# Argumentation Using Temporal Knowledge

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# Arguments

Let $\Delta$ be set of formulas in classical propositional logic

An **argument** is a pair $\langle \Phi, \alpha \rangle$ such that

1. $\Phi \not\vdash \bot$
2. $\Phi \vdash \alpha$
3. $\Phi$ is a minimal subset of $\Delta$ satisfying 2

$\Phi$ is the **support** and $\alpha$ is the **consequent** of the argument

# Using Time in Arguments

House prices have fallen throughout 2008.

- $\forall i. In2008(i) \rightarrow \neg Holds(house\_price\_rise, i)$

During periods of falling house prices, many mortgages are forclosed.

- $\forall i. \neg Holds(house\_price\_rise, i) \rightarrow$
  $Holds(many\_mortgages\_forclosed, i)$

In March, house prices bucked the trend, rising slightly, but April prices were down again. Despite the house price fall, however, few mortgages were forclosed in April

- $Holds(house\_price\_rise, march) \wedge$
  $\neg Holds(house\_price\_rise, april) \wedge$
  $\neg Holds(many\_mortgages\_forclosed, april)$

# What to do?

- Use first order logic?
- Adopt an existing temporal logic?

What we want:

- Simple; as close to propositional logic as possible
- Able to include variable intervals or timepoints.
- Able to handle relationships between intervals or timepoints.

# Talk overview

- $\mathcal{T}$: A calculus which is suitable for the expression of temporal data.
- Define a consequence relationship $\vdash_{\mathcal{T}}$ in terms of $\vdash$.
- Arguments in $\mathcal{T}$; Support and database ground arguments.
- Show how these behave, and compare and contrast the two types of argument.

# Timeline and Intervals

A **timeline** is a tuple $\langle T, \preceq \rangle$, where $T$ is a finite set of **timepoints**, and $\preceq$ is a total linear ordering over these timepoints. The symbol $\prec$ is used such that $t_1 \prec t_2$ iff $t_1 \preceq t_2 \wedge t_1 \neq t_2$.

An **interval** (on a timeline $\langle T, \preceq \rangle$) is a pair $(t_1, t_2)$ where $t_1$ and $t_2$ are both timepoints in $T$, and $t_1 \prec t_2$.

- Given a subset $T \subset \mathbb{Z}$, then $\langle T, \leq \rangle$ is suitable for a timeline.

# Interval relations

Relationships between intervals are defined as follows:

$(a, b)$ **precedes** $(c, d)$ iff $b < c$
$(a, b)$ **meets** $(c, d)$ iff $b = c$
$(a, b)$ **overlaps** $(c, d)$ iff $a < c < b < d$
$(a, b)$ **during** $(c, d)$ iff $c < a < b < d$
$(a, b)$ **starts** $(c, d)$ iff $a = c$ and $b < d$
$(a, b)$ **ends** $(c, d)$ iff $c < a$ and $b = d$
$(a, b)$ **equals** $(c, d)$ iff $a = c$ and $b = d$

- (0,2) **meets** (2,8)
- (3,6) **starts** (3, 12)
- (5,7) **during** (3, 12)

# Syntax of $\mathcal{T}$

The syntax for the letters of $\mathcal{T}$ is defined as below.

$$
\begin{array}{lcl}
\mathit{symbol} & ::= & \alpha \mid \beta \mid \gamma \mid \delta \ldots \\
\mathit{timepoint} & ::= & 0 \mid 1 \mid 2 \mid 3 \ldots \\
\mathit{varinterval} & ::= & i \mid j \mid k \ldots \\
\mathit{interval} & ::= & (\mathit{timepoint}, \mathit{timepoint}) \\
 & \mid & \mathit{varinterval} \\
\mathit{relation} & ::= & \mathbf{precedes} \mid \mathbf{meets} \mid \mathbf{overlaps} \mid \mathbf{during} \\
 & & \quad \mid \mathbf{starts} \mid \mathbf{ends} \mid \mathbf{equals} \\
\mathit{letter} & ::= & \mathit{Holds}(\mathit{symbol}, \mathit{interval}) \\
 & \mid & (\mathit{interval}\ \mathit{relation}\ \mathit{interval})
\end{array}
$$

# Examples of formuale in $\mathcal{T}$

$$Holds(\alpha, i)$$

$$((Holds(\alpha, i) \wedge (i \textbf{ meets } j)) \rightarrow Holds(\beta, j))$$

$$(((0, 1) \textbf{ starts } (4, 7)) \wedge Holds(\alpha, (3, 4)))$$

$$\neg((0, 1) \textbf{ starts } (4, 7))$$

$$((0, 1) \textbf{ starts } (0, 7))$$

# Grounding

A **grounding** is pair $[\alpha, \Theta]$, where $\alpha$ is a formula in $\mathcal{T}$ and $\Theta$ is a set of substitutions.

$$[Holds(\alpha, i), \{i/(2,3)\}]$$

$$[Holds(\alpha, i), \emptyset]$$

$$[Holds(\alpha, i) \wedge (i \textbf{ meets } j) \rightarrow Holds(\beta, j), \{i/(1,2), j/(2,5)\}]$$

If $\Theta$ contains a substitution for all variables in $\alpha$, then we have a **complete grounding**, otherwise it is a **partial grounding**.

