Quadrature errors, discrepancies, and their relations to halftoning on the torus and the sphere

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This paper deals with continuous-domain quantization, which aims to create the illusion of a gray-value image by appropriately distributing black dots. For lack of notation we refer to the process as halftoning which is usually associated with the quantization on a discrete grid. Recently a framework for this task was proposed by minimizing an attraction-repulsion functional consisting of the difference of two continuous, convex functions. The firstone of these functions describes attracting forces caused by the image gray values, the second one enforces repulsion between the dots. In this paper, we generalize this approach by considering quadrature error functionals on reproducing kernel Hilbert spaces (RKHSs) with respect to the quadrature nodes, where we ask for optimal distributions of these nodes. For special reproducing kernels these quadrature error functionals coincide with discrepancy functionals, which leads to a geometric interpretation. It turns out that the original attraction-repulsion functional appears for a special RKHS of functions on R^2. Moreover, our more general framework enables us to consider optimal point distributions not only in R^2 but also on the torus T^2 and the sphere S^2. For a large number of points the computation of such point distributions is a serious challenge and requires fast algorithms. To this end, we work in RKHSs of bandlimited functions on T^2 and S^2. Then the quadrature error functional can be rewritten as a least squares functional. We use a nonlinear conjugate gradient method to compute a minimizer of this functional and show that each iteration step can be computed in an efficient way by fast Fourier transforms at nonequispaced nodes on the torus and the sphere. Finally we consider a relation with kinetic mean field limits of the discrete systems of interacting particles. Numerical examples demonstrate the good quantization results obtained by our method.

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