

---

# A Systematic Classification of Argumentation Frameworks where Semantics Agree

**Pietro Baroni, Massimiliano Giacomin**

{baroni, giacomin}@ing.unibs.it



DEA - Dipartimento di Elettronica per l'Automazione  
Università degli Studi di Brescia (Italy)

# Many semantics

---

A variety of argumentation semantics have been proposed in the context of Dung's framework

Traditional semantics (Dung 95):

- Grounded
- Complete
- Stable
- Preferred

Recent semantics include:

- CF2 (Baroni, Giacomin, Guida 05)
- Semi-stable (Caminada 06)
- Ideal (Dung, Mancarella, Toni 06)

France or Italy?

---

W la différence !

# France or Italy?

---

W la différence !

- alternative intuitions and viewpoints

# France or Italy?

---

## W la différence !

- alternative intuitions and viewpoints
- suitability for different application domains

# France or Italy?

---

## W la différence !

- alternative intuitions and viewpoints
- suitability for different application domains
- fruitful debates opening new research directions

France or Italy?

---

Don't look to what divides you

France or Italy?

---

Don't look to what divides you ...





## France or Italy?

---

Don't look to what divides you, look to what unites you! (Pope John XXIII)

## France or Italy?

---

Don't look to what divides you, look to what unites you! (Pope John XXIII)



## France or Italy?

---

Don't look to what divides you, look to what unites you (Pope John XXIII)

- common behavior in some (many, most?) cases

## France or Italy?

---

Don't look to what divides you, look to what unites you (Pope John XXIII)

- common behavior in some (many, most?) cases
- shared principles behind (partial) differences

## France or Italy?

---

Don't look to what divides you, look to what unites you (Pope John XXIII)

- common behavior in some (many, most?) cases
- shared principles behind (partial) differences
- basic reference behavior

## France or Italy?

---

Don't look to what divides you, look to what unites you (Pope John XXIII)

- common behavior in some (many, most?) cases
- shared principles behind (partial) differences
- basic reference behavior
- (ir)relevance of choosing a specific semantics

# Aim of the work

---

Creating a systematic basis for the study of agreement between argumentation semantics by

providing a classification of argumentation frameworks with respect to the issue of semantics agreement

considering the seven semantics mentioned before:

- **GR**ounded
- **CO**mplete
- **ST**able
- **PR**eferred
- **CF2**
- **SemiST**able
- **ID**eal

# Presentation plan

---

Basic concepts and review of existing results



# Presentation plan

---

Basic concepts and review of existing results

Description of the analysis carried out

# Presentation plan

---

Basic concepts and review of existing results

Description of the analysis carried out

Unique-status agreement

# Presentation plan

---

Basic concepts and review of existing results

Description of the analysis carried out

Unique-status agreement

Multiple-status agreement

# Presentation plan

---

Basic concepts and review of existing results

Description of the analysis carried out

Unique-status agreement

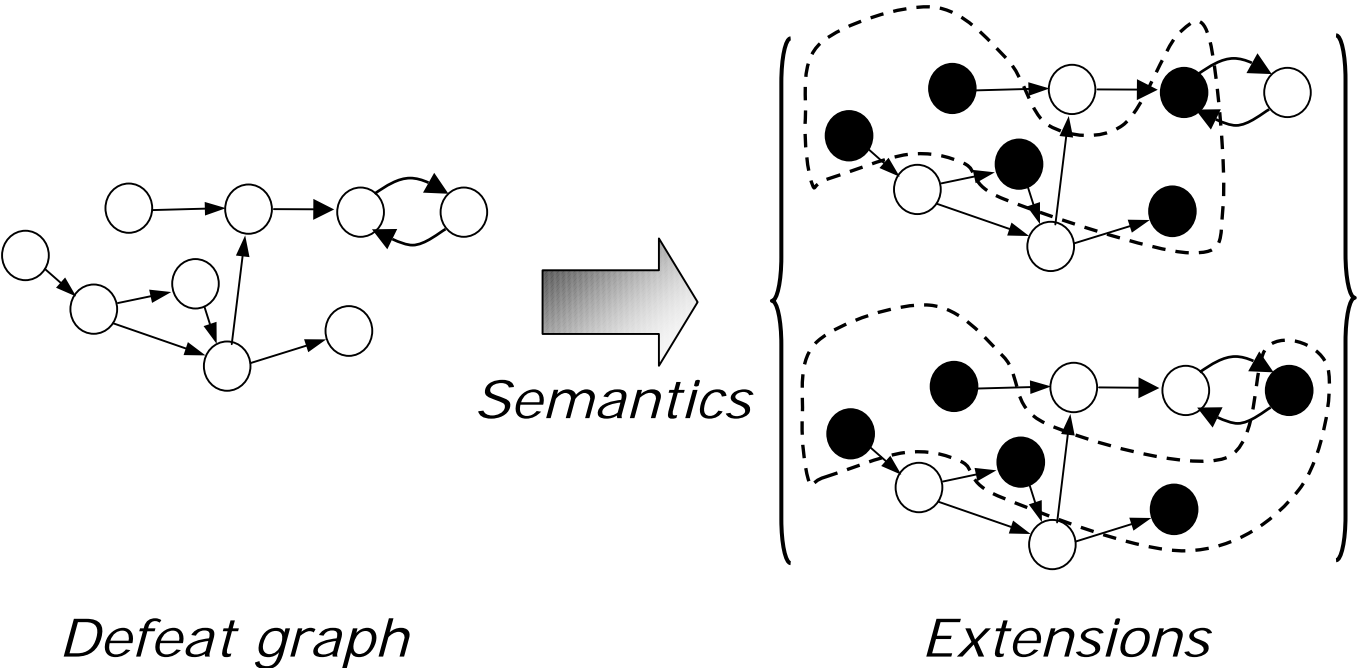
Multiple-status agreement

Conclusions

# Dung's Argumentation Framework

$$AF = \langle A, \rightarrow \rangle$$

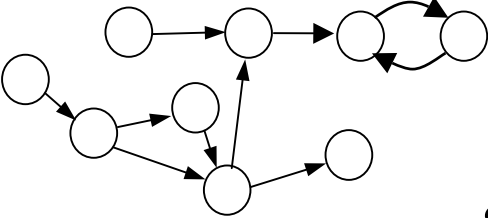
attack relation  
arguments



# Dung's Argumentation Framework

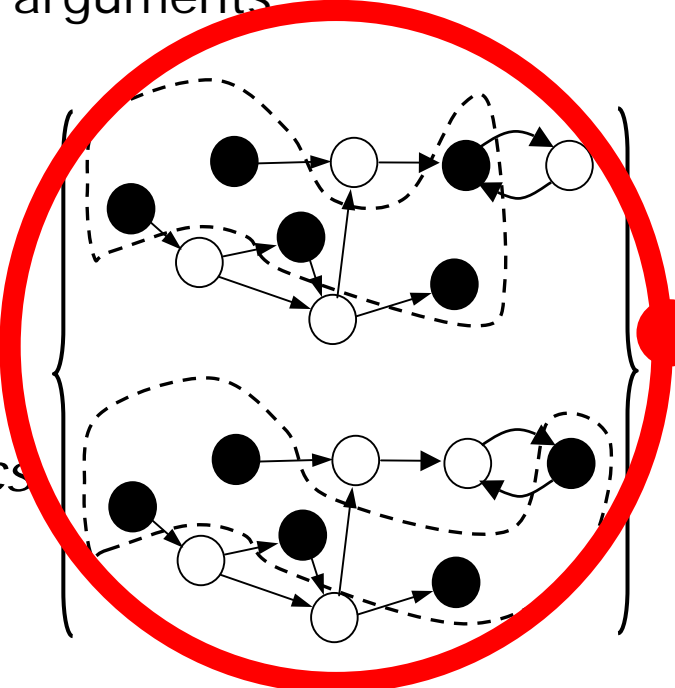
$$AF = \langle A, \rightarrow \rangle$$

attack relation  
arguments



*Defeat graph*

*Semantics*



*Extensions*

$$\mathcal{E}_S(AF)$$

The set of extensions prescribed by semantics  $S$  for AF

## Definition of semantics agreement

---

Two semantics  $\mathcal{S}_1$  and  $\mathcal{S}_2$  are in agreement about an argumentation framework AF if  $\mathcal{E}_{\mathcal{S}_1}(\text{AF}) = \mathcal{E}_{\mathcal{S}_2}(\text{AF})$

We require that both  $\mathcal{S}_1$  and  $\mathcal{S}_2$  admit extensions for AF, namely  $\mathcal{E}_{\mathcal{S}_1}(\text{AF}) \neq \emptyset$  and  $\mathcal{E}_{\mathcal{S}_2}(\text{AF}) \neq \emptyset$

In other words, for a semantics  $\mathcal{S}$ , agreement is evaluated only about argumentation frameworks where  $\mathcal{S}$  is defined

Only **ST** may be undefined for some AF

## Existing results on agreement

---

Grounded, stable and preferred semantics are in agreement about AF if AF is well-founded [Dung, AIJ 95]  
(when AF is finite, well-founded is equivalent to acyclic)

Stable and preferred semantics are in agreement about AF if AF is limited controversial [Dung, AIJ 95]  
(when AF is finite, limited controversial is equivalent to free of odd-length cycles)

Stable, preferred and naïve semantics are in agreement about AF if AF is symmetric (all attacks are mutual)  
[Coste-Marquis et al., ECSQARU 05]

Agreement in three topological classes of argumentation frameworks related with the notion of strongly connected-components investigated in [Baroni&Giacomin, ARGNMR 07]



## Existing results on agreement vs. ...

---

Relatively limited attention to the issue of agreement after Dung's paper, with some recent revival

**A few** agreement classes identified considering mainly **topological properties** of argumentation frameworks

## ... a complementary perspective

---

Relatively limited attention to the issue of agreement after Dung's paper, with some recent revival

A few agreement classes identified considering mainly **topological properties** of argumentation frameworks

Systematic identification of **all possible agreement classes** (given the considered set of semantics)

Based on general **set-theoretical properties of semantics extensions** rather than on topological properties of argumentation frameworks

## Agreement classes: notation and basic properties

---

Given a set  $\mathcal{S}$  of argumentation semantics, the set of argumentation frameworks where all semantics in  $\mathcal{S}$  agree will be denoted as  $AGR(\mathcal{S})$

E.g.  $AGR(\{PR, ST, SST\})$

denotes the set of argumentation frameworks where preferred, stable and semi-stable semantics agree

Clearly  $\mathcal{S}_1 \subseteq \mathcal{S}_2 \Rightarrow AGR(\mathcal{S}_2) \subseteq AGR(\mathcal{S}_1)$

E.g.

