Partial Difference Equations (PdE) on Graphs for Image and Data Processing

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In image processing and computer vision, techniques based on energy minimization and Partial Differential Equations (PDEs) have shown their efficiency in solving many important problems, such as smoothing, denoising, interpolation and segmentation. Solutions of such problems can be obtained by considering the input discrete data as continuous functions defined on a continuous domain, and by designing continuous PDEs whose solutions are discretized in order to fit with the natural discrete domain. An alternative methodology to continuous PDEs-based regularization, is to formalize the problem directly in a discrete setting that is not necessarily a grid. However, PDE-based methods are difficult to adapt for data that live on non Euclidean domains since the discretization of the underlying differential operators is difficult for high dimensional data. Problems involving PDEs can be reduced to algebraic ones of a very much simpler structure by replacing the differentials by difference equations on graphs. As a consequence, it is possible to provide methods that mimic on graphs well-known PDE variational formulations under a functional analysis point of view. One way to tackle this is to use Partial difference Equations (PdE) over graphs. Conceptually, PdEs mimic PDEs in domains having a graph structure. Our proposed PdE framework unifies local and nolocal processing of images and allows most PDEs to be extended to graphs.

In this talk, I will present nonlocal difference operators on graphs and will use the framework of PdEs to transcribe PDEs on graphs:

- a nonlocal discrete regularization on graphs as a framework for data simplification and interpolation [1],
- a multiscale hierarchical decomposition of graph signals [2],
- an adaptation of active contours for data clustering and image segmentation [3].

[1] A. Elmoataz, O. Lézoray, S. Bougleux, Nonlocal Discrete Regularization on Weighted Graphs: a framework for Image and Manifold Processing, IEEE transactions on Image Processing, Vol. 17, no. 7, pp. 1047-1060, 2008.

[2] M. Hidane, O. Lézoray, A. Elmoataz, Nonlinear Multilayered Representation of Graph-Signals, Journal of Mathematical Imaging and Vision, Vol. 45, no. 2, pp. 114-137, 2013.

[3] O. Lézoray, A. Elmoataz, V.-T. Ta, Nonlocal PdEs on graphs for active contours models with applications to image segmentation and data clustering, International Conference on Acoustics, Speech, and Signal Processing (IEEE), pp. 873-876, 2012.

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