Duality between Addition and Removal: a Tool for Studying Change in Argumentation*

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Abstract

In this paper, we address a new problem in the field of argumentation theory: the link between two different change operations, namely addition and removal of an argument. We define two concepts of duality reflecting this link. They are used to propose new results about an operation from existing results concerning its dual operation. Finally, the propositions that are obtained are studied for characterizing the change operations.

Keywords: Argumentation, Dynamics in abstract argumentation.

1 Introduction

Mr Pink knows that a given argument could be fatal to Mr White's argumentation, but this argument is lacking. Another way to win could be to remove one of Mr White's arguments (e.g. by doing an objection). However, Mr Pink does not know all the consequences of this removal. The study of the connection between addition and removal of an argument can help Mr Pink to compute these consequences.

Argumentation is becoming a key approach to deal with incomplete and / or contradictory information, especially for reasoning [9, 1]. It can moreover represent dialogues between several agents by modeling the exchange of arguments in, for example, negotiation between agents [2]. Argumentation usually consists of a set of arguments interacting with each other through a relation reflecting conflicts between them, called *attack*. The issue of argumentation is then to determine "acceptable" sets of arguments (*i.e.*, sets able to defend themselves collectively while avoiding internal attacks), called *"extensions"*. An argumentation system (AS) can also be used for identifying the status of each argument according to its membership to the extensions.

Formal frameworks have greatly eased the modeling and study of ASs. In particular, the formal framework of [9] allows to completely abstract the "real" meaning of the arguments and relies only on binary interactions that may exist between them. This approach enables the user to focus on other aspects of argumentation, including its dynamic side. Indeed, in the course of a discussion or during the acquisition of new information, an AS can undergo changes such as the addition of a new argument or the removal of an argument considered as illegal. The study of these changes leads to characterizations, *i.e.*, necessary and sufficient conditions that hold when adding or removing an argument. This study can rely on the links between addition and removal through the concept of duality; this would enable us to directly circumscribe the characterization

*This work was partially supported by the ANR project LELIE on risk analysis and prevention (http://www.irit.fr/recherches/ILPL/lelie/accueil.html). This paper is a draft version, the original article was published In : International Conference on Information Processing and Management of Uncertainty in Knowledge-based Systems (IPMU 2012), Vol. 297, Springer, Communications in Computer and Information Science, pp. 219-229, July 2012.

of removal through the work previously done on addition, and conversely. Although the research on dynamics of ASs is growing [6, 7, 3, 11, 10], the removal of argument has so far been little considered. An attempt to justify the use of removal may nevertheless be found in [4] (exclusively devoted to removal). *A fortiori*, the relationship between addition and removal of argument has not, to our knowledge, been treated so far. In this work, we therefore initiate a theoretical study of this relationship and examine the impact this may have on the analysis of the dynamics of an AS.

The paper is organized as follows: Sect. 2 recalls some key concepts of the theory of abstract argumentation and introduces new definitions relevant to our study. Sect. 3 displays properties of a change operation reflecting possible modifications of an AS. Various notions of duality, and the results of our study are respectively presented in Sect. 4 and 5. Finally, Sect. 6 concludes and suggests perspectives of our work. The proofs are available in a technical report [5].

2 Formal Framework

Before going further into the subject of this article, we should recall some basic background about argumentation systems, the change operations (addition and removal) and some properties already known for the addition operation.

Argumentation systems The work presented in this paper falls within the formal framework of [9]:

Definition 1 (Argumentation System (AS)) *An* argumentation system (AS) *is a pair* $\langle \mathbf{A}, \mathbf{R} \rangle$*, where* $\mathbf{A}$ *is a finite nonempty set of arguments and* $\mathbf{R}$ *is a binary relation on* $\mathbf{A}$*, called* attack relation*. Let* $A, B \in \mathbf{A}$*,* $A\mathbf{R}B$ *means that* A *attacks* B*.* $\langle \mathbf{A}, \mathbf{R} \rangle$ *will be represented by an* argumentation graph $\mathcal{G}$ *whose vertices are the arguments and whose edges correspond to* $\mathbf{R}$[1]*.*

In the remainder of this article, we will need an extended notion of the attack, namely the attack of an argument to a set and vice versa:

Definition 2 (Attack from and to a set) *Let* $A \in \mathbf{A}$ *and* $\mathcal{S} \subseteq \mathbf{A}$*.* $\mathcal{S}$ *attacks* A *iff*[2] $\exists X \in \mathcal{S}$ *such that* $X\mathbf{R}A$*.* A *attacks* $\mathcal{S}$ *iff* $\exists X \in \mathcal{S}$ *such that* $A\mathbf{R}X$*.*

