Modeling Persuasiveness: change of uncertainty through agents’ interactions

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Outline

1. PERSEUS Project

2. Strength and dynamics of persuasion
   - Example
   - Formal models

3. Formalization
   - Syntax and semantics
   - Axiomatization

4. Investigation of the persuasion systems
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PERSEUS

PERsuasiveness: Studies on the Effective USE of arguments

Figure: The Perseus Project and the Formal Theory of Persuasion.
The notion of persuasion

Definition (Walton and Krabbe)

Persuasion dialogue - dialogue of which initial situation is a conflict of opinion and the aim is to resolve this conflict by verbal means and thereby influence the change of agents’ beliefs

The aspects of persuasion we want to model:

1. **Persuasiveness** - a degree of changes in the agent’s beliefs induced by the persuasion

2. **Dynamics of persuasion** - tracking changes in the belief state of an agent at any intermediate stage of the persuasion
The aim of our theory

Investigation into **properties of persuasion systems** based on existing theories (instead of developing and implementing arguing agents or determining their architecture and specification)

1. **Logic** allowing to express such properties of multi-agent systems
2. **Software system** allowing to examine selected multi-agent systems
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Ann and Paul discuss where John is spending his summer holidays this year.

Ann allows scenarios in which John is in Italy, Spain or Peru.

Paul wants to convince her that John is in Alaska.

**Figure:** Before the persuasion
**Paul**: Last time I met John in a restaurant he told me about great discounts for vacation in Alaska.

**Ann**: Hm, Alaska - I really don't know. But it could be interesting...

**Figure**: An argument $a_1$
- **Paul**: You know that John likes original places.
- **Ann**: Yes, you are right. He wouldn’t choose Italy or Spain - it would be too trivial for him.

**Figure**: An argument $a_2$
Paul: Do you know that he spent whole month in Peru last year?

Ann: Really? He wouldn’t visit the same place twice!

Figure: An argument $a_3$
Assumptions

The thesis T: "John spends his summer holidays in Alaska"

1. START: Ann is absolutely sure that T is false
2. Intermediate stages: each successive argument increases her certainty that T is true
3. END: after $a_3$ Ann is absolutely sure that T is true
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Motivation

The **formal tool** that allows to:

1. **express persuasiveness**, i.e. a *degree of changes* in Ann’s beliefs
   - in what degree Ann is convinced of T after the given argumentation
   - one argumentation may be more persuasive than the other one

2. **track the changes** in her belief state at *any intermediate stage* of the persuasion
   - how Ann reacts after each successive argument
   - the changes in her beliefs after $a_1$, then after $a_2$ and finally after $a_3$
NON-GRADED DOXASTIC LOGIC
The **degrees of belief** of an agent with respect to a thesis T:

1. **$B(\neg T)$** - a negative belief
   - the agent believes T is false

2. **$N(T)$** - a neutral belief
   - the agent is not sure if T is true or false
   - $N(T) \text{ wtw } \neg B(T) \land \neg B(\neg T)$

3. **$B(T)$** - a positive belief
   - the agent believes T is true
Dynamics in non-graded logic

Figure: Dynamics of persuasion
The "Alaska" example

Figure: The change of beliefs induced by Paul’s argumentation
GRADED BELIEFS
If we wanted to describe **three types of uncertainty**, our model should include five belief states:

1. 0 - absolutely negative beliefs
2. \(\frac{1}{4}\) - rather negative beliefs
3. \(\frac{1}{2}\) - "fifty-fifty"
4. \(\frac{3}{4}\) - rather positive beliefs
5. 1 - absolutely positive beliefs
Dynamics in the model of graded beliefs

Figure: Dynamics of persuasion
The "Alaska" example

Figure: The change of beliefs induced by Paul’s argumentation
The extension

Figure: The extension of the model of beliefs’ change
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GRADED BELIEFS
Logic of graded modalities:
Wiebe van der Hoek, Modalities for reasoning about knowledge and quantities, Amsterdam, 1992
The basic formula we use for expressing *uncertainty* is:

$$M^T_{j ! d_1, d_2}$$

where $d_1, d_2$ are natural numbers.

- Intuitively: in exactly $d_1$ doxastic alternatives the thesis $T$ is true among $d_2$ doxastic alternatives the agent $j$ considers as possible.
- We say that $j$ believes $T$ with degree $\frac{d_1}{d_2}$. 
The "Alaska" example

\[ \mathcal{M}, s_1 \models M_{\text{aud}}^{0,3} T \] since exactly 0 states satisfy T among 3 accessible states considered by the audience.

