i) our BLF is a kind of CUF, but it is a richer qualitative bipolar structure that takes into account available knowledge and not only agents' utilities; (ii) our BLF is based on justifications of why we should take one decision (since a DP contains the reasons for saying that a given goal is achieved) while in standard CUF, utilities are given with no structured explanation; (iii) we assume that agents have no uncertainty about what they know but they are uncertain about what the other agents know.

In this paper, we consider that agents can provide only factual information and no information about their preferences, since the BLF contains already importance levels and polarities that are assumed to reflect the group utility. Hence the agents has only access to the information describing the candidates which differs from classical utilitarian approaches. In this context, our paper studies what happens when agents are not purely acting for the wellfare of the group, hence we take into account a second type of private utility: the fact that an agent wants secretely to accept a candidate without considering the common BLF. In this case, this kind of utility is personal and the agent does not want to agregate this information. This differs from the aim to aggregate the utilities of the different agents used in the literature about collective decision making. Our study shows that the BLF may help to reduce the possibilities of agents to influence the decision that would have been taken for the wellfare of the group. Indeed for an agent it is very difficult in our context to know what the other agents know and what they will utter hence to compute what goal will be achieved after the vote of the group, and moreover to know the status that will get the candidate.

Concerning vote and manipulability, we plan to introduce reputation and blame, i.e., if an agent has always access to some information and does not deliver it for a given candidate, then he can be accused of attempting to manipulate the group decision. This could enforce agents to utter all the information they know in order to behave better for the group. Another direction would be to extend the expressivity of a BLF in order to incorporate the strength of a decision principle (which will both have an importance relative to its goal, and a strength relative to the certainty of its conclusion when the reasons hold). A possible extension would be to handle the agent's uncertain knowledge about the situation, hence to allow for features associated with uncertainty degrees. The question of how to build a BLF from different points of view has been evoked in [12] but has not been developed yet. We plan to work on the subject of BLF building with health decision makers in order to see how complicated it is for them to define decision principles, inhibitors and importance relations.

Annex: Proofs

Proof of Proposition 1: In both definitions (Definition 8 and Definition 9) $\forall f \in Voted_v^s$ it holds that $K_v \vdash f$. Moreover K is supposed to be consistent and $K_v \subseteq K$. $\square$

Proof of Proposition 2: It is straightforward from the definition of the pessimistic strategy that is based on the optimistic one on which some more constraints are added. $\square$

Proof of Proposition 3: Let us show that $\mathtt{R}_K^{\oplus} \subseteq \mathtt{R}_{Voted}^{\oplus}$, if $g \in \mathtt{R}_K^{\oplus}$ then there exists an unhibited positive DP with conclusion g, i.e., $\exists(\varphi, g) \in \mathcal{P}_K^{\oplus}$ s.t. $K \vdash \varphi$ and $g \in \oplus$ and $\nexists(\varphi', (\varphi, g)) \in \mathcal{R}_K$ s.t. $K \vdash \varphi'$, hence, $\forall l \in LIT_K$, s.t. $K \vdash l$, if $l \in \varphi$ then $l \wedge \varphi \nvdash \bot$ (since K is consistent), so l is voted with the optimistic strategy. Moreover if $\nexists(\varphi', (\varphi, g)) \in \mathcal{R}_K$ s.t. $K \vdash \varphi'$ then it is not possible that $Voted \vdash \varphi'$ s.t. φ' inhibits (φ, g) since $K \vdash Voted$ and K is consistent. $\square$

Proof of Proposition 4: If g is in $\mathtt{R}_{Voted}^{\ominus}$ then it means that g is the conclusion of a negative DP (φ, g) such that $Voted \vdash \varphi$ and forall inhibitor φ' s.t. $(\varphi', (\varphi, g)) \in R_K$, $Voted \nvdash \varphi'$, which means that $K \nvdash \varphi'$ unless by strategy agents would have inhibit the negative DP. It means that $K \vdash \varphi$ since $K \vdash Voted$, and $K \nvdash \varphi'$ for any inhibitor φ' of the negative DP. Hence $g \in \mathtt{R}_K$. $\square$