The acceptable sets of arguments ("extensions") are determined according to a given semantics whose definition is usually based on the following concepts:

Definition 3 (Conflict-free set, defense and admissibility) *Let* $A \in \mathbf{A}$ *and* $\mathcal{S} \subseteq \mathbf{A}$*.* $\mathcal{S}$ *is* conflict-free *iff there does not exist* $A, B \in \mathcal{S}$ *such that* $A\mathbf{R}B$*.* $\mathcal{S}$ defends *an argument* A *iff each attacker of* A *is attacked by an argument of* $\mathcal{S}$*. The set of the arguments defended by* $\mathcal{S}$ *is denoted by* $\mathcal{F}(\mathcal{S})$*;* $\mathcal{F}$ *is called the* characteristic function *of* $\langle \mathbf{A}, \mathbf{R} \rangle$*.* $\mathcal{S}$ indirectly defends A *iff* $A \in \bigcup_{i \geq 1} \mathcal{F}^i(\mathcal{S})$*.* $\mathcal{S}$ *is an* admissible *set iff it is conflict-free and it defends all its elements.*

The set of extensions of $\langle \mathbf{A}, \mathbf{R} \rangle$ is denoted by $\mathbf{E}$ (with $\mathcal{E}_1, \ldots, \mathcal{E}_n$ standing for the extensions). For instance, for the *grounded semantics*, one of the most traditional semantics proposed by [9], we have:

Definition 4 (Grounded semantics) *Let* $\mathcal{E} \subseteq \mathbf{A}$*,* $\mathcal{E}$ *is* the only grounded extension *iff* $\mathcal{E}$ *is the least fixed point (with respect to* $\subseteq$*) of* $\mathcal{F}$*.*

The status of an argument is determined by its presence in the extensions of the selected semantics. For example, an argument can be "skeptically accepted" (resp. "credulously") if it belongs to all the extensions (resp. at least to one extension) and be "rejected" if it does not belong to any extension.

[1] In this paper we use freely $\langle \mathbf{A}, \mathbf{R} \rangle$ or $\mathcal{G}$ to refer to an AS. Similarly, if there is no ambiguity, we use without distinction $\mathbf{A}$ and $\mathcal{G}$.

[2] iff = if and only if.

Change operations: addition and removal We rely on the work of [8] which have distinguished four change operations; in this paper, we only use the operations of addition and removal of an argument and its interactions:

Definition 5 (Change operations) *Let $\langle \mathbf{A}, \mathbf{R} \rangle$ be an AS, Z be an argument and $\mathcal{I}_z$ be a set of interactions concerning Z.*

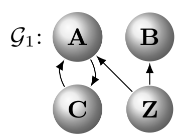
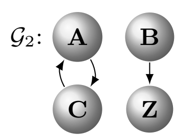
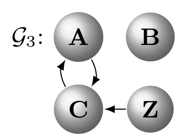
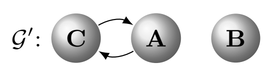
- Adding $Z \notin \mathbf{A}$ *and* $\mathcal{I}_z \nsubseteq \mathbf{R}$ *is a change operation, denoted by* $\oplus$*, providing a new AS such that:* $\langle \mathbf{A}, \mathbf{R} \rangle \oplus (Z, \mathcal{I}_z) = \langle \mathbf{A} \cup \{Z\}, \mathbf{R} \cup \mathcal{I}_z \rangle$.
- Removing $Z \in \mathbf{A}$ *and* $\mathcal{I}_z \subseteq \mathbf{R}$ *is a change operation, denoted by* $\ominus$*, providing a new AS such that:* $\langle \mathbf{A}, \mathbf{R} \rangle \ominus (Z, \mathcal{I}_z) = \langle \mathbf{A} \setminus \{Z\}, \mathbf{R} \setminus \mathcal{I}_z \rangle$.