**Figure:** Uncertainty of Ann about the place where John is spending holidays.
Graded modalities

Other doxastic operators

- $M^d_i \alpha$ - agent $i$ considers more than $d$ accessible worlds verifying $\alpha$
- $B^d_i \alpha$ - agent $i$ reckons with at most $d$ exceptions for $\alpha$
- $M!^d_i \alpha$ - agent $i$ considers exactly $d$ accessible worlds verifying $\alpha$
CHANGE OF GRADED BELIEFS
Inspiration

Dynamic logic:

Algorithmic logic:
Basic formula

The basic formula which expresses the change of uncertainty is:

$$\Diamond (i : P) M!_{j}^{d_1,d_2} T$$

Intuitively: after execution of a sequence of arguments $P$ performed by $i$ it is possible that $j$ will believe $T$ with degree $\frac{d_1}{d_2}$. 
The "Alaska" example

\[ \mathcal{M}, s_1 \models \diamond (\text{prop} : a_1 ; a_2 ; a_3 ) \mathcal{M}^{1,1}_{\text{aud}} T \]

**Figure:** The change of Ann’s uncertainty during the persuasion.
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Syntax

The set $F$ of all well-formed expressions of $AG_n$ is given by the following Backus-Naur Form (BNF):

$$\alpha ::= p | \neg \alpha | \alpha \lor \alpha | M^d_i \alpha | \Diamond (i : P) \alpha,$$

where $p$ is a propositional variable, $d$ is a natural number, $P$ is a program scheme, $i \in \{1, \ldots, n\}$ is a name of an agent.
Model

Definition

Let $Agt = \{1, 2, \ldots, n\}$ be a finite set of agents. By a semantic model we mean a Kripke structure $\mathcal{M} = (S, RB, I, v)$ where

- $S$ is a non-empty set of states,
- $RB$ is a doxastic function, $RB : Agt \rightarrow 2^{S \times S}$, where for every $i \in Agt$, the relation $RB(i)$ is serial, transitive and euclidean,
- $I$ is an interpretation of the program variables, $I : \Pi_0 \rightarrow (Agt \rightarrow 2^{S \times S})$, where for every $a \in \Pi_0$ and $i \in Agt$, the relation $I(a)(i)$ is serial, and $I(Id)(i) = \{(s, s) : s \in S\}$, where $Id$ is a program constant which means identity,
- $v : S \rightarrow \{0, 1\}^{V_0}$ is a valuation function.
Formalization
Syntax and semantics

Semantics

Definition

For a given structure $\mathcal{M} = (S, RB, I, \nu)$ and a given state $s \in S$ the boolean value of the formula $\alpha$ is denoted by $\mathcal{M}, s \models \alpha$ and is defined inductively as follows:

- $\mathcal{M}, s \models p$ iff $\nu(s)(p) = 1$, for $p \in V_0$.
- $\mathcal{M}, s \models \neg \alpha$ iff $\mathcal{M}, s \not\models \alpha$.
- $\mathcal{M}, s \models \alpha \lor \beta$ iff $\mathcal{M}, s \models \alpha$ or $\mathcal{M}, s \models \beta$.
- $\mathcal{M}, s \models M^d_i \alpha$ iff $|\{s' \in S : (s, s') \in RB(i) \text{ and } \mathcal{M}, s' \models \alpha\}| > d$, $d \in \mathbb{N}$.
- $\mathcal{M}, s \models \Diamond (i : P) \alpha$ iff $\exists s' \in S ((s, s') \in I\Pi(P)(i) \text{ and } \mathcal{M}, s' \models \alpha)$. 
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### Inference rules

<table>
<thead>
<tr>
<th>Rule</th>
<th>Premises</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>R1</strong></td>
<td>$\alpha, \alpha \rightarrow \beta$</td>
<td>$\beta$</td>
</tr>
<tr>
<td><strong>R2</strong></td>
<td>$\alpha$</td>
<td>$B^0_i \alpha$</td>
</tr>
<tr>
<td><strong>R3</strong></td>
<td>$\alpha$</td>
<td>$\Box(i:P)\alpha$</td>
</tr>
</tbody>
</table>

