On the Relationship of Defeasible Argumentation and Answer Set Programming

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May 29, 2008

Outline

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The motivation

- Both, Defeasible Logic Programming and Answer Set Programming use logic programming as a representation mechanism
- While logic programming in general is a well understood framework, argumentation frameworks are still under heavy development
- Although the relationship of argumentation and default logic has been investigated using abstract argumentation frameworks, we are trying to investigate a direct link between DeLP and ASP

Our aim is to express the set of warranted literals of a defeasible logic program directly in terms of answer set semantics to get a better understanding of the relationships of their inference mechanisms.

Outline

A very brief overview

- in DeLP (*Defeasible Logic Programming*) we are dealing with facts, strict rules and defeasible rules.
- A defeasible logic program (*de.l.p.*) $\mathcal{P}$ is a tuple $\mathcal{P} = (\Pi, \Delta)$ with a set Π of facts and strict rules and a set Δ of defeasible rules.
- Using defeasible argumentation via a dialectical analysis one can determine warrants and warranted literals.

Definition (Warrant)

A literal h is *warranted*, iff there exists an argument $\langle \mathcal{A}, h \rangle$ for h, such that the root of the marked dialectical tree $\mathcal{T}^*_{\langle \mathcal{A}, h \rangle}$ is marked "undefeated". Then $\langle \mathcal{A}, h \rangle$ is a *warrant* for h.

Outline

Warranting arguments

In general, a warrant $\langle \mathcal{A}, h \rangle$ is not unbeatable, i. e. it does not hold: “If an argument $\langle \mathcal{A}, h \rangle$ is undefeated in the dialectical tree $\mathcal{T}_{\langle \mathcal{A}, h \rangle}$, then it is undefeated in every dialectical tree”.

Warranting arguments

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But

Proposition

If an argument $\langle \mathcal{A}, h \rangle$ is undefeated in the dialectical tree $\mathcal{T}_{\langle \mathcal{A}, h \rangle}$, then it is undefeated in every dialectical tree $\mathcal{T}_{\langle \mathcal{A}', h' \rangle}$, where $\langle \mathcal{A}, h \rangle$ is a child of $\langle \mathcal{A}', h' \rangle$.

and therefore

Proposition

If h and h' are warranted literals in a de.l.p. $\mathcal{P}$, then h and h' cannot disagree.

Joint disagreement 1/2

Although two warranted literals are consistent, this is not always the case for sets of more than two warranted literals.

Definition (Joint disagreement)

If $\{h_1, \ldots, h_n\} \cup \Pi \mathrel{|\!\!\sim} \bot$, then $h_1, \ldots, h_n$ are in *joint disagreement*.

Example

Let *de.l.p.* $\mathcal{P} = (\Pi, \Delta)$ with

$$\begin{aligned} \Pi &= \{a, (h \leftarrow c, d), (\neg h \leftarrow e, f)\} \\ \Delta &= \{(c \prec a), (d \prec a), (e \prec a), (f \prec a)\} \end{aligned}$$

$\Rightarrow c, d, e, f$ are warranted (assuming a suitable preference relation under arguments) and in joint disagreement.

Joint disagreement 2/2

Some sets of warranted literals can never be in joint disagreement as the following two propositions show.

Proposition

Let $\langle \mathcal{A}, h \rangle$ *be an argument such that* $\{h, h_1, \ldots, h_n\} = \{head(r) \mid r \in \mathcal{A}\}$. *Then* $h, h_1, \ldots, h_n$ *do not jointly disagree.*

It follows

Proposition

Let $\mathcal{P}$ *be a de.l.p. If* h *is a warranted literal in* $\mathcal{P}$ *and* $\langle \mathcal{A}, h \rangle$ *is a warrant for* h, *then* h' *is warranted in* $\mathcal{P}$ *for every subargument* $\langle \mathcal{B}, h' \rangle$ *of* $\langle \mathcal{A}, h \rangle$.