We denote by $\mathcal{O}$ a change operation ($\oplus$ or $\ominus$). The new AS $\langle \mathbf{A}', \mathbf{R}' \rangle$ obtained by the application of $\mathcal{O}$ will be represented by the argumentation graph $\mathcal{G}' = \mathcal{O}(\mathcal{G})$.[3]

The set of extensions of $\langle \mathbf{A}', \mathbf{R}' \rangle$ is denoted by $\mathbf{E}'$ (with $\mathcal{E}'_1, \ldots, \mathcal{E}'_n$ standing for the extensions). In this work, we will only consider cases where the semantics remains the same before and after a change. Note also that a change operation is a non injective application (thanks to Def. 5, we know that $\forall \mathcal{G},\ \mathcal{G}' = \mathcal{O}(\mathcal{G})$ is unique; however, for a given $\mathcal{G}'$, there may be several $\mathcal{G}$):

Example 1 *With $\mathcal{O} = \ominus$, three systems can be changed into $\mathcal{G}'$, such that $\mathcal{O}(\mathcal{G}_1) = \mathcal{O}(\mathcal{G}_2) = \mathcal{O}(\mathcal{G}_3) = \mathcal{G}'$ (see Table 1 which also gives the grounded extension of each system).*

Table 1: On the non injective nature of the removal operation.

Systems before the removal of Z:			
	$\mathbf{E}_{\mathcal{G}_1} = \{\{C, Z\}\}$	$\mathbf{E}_{\mathcal{G}_2} = \{\{B\}\}$	$\mathbf{E}_{\mathcal{G}_3} = \{\{A, B, Z\}\}$
System after the removal of Z:			$\mathbf{E}'_{\mathcal{G}'} = \{\{B\}\}$

The impact of a change operation will be studied through the notion of change property. A change property $\mathcal{P}$ can be seen as a set of pairs $(\mathcal{G}, \mathcal{G}')$, where $\mathcal{G}$ and $\mathcal{G}'$ are argumentation graphs:

Example 1 (continued) *Let $\mathcal{P}$ be the property defined by "$\mathcal{P}(\mathcal{G}, \mathcal{G}')$ holds iff any extension of $\mathcal{G}'$ is included in at least one extension of $\mathcal{G}$". Thus, $\mathcal{P}(\mathcal{G}_1, \mathcal{G}')$ does not hold while $\mathcal{P}(\mathcal{G}_2, \mathcal{G}')$ and $\mathcal{P}(\mathcal{G}_3, \mathcal{G}')$ hold.*

Definition 6 (Operation satisfying a property) *A change operation $\mathcal{O}$* satisfies *a property $\mathcal{P}$ iff $\forall \mathcal{G},\ \mathcal{P}(\mathcal{G}, \mathcal{O}(\mathcal{G}))$ holds.*

Example 1 (continued) *$\mathcal{P}(\mathcal{G}_1, \mathcal{G}')$ does not hold. Thus, $\mathcal{O} = \ominus$ does not satisfy $\mathcal{P}$.*

[3]
- We assume that Z does not attack itself and $\forall (X, Y) \in \mathcal{I}_z$, we have either ($X = Z$ and $Y \neq Z$, $Y \in \mathbf{A}$) or ($Y = Z$ and $X \neq Z$, $X \in \mathbf{A}$).
- In case of removing, $\mathcal{I}_z$ is the set of all the interactions concerning Z in $\langle \mathbf{A}, \mathbf{R} \rangle$.
- The symbols $\oplus$ and $\ominus$ used here correspond to the symbols $\oplus^a_I$ and $\ominus^a_I$ of [8], where a stands for "argument" and I for "interactions", meaning that the operation concerns an argument and its interactions.

Some propositions about addition The following propositions list the main results obtained for characterizing the operation of addition under the grounded semantics. Prop. 1 is directly taken from [8]:

Proposition 1 *When adding an argument Z under the grounded semantics,*

1. *(Prop. 7) If $X \in \mathcal{E}$ and Z does not indirectly attack X, then $X \in \mathcal{E}'$.*
2. *(Prop. 9, item 3) If $\mathcal{E} = \varnothing$ and Z is attacked by $\mathcal{G}$, then $\mathcal{E}' = \varnothing$.*
3. *(Prop. 9, item 4) If $\mathcal{E} = \varnothing$ and Z is not attacked by $\mathcal{G}$, then $\mathcal{E}' = \{Z\} \cup \bigcup_{i \geq 1} \mathcal{F}'^i(\{Z\})$.*

Prop. 2 is a generalization of some propositions given in [8] (the condition $\mathcal{E} \neq \varnothing$ can be removed from the initial propositions – see proof in [5]):