$AGR(\{ID, ST, PR, SST\}) \subseteq AGR(\{PR, ST, SST\})$

It may be (and it is) the case that for some different sets of semantics  $\mathcal{S}_1$  and  $\mathcal{S}_2$  it holds that  $AGR(\mathcal{S}_1) = AGR(\mathcal{S}_2)$

## Agreement classes: how many?

---

Given that we consider a set of 7 argumentation semantics,

$$\Omega = \{GR, ID, CO, PR, ST, CF2, SST\}$$

any subset  $\mathcal{S}$  of  $\Omega$  such that  $|\mathcal{S}| \geq 2$  gives rise, in principle, to an agreement class (**120** classes in total)

We have proved that most of these 120 classes are not actually different: only **14** distinct classes exist

# Agreement classes: which kind of analysis?

---

Agreement classes are denoted as  $\Sigma_1 \dots \Sigma_{14}$

We proceed by partial order of inclusion: if  $\Sigma_i \subset \Sigma_j$  then  $j > i$

For each class  $\Sigma_i$  three main steps have been carried out:

1. identifying which classes  $\Sigma_k$ , with  $k < i$ , are included in  $\Sigma_i$
2. for each of these classes  $\Sigma_k$ , showing that  $\Sigma_i \setminus \Sigma_k \neq \emptyset$
3. for any  $\Sigma_h$  with  $h < i$  and  $\Sigma_h \not\subset \Sigma_i$ , examining  $\Sigma_i \cap \Sigma_h$

For any set  $\mathcal{S}$  of semantics not directly corresponding to any of  $\Sigma_1 \dots \Sigma_{14}$  it is shown that  $AGR(\mathcal{S})$  coincides with one of them

## Agreement classes: which (known) properties we use?

---

Several kinds of inclusion relations between (sets of) extensions:  
for any argumentation framework AF

- $\mathcal{E}_{\mathcal{PR}}(\text{AF}) \subseteq \mathcal{E}_{\mathcal{CO}}(\text{AF})$
- $\mathcal{E}_{\mathcal{SST}}(\text{AF}) \subseteq \mathcal{E}_{\mathcal{PR}}(\text{AF})$
- $\text{GE}(\text{AF}) \in \mathcal{E}_{\mathcal{CO}}(\text{AF})$
- $\forall E \in \mathcal{E}_{\mathcal{S}}(\text{AF})$   
 $\text{GE}(\text{AF}) \subseteq E$  with  $\mathcal{S} \in \{\mathcal{CO}, \mathcal{PR}, \mathcal{ST}, \mathcal{SST}, \mathcal{CF2}\}$
- $\forall E \in \mathcal{E}_{\mathcal{CO}}(\text{AF}) \exists E' \in \mathcal{E}_{\mathcal{PR}}(\text{AF}) : E \subseteq E'$
- $\forall E \in \mathcal{E}_{\mathcal{PR}}(\text{AF}) \exists E' \in \mathcal{E}_{\mathcal{CF2}}(\text{AF}) : E \subseteq E'$

## Agreement classes: which (known) properties we use?

---

Several kinds of inclusion relations between (sets of) extensions:  
for any argumentation framework AF

$$\mathcal{E}_{\mathcal{PR}}(\text{AF}) \subseteq \mathcal{E}_{\mathcal{CO}}(\text{AF})$$

$$\bullet \mathcal{E}_{\mathcal{SST}}(\text{AF}) \subseteq \mathcal{E}_{\mathcal{PR}}(\text{AF})$$

$$\text{GE}(\text{AF}) \in \mathcal{E}_{\mathcal{CO}}(\text{AF})$$

$$\bullet \forall E \in \mathcal{E}_{\mathcal{S}}(\text{AF})$$

$$\text{GE}(\text{AF}) \subseteq E \text{ with } \mathcal{S} \in \{\mathcal{CO}, \mathcal{PR}, \mathcal{ST}, \mathcal{SST}, \mathcal{CF2}\}$$

$$\bullet \forall E \in \mathcal{E}_{\mathcal{CO}}(\text{AF}) \exists E' \in \mathcal{E}_{\mathcal{PR}}(\text{AF}) : E \subseteq E'$$

$$\bullet \forall E \in \mathcal{E}_{\mathcal{PR}}(\text{AF}) \exists E' \in \mathcal{E}_{\mathcal{CF2}}(\text{AF}) : E \subseteq E'$$

**Inclusion of the whole  
set of extensions**

## Agreement classes: which (known) properties we use?

---

Several kinds of inclusion relations between (sets of) extensions:  
for any argumentation framework AF

$$\bullet \mathcal{E}_{\mathcal{PR}}(\text{AF}) \subseteq \mathcal{E}_{\mathcal{CO}}(\text{AF})$$

$$\bullet \mathcal{E}_{\mathcal{SST}}(\text{AF}) \subseteq \mathcal{E}_{\mathcal{PR}}(\text{AF})$$

$$\bullet \text{GE}(\text{AF}) \in \mathcal{E}_{\mathcal{CO}}(\text{AF})$$

**The grounded  
extension is included  
in many kinds of  
extensions**

$$\bullet \forall E \in \mathcal{E}_{\mathcal{S}}(\text{AF})$$

$$\text{GE}(\text{AF}) \subseteq E \text{ with } \mathcal{S} \in \{\mathcal{CO}, \mathcal{PR}, \mathcal{ST}, \mathcal{SST}, \mathcal{CF}_2\}$$

$$\bullet \forall E \in \mathcal{E}_{\mathcal{CO}}(\text{AF}) \exists E' \in \mathcal{E}_{\mathcal{PR}}(\text{AF}) : E \subseteq E'$$

$$\bullet \forall E \in \mathcal{E}_{\mathcal{PR}}(\text{AF}) \exists E' \in \mathcal{E}_{\mathcal{CF}_2}(\text{AF}) : E \subseteq E'$$



## Agreement classes: which (known) properties we use?

---

Several kinds of inclusion relations between (sets of) extensions:  
for any argumentation framework AF

- $\mathcal{E}_{\mathcal{PR}}(\text{AF}) \subseteq \mathcal{E}_{\mathcal{CO}}(\text{AF})$

- $\mathcal{E}_{\mathcal{SST}}(\text{AF}) \subseteq \mathcal{E}_{\mathcal{PR}}(\text{AF})$

- $\text{GE}(\text{AF}) \in \mathcal{E}_{\mathcal{CO}}(\text{AF})$

- $\forall E \in \mathcal{E}_{\mathcal{S}}(\text{AF})$

- $\text{GE}(\text{AF}) \subseteq E$  with  $\mathcal{S} \in \{\mathcal{CO}, \mathcal{PR}, \mathcal{ST}, \mathcal{SST}, \mathcal{CF2}\}$

- $\forall E \in \mathcal{E}_{\mathcal{CO}}(\text{AF}) \exists E' \in \mathcal{E}_{\mathcal{PR}}(\text{AF}) : E \subseteq E'$

- $\forall E \in \mathcal{E}_{\mathcal{PR}}(\text{AF}) \exists E' \in \mathcal{E}_{\mathcal{CF2}}(\text{AF}) : E \subseteq E'$

**Any extension of one kind  
is included in an extension  
of another kind**

## Agreement classes: which kind of properties we prove?

---

As intermediate steps, we have proved several lemmata based on the inclusion relationships like the following

**Lemma 3** *If  $|\mathcal{E}_{CO}(AF)| = 1$ , then  $\mathcal{E}_{CO}(AF) = \mathcal{E}_{GR}(AF) = \mathcal{E}_{PR}(AF)$*

**Lemma 4** *Let  $S \in \{\mathcal{PR}, \mathcal{ST}, \mathcal{SST}, \mathcal{CF2}\}$ , if  $GE(AF) \in \mathcal{E}_S(AF)$   
then  $\mathcal{E}_S(AF) = \{GE(AF)\}$*

**Lemma 6** *If  $\mathcal{E}_{ST}(AF) = \{GE(AF)\}$  then  $\mathcal{E}_{CF2}(AF) = \{GE(AF)\}$*

**Lemma 8** *If  $\mathcal{E}_{CF2}(AF) = \mathcal{E}_{ID}(AF) = \{ID(AF)\}$   
then  $\mathcal{E}_{CF2}(AF) = \mathcal{E}_{PR}(AF)$*

**Lemma 10** *If  $\mathcal{E}_{CF2}(AF) \subseteq \mathcal{AS}(AF)$  then  $\mathcal{E}_{CF2}(AF) = \mathcal{E}_{PR}(AF)$*

## Agreement classes: which kind of properties we prove?

---

As intermediate steps, we have proved several lemmata based on the inclusion relationships like the following

**Lemma 3** *If  $|\mathcal{E}_{CO}(AF)| = 1$  then  $\mathcal{E}_{CO}(AF) = \mathcal{E}_{GR}(AF) = \mathcal{E}_{PR}(AF)$*

**Lemma 4** *Let  $S \in \{PR, ST, SST, CF2\}$ , if  $GE(AF) \in \mathcal{E}_S(AF)$  then  $\mathcal{E}_S(AF) = \{GE(AF)\}$*  **Implications of cardinality**

**Lemma 6** *If  $\mathcal{E}_{ST}(AF) = \{GE(AF)\}$  then  $\mathcal{E}_{CF2}(AF) = \{GE(AF)\}$*

**Lemma 8** *If  $\mathcal{E}_{CF2}(AF) = \mathcal{E}_{ID}(AF) = \{ID(AF)\}$  then  $\mathcal{E}_{CF2}(AF) = \mathcal{E}_{PR}(AF)$*

**Lemma 10** *If  $\mathcal{E}_{CF2}(AF) \subseteq \mathcal{AS}(AF)$  then  $\mathcal{E}_{CF2}(AF) = \mathcal{E}_{PR}(AF)$*

## Agreement classes: which kind of properties we prove?

---

As intermediate steps, we have proved several lemmata based on the inclusion relationships like the following

**Lemma 3** *If  $|\mathcal{E}_{CO}(AF)| = 1$ , then  $\mathcal{E}_{CO}(AF) = \mathcal{E}_{GR}(AF) = \mathcal{E}_{PR}(AF)$*

**Lemma 4** *Let  $S \in \{PR, ST, SST, CF2\}$ , if  $GE(AF) \in \mathcal{E}_S(AF)$  then  $\mathcal{E}_S(AF) = \{GE(AF)\}$*

**Lemma 6** *If  $\mathcal{E}_{ST}(AF) = \{GE(AF)\}$  then  $\mathcal{E}_{CF2}(AF) = \{GE(AF)\}$*

**Lemma 8** *If  $\mathcal{E}_{CF2}(AF) = \mathcal{E}_{ID}(AF) = \{$*  **Implications of inclusion in the set of extensions** *then  $\mathcal{E}_{CF2}(AF) = \mathcal{E}_{PR}(AF)$*

**Lemma 10** *If  $\mathcal{E}_{CF2}(AF) \subseteq \mathcal{AS}(AF)$  then  $\mathcal{E}_{CF2}(AF) = \mathcal{E}_{PR}(AF)$*

## Agreement classes: which kind of properties we prove?

---

As intermediate steps, we have proved several lemmata based on the inclusion relationships like the following

**Lemma 3** *If  $|\mathcal{E}_{CO}(AF)| = 1$ , then  $\mathcal{E}_{CO}(AF) = \mathcal{E}_{GR}(AF) = \mathcal{E}_{PR}(AF)$*

**Lemma 4** *Let  $S \in \{PR, ST, SST, CF2\}$ , if  $GE(AF) \in \mathcal{E}_S(AF)$   
then  $\mathcal{E}_S(AF) = \{GE(AF)\}$*

**Some agreements  
imply others**

**Lemma 6** *If  $\mathcal{E}_{ST}(AF) = \{GE(AF)\}$  then  $\mathcal{E}_{CF2}(AF) = \{GE(AF)\}$*

**Lemma 8** *If  $\mathcal{E}_{CF2}(AF) = \mathcal{E}_{ID}(AF) = \{ID(AF)\}$   
then  $\mathcal{E}_{CF2}(AF) = \mathcal{E}_{PR}(AF)$*

**Lemma 10** *If  $\mathcal{E}_{CF2}(AF) \subseteq \mathcal{AS}(AF)$  then  $\mathcal{E}_{CF2}(AF) = \mathcal{E}_{PR}(AF)$*

## Agreement classes: which kind of properties we prove?

---

As intermediate steps, we have proved several lemmata based on the inclusion relationships like the following

**Lemma 3** *If  $|\mathcal{E}_{CO}(AF)| = 1$ , then  $\mathcal{E}_{CO}(AF) = \mathcal{E}_{GR}(AF) = \mathcal{E}_{PR}(AF)$*