Outline

Overview

Extended logic programs (Gelfond, Lifschitz) use default negation to handle uncertainty and to realize non-monotonic reasoning.

Definition (Extended logic program)

An *extended logic program* (*program* for short) P is a finite set of rules of the form

$$h \leftarrow a_1, \ldots, a_n, \text{not } b_1, \ldots, \text{not } b_m$$

Answer sets

Let X be a set of literals.

Definition (Reduct)

The *X-reduct* of a program P (P^X) is the union of all rules $h \leftarrow a_1, \ldots, a_n$ such that $h \leftarrow a_1, \ldots, a_n, \text{not } b_1, \ldots, \text{not } b_m \in P$ and $X \cap \{b_1, \ldots, b_m\} = \emptyset$.

The reduct is used to characterize a set of literals as an answer set:

Definition (Answer set)

A consistent set of literals S is an *answer set* of a program P, iff S is the minimal model of P^S.

Outline

Minimal disagreement, guard rules

To preserve consistency in answer sets, sets of warranted literals that are in joint disagreement have to be handled appropriately.

Definition (Minimal disagreement set)

A *minimal disagreement set* $\mathcal{X}$ is a set of derivable literals such that $\mathcal{X} \cup \Pi \mathrel{|\!\!\sim} \bot$ and there is no proper subset $\mathcal{X}'$ of $\mathcal{X}$ with $\mathcal{X}' \cup \Pi \mathrel{|\!\!\sim} \bot$. Let $\mathfrak{X}(\mathcal{P})$ be the set of all minimal disagreement sets of $\mathcal{P}$.

Definition (Guard literals, guard rules)

The set of *guard literals* $GuardLit(\mathcal{P})$ for $\mathcal{P}$ is defined as $GuardLit(\mathcal{P}) = \{\alpha_h | h \text{ is a literal in } \mathcal{P}\}$ with new symbols α_h. The set of *guard rules* $GuardRules(\mathcal{P})$ of $\mathcal{P}$ is defined as $GuardRules = \{\alpha_h \leftarrow h_1, \ldots, h_n | \{h, h_1, \ldots, h_n\} \in \mathfrak{X}(\mathcal{P})\}$.

Induced answer set programs

Definition (*de.lp*-induced answer set program)

The $\mathcal{P}$*-induced answer set program* $\mathrm{ASP}(\mathcal{P})$ is defined as the minimal extended logic program satisfying

1. for every $a \in \Pi$ it is $a \in \mathrm{ASP}(\mathcal{P})$,
2. for every $r : h \leftarrow b_1, \ldots, b_n \in \Pi$ it is $r \in \mathrm{ASP}(\mathcal{P})$,
3. for every $h \prec b_1, \ldots, b_n \in \Delta$ it is $h \leftarrow b_1, \ldots, b_n, \text{not } \alpha_h \in \mathrm{ASP}(\mathcal{P})$ and
4. $\mathit{GuardRules}(\mathcal{P}) \subseteq \mathrm{ASP}(\mathcal{P})$.

An example

Example

Let $\mathcal{P} = (\Pi, \Delta)$ with

$$\begin{aligned} \Pi &= \{a, b, (h \leftarrow c, d), (\neg h \leftarrow e)\} \\ \Delta &= \{(p \prec a), (\neg p \prec b), (c \prec b), (d \prec b), (e \prec a)\} \end{aligned}$$

Here we have $\{(\alpha_h \leftarrow \neg h), (\alpha_{\neg h} \leftarrow c, d), (\alpha_c \leftarrow d, \neg h), (\alpha_c \leftarrow d, e), (\alpha_d \leftarrow c, e)\} \subseteq \mathit{GuardRules}(\mathcal{P})$.

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Here we have $\{(\alpha_h \leftarrow \neg h), (\alpha_{\neg h} \leftarrow c, d), (\alpha_c \leftarrow d, \neg h), (\alpha_c \leftarrow d, e), (\alpha_d \leftarrow c, e)\} \subseteq \mathit{GuardRules}(\mathcal{P})$.