Proposition 2 *When adding an argument Z under the grounded semantics,*

1. *(Prop. 10) If Z does not attack $\mathcal{E}$, then $\mathcal{E} \subseteq \mathcal{E}'$.*
2. *(Prop. 11, item 1) If Z does not attack $\mathcal{E}$ and $\mathcal{E}$ does not defend Z, then $\mathcal{E}' = \mathcal{E}$.*
3. *(Prop. 11, item 2 part 1) If Z does not attack $\mathcal{E}$ and $\mathcal{E}$ defends Z, then $\mathcal{E}' = \mathcal{E} \cup \{Z\} \cup \bigcup_{i \geq 1} \mathcal{F}'^i(\{Z\})$.*
4. *(Prop. 11, item 2 part 2) If Z does not attack $\mathcal{G}$ and $\mathcal{E}$ defends Z, then $\mathcal{E}' = \mathcal{E} \cup \{Z\}$.*
5. *(Prop. 13) If Z attacks each unattacked argument of $\mathcal{G}$ and Z is attacked by $\mathcal{G}$, then $\mathcal{E}' = \varnothing$.*

Prop. 3 is a new proposition about the conservation of the "rejected" status of an argument (see proof in [5]):

Proposition 3 *When adding an argument Z under the grounded semantics, $\forall X \in \mathcal{G}$, if $X \notin \mathcal{E}$ and Z does not indirectly defend X, then $X \notin \mathcal{E}'$.*

3 Some Change Properties

Change properties express structural modifications of an AS that are caused by a change operation. They are defined in order to obtain a clear and accurate classification. So, a new partition, inspired by the work of [8] and based on three possible cases of evolution of the set of extensions, has been defined:

- the *extensive* case, in which the number of extensions increases,
- the *restrictive* case, in which the number of extensions decreases,
- the *constant* case, in which the number of extensions remains the same.

In this article, due to space limitations, we only address the *constant* case[4]. For the sake of clarity, a change satisfying a property $\mathcal{P}$ is called a "$\mathcal{P}$ *change*"; for example, a change satisfying the *constant* property is called a *constant* change.

Definition 7 (Constant change) *The change from $\mathcal{G}$ to $\mathcal{G}'$ is* constant *iff $|\mathbf{E}| = |\mathbf{E}'|$.*

Restricting our scope to the *constant* case allows us to focus on other criteria than the number of extensions of $\mathcal{G}$ and $\mathcal{G}'$, namely inclusions between the various possible extensions ($\mathcal{G}$ to $\mathcal{G}'$ and vice versa), emptiness of these extensions, etc. Here are the definitions of these various sub-cases[5]:

[4]For the same reason, we do not address properties related to the status of a specific argument.

[5]Note that the names of these sub-cases are prefixed with the letter c to highlight the fact that they follow from the *constant* property.

Definition 8 *The change from $\mathcal{G}$ to $\mathcal{G}'$ is:*

1. **c-conservative** *iff* $\mathbf{E} = \mathbf{E}'$.
2. **c-decisive** *iff* $\mathbf{E} = \{\{\}\}$ *and* $\mathbf{E}' = \{\mathcal{E}'\}$, *with* $\mathcal{E}' \neq \varnothing$.
3. **c-destructive** *iff* $\mathbf{E} = \{\mathcal{E}\}$, *with* $\mathcal{E} \neq \varnothing$ *and* $\mathbf{E}' = \{\{\}\}$.
4. **c-expansive** *iff* $\mathbf{E} \neq \varnothing$, $|\mathbf{E}| = |\mathbf{E}'|$, $\forall \mathcal{E}_i \in \mathbf{E}, \exists \mathcal{E}'_j \in \mathbf{E}', \varnothing \neq \mathcal{E}_i \subset \mathcal{E}'_j$ *and* $\forall \mathcal{E}'_j \in \mathbf{E}', \exists \mathcal{E}_i \in \mathbf{E}, \varnothing \neq \mathcal{E}_i \subset \mathcal{E}'_j$.
5. **c-narrowing** *iff* $\mathbf{E} \neq \varnothing$, $|\mathbf{E}| = |\mathbf{E}'|$, $\forall \mathcal{E}_i \in \mathbf{E}, \exists \mathcal{E}'_j \in \mathbf{E}', \varnothing \neq \mathcal{E}'_j \subset \mathcal{E}_i$ *and* $\forall \mathcal{E}'_j \in \mathbf{E}', \exists \mathcal{E}_i \in \mathbf{E}, \varnothing \neq \mathcal{E}'_j \subset \mathcal{E}_i$.
6. **c-altering** *iff* $|\mathbf{E}| = |\mathbf{E}'|$ *and it is neither* c-conservative*, nor* c-decisive*, nor* c-destructive*, nor* c-expansive*, nor* c-narrowing.

