

# On Two Results in Contemporary Modal Logic: Local Definability and Succinctness

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In this invited talk, I present two kinds of results and methods in modal logic. The first concerns *local definability*, and is joint work with Hans van Ditmarsch and Barteld Kooi. In modal logic, when adding a syntactic property to an axiomatisation, this property becomes true in all models, in all situations, under all circumstances. For instance, adding a property like  $K_ap \rightarrow K_bp$  (agent  $b$  knows at least what agent  $a$  knows) to an axiomatisation of some epistemic logic has as an effect that such a property becomes *globally* true, i.e., it will hold in all states, at all time points (in a temporal setting), after every action (in a dynamic setting) and after any communication (in an update setting), and every agent will know that it holds, it will even be common knowledge. We propose a way to express that a property like the above only needs to hold *locally*: it may hold in the actual state, but not in all states. We achieve this by adding relational atoms to the language that represent (implicitly) quantification over all formulas, as in  $\forall p(K_ap \rightarrow K_bp)$ . We show how this can be done for a rich class of modal logics and a variety of syntactic properties.

The second theme concerns that of succinctness, and is joint work with Tim French, Petar Iliev and Barteld Kooi. One way of comparing knowledge representation formalisms is in terms of *representational succinctness*, i.e., we can ask whether one of the formalisms allows for a more ‘economical’ encoding of information than the other. Proving that one language is more succinct than another becomes harder when the underlying semantics is stronger. We propose to use *Formula Size Games* (as put forward by Adler and Immerman), games that are played on two *sets* of models, and that directly link the *length* of play with the *size* of the formula. Using Formula Size Games, we prove the following succinctness results for  $m$ -dimensional modal logic: (1) on general Kripke models, a notion of ‘everybody knows’ makes the resulting language exponentially more succinct for  $m > 1$ ; (2) on epistemic models, the same language becomes more succinct for  $m > 3$ , (3) the results for the language with ‘everybody knows’ also hold for a language with ‘somebody knows’, and (4) on epistemic models, Public Announcement Logic is exponentially more succinct than epistemic logic, if  $m > 3$ . The latter settles an open problem raised by Lutz.

## References

1. van Ditmarsch, H., van der Hoek, W., and Kooi, B: Local Properties in Modal Logic. *Artificial Intelligence*, 187–188, pp. 133–155, 2012.
2. French, T., van der Hoek, W., Iliev, P., and Kooi, B: Succinctness of Epistemic Languages. In: Walsh, T. (ed), Proc IJCAI, pp. 881–886, 2011.