Majorize-Minimize stepsize strategies for image restoration

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Many inverse problems can be coped with by the minimization of a penalized criterion that jointly accounts for the available data and the prior information. Here, we address linear operator inversion such as image restoration, and we mainly consider the case of differentiable criteria. More precisely, we deal with efficient stepsize strategies within iterative descent algorithms, and we propose strategies based on the majorize-minimize (MM) principle.

MM algorithms rely on the iterative minimization of a majorizing approximation that coincides with the criterion at the current point. In practice, existing MM algorithms can be split into two groups, whether the majorizing approximation is separable (e.g., iterative thresholding), or quadratic (e.g. reweighted least square algorithms).

We have focused on the second group. In the case of large scale problems such as image restoration, a drawback of such algorithms is that each iteration amounts to solving a large dimension linear system, which is often computationally costly. One of our contributions has been to show that truncated versions are still provably convergent, while being much faster in practice.

Then we have considered several descent strategies that are more naturally suited to large scale problems, i.e., conjugate gradient, LBFGS, ..., with a view to combine them to a stepsize algorithm defined as a one-dimensional quadratic majorize-minimize strategy. We have shown that the resulting schemes (i) are convergent under

broad mathematical conditions, (ii) have a simple structure, (iii) are fast converging in practice. A specifically efficient scheme is a memory gradient algorithm where each update is performed in a subspace of given dimension using the MM principle [2].

Our approach is not limited to convex criteria [1,2,3], and neither is it limited to quadratic majorizing approximations. In particular, we have developped a specific stepsize algorithm based on a non-quadratic one-dimensional majorizing function, for the case where the criterion holds a barrier function (e.g., entropy-based criterion, Poisson likelihoods) [5,6].

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