Re-examining the Foundations of Voting Theory

Hannu Nurmi

Department of Philosophy, Contemporary History and Political Science
University of Turku

TeSS, June 27 – July 4, 2017

Issues touched upon in this presentation

- collective choices are essentially procedure-dependent
- choice rules have reasons
- classic voting paradoxes and many theorems demonstrate value incompatibilities
- are some values more important than others?
- how often will we encounter problems in preference aggregation?
- are the assumptions underlying the social choice theory plausible?
- is internet voting a way to alleviate democratic deficit?
- how should the choice setting be taken into account in choosing the voting rules?
- what lessons can we draw from social choice theory to the design of real world voting procedures?

Same opinions, different outcomes

4 voters	3 voters	2 voters
A	E	D
B	D	C
C	B	B
D	C	E
E	A	A

Table: 5 candidates, 5 winners

plurality voting: A; plurality runoff voting: E, Condorcet extensions: D, Borda count: B; approval voting (with additional assumptions): C.

This holds for Condorcet extensions as well

. . . assuming, of course, that there is no Condorcet winner in the profile under scrutiny.

10 voters	7 voters	1 voter	7 voters	4 voters
D	B	B	C	D
A	C	A	A	C
B	A	C	B	A
C	D	D	D	B

Table: Discrepancy among some Condorcet extensions

Copeland: A, B, C
Dodgson: D
Max-min: D

Discrepancy among positional procedures

2	2	2	3
A	A	C	D
B	D	B	B
C	C	D	C
D	B	A	A

Here the plurality winner is A, vote-for-two winner is B, vote-for-three winner is C, and the Borda winner D.

Theorem

Saari 1992. Consider the alternative set $c_1, \ldots, c_K$ of at least three elements. Then such a profile exists that alternative c_j wins when the voting rule is vote-for-j and this holds for $j = 1, \ldots, K-1$. Moreover, c_K is the Borda winner.

Does unanimity guarantee the same outcome?

No.

1 voter	1 voter	1 voter
A	A	A
B	B	B
C	C	C

Table: A unanimous profile

If vote-for-two system is used or approval voting with everyone approving of their two highest ranked alternatives, the outcome is an A-B tie, not A.

Reasons for rules

- The plurality rule has a straight-forward rationale: if only one alternative is to be chosen, it makes sense to ask each voter for his/her most preferred alternative. Whichever alternative is reported as the most preferred by more voters than any other alternative is then regarded as the social choice. Choosing any other alternative could be criticized by pointing out that the plurality winner was viewed the best by more voters than the chosen alternative.
- In some contexts the support of the majority is deemed important for legitimacy. This consideration seems to underlie the plurality runoff system which is resorted to in many elections of the head of state around the world.

Reasons, cont'd

- The motivation for electing the Condorcet winner when one exists is obvious: if an alternative defeats each of its competitors in pairwise majority comparisons, it deserves to be elected. Things get more complicated when no Condorcet winner exists. Various Condorcet extensions address this problem in different ways. Copeland's rule takes its cue in the notion of pairwise majority winning. Since the Condorcet winner defeats the largest possible $(k-1)$ other alternatives in pairwise majority comparisons, it seem plausible to use this fact in devising a system that elects the alternative that defeats more other alternatives than any of its contestants in pairwise majority comparisons.

Reasons, cont'd

- Dodgson's rule performs the smallest number of modifications (pairwise preference switches) in the observed preference profile needed to make any given alternative the Condorcet winner.
- The max-min procedure can be justified by considering the performance of each alternative when compared with its toughest competitor. If one wishes to choose the alternative that does best in this comparison, the max-min rule is an appropriate method for this purpose.

Reasons, cont'd

- The Borda count can be given two justifications. Firstly, this procedure results in the alternative that has the highest average ranking in the individual preference rankings. Secondly, by tallying the number of voters supporting a given alternative in all its $(k-1)$ pairwise comparisons one ends up with its Borda score. So, the Borda count can find its justification in the fact that the Borda winner receives the maximum number of votes when these are summed over all $k-1$ pairwise comparisons where it is present.