A **zero grounding** is when the set is empty.

# Grounding Functions

The **complete grounding function** $G_C(\Phi)$ results in the set containing all complete groundings $[\phi, \Theta]$ where $\phi \in \Phi$.

$G_C(Holds(\alpha, i))$ with timepoints (0,1,2) is
$\{[Holds(\alpha, i), \{i/(0,1)\}], [Holds(\alpha, i), \{i/(0,2)\}], [Holds(\alpha, i), \{i/(1,2)\}]\}$

$G_C(Holds(\alpha, (2,3)))$ is $\{[Holds(\alpha, (2,3)), \emptyset]\}$

$G_C([Holds(\alpha, i) \wedge Holds(\beta, j), \{i/(0,2)\}])$ contains
$[Holds(\alpha, i) \wedge Holds(\beta, j), \{i/(0,2), j/(0,1)\}]$,
$[Holds(\alpha, i) \wedge Holds(\beta, j), \{i/(0,2), j/(0,2)\}]$ etc

The **zero grounding function** $G_0(\Phi)$ results in the set of all zero groundings of each formula in $\Phi$.

# Intervals Revisited

The set $I$ of interval relations is $I^{\top} \cup I^{\perp}$, where:

- $I^{\top}$ is the set of all formulae $((a, b)$ **relation** $(c, d))$ where the conditions on $a$, $b$, $c$ and $d$ for that relation are met.

$((1, 2)$ **meets** $(2, 3)) \in I^{\top}$

- $I^{\perp}$ is the set of all formulae $\neg((a, b)$ **relation** $(c, d))$ the conditions on $a$, $b$, $c$ and $d$ are for that relation not met.

$\neg((6, 7)$ **starts** $(1, 3)) \in I^{\perp}$

# The $\vdash_{\mathcal{T}}$ relation

Let $\Phi$ be a set of formulae in $\mathcal{T}$ and $\alpha$ a formula in $\mathcal{T}$. Then $\Phi \vdash_{\mathcal{T}} \alpha$ iff

$$G_C(\Phi) \cup I \vdash \bigwedge G_C(\alpha)$$

where $I$ is the interval lookup set

$Holds(\alpha, i) \vdash_{\mathcal{T}} Holds(\alpha, (0,1))$

- $\alpha_{(0,1)}$ is present in $G_C(Holds(\alpha, i))$

$Holds(\alpha, i) \vdash_{\mathcal{T}} Holds(\alpha, i)$

- Every $\alpha_{(a,b)}$ is present in $G_C(Holds(\alpha, i))$

$\vdash_{\mathcal{T}} (0,1)$ **meets** $(1,2)$

- $(0,1)$ **meets** $(1,2) \in I$

# Example of an Invalid Argument

Given the following database

(1) $\neg Holds(house\_price\_rise, i) \rightarrow$
$Holds(many\_mortgages\_forclosed, i)$

(2) $\neg Holds(house\_price\_rise, (0, 1)) \wedge$
$\neg Holds(house\_price\_rise, (9, 10)) \wedge$
$\neg Holds(many\_mortgages\_forclosed, (9, 10))$

$\langle\{(1), (2)\}, Holds(many\_mortgages\_forclosed, (0, 1))\rangle$ is not an argument.

- (1) and (2) are inconsistent in time period (9,10)

# Database Ground Arguments

A **database ground argument** is a pair $\langle \Phi, \alpha \rangle$, assuming a database $\Delta$, such that:

(1) $\Phi \not\vdash_{\mathcal{T}} \perp$

(2) $\Phi \vdash_{\mathcal{T}} \alpha$

(3) $\Phi$ is a minimal subset of $G_C(\Delta)$ satisfying (1) and (2)

With the database:

(1) $Holds(high\_sales, i) \wedge (j \textbf{ during } i) \rightarrow Holds(high\_sales, j)$

(2) $Holds(high\_sales, (0,8)) \wedge \neg Holds(high\_sales, (6,7))$

We have a database ground argument:
$\langle \{(2), (1)_{i=(0,8), j=(3,4)}\}, Holds(high\_sales, (3,4)) \rangle$

# Support Ground Arguments

A **support ground argument** is a pair $\langle \Phi, \alpha \rangle$, assuming a database $\Delta$, such that:

(1) $\Phi \not\vdash_{\mathcal{T}} \perp$

(2) $\Phi \vdash_{\mathcal{T}} \alpha$

(3) $\Phi$ is a minimal subset of $G_0(\Delta)$ satisfying (1) and (2)

With the database:

(1) $Holds(high\_sales, i) \land (j\ \mathbf{during}\ i) \rightarrow Holds(high\_sales, j)$

(3) $Holds(high\_sales, (0, 5))$

We have a support ground argument:
$\langle \{(3), (1)\}, Holds(high\_sales, (3, 4)) \rangle$

# Examples

With the database:

(1) $Holds(high\_sales, i) \wedge (j \textbf{ during } i) \rightarrow Holds(high\_sales, j)$

(2) $Holds(high\_sales, (0, 8)) \wedge \neg Holds(high\_sales, (6, 7))$

(3) $Holds(high\_sales, (0, 5))$

We have three arguments for $Holds(high\_sales, (3, 4))$

- S1: $\langle \{(3), (1)\}, Holds(high\_sales, (3, 4))\rangle$
- D1: $\langle \{(3), (1)_{i=(0,5),j=(3,4)}\}, Holds(high\_sales, (3, 4))\rangle$
- D2: $\langle \{(2), (1)_{i=(0,8),j=(3,4)}\}, Holds(high\_sales, (3, 4))\rangle$

# Relationships

**Theorem**

Given a finite database $\Delta$, for every support ground argument $\langle \Phi, \alpha \rangle$, there is at least one database ground argument $\langle \Psi, \alpha \rangle$, such that $\Psi \subseteq G_C(\Phi)$

- We have already shown the reverse does not hold; there are database ground arguments which have no comparable support ground argument.

# Undercuts

Returning to the database

(1) $Holds(high\_sales, i) \wedge (j \textbf{ during } i) \rightarrow Holds(high\_sales, j)$

(2) $Holds(high\_sales, (0, 8)) \wedge \neg Holds(high\_sales, (6, 7))$

(3) $Holds(high\_sales, (0, 5))$

And the arguments:

- S1: $\langle\{(3), (1)\}, Holds(high\_sales, (3, 4))\rangle$
- D1: $\langle\{(3), (1)_{i=(0,5),j=(3,4)}\}, Holds(high\_sales, (3, 4))\rangle$

$\langle\{(2)\}, \diamond\rangle$ is a undercut for S1, but not for D1.

# Argument Trees

Considering the following database:

(1) $Holds(\beta, i) \rightarrow Holds(\alpha, i)$

(2) $Holds(\alpha, (1,2)) \land Holds(\beta, (2,3)) \land \neg Holds(\alpha, (2,3))$

(3) $Holds(\beta, (1,2)) \land Holds(\beta, (3,4)) \land \neg Holds(\alpha, (3,4))$

(4) $Holds(\beta, (5,6)) \land \neg Holds(\alpha, (5,6))$

We wish to argue for $Holds(\alpha, (1,2))$.

We have the following support ground argument tree:

$$\langle \{(2)\}, Holds(\alpha, (1,2)) \rangle$$

$$\uparrow$$

$$\langle \{(1)\}, \diamond \rangle$$

$$\nearrow \qquad\qquad \nwarrow$$

$$\langle \{(3)\}, \diamond \rangle \qquad\qquad\qquad\qquad \langle \{(4)\}, \diamond \rangle$$

# Example Trees, continued

We have the following pair of database ground argument trees,

- One related to the support ground argument tree

$$\langle \{(2)\}, Holds(\alpha, (1,2)) \rangle$$
$$\uparrow$$
$$\langle \{(1)_{i=(2,3)}\}, \Diamond \rangle$$

- One which does not have a corresponding support ground argument tree.

$$\langle \{(3), (1)_{i=(1,2)}\}, Holds(\alpha, (1,2)) \rangle$$
$$\uparrow$$
$$\langle \{(1)_{i=(3,4)}\}, \Diamond \rangle$$

# Problem trees

(1) $Holds(\alpha, i)$

(2) $Holds(\beta, i)$

(3) $Holds(\beta, i) \rightarrow \neg Holds(\alpha, i)$

The support ground argument tree for $Holds(\alpha, i)$.

$$\langle \{(1)\}, Holds(\alpha, i) \rangle$$
$$\uparrow$$
$$\langle \{(2), (3)\}, \diamond \rangle$$

The database ground argument tree for $Holds(\alpha, i)$.

$$\langle G_C((1)), Holds(\alpha, i) \rangle$$
$$\nearrow \qquad\qquad\qquad \uparrow$$
$$\langle \{(2)_{i=(1,2)}, (3)_{i=(1,2)}\}, \diamond \rangle \quad \langle \{(2)_{i=(2,3)}, (3)_{i=(2,3)}\}, \diamond \rangle \quad \ldots$$

- Leaves are $\langle \{(2)_{i=(a,b)}, (3)_{i=(a,b)}\}, \diamond \rangle$ where $a \in T, b \in T, a \prec b$

# Conclusions

- There is a need for temporal knowledge in argumentation
- A calculus $\mathcal{T}$ can be defined in terms of propositional logic
- There is the possibility of support ground and database ground arguments
- Each type of argument has its advantages and disadvantages.