Proof of Theorem 1:
- $c \in N_{ad}$ under total knowledge implies $\mathtt{R}_{eK}^{\oplus} \neq \varnothing$ and $\mathtt{R}_{eK}^{\ominus} = \varnothing$. Using Proposition 3 we have $\mathtt{R}_{eVoted}^{\oplus} \neq \varnothing$ and due to Proposition 4, $\mathtt{R}_{eVoted}^{\ominus} = \varnothing$, hence the result.
- $c \in Ct$ under total knowledge implies $\mathtt{R}_{eK}^{\oplus} \neq \varnothing$ and $\mathtt{R}_{eK}^{\ominus} \neq \varnothing$. Using Proposition 3, we have $\mathtt{R}_{eVoted}^{\oplus} \neq \varnothing$ and due to Proposition 4, $\mathtt{R}_{eVoted}^{\ominus} = \varnothing$ or $\mathtt{R}_{eVoted}^{\ominus} \neq \varnothing$, hence the result.

- $c \in Id$ under total knowledge implies $\mathtt{R}^{\oplus}_{K} = \varnothing$ and $\mathtt{R}^{\ominus}_{K} = \varnothing$. Using Proposition 3, we have $\mathtt{R}^{\oplus}_{Voted} \neq \varnothing$ or $\mathtt{R}^{\oplus}_{Voted} = \varnothing$ and due to Proposition 4, $\mathtt{R}^{\ominus}_{Voted} = \varnothing$, hence the result.
- $c \in N_{\neg ad}$ under total knowledge implies $\mathtt{R}^{\oplus}_{eK} = \varnothing$ and $\mathtt{R}^{\ominus}_{eK} \neq \varnothing$. Using Proposition 3 we have $\mathtt{R}^{\oplus}_{eVoted} \neq \varnothing$ or $\mathtt{R}^{\oplus}_{eVoted} = \varnothing$ and due to Proposition 4 $\mathtt{R}^{\ominus}_{eVoted} = \varnothing$ or $\mathtt{R}^{\ominus}_{eVoted} \neq \varnothing$. □

Proof of Proposition 5: If $g \in \mathtt{R}^{\ominus}_{Voted}$ then $\exists(\varphi, g) \in \mathcal{P}^{\ominus}_{K}$ s.t. $K \vdash \varphi$ hence for all $f \in LIT_K$ s.t. $K \vdash f$, if $f \in \varphi$ then $f \wedge \varphi \nvdash \perp$ hence f is not voted, hence $Voted \nvdash \varphi$ i.e., $g \notin \mathtt{R}^{\ominus}_{Voted}$. □

Proof of Corollary 1: From proposition 5 we know that $\mathtt{R}^{\ominus}_{Voted} = \varnothing$, hence the result. □

Proof of Corollary 3: Follows from Corollary 1. □

Proof of Prop 6: Since v and v' have the same knowledge K, and v is pro c it means that all facts that appear in a positive DP or that inhibits a negative DP will be uttered. Symmetrically, v' being con c' means that all facts appearing in a negative DP or inhibiting a positive one will be uttered as well. Hence $Voted = K$. □

Proof of Proposition 7: The proof consists in giving examples of all the possibilities. □

Proof of Proposition 8: Let us proove that $\mathtt{R}^{\oplus}_{K} \subseteq \mathtt{R}^{\oplus}_{Voted}$, if $g \in \mathtt{R}^{\oplus}_{K}$ then it means that $\exists(\varphi, g) \in \mathcal{P}^{\oplus}_{K}$ s.t. (φ, g) is not inhibited in the K-valid BLF. Hence $v1$ has voted every literal in φ and (φ, g) is not inhibited in $Voted$ (since $v2$ has no way to inhibit it). Thus, $g \in \mathtt{R}^{\oplus}_{Voted}$.

Let us proove that $\mathtt{R}^{\ominus}_{Voted} \subseteq \mathtt{R}^{\ominus}_{K}$, if $g \in \mathtt{R}^{\ominus}_{Voted}$ then $\exists(\varphi, g) \in \mathcal{P}_{Voted}o^{\ominus}$ s.t. (φ, g) is not inhibited in the $Voted$-valid BLF. This DP can come from v_2's or v_1's votes since v_2 may have utter facts against c when none of those facts can be used positively, but it can also come from v_1 if these facts belong also to positive DPs. There are two possibilities when (φ, g) is not inhibited in the Voted-valid BLF, either there is no inhibitor in K then $g \in \mathtt{R}^{\ominus}_{K}$ or the inhibitor exists in K but it has not been voted. However, since v_1 has an optimistic strategy and is pro c, v_1 would have voted any fact that appear in an inhibitor of a negative DP, hence this case is impossible. □

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