Arrow's theorem

Theorem

Arrow'(1963): *No social welfare function satisfies the following conditions:*

1. *unrestricted domain (U)*
2. *independence of irrelevant alternatives (IIA)*
3. *Pareto (P)*
4. *non-dictatorship (D)*

Remark: social welfare functions assigns to each n-tuple of connected and transitive individual preference relations a (collective) connected and transitive preference relation.

Gibbard-Satterthwaite theorem

Definition

A social choice function is manipulable (by individuals) iff there is a situation and an individual so that the latter can bring about a preferable outcome by preference misrepresentation than by truthful revelation of his/her preference ranking, *ceteris paribus*.

Definition

A social choice function is non-trivial (non- degenerate) iff for each alternative x, there is a preference profile so that x is chosen.

Gibbard-Satterthwaite theorem, cont'd

Theorem

*(*Gibbard-Satterthwaite 1973-75). Every universal and non-trivial resolute social choice function is either manipulable or dictatorial.

Gärdenfors' theorem

Theorem

Gärdenfors 1976. *If a social choice function is anonymous and neutral and satisfies the Condorcet winning criterion, then it is manipulable.*

Examples of non-manipulable SCF's:

- If every voter's preference ranking is strict (no ties), then SCF that chooses the Condorcet winner when one exists and all alternatives, otherwise, is non-manipulable.
- Under the same assumption concerning voter preferences any SCF that chooses the Condorcet winner when one exists and the set of Pareto-undominated outcomes, otherwise, is also non-manipulable.

Young and Moulin

Theorem

Young: *all consistent methods are incompatible with the Condorcet winning criterion.*

Theorem

Moulin: *all procedures that satisfy the Condorcet winning criterion are vulnerable to no-show paradox.*

Theorem

Muller and Satterthwaite: *if there are at least three alternatives, then any procedure that satisfies citizen's sovereignty and (Maskin) monotonicity is dictatorial.*

Some systems and their properties

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
Amendment	1	1	1	1	0	0	0	0	0
Copeland	1	1	1	1	1	0	0	0	0
Dodgson	1	0	1	0	1	0	0	0	0
Maximin	1	0	1	1	1	0	0	0	0
Kemeny	1	1	1	1	1	0	0	0	0
Plurality	0	0	1	1	1	1	0	0	1
Borda	0	1	0	1	1	1	0	0	1
Approval	0	0	0	1	0	1	1	0	1
Black	1	1	1	1	1	0	0	0	0
Pl. runoff	0	1	1	0	1	0	0	0	0
Nanson	1	1	1	0	1	0	0	0	0
Hare	0	1	1	0	1	0	0	0	0

The selected criteria

a the Condorcet winner criterion
b the Condorcet loser criterion
c the strong Condorcet criterion
d monotonicity
e Pareto
f consistency
g Chernoff property
h independence of irrelevant alternatives
i invulnerability to the no-show paradox

Cyclic preferences may make sense

Example

Three universities A, B and C are being compared along three criteria: (i) research output (scholarly publications), (ii) teaching output (degrees), (iii) external impact (expert assignments, media visibility, R& D projects, etc.)

crit. (i)	crit. (ii)	crit. (iii)
A	B	C
B	C	A
C	A	B

Cycle: $A \succ B \succ C \succ A \succ \ldots$.

Ostrogorski's paradox

Example

issue	*issue 1*	*issue 2*	*issue 3*	*the criterion chooses*
crit. A	X	X	Y	X
crit. B	X	Y	X	X
crit. C	Y	X	X	X
crit. D	Y	Y	Y	Y
crit. E	Y	Y	Y	Y
issue-wise choice	Y	Y	Y	*overall choice* ?

Reinterpretation

- criterion A: relevant educational background
- criterion B: political experience
- criterion C: negotiation skills
- criterion D: substance expertise
- criterion E: relevant political connections

Suppose that the criterion-wise preference is formed on the basis of which alternative is better on more issues than the other. If all issues and criteria are deemed importance, the decision of which candidate the individual should vote is ambiguous: the row-column aggregation with the majority principle suggests X, but the column-row aggregation with the same principle yields Y.