**Lemma 4** *Let  $S \in \{PR, ST, SST, CF2\}$ , if  $GE(AF) \in \mathcal{E}_S(AF)$  then  $\mathcal{E}_S(AF) = \{GE(AF)\}$*

**Lemma 6** *If  $\mathcal{E}_{ST}(AF) = \{GE(AF)\}$  then  $\mathcal{E}_{CF2}(AF) = \{GE(AF)\}$*

**Lemma 8** *If  $\mathcal{E}_{CF2}(AF) = \mathcal{E}_{ID}(AF) = \{ID(AF)\}$*

*then  $\mathcal{E}_{CF2}(AF) = \mathcal{E}_{PR}(AF)$*      **Implications of extension properties**

**Lemma 10** *If  $\mathcal{E}_{CF2}(AF) \subseteq \mathcal{AS}(AF)$  then  $\mathcal{E}_{CF2}(AF) = \mathcal{E}_{PR}(AF)$*

## Agreement classes: coincidence

---

Coincidence of agreement classes follows (almost directly) from the lemmata. Examples are:

If  $\{GR, ST\} \subseteq \mathbb{S}$  then  $AGR(\mathbb{S}) = \Sigma_1$

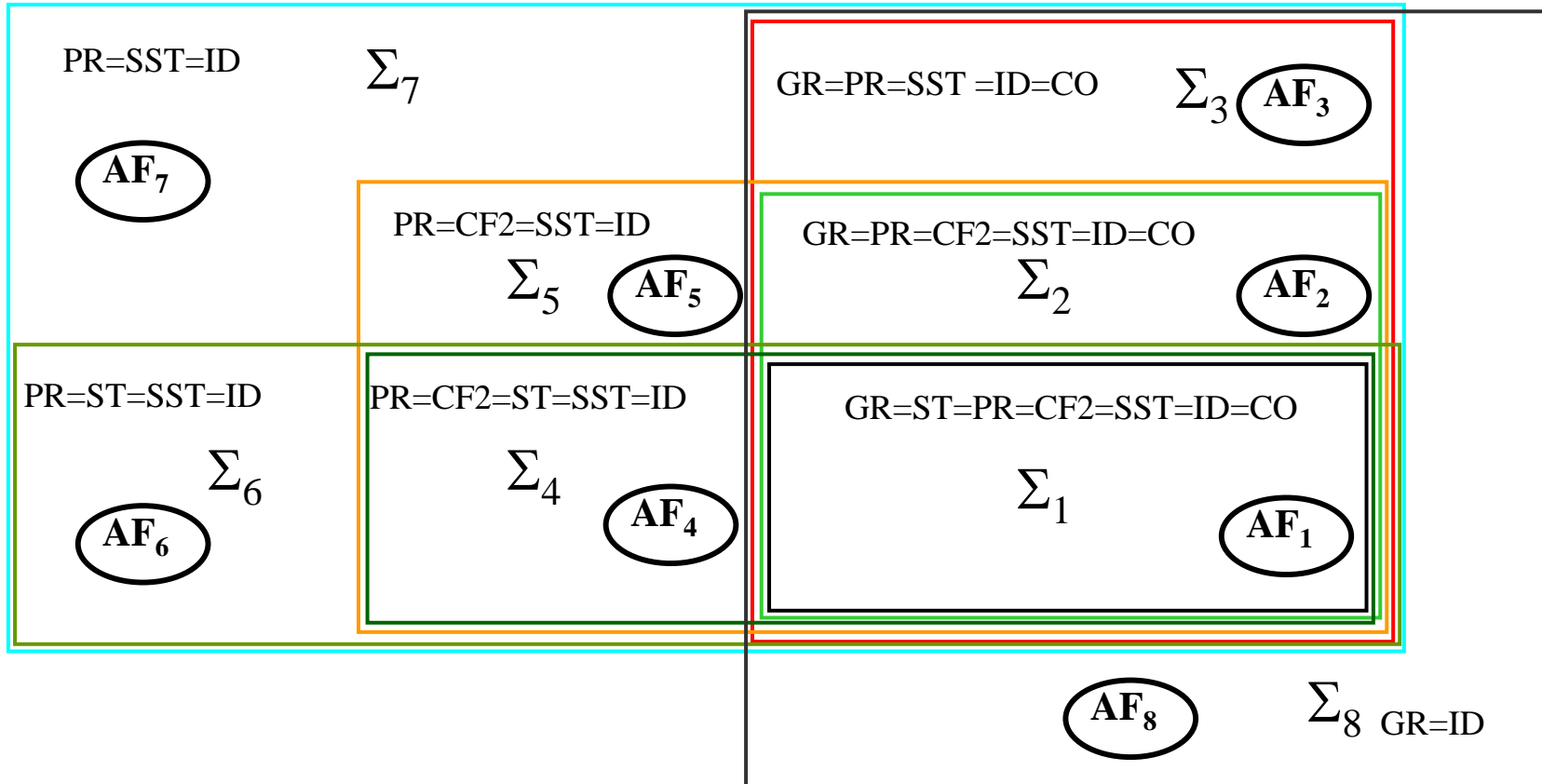
If  $\{ID, CF2\} \subseteq \mathbb{S}$  then  $AGR(\mathbb{S}) \in \{\Sigma_4, \Sigma_5\}$

If  $\{GR, SST\} \subseteq \mathbb{S} \wedge (\{ST, CF2\} \cap \mathbb{S}) = \emptyset$   
then  $AGR(\mathbb{S}) = \Sigma_3$

$\{CF2, ST\} \subseteq \mathbb{S} \subseteq \{CF2, ST, PR, SST\} \Rightarrow AGR(\mathbb{S}) = \Sigma_9$

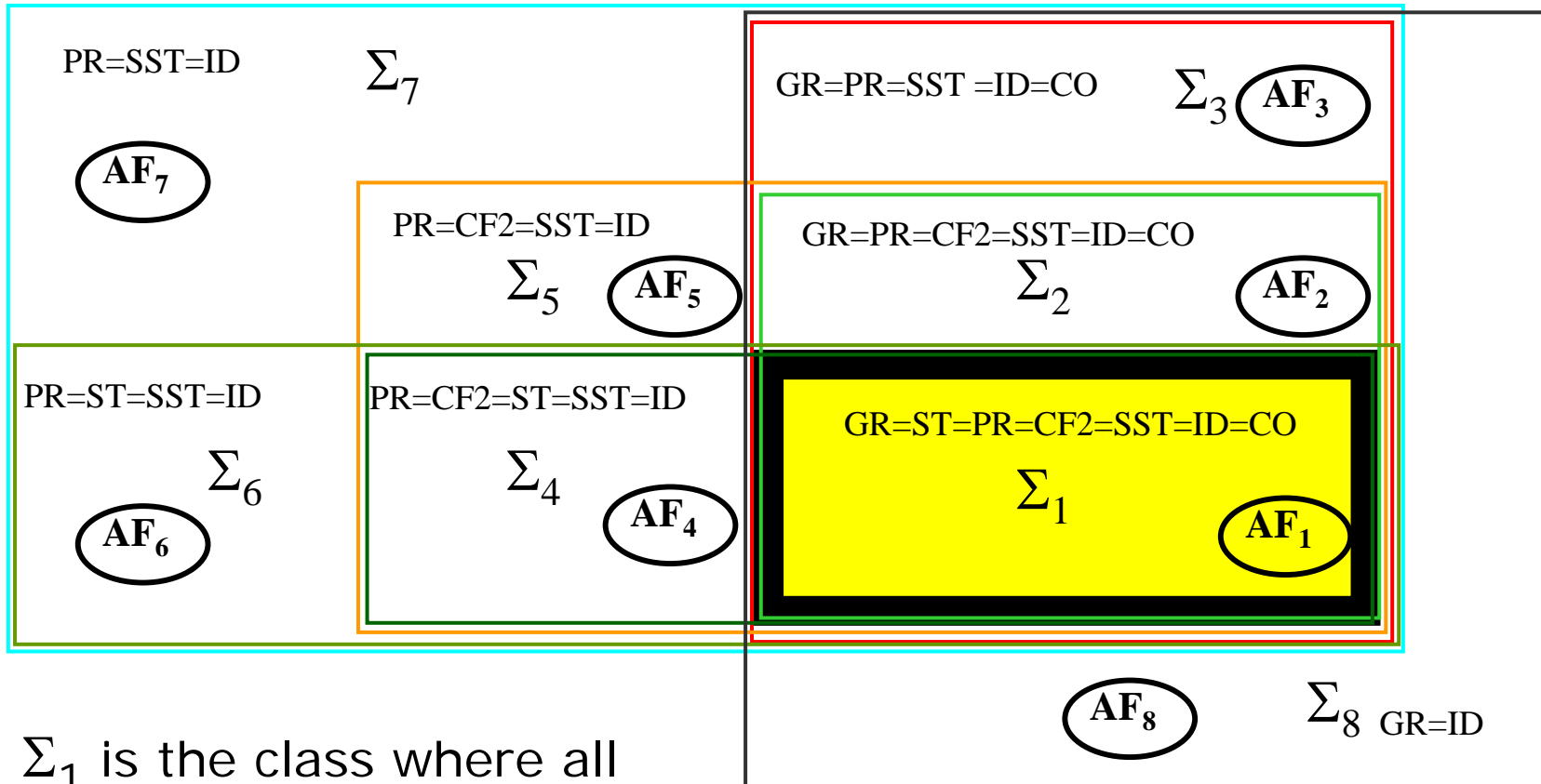
$\{CF2, SST\} \subseteq \mathbb{S} \subseteq \{CF2, PR, SST\} \Rightarrow AGR(\mathbb{S}) = \Sigma_{10}$

# Agreement classes: unique-status behavior





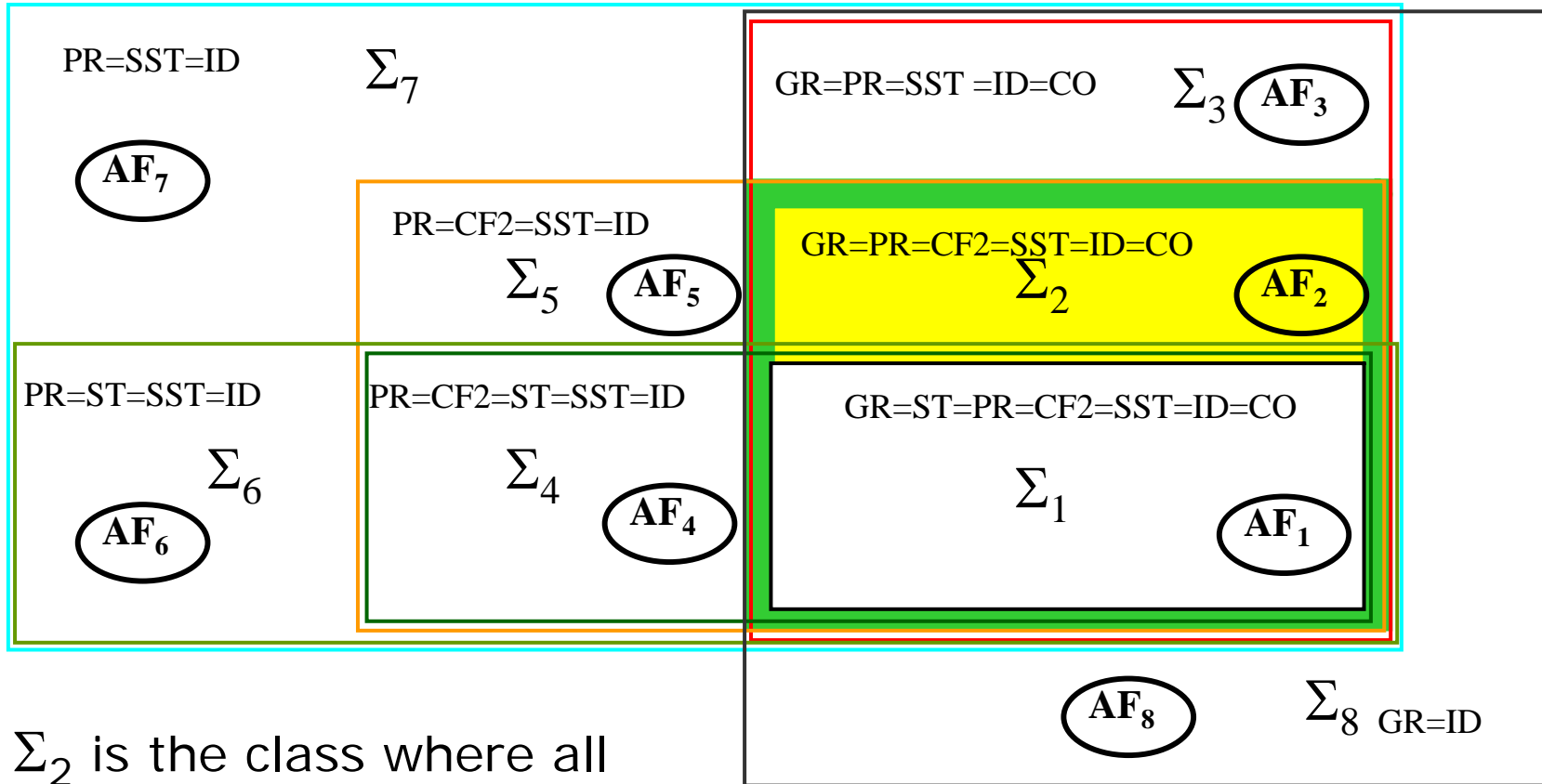
# Agreement classes: **GR** unique-status behavior