Definitions 8.1, 8.2, 8.3 and 8.6 are fairly straightforward. Def. 8.4 states that a *c-expansive* change is a change where all the extensions of $\mathcal{G}$, which are not initially empty, are increased by some arguments. A *c-narrowing* change, according to Def. 8.5, is a change where all the extensions of $\mathcal{G}$ are reduced by some arguments without becoming empty.

4 Usefulness of Duality

Two definitions of duality As far as we know, the problem of removing an argument and, *a fortiori*, the link between addition and removal of an argument have been little discussed. However, it can be worthy to use the links between these operations in order to study the properties characterizing the changes that may impact an AS. For that purpose, the notion of duality seems pertinent. We focus on two concepts of duality: first, a duality at the level of change operations, *based on the notion of inverse*, expressing the opposite nature of two operations, then a duality at the level of change properties, *based on the notion of symmetry*, conveying a correspondence between two properties.

Definition 9 (Duality based on the notion of inverse) *Two change operations $\mathcal{O}$ and $\mathcal{O}'$ are the* inverse *of each other iff: "$\forall \mathcal{G}, \forall \mathcal{G}'$, $\mathcal{O}(\mathcal{G}) = \mathcal{G}'$ iff $\mathcal{O}'(\mathcal{G}') = \mathcal{G}$".*

Obviously, following Def. 9, it is clear that the operations of addition and removal of an argument defined in Sect. 2 are the inverse of each other.

Definition 10 (Duality based on the notion of symmetry) *Two properties $\mathcal{P}$ and $\mathcal{P}'$ are* symmetric *iff: "$\forall \mathcal{G}$, $\forall \mathcal{G}'$, $\mathcal{P}'(\mathcal{G}', \mathcal{G})$ holds iff $\mathcal{P}(\mathcal{G}, \mathcal{G}')$ holds".*

From these definitions, we can draw a condition for the satisfaction of a property by a change operation:

Proposition 4 *Let $\mathcal{O}$ and $\mathcal{O}'$ two inverse change operations and $\mathcal{P}$ and $\mathcal{P}'$ two symmetric properties. $\mathcal{O}$ satisfies $\mathcal{P}$ iff $\mathcal{O}'$ satisfies $\mathcal{P}'$.*

Both concepts of duality can be used for linking the change properties:

Proposition 5
A change is constant *iff the inverse change is* constant *as well.*
A change is c-destructive *iff the inverse change is* c-decisive.
A change is c-conservative *iff the inverse change is* c-conservative *as well.*
A change is c-narrowing *iff the inverse change is* c-expansive.
A change is c-altering *iff the inverse change is* c-altering *as well.*

Figure 1 graphically summarizes the above results (proofs are in [5]).

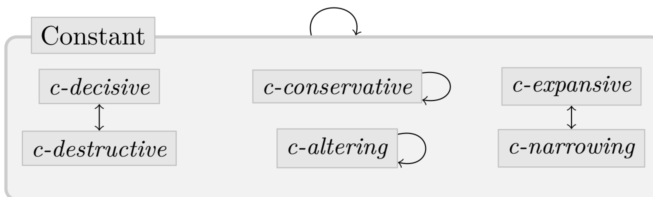


Figure 1: Presentation of the duality of the *constant* property and its sub-cases.

Methodology for Using Duality This part describes how to use duality in order to obtain new propositions for the operation of removal, starting from propositions relating to addition[6]. Note first that we restrict our study to the grounded semantics (see Def. 4). Let us describe this methodology using Prop. 3. Let us first proceed to a renaming in order to clarify the presentation. The graphs and the extensions are going to be indexed by two capital letters - IA, OA, IR and OR - representing respectively the **I**nput system for the **A**ddition, the **O**utput system for the **A**ddition, the **I**nput system for the **R**emoval and the **O**utput system for the **R**emoval. Thus, Prop. 3 can be rewritten as follows:

Proposition 1 (3.1) *When adding an argument Z under the grounded semantics, if $X \notin \mathcal{E}_{IA}$ and Z does not indirectly defend X, then $X \notin \mathcal{E}_{OA}$.*

Let $\mathcal{P}$ be a property and $\mathcal{P}^{-1}$ its symmetric. Thanks to Prop. 4, we can write: $\oplus$ *satisfies* $\mathcal{P}$ *iff* $\ominus$ *satisfies* $\mathcal{P}^{-1}$.