### Axioms

<table>
<thead>
<tr>
<th>Axiom</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>[A0]</td>
<td>classical propositional tautologies</td>
</tr>
<tr>
<td>[A1]</td>
<td>$M^d_{i+1}\alpha \rightarrow M^d_i\alpha$ (analogue of modal system K)</td>
</tr>
<tr>
<td>[A2]</td>
<td>$B^0_i(\alpha \rightarrow \beta) \rightarrow (M^d_i\alpha \rightarrow M^d_i\beta)$</td>
</tr>
<tr>
<td>[A3]</td>
<td>$M!^0_i(\alpha \land \beta) \rightarrow ((M!^d_1\alpha \land M!^d_2\beta) \rightarrow M!^d_{i+1} \land M!^d_{i+2} (\alpha \lor \beta))$</td>
</tr>
<tr>
<td>[A4]</td>
<td>$M^d_i\alpha \rightarrow B^0_i M^d_i\alpha$ (negative introspection)</td>
</tr>
<tr>
<td>[A5]</td>
<td>$M^0_i M^d_i\alpha \rightarrow M^d_i\alpha$ (positive introspection)</td>
</tr>
<tr>
<td>[A6]</td>
<td>$M^0_i(\text{true})$ (consistency of beliefs)</td>
</tr>
<tr>
<td>[A7]</td>
<td>$\Box(i : P)(\alpha \rightarrow \beta) \rightarrow (\Box(i : P)\alpha \rightarrow \Box(i : P)\beta)$</td>
</tr>
<tr>
<td>[A8]</td>
<td>$\Box(i : P)(\alpha \land \beta) \leftrightarrow (\Box(i : P)\alpha \land \Box(i : P)\beta)$</td>
</tr>
<tr>
<td>[A9]</td>
<td>$\Box(i : P_1; P_2)\alpha \leftrightarrow \Box(i : P_1)(\Box(i : P_2)\alpha)$</td>
</tr>
<tr>
<td>[A10]</td>
<td>$\Box(i : P)\alpha \rightarrow \Diamond(i : P)\alpha$</td>
</tr>
<tr>
<td>[A11]</td>
<td>$\Box(i : P)\text{true}$</td>
</tr>
<tr>
<td>[A12]</td>
<td>$\Box(i : ld)\alpha \leftrightarrow \alpha$</td>
</tr>
</tbody>
</table>
Soundness and completeness

Theorem

$\mathcal{AG}_n$ is sound and complete with respect to $\mathcal{M}$.

The proof is based on the completeness results for normal modal logics with graded modalities, epistemic logics, and dynamic logics (the technique of the canonical models for classical modal logics).

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We would like to learn about properties of the persuasion systems such as:

- "What chances has a persuader to influence a degree of others’ beliefs about a given thesis?",
- "How significant will be such a change?",
- "Would rearrangement of arguments give better or worse effect?",
etc.
Questions’ grammar

Context-free grammar

\[ \phi ::= \omega | \neg \phi | \phi \lor \phi | M_i^d \phi | \Diamond (i : P) \phi | M_i? \omega | \Diamond (i : ?) \omega \]

where \( \omega \) is defined as follows

\[ \omega ::= p | \neg \omega | \omega \lor \omega | M_i^d \omega | \Diamond (i : P) \omega \]

and \( p \in V_0, \; d \in \mathbb{N}, \; i \in \text{Agt}. \)
Examples of questions

Verification of a property

\[ \mathcal{M}, s \models \Diamond (ag1 : a1; a2; a3) M_{ag2}^{2,3} p \]

Question about the degree of beliefs

\[ \mathcal{M}, s \models \Diamond (ag1 : a1; a2; a3) M_{ag2}^{?,?} p \]

Question about arguments

\[ \mathcal{M}, s \models \Diamond (ag1 : ?) M_{ag2}^{2,3} p \]
Investigation of the persuasion systems

Figure: PERSEUS - the program window
Investigation of the persuasion systems

Figure: PERSEUS generates the graph of the model
Investigation of the persuasion systems

Figure: PERSEUS verifies the property
Investigation of the persuasion systems

Figure: PERSEUS solves the question

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Modeling Persuasiveness
Summary

- Formal model of persuasion including *dynamics* of this process and *uncertainty* of beliefs.
- Logic in which we can express the properties of persuasion.
- Investigation of persuasion systems.
Thank you.

Figure: to be continued...
For Further Reading

K. Budzyńska and M. Kacprzak.
A logic for reasoning about persuasion.
*Fundamenta Informaticae*, IOS Press 85(2008).

K. Budzyńska and M. Kacprzak and P. Rembelski
Investigation into properties of persuasion systems.
*Proc. of Workshop on Logics for Agents and Mobility (LAM’08)* 2008.

K. Budzyńska and M. Kacprzak
Aristotle, Rhetoric and Probability.

http://perseus.ovh.org/