Σ<sub>1</sub> is the class where all the considered semantics agree (in particular with **GR**)

It includes (for instance) acyclic argumentation frameworks like AF<sub>1</sub> =  $\alpha$

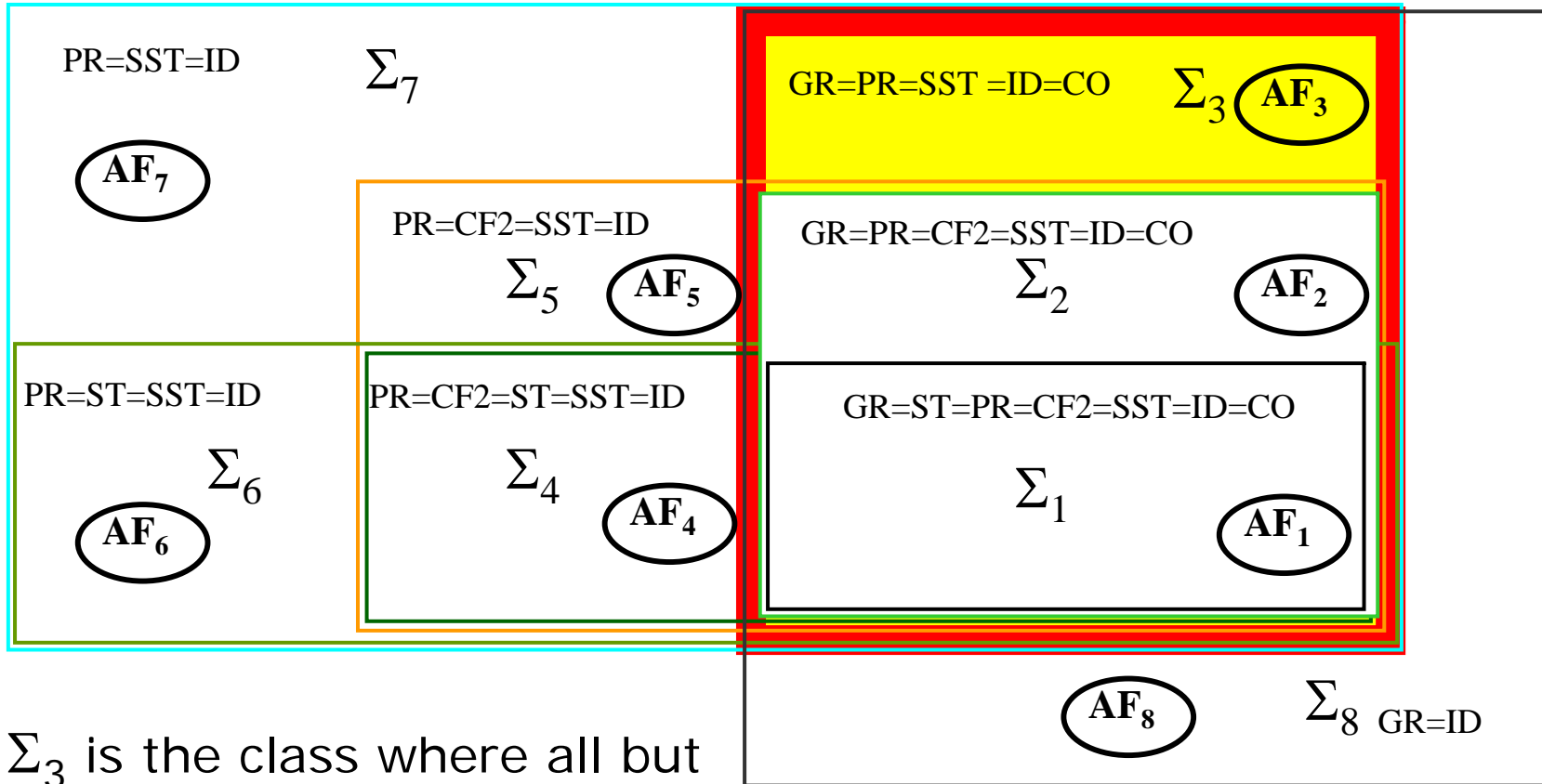
# Agreement classes: **GR** unique-status behavior



$\Sigma_2$  is the class where all the considered semantics agree (but **ST** may be undefined)

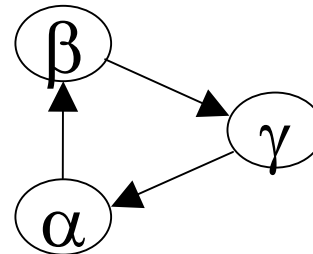
$\Sigma_2 \setminus \Sigma_1 \neq \emptyset$  as it includes AF<sub>2</sub> =  $\alpha$   $\beta$

# Agreement classes: **GR** unique-status behavior



$\Sigma_3$  is the class where all but **CF2** semantics agree (while **ST** may be undefined)

$\Sigma_3 \setminus \Sigma_2 \neq \emptyset$  as it includes  $AF_3 =$



## A note on complete semantics

---

The grounded extension belongs to the set of complete extensions (is the least complete extension)

For all semantics considered in this paper, except **CO**, it holds that no extension can be a proper subset of another extension

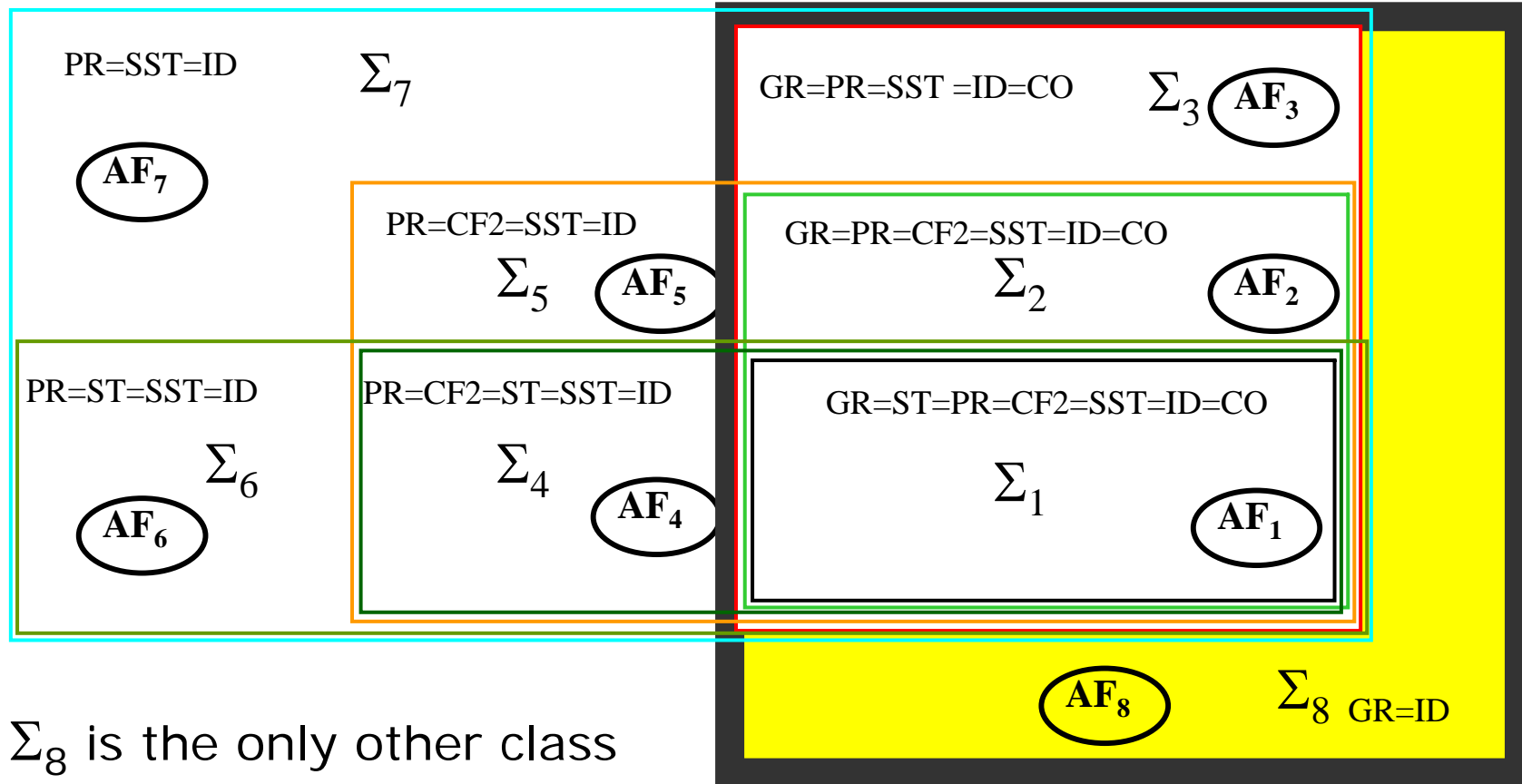
For all semantics considered in this paper, any extension is a superset of the grounded extension

It follows that agreement with **CO** is possible for a multiple-status semantics only if also agreement with **GR** holds

