

# Heuristics in Argumentation: A Game-Theoretical Investigation

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May 30, 2008

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**We are interested here in the heuristic layer.**

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- ▶ Problem: how to determine optimal strategies in a dialogue games for argumentation?
- ▶ Solution: we propose the use of game-theoretical tools.

# Adjudication debates

We focus on 'adjudication debates':

1. Two parties argue on a claim,
2. A neutral party decides whether to accept the statements stated during the debate.

Introduction

Outline

Dialectical setting

Game-theoretical model

Preference specifications

Conclusion

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2. Opposing arguers make estimates how likely it is that the premises of their arguments will be accepted by the adjudicator.

## Introduction

### Dialectical setting

- Assumptions on the logic

- Assumptions on the game protocol

- Assumptions on argument games

- Four structures

### Game-theoretical model

- Game-theoretical assumptions

- Dialogue games as extensive games

### Preference specifications

- Expected utility

- Outcomes of a game

- Probability of success

- Utility values

## Conclusion

# Assumptions on the logic

1. Arguments have a finite nonempty set of premises and one conclusion.
2. There is a binary relation of defeat between arguments.

# Assumptions on the game protocol

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5. The turn shifts after a player has made 1 or at maximum  $m$  moves in a row and indicates explicitly that she has ended her turn.
6. Each argument move other than the first one defeats its target argument.

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4. An argument move in a reply tree *favours Pro* if the argument move is *in*; otherwise it favours *Opp*.
5. A game is *won* by a player if at termination the initial move favours the player.

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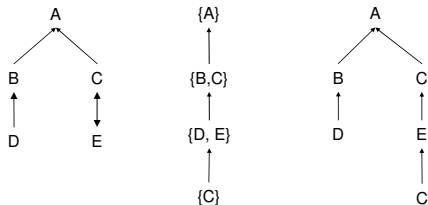
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3. A multi-move argument game which is a sequence of turns by two players *Pro* and *Opp*. Each turn consists of zero or more arguments;
4. A game tree of all possible turn games in which the nodes are turns and the links express their temporal order in a game.

# Example



**Figure:** In the middle, a single terminated argument game based on the defeat graph on the left, and its reply graph on the right.

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2. Arguers are perfectly informed about the arguments previously advanced by the other arguer: *extensive games with perfect information*.
3. The set of all arguments and their defeat relations is given in advance, is finite, stays fixed during a game and is known by both players between the games: *extensive games with perfect and complete information*.

# Dialogue games as extensive games

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1. Players: opponent and proponent.
2. Histories: sequences of turns.
3. A player turn function: the arguer turn function.
4. A preference relation for each player over terminated histories.



# Strategies

The strategy of an arguer is the specification of the sequences of arguments chosen by the arguer for every history after which it is her turn to move.

## Definition

A strategy of arguer  $i \in N$  in an extensive argumentation game with perfect information  $\langle N, H, P, (\succeq_i) \rangle$  is a function that assigns a move  $M(h)$  to each nonterminal history  $h \in H - Z$  for which  $P(h) = i$ .



# Equilibrium

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- ▶ In extensive game, we consider *subgame perfect equilibrium*: a subgame perfect equilibrium is a Nash equilibrium of every subgame of the original game.

# Equilibrium

## Definition

A subgame perfect equilibrium of an extensive argumentation game with perfect information  $\Gamma = \langle N, H, P, (\succeq_i) \rangle$  is a strategy profile  $s^*$  such that for every nonterminal history  $h \in H - Z$  for which  $P(h) = i$ ,  $i \in \{Opp, Pro\}$ , we have:

$$Out_h(s_{Pro}^*|h, s_{Opp}^*|h) \succeq_{Opp|h} Out_h(s_{Pro}^*|h, s_{Opp})$$

$$Out_h(s_{Pro}^*|h, s_{Opp}^*|h) \succeq_{Pro|h} Out_h(s_{Pro}, s_{Opp}^*|h)$$

for every  $s_{Pro}$  and  $s_{Opp}$  in the subgame  $\Gamma(h)$ .

# Backwards induction

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- ▶ Backward induction: start at a player's final decision nodes to see what a player will do there, and then reasons backwards to tell which action is best for the other player.

## Preferences specifications

The preference relation is defined by means of an utility function  $EU_i : Out(s) \rightarrow \mathbb{R}$  such that:

$$Out(s) \succeq_i Out(s') \text{ if and only if } EU_i(Out(s)) \geq EU_i(Out(s')).$$

# Expected utility

The utility function is specified in terms of expected utility.

$$EU(X) = \sum_{i=1}^n Pr(o_i) \cdot u(o_i)$$

where  $o_1, \dots, o_n$  are the possible (and mutually exclusive) outcomes of  $X$ .

## Outcomes of a game

The game-theoretical outcome  $Out(s)$  of a strategy profile  $s$  is a terminal history, i.e. the dialogue resulting from  $s$ . For each terminated game associated to a strategy profile  $s$ , we have two mutually exclusive utility outcomes: an arguer can win or lose. In other words, the initial argument is successful or not.

$$EU_i(Out(s)) = Pr(Succ(A, Out(s))) \times u_i(Succ(A, Out(s))) + Pr(\neg Succ(A), Out(s)) \times u_i(\neg Succ(A, Out(s))) \quad (1)$$

- ▶  $Pr(Succ(A, Out(s)))$ : the probability of success of the initial argument  $A$  w.r.t. the dialogue  $Out(s)$
- ▶  $u_i(Succ(A, Out(s)))$  is the utility value of the success of  $A$  w.r.t. the dialogue  $Out(s)$ .



## Probability of success of an argument

The probability of success of an argument is intended to mean the probability that the argument is accepted as justified given a knowledge base of which the statements are assigned a probability of acceptance by the adjudicator.

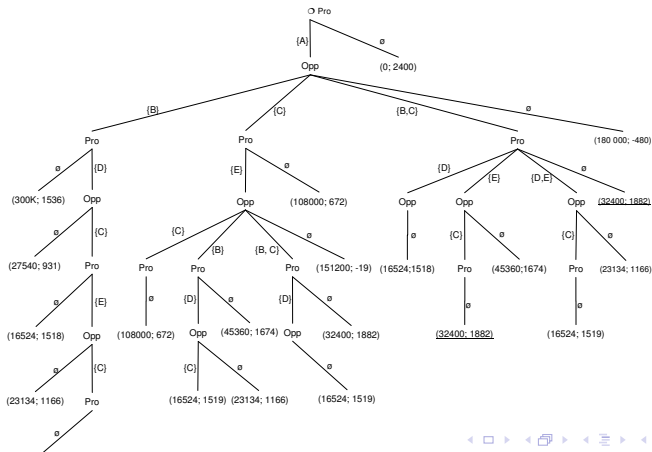
## Utility values

The utility values  $u_i(\text{Succ}(A, \text{Out}(s)))$  and  $u_i(\neg\text{Succ}(A, \text{Out}(s)))$  incorporate costs and benefits of moves.

We distinguish:

1. Fixed costs/benefits capture costs/benefits independent of the success of the player (e.g. trial expenses).
2. Costs/benefits of moves dependant upon success.

# Dialogue games as extensive games



# Conclusion

- ▶ An interpretation of a dialectical setting in game-theoretical terms.
- ▶ A specification of preferences over outcomes has been provided in terms of expected utility combining the probability of success of arguments, costs and benefits of arguments.