The $\mathcal{P}$-induced answer set program $\mathrm{ASP}(\mathcal{P})$ arises as

$$\begin{aligned} \mathrm{ASP}(\mathcal{P}) = \; & \{a, b, (h \leftarrow c, d), (\neg h \leftarrow e), (p \leftarrow a, \text{not } \alpha_p), \\ & (\neg p \leftarrow b, \text{not } \alpha_{\neg p}), (c \leftarrow b, \text{not } \alpha_c), \\ & (d \leftarrow b, \text{not } \alpha_d), (e \leftarrow a, \text{not } \alpha_e)\} \cup \mathit{GuardRules}(\mathcal{P}) \end{aligned}$$

Results

It can be shown that sets of warranted literals and answer sets are related:

Theorem

Let $\mathcal{P} = (\Pi, \Delta)$ *be a de.l.p. and* $\mathrm{ASP}(\mathcal{P})$ *the* $\mathcal{P}$*-induced answer set program. If* h *is warranted in* $\mathcal{P}$ *then there exists at least one answer set* M *of* $\mathrm{ASP}(\mathcal{P})$ *with* $h \in M$.

Results

It can be shown that sets of warranted literals and answer sets are related:

Theorem

Let $\mathcal{P} = (\Pi, \Delta)$ be a de.l.p. and $\mathrm{ASP}(\mathcal{P})$ the $\mathcal{P}$-induced answer set program. If h is warranted in $\mathcal{P}$ then there exists at least one answer set M of $\mathrm{ASP}(\mathcal{P})$ with $h \in M$.

For a special case it follows

Corollary

Let $\mathcal{P} = (\Pi, \Delta)$ be a de.l.p. and $\mathrm{ASP}(\mathcal{P})$ the $\mathcal{P}$-induced answer set program. If Π does not contain any strict rule and M is the set of all warranted literals of $\mathcal{P}$ then there exists an answer set M' of $\mathrm{ASP}(\mathcal{P})$ with $M \subseteq M'$.

Induced* answer set programs 1/3

Definition (*de.l.p**-induced answer set program)

The *$\mathcal{P}^*$-induced answer set program* $\mathrm{ASP}^*(\mathcal{P})$ is defined as the minimal extended logic program satisfying

1. for every $a \in \Pi$ it is $a \in \mathrm{ASP}^*(\mathcal{P})$ and
2. for every (strict or defeasible) rule $h \leftarrow\!\!\!-\!\!-\; b_1, \ldots, b_n \in \Pi \cup \Delta$ it is $h \leftarrow b_1, \ldots, b_n, \text{not } b'_1, \ldots, \text{not } b'_m \in \mathrm{ASP}^*(\mathcal{P})$ where $\{b'_1, \ldots, b'_m\} = \{b | b \text{ and } h \text{ disagree}\}$.

Theorem

Let $\mathcal{P} = (\Pi, \Delta)$ be a de.l.p.. Let furthermore $\mathrm{ASP}^*(\mathcal{P})$ be the $\mathcal{P}^*$-induced answer set program. If M is the set of all warranted literals of $\mathcal{P}$, then there exists an answer set M' of $\mathrm{ASP}^*(\mathcal{P})$ with $M \subseteq M'$.

Outline

Conclusion

- we studied transformations of defeasible logic programs into answer set programs in order to make relationships between their inference mechanisms explicit
- we proved that for our conversion, warrant implies credulous inference
- for the second type of conversion, all warranted literals are in one answer set

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- we studied transformations of defeasible logic programs into answer set programs in order to make relationships between their inference mechanisms explicit
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Thank you for your attention

Appendix I: Comparing arguments

Arguments can be compared, e. g. using *Generalized Specificity*.