As a consequence **CO** only appears in agreement classes

$\Sigma_1, \Sigma_2, \Sigma_3$

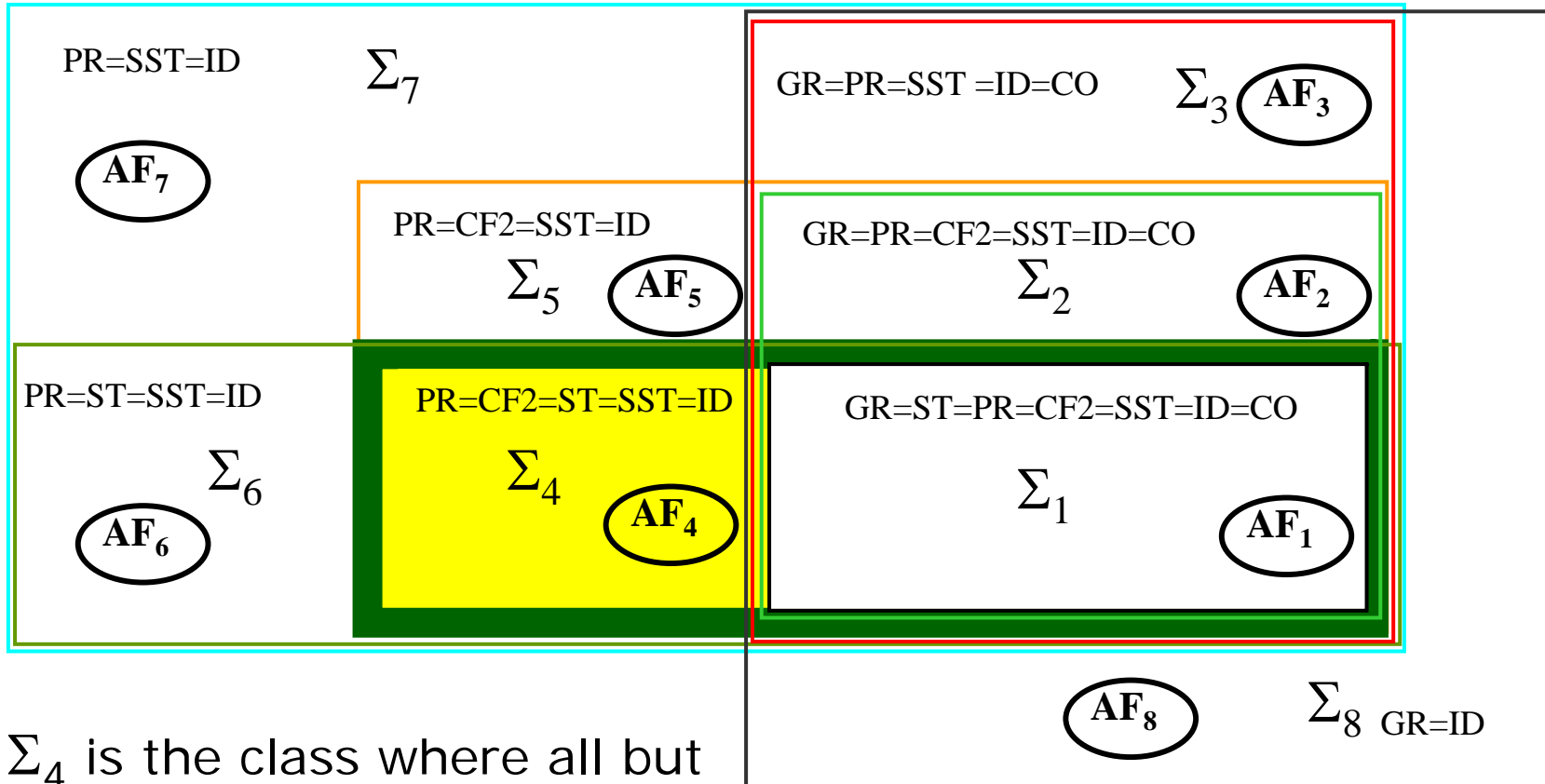
# Agreement classes: **GR** unique-status behavior



Σ<sub>8</sub> is the only other class of agreement involving **GR**

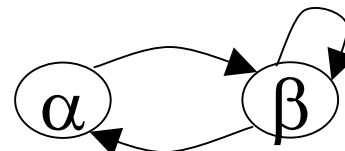
Examples of argumentation frameworks in Σ<sub>8</sub> \ Σ<sub>7</sub> will be given when examining its intersections with multiple-status agreement classes

# Agreement classes: *ID* unique-status behavior

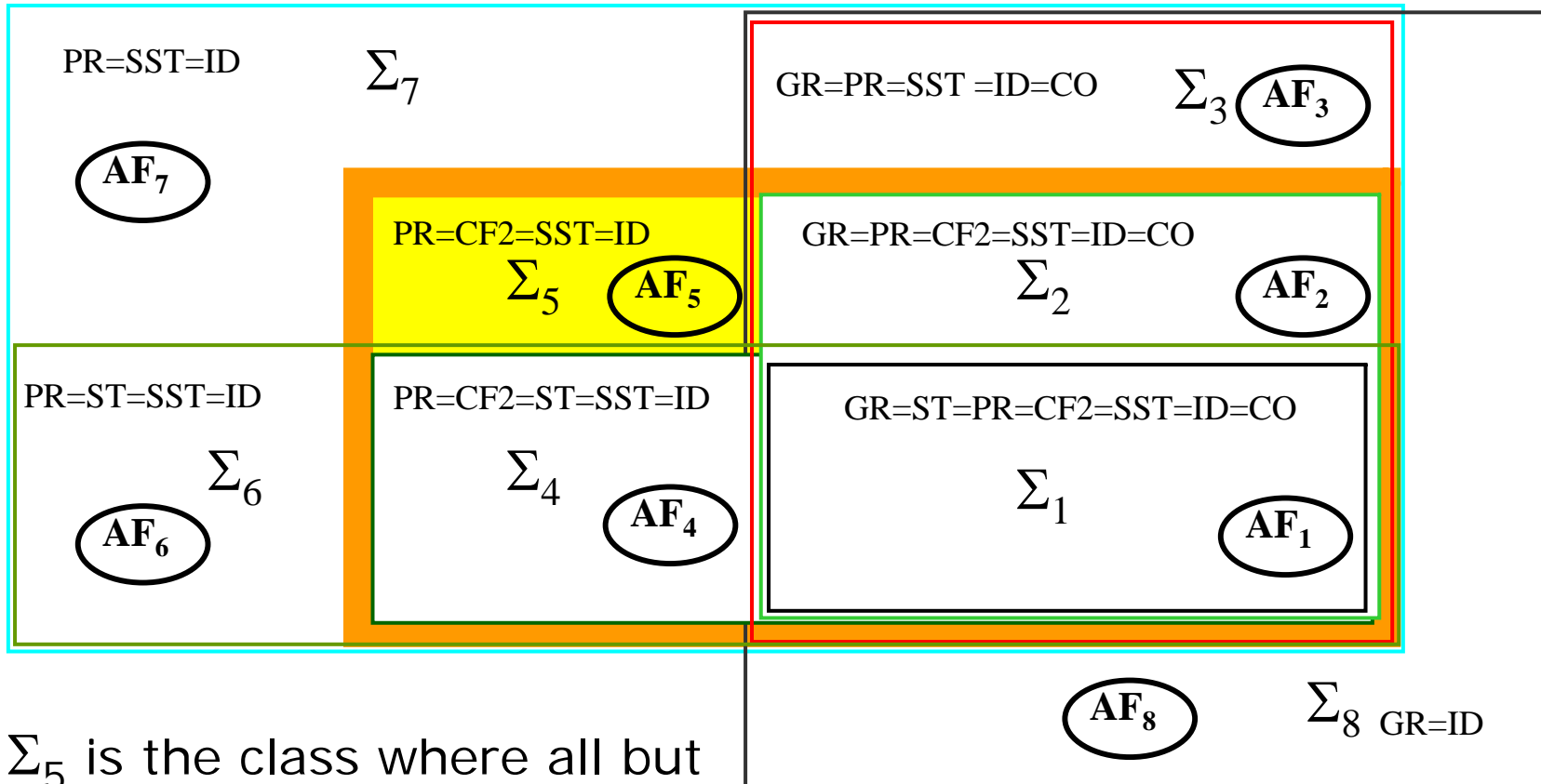


$\Sigma_4$  is the class where all but **CO** and **GR** semantics agree

$\Sigma_4 \setminus \Sigma_1 \neq \emptyset$  as it includes  $AF_4 =$

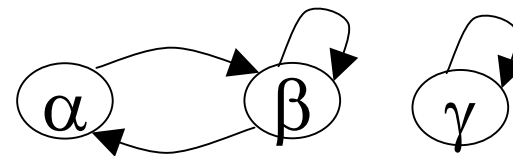


# Agreement classes: *ID* unique-status behavior

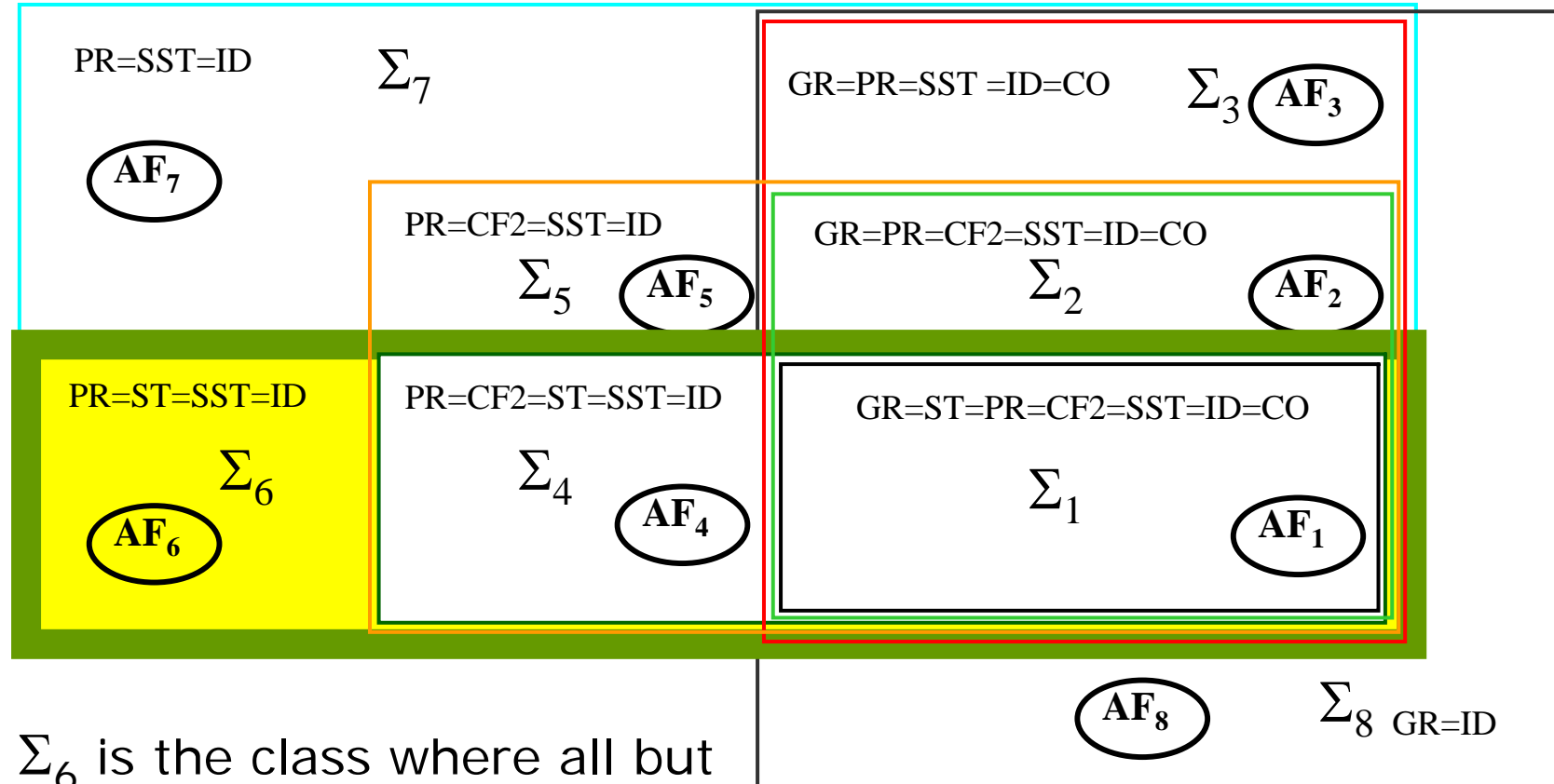


$\Sigma_5$  is the class where all but **CO** and **GR** semantics agree, while **ST** may be undefined