Anscombe's paradox and criterial relevance

Example

issue	*issue 1*	*issue 2*	*issue 3*
crit. 1	Y	Y	X
crit. 2	X	X	X
crit. 3	X	Y	Y
crit. 4	Y	X	Y
crit. 5	Y	X	Y

MCDM interpretation: a majority of criteria – here 1, 2 ,3 – may suggest a choice that is not "elected" on a majority of issues. Should these criteria suggest the same alternative, it would necessarily be chosen.

. . . and so on

- rankings over risky prospects (Tversky and Kahneman)
- “holistic” effects (compromise, asymmetric domination, framing)
- preference reversal

Monotonicity of Dodgson's rule

42 voters	*26 voters*	*21 voters*	*11 voters*
b	a	e	e
a	e	d	a
c	c	b	b
d	b	a	d
e	d	c	c

Alternative a needs 14 binary preference reversals to become the Condorcet winner, other alternatives need more. Hence a wins. Now, suppose that the 11 right-most voters increase the support of a by ranking it first, *ceteris paribus*. After the change, b is immediately below e in the 11-voter ranking and b needs only 9 binary preference changes to become the Condorcet winner, while a still needs 14. Therefore, the new winner is b.

Upshot

As a ranking rule the Dodgson function is nonmonotonic (Fishburn).
As a tournament aggregation rule it is monotonic.

Consistency of Kemeny's rule

As a preference function Kemeny's rule is consistent (Young and Levenglick 1978), but as a choice rule it isn't (Fishburn 1977). Choice functions map preference profiles into subsets of alternatives. Denoting by Φ the set of all preference profiles and by A the set of alternatives, we thus have

$$f : \Phi \longrightarrow 2^A$$

for social choice functions.

Kemeny, cont'd

Preference functions, in contradistinction, map preference profiles into rankings over alternatives (cf. social welfare functions). I.e.

$$F : \Phi \rightarrow \mathcal{R}$$

where $\mathcal{R}$ denotes the set of all preference rankings over A.

Kemeny cont'd

Consider now a partition of a set N of individuals with preference profile ϕ into two separate sets of individuals N_1 and N_2 with corresponding profiles ϕ_1 and ϕ_2 over A and assume that $f(\phi_1 \cap \phi_2) \neq \emptyset$. The social choice function f is consistent iff $f(\phi_1 \cap \phi_2) = f(\phi)$, for all partitionings of the set of individuals.
The same definition can be applied to social preference functions. F is consistent iff whenever $F(\phi_1) \cap F(\phi_2) \neq \emptyset$ implies that $F(\phi_1) \cap F(\phi_2) = F(\phi)$.

It turns out that, like all Condorcet extensions, Kemeny's rule is an inconsistent social choice function. An example is provided by Fishburn (1977, 484). However, as a preference function it is consistent.

Agenda control

- referenda can be seen as obvious ways of minimizing the democratic deficit
- while the number of referenda can be essentially increased with internet voting technologies, representative forms will undoubtedly remain the main mechanism
- the main problems with direct forms of democracy pertain to agenda control
- deliberative mechanisms may be useful in setting the agenda of referenda (possibly through profile restrictions)

The limits of agenda-control?

Plott and Levine (1978):

> *Experimental results indicate that within a range of circumstances the agenda can indeed be used to influence the outcome of a committee decision.*

Saari (2001):

> *For a price, I will come to your organization to design your election procedure. You tell me who you want to win. After talking with the members of your organization to ascertain their preferences, I will construct a 'democratic voting procedure' which will ensure the victory of your candidate.*

Some formal results: McKelvey

Theorem

McKelvey (1976, 1979). Suppose that the voter ideal points are located in the $\mathcal{R}^m$ and that the preferences have continuous utility representations. Suppose moreover that the core is empty. Take now any two points x and y. Then an agenda $a_1, \ldots, a_k$ can be built so that for all $s = 2, \ldots, k$, the following holds: $a_s P a_{s-1}$, $a_1 = x$, and $a_k = y$.

Here P denotes the majority preference relation. I.e. in the absence of the core, any point in $\mathcal{R}^m$ can be made the majority voting winner. N.B. this does not exclude outcomes outside the Pareto frontier.

Banks

Given a set of alternatives $a_1, \ldots, a_k$, a chain from a_i consists of all sequences of alternatives such that each alternative in the sequence defeats all the previous ones. The last alternative in the sequence is the end point of the (Banks) chain emanating from a_i.

