Inverse Problem Regularization with Weak Decomposable Priors. Part II: Sensitivity Analysis and Parameter Selection

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This is the second part of a series of two talks, where we investigate in a unified way the structural properties of a large class of convex regularizers for linear inverse problems.

This second talk is dedicated to analyzing the local sensitivity of any regularized solution to perturbations on the observations, and as a by-product, its implications to construct unbiased risk estimators and parameters selection procedures. We consider regularizations with convex positively 1-homogenous functionals (in fact gauges) which obey a weak decomposability property and belong to an o-minimal structure. The weak decomposability will promote solutions of the inverse problem conforming to some notion of simplicity/low complexity by living on a low dimensional subspace. This family of priors encompasses many special instances routinely used in regularized inverse problems such as l^1, l^1-l^2 (group sparsity), nuclear norm, or the l^\infty norm. The weak decomposability requirement is flexible enough to cope with analysis-type priors that include a pre-composition with a linear operator, such as for instance the total variation and polyhedral gauges. Weak decomposability is also stable under summation of regularizers, thus enabling to handle mixed regularizations.

In this talk, owing to weak decomposability, we give a precise local parameterization and establish local differentiability of any regularized solution as a function of the observations. We also give an expression of the Jacobian of a solution mapping w.r.t. to the observations. Using tools from o-minimal geometry, we prove that the set of non-differentiability points is of zero Lebesgue measure, hence showing that the expression of the Jacobian is valid Lebesgue a.e.. These results are achieved without requiring that the forward-model linear operator is injective, and extend and unify previous results known for analysis l^1 sparsity, see [1] and references therein. When the noise in the observations is Gaussian, these results allow us to derive expressions of the Generalized Stein Unbiased Risk Estimator (GSURE) [2]. In turn, this provides a principled way to automatically select the hyperparameters involved, such as the regularization parameter.

This is a joint work with S. Vaiter, C. Deledalle, G. Peyré and C. Dossal.

Bibliography:

[1] S. Vaiter, C. Deledalle, G. Peyre, J. Fadili, and C. Dossal, Local behavior of sparse analysis regularization: Applications to risk estimation, to appear in Applied and Computational Harmonic Analysis, 2013.

[2] Y. C. Eldar, Generalized sure for exponential families: Applications to regularization, IEEE Transactions on Signal Processing, vol. 57, pp. 471--481, 2009.