$\Sigma_5 \setminus (\Sigma_4 \cup \Sigma_2) \neq \emptyset$  as it includes  $AF_5 =$

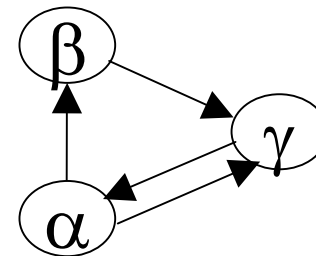


# Agreement classes: *ID* unique-status behavior



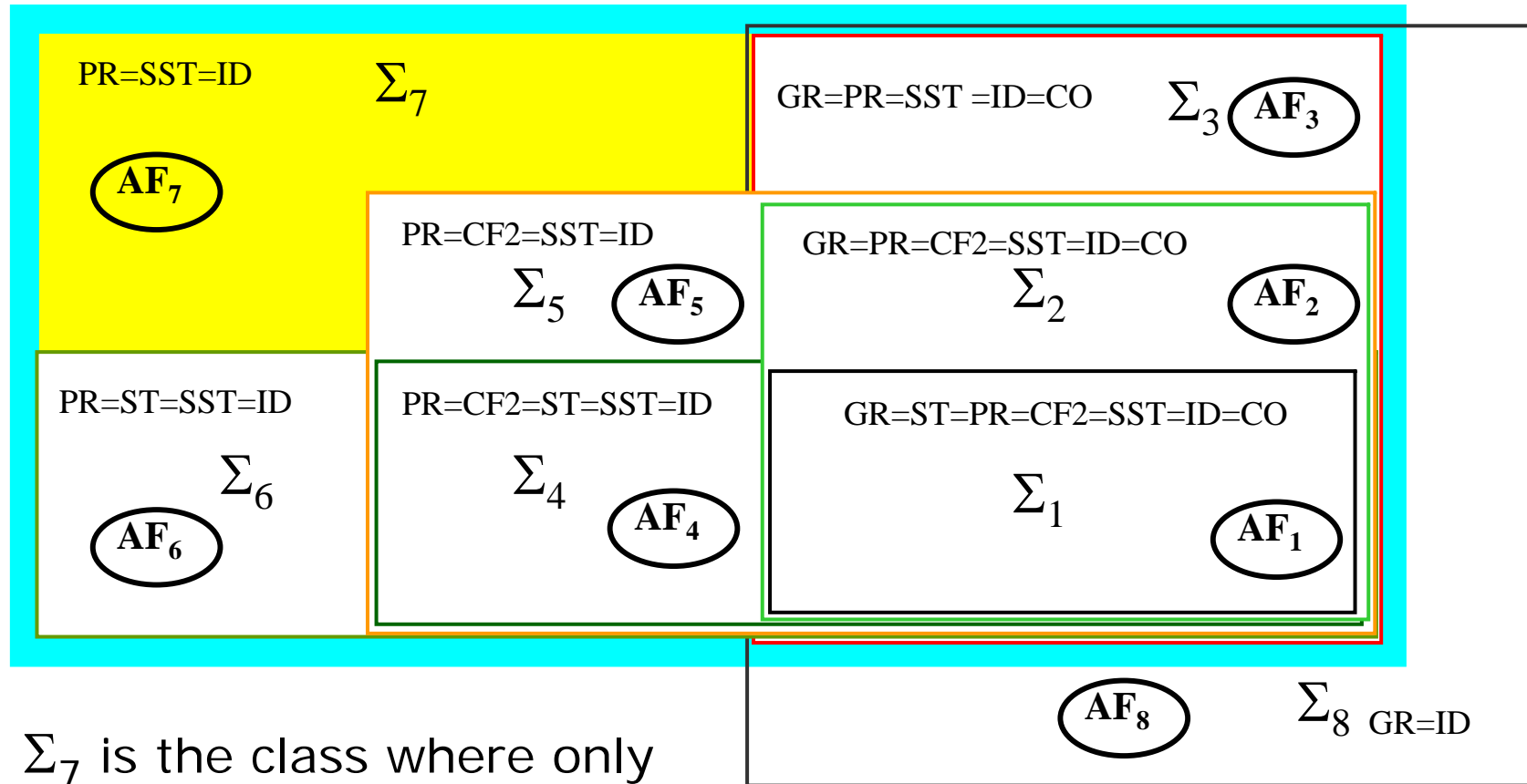
$\Sigma_6$  is the class where all but **CO**, **GR**, and **CF2** semantics agree

$\Sigma_6 \setminus (\Sigma_4 \cup \Sigma_5) \neq \emptyset$  as it includes  $AF_6 =$





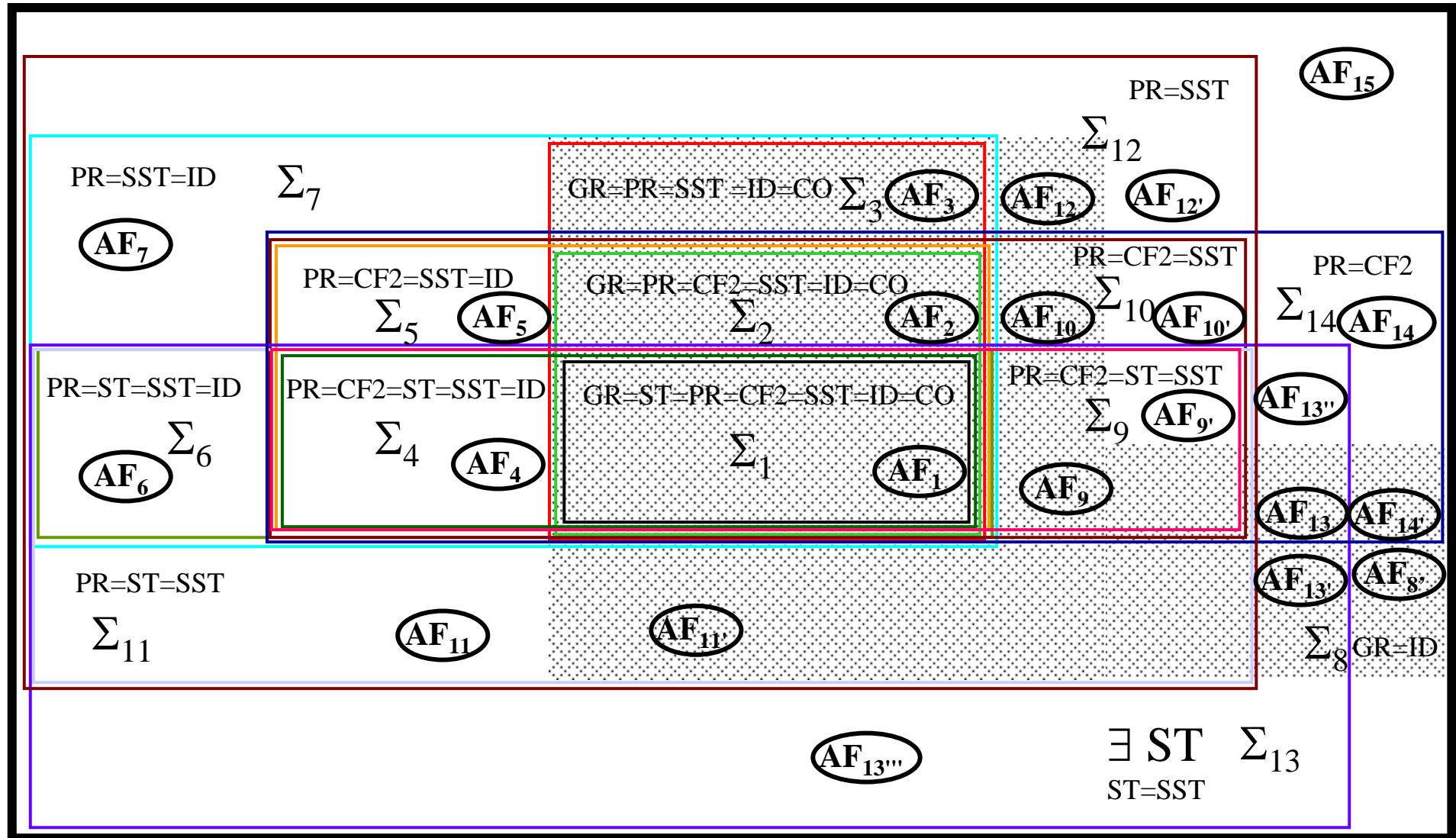
# Agreement classes: *ID* unique-status behavior



$\Sigma_7$  is the class where only **PR**, **SST**, and **ID** semantics agree

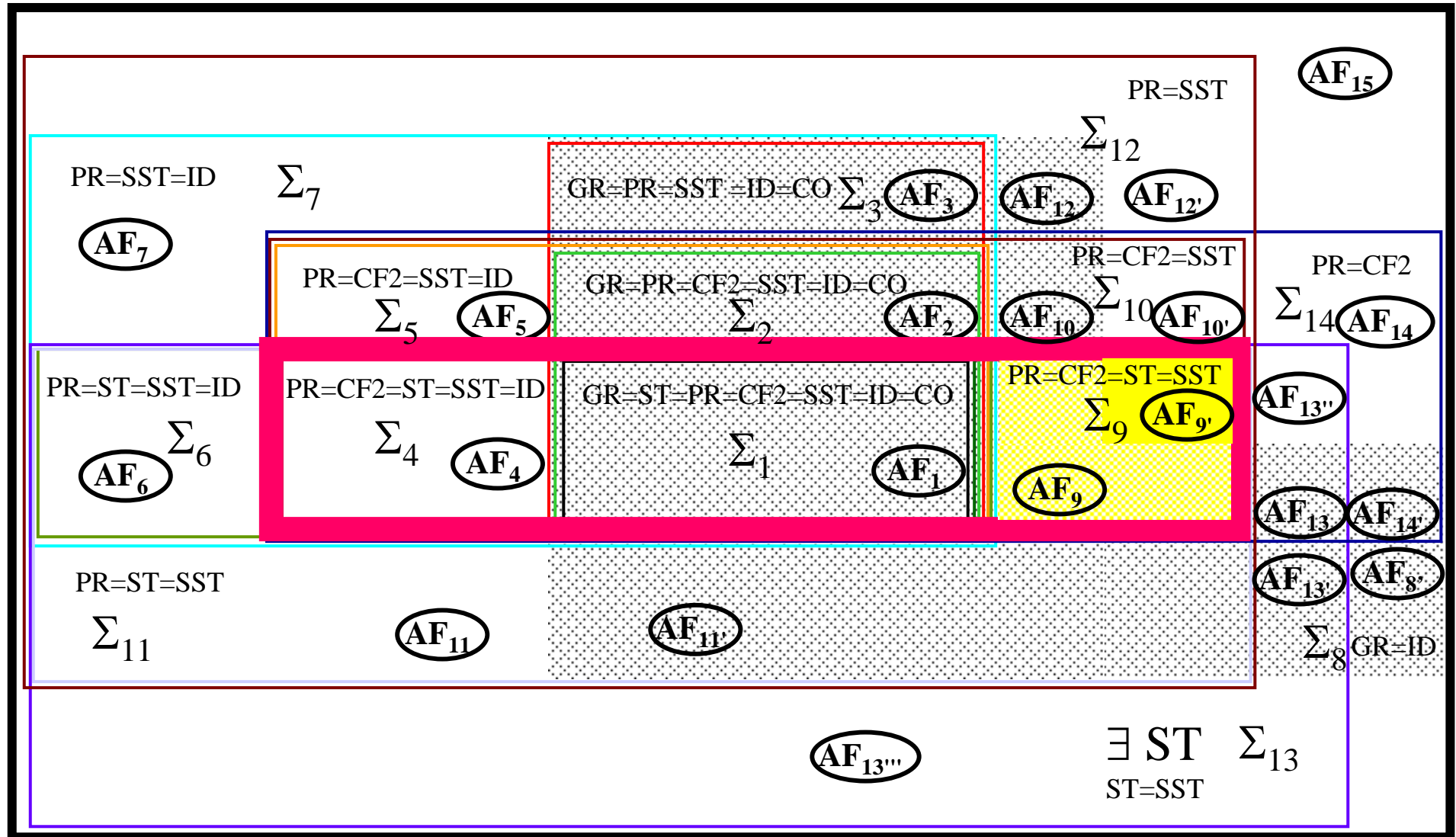
$\Sigma_7 \setminus (\Sigma_6 \cup \Sigma_5 \cup \Sigma_3) \neq \emptyset$  as it includes  $AF_7 =$

