Inverse Problem Regularization with Weak Decomposable Priors. Part I: Recovery Guarantees

Gabriel Peyré

This is the first part of a series of two talks, where we investigate in a unified way the structural properties of a large class of convex regularizers for linear inverse problems.

This first talk is dedicated to assessing the theoretical recovery performance of this class of regularizers. We consider regularizations with convex positively 1-homogenous functionals (in fact gauges) which obey a weak decomposability property. The weak decomposability will promote solutions of the inverse problem conforming to some notion of simplicity/low complexity by living on a low dimensional sub-space. This family of priors encompasses many special instances routinely used in regularized inverse problems such as l^1, l^1-l^2 (group sparsity), nuclear norm, or the l^\infty norm. The weak decomposability requirement is flexible enough to cope with analysis-type priors that include a pre-composition with a linear operator, such as for instance the total variation and polyhedral gauges. Weak decomposability is also stable under summation of regularizers, thus enabling to handle mixed regularizations.

In this talk we provide sufficient conditions that allow to provably control the deviation of the recovered solution from the true underlying object, as a function of the noise level. More precisely we establish two main results. The first one ensures that the solution to the inverse problem is unique and lives on the same low dimensional sub-space as the true vector to recover, with the proviso that the minimal signal to noise ratio is large enough. This extends previous results well-known for the l^1 norm [1], analysis l^1 semi-norm [2], and the nuclear norm [3] to the general class of weakly decomposable gauges. In the second result, we establish l^2 stability by showing that the l^2 distance between the recovered and true vectors is within a factor of the noise level, thus extending results that hold for coercive convex positively 1-homogenous functionals [4].

This is a joint work with S. Vaiter, M. Golbabaee and J. Fadili.

Bibliography:

[1] J.J. Fuchs, On sparse representations in arbitrary redundant bases. IEEE Transactions on Information Theory, 50(6):1341-1344, 2004.

[2] S. Vaiter, G. Peyré, C. Dossal, J. Fadili, Robust Sparse Analysis Regularization, to appear in IEEE Transactions on Information Theory, 2013.

[3] F. Bach, Consistency of trace norm minimization, Journal of Machine Learning Research, 9, 1019-1048, 2008.

[4] M. Grasmair, Linear convergence rates for Tikhonov regularization with positively homogeneous functionals. Inverse Problems, 27(7):075014, 2011.20