A Sample Distortion Analysis for Compressed Imaging

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We propose the notion of sample distortion function for i.i.d compressive distributions with the aim to fundamentally quantify the achievable reconstruction performance of compressed sensing for certain encoder-decoder pairs at a given undersampling ratio. The theoretical SD function is derived for the Gaussian encoder and Bayesian optimal approximate message passing (AMP) decoder thanks to the rigorous analysis of the AMP algorithm. We also show the convexity of the general SD function and derive two lower bounds. We then apply the SD framework to analyse compressed sensing for natural images using a multi-resolution statistical image model with either a generalized Gaussian distribution and the two-state Gaussian mixture distribution. For these scenarios we are able to achieve an optimal bandwise sample allocation and show that the corresponding SD function for natural images accurately predicts the observed compressed sensing performance gains. We further adopt Som and Schniter's turbo message passing approach to integrate the bandwise sampling with the exploitation of the hidden Markov tree structure of wavelet coefficients. Natural image simulation confirms the theoretical improvements and the effectiveness of bandwise sampling.