# Agreement classes: multiple-status behavior



$\Sigma_8$  GR=ID

# Agreement classes: multiple-status behavior



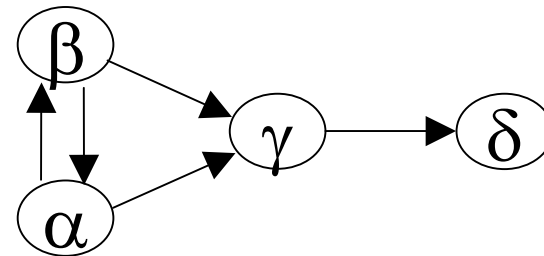
$\Sigma_8$  GR=ID

# The $\Sigma_9$ class

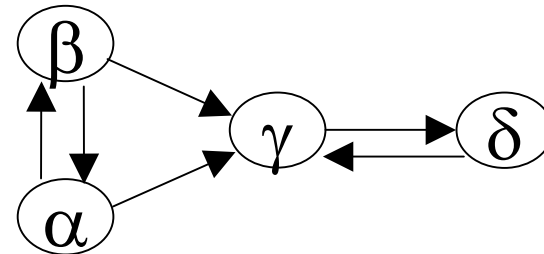
---

$\Sigma_9$  is the class where all multiple-status semantics (except **CO**) agree

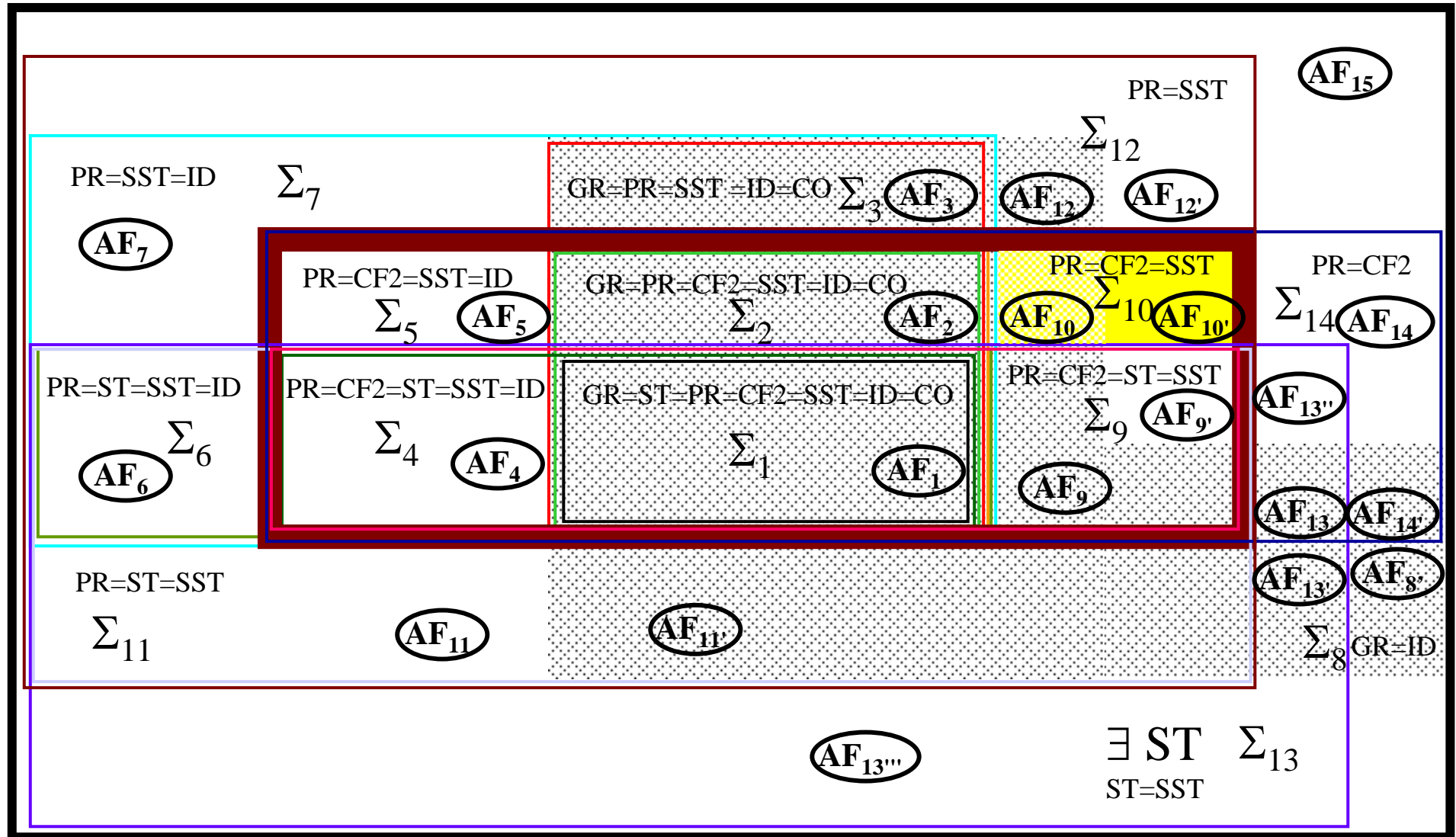
$(\Sigma_9 \setminus \Sigma_4) \cap \Sigma_8 \neq \emptyset$  as it includes  $AF_9$



$\Sigma_9 \setminus (\Sigma_4 \cup \Sigma_8) \neq \emptyset$  as it includes  $AF_9$ ,



# Agreement classes: multiple-status behavior



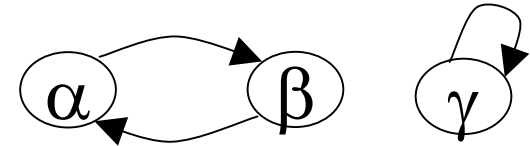
$\Sigma_8$  GR=ID

# The $\Sigma_{10}$ class

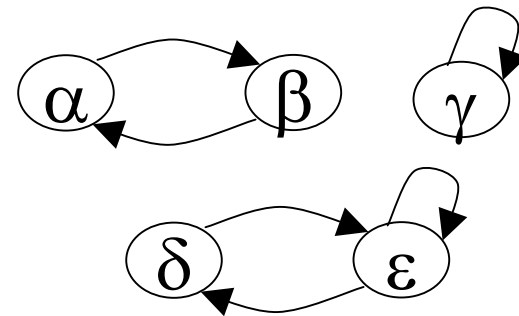
---

$\Sigma_{10}$  is the class where all multiple-status semantics (except **CO**) agree on a multiple-status behavior but **ST** may be undefined

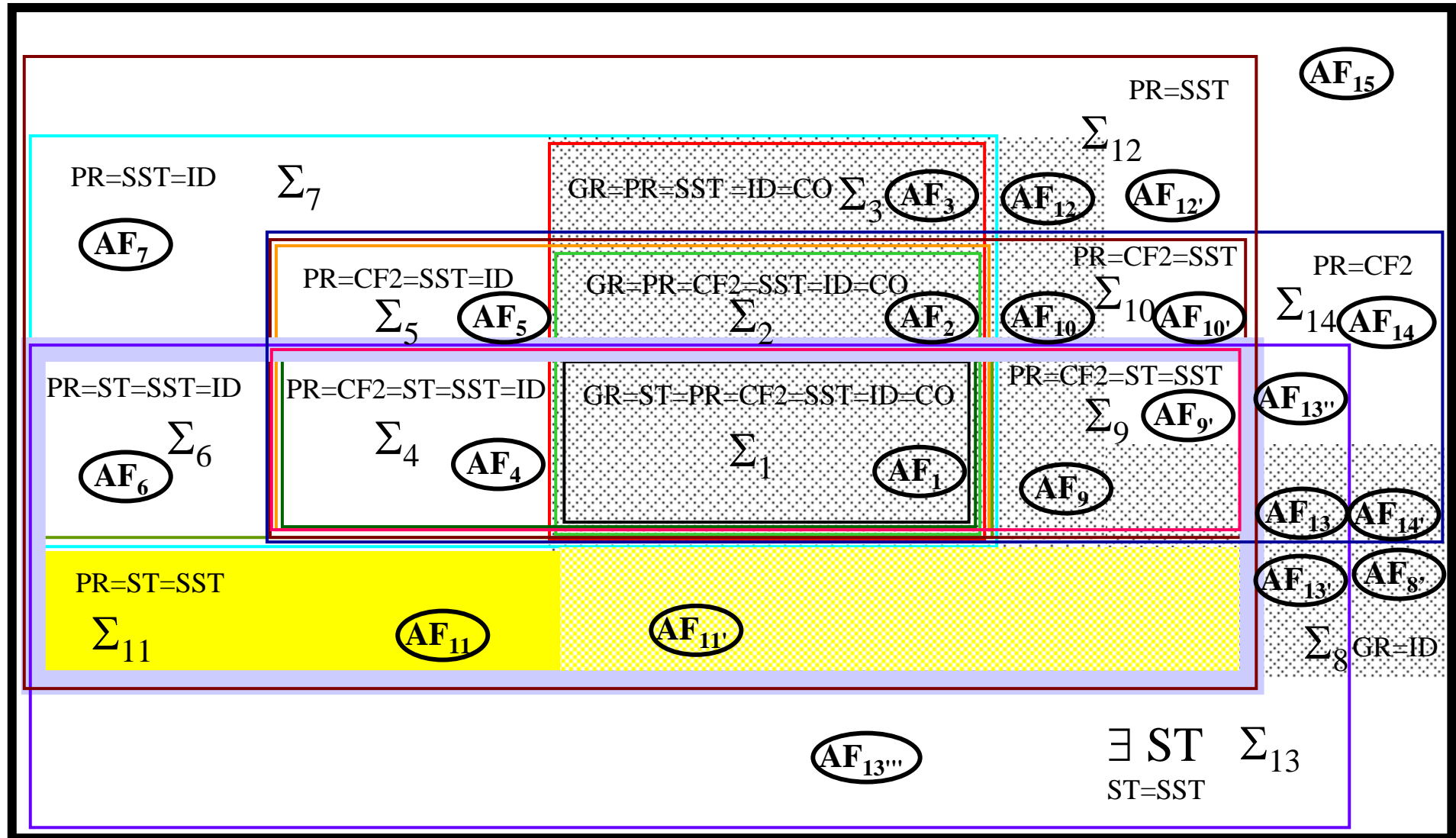
$(\Sigma_{10} \cap \Sigma_8) \setminus (\Sigma_9 \cup \Sigma_5) \neq \emptyset$  as it includes  $AF_{10}$