Example

- Let a, b be facts. Then

$$\begin{array}{rcl} \langle\{(c \prec a, b)\}, c\rangle & \succ_{spec} & \langle\{(\neg c \prec a)\}, \neg c\rangle \\ \langle\{(d \prec a)\}, d\rangle & \succ_{spec} & \langle\{(c \prec a), (\neg d \prec c)\}, \neg d\rangle \end{array}$$

$\rightarrow$ *proper attacks*

- Arguments might be incomparable

$$\begin{array}{rcl} \langle\{(c \prec a)\}, c\rangle & \not\succ_{spec} & \langle\{(\neg c \prec b)\}, \neg c\rangle \\ \langle\{(c \prec a)\}, c\rangle & \not\prec_{spec} & \langle\{(\neg c \prec b)\}, \neg c\rangle \end{array}$$

$\rightarrow$ *blocking attacks*

Appendix II: Induced* answer set programs

Example

Let $\mathcal{P} = (\Pi, \Delta)$ with

$$\begin{aligned}\Pi &= \{a, b, (h \leftarrow c, d), (\neg h \leftarrow e)\}\\ \Delta &= \{(p \prec a), (\neg p \prec b), (c \prec b), (d \prec b), (e \prec a)\}\end{aligned}$$

$\rightarrow \{a, b, c, d\}$ are warranted (using *Generalized Specificity*)

The $\mathcal{P}^*$-induced answer set program $\mathrm{ASP}^*(\mathcal{P})$ arises as

$$\begin{aligned}\mathrm{ASP}^*(\mathcal{P}) = \{ & a, b, (h \leftarrow c, d, \text{not } \neg h, \text{not } e), (\neg h \leftarrow e, \text{not } h,),\\ & (p \prec a, \text{not } \neg p), (\neg p \prec b, \text{not } p), (c \prec b),\\ & (d \prec b), (e \prec a, \text{not } h)\}\end{aligned}$$

$\rightarrow$ The answer sets of $\mathrm{ASP}^*(\mathcal{P})$ are $\{a, b, c, d, e, \neg h, p\}$, $\{a, b, c, d, e, \neg h, \neg p\}$, $\{a, b, c, d, h, p\}$, $\{a, b, c, d, h, \neg p\}$

Appendix III: Proofs 1/7

Proposition

If an argument $\langle \mathcal{A}, h \rangle$ is undefeated in the dialectical tree $\mathcal{T}_{\langle \mathcal{A}, h \rangle}$, then it is undefeated in every dialectical tree $\mathcal{T}_{\langle \mathcal{A}', h' \rangle}$, where $\langle \mathcal{A}, h \rangle$ is a child of $\langle \mathcal{A}', h' \rangle$.

Proof.

- the subtree rooted at $\langle \mathcal{A}, h \rangle$ after $\langle \mathcal{A}', h' \rangle$ is a subtree of $\mathcal{T}_{\langle \mathcal{A}, h \rangle}$
- every "needed" supporting argument of $\langle \mathcal{A}, h \rangle$ in $\mathcal{T}_{\langle \mathcal{A}, h \rangle}$ is in $\mathcal{T}_{\langle \mathcal{A}', h' \rangle}$
- $\langle \mathcal{A}, h \rangle$ is undefeated in $\mathcal{T}_{\langle \mathcal{A}', h' \rangle}$

$\square$

Appendix III: Proofs 2/7

Proposition

If h and h' are warranted literals in a de.l.p. $\mathcal{P}$, then h and h' cannot disagree.

Proof.

- suppose h, h' disagree
- let $\langle \mathcal{A}, h \rangle$, $\langle \mathcal{A}', h' \rangle$ be warrants
- wlog $\langle \mathcal{A}, h \rangle$ attacks $\langle \mathcal{A}', h' \rangle$
- due to last proposition, $\langle \mathcal{A}, h \rangle$ is undefeated in dial. tree of $\langle \mathcal{A}', h' \rangle$
- $\langle \mathcal{A}', h' \rangle$ is defeated, hence no warrant.

□

Appendix III: Proofs 3/7

Proposition

Let $\langle \mathcal{A}, h \rangle$ *be an argument such that* $\{h, h_1, \ldots, h_n\} = \{head(r) \mid r \in \mathcal{A}\}$. *Then* $h, h_1, \ldots, h_n$ *do not jointly disagree.*

Proof.