Theorem

Banks (1985). Sophisticated voting outcomes are can be found as the set of end points of all (Banks) chains. This is always a subset of the Pareto set and of the uncovered set.

Marengo and Settepanella

Choices are made from bundles of elements called *features*, i.e. $F = \{f_1, \ldots, f_n\}$. Each feature may take one value out of a finite set of alternatives. If all features may take $m+1$ values, there are $(m+1)^n$ social outcomes, i.e. $n-$ tuples of feature values. An object scheme is a bundling of features into subsets (not necessarily distinct).

Example

How to spend an evening? Features: where to go, when to go, how to go. Values: $\{restaurant, cinema\}$, $\{7PM, 8PM\}$, $\{car, walk\}$. One outcome would be $(cinema, 7PM, car)$.

It is always possible to manipulate the the objects scheme in such a way that the median voter theorem does not apply and the social choice may converge to social outcomes very distant from the median voter's preferred one.

Internet elections

- cryptographic protocols can be used to achieve goals that are not achievable under traditional paper ballot elections (e.g. checking that one's ballot has been correctly assigned by the ballot counting system)
- however, when the voting takes places in uncontrolled environments, new problems arise (pressurising or bribing voters)
- there seems to be a wide-spread opinion among experts of computerized election system that no existing system (E2E VIV, end-to-end verifiable internet voting) is safe as a whole
- even with internet voting, the problems of agenda control remain

Alleviating democratic deficit

- the amount or degree of democratic deficit depends on how the popular opinions are represented (distributions of first ranked alternatives, rankings, paired comparisons)
- if the rankings are used as the starting point, then rank-distance minimizing methods – notably Kemeny's rule and the Borda count – would seem most appropriate ways of teasing out the collective 'will', given the individual opinions
- Condorcet extensions methods are incompatible with the quite obvious rationale of voting, viz. that the voters cannot lose by voting according to their opinions as compared to not voting at all

Alleviating democratic deficit, cont'd

- domain restrictions play a major role in securing stable collective decisions
- the deliberative mechanisms might have a useful role in setting the agendas of referenda
- internet voting when combined with cryptographic protocols can solve some problems related to traditional balloting, but uncontrolled balloting environments remain a major problem

Some choice settings

- choosing a candidate
- choosing an assembly
- issuing a verdict
- choosing a policy

From sets to individuals

- binary procedures. Advantage: typically elect Condorcet winners when they exist and are robust under modifications of the candidate set. Disadvantage: are not robust under modifications of the voter set and are vulnerable to no-show paradoxes
- positional procedures. Advantage: typically avoid no-show paradoxes. Disadvantage: vulnerable to modifications of the candidate set.
- hybrid procedures: typically fix one problem of the above procedures but introduce another one (not present in the original one)

From sets to assemblies

New questions (in addition to those related to electing individuals):

- do we want proportionality and if we do, what kind?
- should people vote for single members or entire committees?
- is minimizing misrepresentation the right way to compose assemblies?
- should we strive at the widest spectrum of views or maximum proportionality?
- how much distortion between representatives' decisions and popular views is tolerable?

Searching for the right decision

- Condorcet's Jury Theorem as a benchmark
- can we do better than just increase the crowd?
- is probability of being right the appropriate measure of expertise or wisdom?
- is the best judge always better than the majority of almost as good judges?

Choosing a policy

- in many-dimensional policy (real) policy-space, the core is typically empty
- with sincere voting the agenda-setter effectively decides the outcome (if perfectly knowledgeable)
- with sophisticated voting the outcomes converge to a subset of the uncovered set which is a subset of the Pareto set
- these results pertain to pairwise amendment agendas
- not much is known about policy outcome convergence with other voting systems

Concluding questions:

- are the basic ‘givens’ of the social choice theory plausible (fruitful, realistic, appropriate)?
- is the focus on norms satisfied by the procedures the best way to dictate the choice of a rule?
- what do we learn from paradoxes and incompatibilities?
- should consensus-orientation rather than collective utility maximization be the main consideration?
- what is the proper role of deliberation in collective decision making?