

ℓ_0 norm as a regularizer or as a constraint for least squares: analytical comparison of the optimal solutions

Mila NIKOLOVA

CMLA, ENS Cachan, CNRS, 61 Avenue du President Wilson, F-94230 Cachan,
France

nikolova@cmla.ens-cachan.fr

<http://mnikolova.perso.math.cnrs.fr>

Abstract

We have an $M \times N$ real-valued arbitrary matrix A (e.g. a dictionary) with $M < N$ and data d describing the sought-after unknown object u with the help of A . When the sought-after u is sparse, two popular models to recover it are the variational formulation $\|Au - d\|^2 + \beta\|u\|_0$ for $\beta > 0$ a parameter and the sparsity constrained formulation where $\|Au - d\|^2$ is minimized under the constraint $\|u\|_0 \leq K$ for an integer K . Many papers mention that these models are “equivalent” via the choice of β / K . We analyze in depth the relationship between the optimal solutions of these models. We prove that a *true equivalence* (i.e. for each K there is a corresponding β and vice-versa) occurs only under specific conditions that depend mainly on the data d . In more general conditions we exhibit the similarities and the differences between their optimal solutions.