And thanks to Def. 6, we know that a change operation $\mathcal{O}$ satisfies a property $\mathcal{P}$ if and only if $\forall \mathcal{G}$, it holds that $\mathcal{P}(\mathcal{G}, \mathcal{O}(\mathcal{G}))$. So we can write: $\forall \mathcal{G}_{IA}$, $\mathcal{P}(\mathcal{G}_{IA}, \oplus(\mathcal{G}_{IA}))$ *holds iff* $\forall \mathcal{G}_{IR}$, $\mathcal{P}^{-1}(\mathcal{G}_{IR}, \ominus(\mathcal{G}_{IR}))$ *holds.*

Moreover, thanks to Def. 10, we have: $\forall \mathcal{G}_{IR}$, $\mathcal{P}^{-1}(\mathcal{G}_{IR}, \ominus(\mathcal{G}_{IR}))$ *holds iff* $\mathcal{P}(\ominus(\mathcal{G}_{IR}), \mathcal{G}_{IR})$ *holds.*

And so, we have: $\forall \mathcal{G}_{IA}$, $\forall \mathcal{G}_{IR}$, $\mathcal{P}(\mathcal{G}_{IA}, \oplus(\mathcal{G}_{IA}))$ *holds iff* $\mathcal{P}(\ominus(\mathcal{G}_{IR}), \mathcal{G}_{IR})$ *holds.*

Let $\mathcal{G}_{OA} = \oplus(\mathcal{G}_{IA})$ and $\mathcal{G}_{OR} = \ominus(\mathcal{G}_{IR})$. Since we know that Property $\mathcal{P}$ holds for the operation of addition, we can rewrite it for the operation of removal:

Proposition 2 (3.2) *When removing an argument Z under the grounded semantics, if $X \notin \mathcal{E}_{OR}$ and Z does not indirectly defend X, then $X \notin \mathcal{E}_{IR}$.*

Which is equivalent to:

Proposition 3 (3.3) *When removing an argument Z under the grounded semantics, if $X \in \mathcal{E}_{IR}$ and Z does not indirectly defend X, then $X \in \mathcal{E}_{OR}$.*

Thus, for the operation of removal, we obtain a proposition analogous to Prop. 3 denoted by Prop. $3^{\ominus}$; in the remainder of this article, the exponent ($^{\oplus}$ or $^{\ominus}$) will represent the correspondence between a proposition and the one obtained by applying the duality methodology:

Proposition 4 ($3^{\ominus}$) *When removing an argument Z under the grounded semantics, if $X \in \mathcal{E}$ and Z does not indirectly defend X, then $X \in \mathcal{E}'$.*

Using the methodology presented here, in the next section, we show how the propositions of [8] summarized by Prop. 1 and Prop. 2 can also be translated.

5 Propositions Obtained by Duality

This section is divided into two parts: the first part concerns the propositions on which the application of the methodology is directly meaningful, and the second part concerns those that should be transformed in order to make sense.

[6]This methodology can also be used the other way round from removal to addition.

Straightforward Application Here, we deal with the propositions on which our methodology gives analogous propositions that can be used directly.

From Prop. 1.1, we obtain a proposition that gives a sufficient condition for the conservation of the rejection of an argument X when Z is removed:

Proposition 5 ($\mathbf{1.1}^{\ominus}$) *When removing an argument Z under the grounded semantics, if $X \notin \mathcal{E}$ and Z does not indirectly attack X, then $X \notin \mathcal{E}'$.*

Prop. 1.2 and 1.3 give sufficient conditions for a non *c-destructive* change:

Proposition 6 ($\mathbf{1.2}^{\ominus}$) *When removing an argument Z under the grounded semantics, if $\mathcal{E} \neq \varnothing$ and Z is attacked by $\mathcal{G}$, then $\mathcal{E}' \neq \varnothing$.*

Proposition 7 ($\mathbf{1.3}^{\ominus}$) *When removing an argument Z under the grounded semantics, if $\mathcal{E} \neq \{Z\} \cup \bigcup_{i \geq 1} \mathcal{F}^i(\{Z\})$ and Z is not attacked by $\mathcal{G}$, then $\mathcal{E}' \neq \varnothing$.*

Prop. 1.2 and 1.3 also give a necessary condition for a *c-destructive* change:

Corollary 1 *When removing an argument Z under the grounded semantics, if the change is* c-destructive*, then Z is not attacked by $\mathcal{G}$ and $\mathcal{E} = \{Z\} \cup \bigcup_{i \geq 1} \mathcal{F}^i(\{Z\})$.*

From Prop. 2.5, we obtain a proposition that characterizes a change either *c-conservative* or *c-decisive*:

Proposition 8 ($\mathbf{2.5}^{\ominus}$) *When removing an argument Z under the grounded semantics, if Z attacks each unattacked argument of $\mathcal{G} \setminus \{Z\}$ and Z is attacked by $\mathcal{G} \setminus \{Z\}$, then $\mathcal{E} = \varnothing$.*

Not So Straightforward Application Now, we deal with propositions requiring additional work to be usable; for example when they use a condition on the output system, we must consider what it means on the input system.