As $\langle \mathcal{A}, h \rangle$ is an argument, $\Pi \cup \mathcal{A}$ is non-contradictory and thus does not cause the derivation of complementary literals. As $\Pi \cup \mathcal{A} \mid\!\sim h, h_1, \ldots h_n$ the literals $h, h_1, \ldots, h_n$ do not jointly disagree. □

Appendix III: Proofs 4/7

Proposition

Let $\mathcal{P}$ be a de.l.p. If h is a warranted literal in $\mathcal{P}$ and $\langle \mathcal{A}, h \rangle$ is a warrant for h, then h' is warranted in $\mathcal{P}$ for every subargument $\langle \mathcal{B}, h' \rangle$ of $\langle \mathcal{A}, h \rangle$.

Show the contraposition:

Proposition

Let $\mathcal{P}$ be a de.l.p. and $\langle \mathcal{B}, h' \rangle$ an argument. If $\langle \mathcal{B}, h' \rangle$ is defeated in a dialectial process, every argument $\langle \mathcal{A}, h \rangle$, such that $\langle \mathcal{B}, h' \rangle$ is a subargument of $\langle \mathcal{A}, h \rangle$, is also defeated in a dialectical process.

Appendix III: Proofs 5/7

Proof.

- let $\langle \mathcal{B}, h' \rangle$ be defeated in its dialectical tree and $\langle \mathcal{C}, h'' \rangle$ a defeater
- $\langle \mathcal{C}, h'' \rangle$ is also an attack on $\langle \mathcal{A}, h \rangle$
- the tree rooted at $\langle \mathcal{C}, h'' \rangle$ under $\langle \mathcal{A}, h \rangle$ is a subtree of the tree rooted at $\langle \mathcal{C}, h'' \rangle$ under $\langle \mathcal{B}, h' \rangle$
- there is no $\langle \mathcal{D}, g \rangle$ in the tree rooted at $\langle \mathcal{C}, h'' \rangle$ and interfering with $\langle \mathcal{B}, h' \rangle$ in the dial. tree of $\langle \mathcal{B}, h' \rangle$ that is not in the dial. tree of $\langle \mathcal{A}, h \rangle$, provided its parentnode exists in the dial. tree of $\langle \mathcal{A}, h \rangle$
- hence the subtree rooted at $\langle \mathcal{C}, h'' \rangle$ under $\langle \mathcal{A}, h \rangle$ “loses” no needed interfering arguments
- hence $\langle \mathcal{C}, h'' \rangle$ defeats $\langle \mathcal{A}, h \rangle$

□

Appendix III: Proofs 6/7

Theorem

Let $\mathcal{P} = (\Pi, \Delta)$ be a de.l.p. and $\mathrm{ASP}(\mathcal{P})$ the $\mathcal{P}$-induced answer set program. If h is warranted in $\mathcal{P}$ then there exists at least one answer set M of $\mathrm{ASP}(\mathcal{P})$ with $h \in M$.

Proof.

- the set S of literals appearing in a warrant $\langle \mathcal{A}, h \rangle$ do not jointly disagree
- hence S can be extended to a consistent set M, such that M is an answer set of $\mathrm{ASP}(\mathcal{P})$

□

Appendix III: Proofs 7/7

Corollary

Let $\mathcal{P} = (\Pi, \Delta)$ be a de.l.p. and $\mathrm{ASP}(\mathcal{P})$ the $\mathcal{P}$-induced answer set program. If Π does not contain any strict rule and M is the set of all warranted literals of $\mathcal{P}$ then there exists an answer set M' of $\mathrm{ASP}(\mathcal{P})$ with $M \subseteq M'$.

Proof.

- there can be no disagreement sets with cardinality > 2
- no two warranted literals can disagree
- hence M is consistent and can consistently be extended to an answer set M'

□