From Prop. 2.1, we obtain:

Proposition 9 ($\mathbf{2.1}^{\ominus}$) *When removing an argument Z under the grounded semantics, if Z does not attack $\mathcal{E}'$ in $\mathcal{G}$, then $\mathcal{E}' \subseteq \mathcal{E}$.*

This proposition uses a condition on the output AS (*Z does not attack $\mathcal{E}'$*). Lem. 1 expresses the meaning of this condition for the input AS in the case of a removal. For this lemma, we need a new notation:

Notation 1 *Let $\mathcal{U} \subseteq \mathcal{G}$, $\mathcal{U}$ is the set of unattacked arguments in $\mathcal{G} \setminus \{Z\}$.*

Informally, Lem. 1 means that if an argument X is attacked by Z, X is also attacked by another argument $Y \neq Z$ which prevents X to belong to the grounded extension $\mathcal{E}'$:

Lemma 1 *When removing an argument Z under the grounded semantics, Z does not attack $\mathcal{E}'$ in $\mathcal{G}$ iff $\forall X \in \mathcal{G}'$, if Z attacks X then (X is attacked by $\mathcal{G} \setminus \{Z\}$ and X is not indirectly defended by $\mathcal{U}$ in $\mathcal{G} \setminus \{Z\}$).*

Using Lem. 1, we can rewrite the proposition, which gives us a sufficient condition for the fact that no argument non accepted before the change is accepted after. Hence, the change is either *c-conservative*, *c-destructive* or *c-narrowing*:

Proposition 10 ($\mathbf{2.1}^{\ominus}$ (v2)) *When removing an argument Z under the grounded semantics, if $\forall X \in \mathcal{G}$, if Z attacks X then (X is attacked by $\mathcal{G} \setminus \{Z\}$ and X is not indirectly defended by $\mathcal{U}$ in $\mathcal{G} \setminus \{Z\}$), then $\mathcal{E}' \subseteq \mathcal{E}$.*

From Prop. 2.2, we obtain:

Proposition 11 (2.2$^\ominus$) *When removing an argument Z under the grounded semantics, if Z does not attack $\mathcal{E}'$ in $\mathcal{G}$ and $\mathcal{E}'$ does not defend Z in $\mathcal{G}$, then $\mathcal{E} = \mathcal{E}'$.*

Similarly, we need to express the condition *$\mathcal{E}'$ does not defend Z in $\mathcal{G}$* by a condition for the input AS. Such a condition is given by Lem. 2:

Lemma 2 *When removing an argument Z under the grounded semantics, if Z does not attack $\mathcal{E}'$, then $Z \in \bigcup_{i \geq 1} \mathcal{F}^i(\mathcal{U})$ iff $\mathcal{E}'$ defends Z in $\mathcal{G}$.*

Thanks to Lem. 1 and 2, we can rewrite the proposition, which gives us a sufficient condition for the *c-conservative* change:

Proposition 12 (2.2$^\ominus$ (v2)) *When removing an argument Z under the grounded semantics, if (1) $\forall X \in \mathcal{G}$, if Z attacks X then (X is attacked by $\mathcal{G} \setminus \{Z\}$ and X is not indirectly defended by $\mathcal{U}$ in $\mathcal{G} \setminus \{Z\}$) and (2) $Z \notin \bigcup_{i \geq 1} \mathcal{F}^i(\mathcal{U})$, then $\mathcal{E} = \mathcal{E}'$.*

From Prop. 2.3, we obtain:

Proposition 13 (2.3$^\ominus$) *When removing an argument Z under the grounded semantics, if Z does not attack $\mathcal{E}'$ in $\mathcal{G}$ and $\mathcal{E}'$ defends Z in $\mathcal{G}$, then $\mathcal{E} = \mathcal{E}' \cup \{Z\} \cup \bigcup_{i \geq 1} \mathcal{F}^i(\{Z\})$.*

Let $N_Z = (\bigcup_{i \geq 1} \mathcal{F}^i(\{Z\}) \cup \{Z\}) \setminus \mathcal{E}'$; thus, we have $N_Z \subseteq \mathcal{E}$. N_Z contains Z and the arguments of $\mathcal{G} \setminus \{Z\}$ which could not be defended without using Z. In other words, if $X \neq Z$, $X \in N_Z$ if and only if Z is required for proving that $X \in \mathcal{E}$. Obviously, the pair $(\mathcal{E}', N_Z)$ constitutes a partition of $\mathcal{E}$. So, $\mathcal{E}' = \mathcal{E} \setminus N_Z$. Thanks to Lem. 1 and 2, we can rewrite the proposition, which gives us a sufficient condition for the *c-narrowing* change:

Proposition 14 (2.3$^\ominus$ (v2)) *When removing an argument Z under the grounded semantics, if (1) $\forall X \in \mathcal{G}$, if Z attacks X then (X is attacked by $\mathcal{G} \setminus \{Z\}$ and X is not indirectly defended by $\mathcal{U}$ in $\mathcal{G} \setminus \{Z\}$) and (2) $Z \in \bigcup_{i \geq 1} \mathcal{F}^i(\mathcal{U})$, then $\mathcal{E}' = \mathcal{E} \setminus N_Z$.*

From Prop. 2.4, we obtain:

Proposition 15 (2.4$^\ominus$) *When removing an argument Z under the grounded semantics, if Z does not attack $\mathcal{G}'$ and $\mathcal{E}'$ defends Z in $\mathcal{G}$, then $\mathcal{E} = \mathcal{E}' \cup \{Z\}$.*

Thanks to Lem. 2, we can rewrite the proposition, which also gives us a sufficient condition for the *c-narrowing* change:

Proposition 16 (2.4$^\ominus$ (v2)) *When removing an argument Z under the grounded semantics, if (1) $Z \in \bigcup_{i \geq 1} \mathcal{F}^i(\mathcal{U})$ and (2) Z does not attack $\mathcal{G} \setminus \{Z\}$, then $Z \in \mathcal{E}$ and $\mathcal{E}' = \mathcal{E} \setminus \{Z\}$.*

6 Discussion and Conclusion

In this paper, we have studied the link between addition and removal of an argument. To this end, we first took up and refined the change properties of [8] into a clear partition for a special case (when the cardinality of the set of extensions remains unchanged). We then defined two notions of duality, namely the *duality based on the notion of inverse* and the *duality based on the notion of symmetry*, in order to connect these change properties and change operations. This allowed us to discover propositions for an operation (the removal operation) thanks to the propositions already known for its dual operation (the addition operation) – see Table 2. Despite the interest of such a methodology allowing to get new propositions for an operation in a easy way, a “post-processing” is sometimes necessary in order to ensure that the result makes sense.

Let us come back to the example presented in the introduction. For Mr Pink, adding a new argument attacking a specific argument of Mr White without threatening his own accepted arguments corresponds to Prop. 1.1. Moreover, Prop. 3$^\ominus$ ensures him that the removal of his opponent’s argument achieves the same result if this argument is not giving assistance to any of his own accepted arguments. Thereby, instead of using Prop. 1.1, Mr Pink can benefit from Prop. 3$^\ominus$ thanks to our methodology.

Table 2: Synthesis of the necessary and sufficient conditions for *constant* properties under the grounded semantics.

Propositions	**Change properties**
Prop. 1.2$^{\ominus}$	CS for non *c-destructive*
Prop. 1.3$^{\ominus}$	CS for non *c-destructive*
Coro. 1	CN for *c-destructive*
Prop. 2.1$^{\ominus}$	CS for • *c-conservative* **or** • *c-destructive* **or** • *c-narrowing*

Propositions	**Change properties**
Prop. 2.2$^{\ominus}$	CS for *c-conservative*
Prop. 2.3$^{\ominus}$	CS for *c-narrowing*
Prop. 2.4$^{\ominus}$	CS for *c-narrowing*
Prop. 2.5$^{\ominus}$	CS for • *c-conservative* **or** • *c-decisive*

Some issues seem important for future works:

- In this paper, we have focused on the grounded semantics and studied only two of the four operations of [8]. A first issue is to extend our work to the two missing operations, addition and removal of an interaction, and also to other semantics.
- Moreover, we could consider the addition or removal of a set of arguments. These special operations may be seen as a sequence of change operations and their study seems essential in order to approach minimal change problems.
- Due to lack of space, we have outlined here a small subset of the possible change properties. It would be interesting to study and evaluate all the remaining properties through the duality methodology.
- The obligation to perform a post-processing for some of the propositions obtained by our approach is a sore point. We should find new ways to avoid this post-processing or at least find criteria for identifying the propositions that would require transformations.

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