$\Sigma_9 \setminus (\Sigma_9 \cup \Sigma_5 \cup \Sigma_8) \neq \emptyset$  as it includes  $AF_{10}$ ,



# Agreement classes: multiple-status behavior



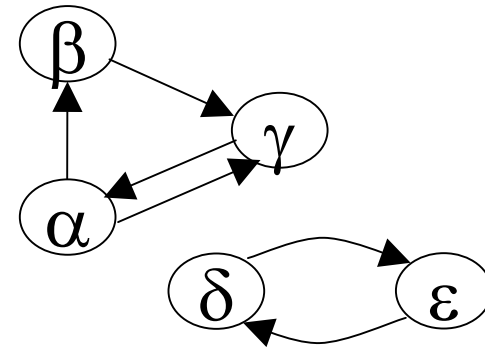
$\Sigma_8$  GR-ID

# The $\Sigma_{11}$ class

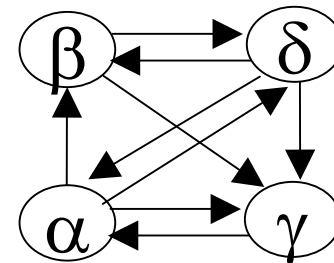
---

$\Sigma_{11}$  is the class where only traditional **PR** and **ST** (and hence **SST**) agree while **CF2** may differ

$\Sigma_{11} \setminus (\Sigma_6 \cup \Sigma_9 \cup \Sigma_8) \neq \emptyset$  as it includes  $AF_{11}$

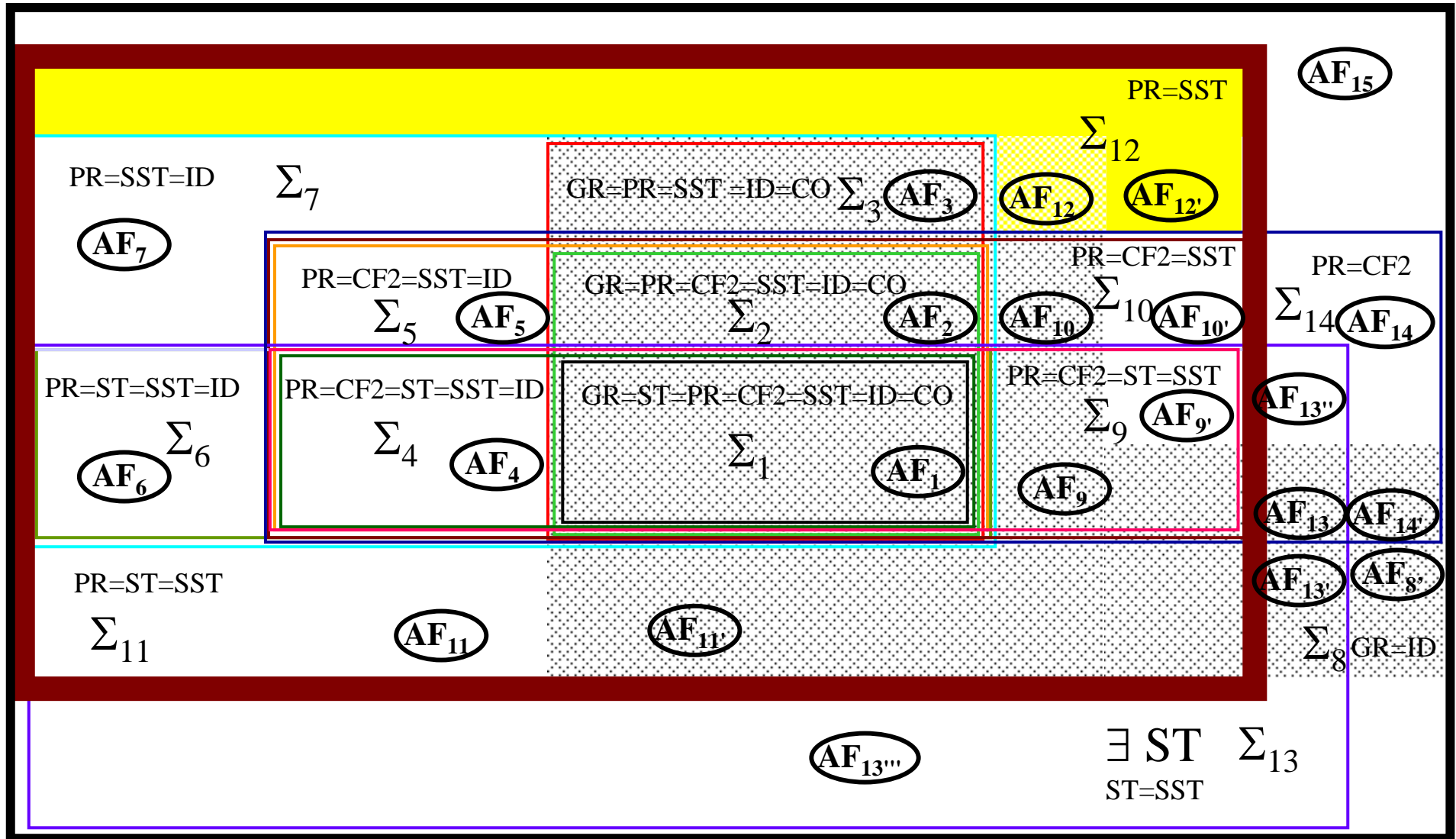


$(\Sigma_{11} \cap \Sigma_8) \setminus (\Sigma_6 \cup \Sigma_9) \neq \emptyset$  as it includes  $AF_{11}'$





# Agreement classes: multiple-status behavior



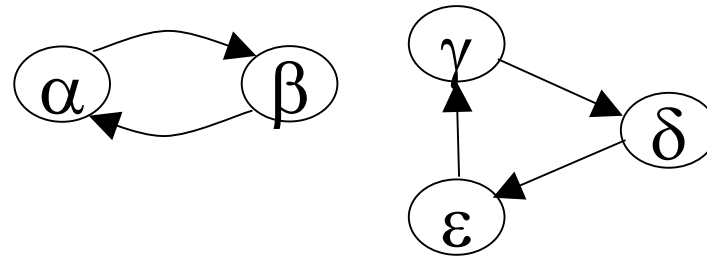
$\Sigma_8$  GR-ID

# The $\Sigma_{12}$ class

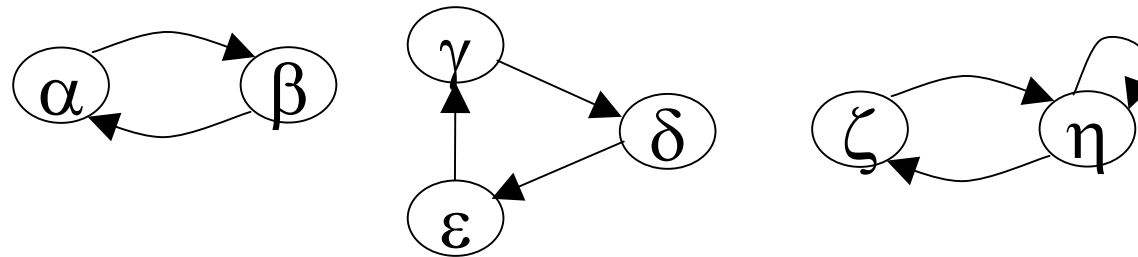
---

$\Sigma_{12}$  is the class where **PR** and **SST** agree and may differ from **CF2** while **ST** is undefined

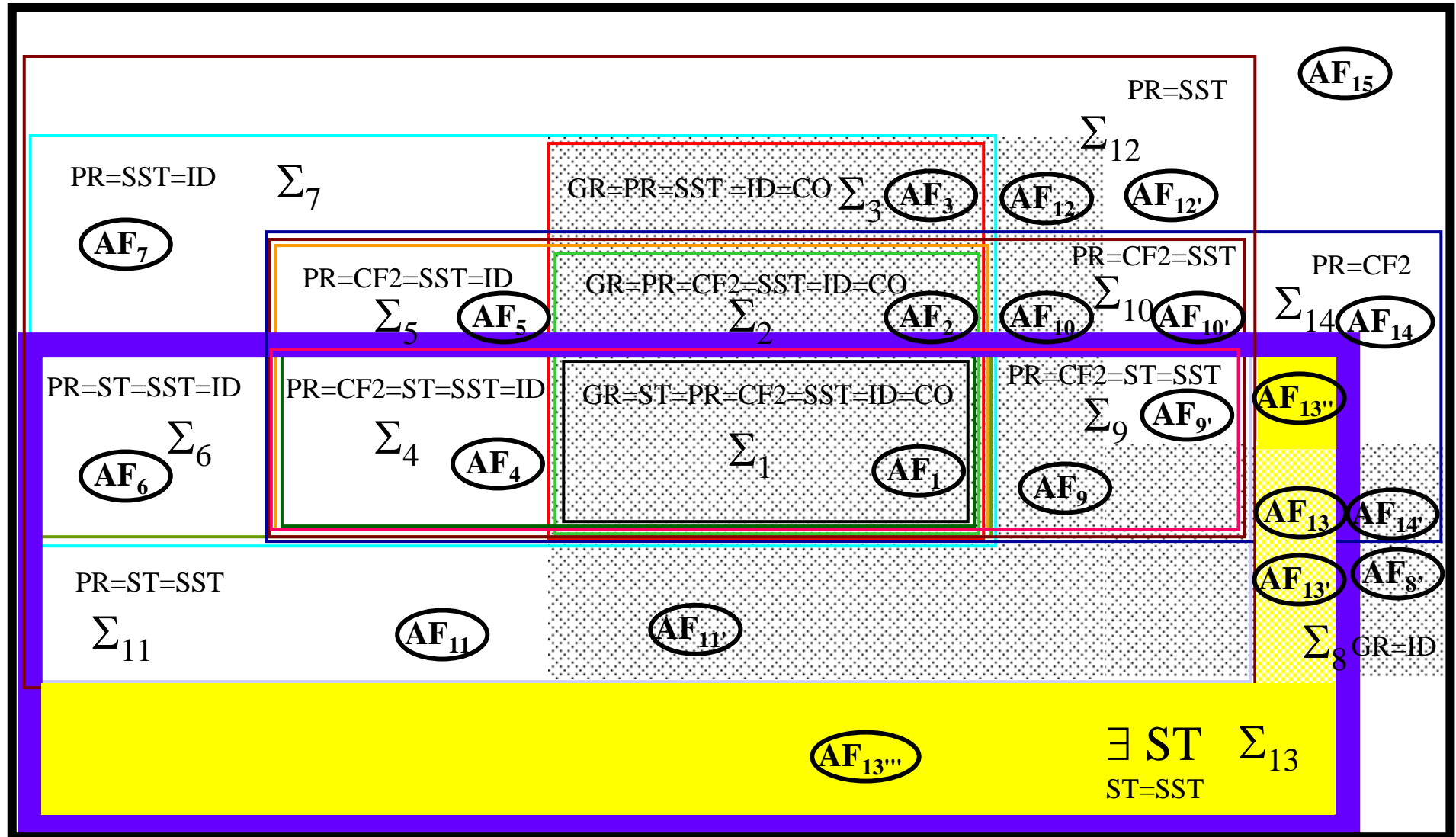
$(\Sigma_{12} \cap \Sigma_8) \setminus (\Sigma_7 \cup \Sigma_{10} \cup \Sigma_{11}) \neq \emptyset$  as it includes  $AF_{12}$



$\Sigma_{12} \setminus (\Sigma_7 \cup \Sigma_{10} \cup \Sigma_{11} \cup \Sigma_8) \neq \emptyset$  as it includes  $AF_{12}'$



# Agreement classes: multiple-status behavior



$\Sigma_8$  GR=ID

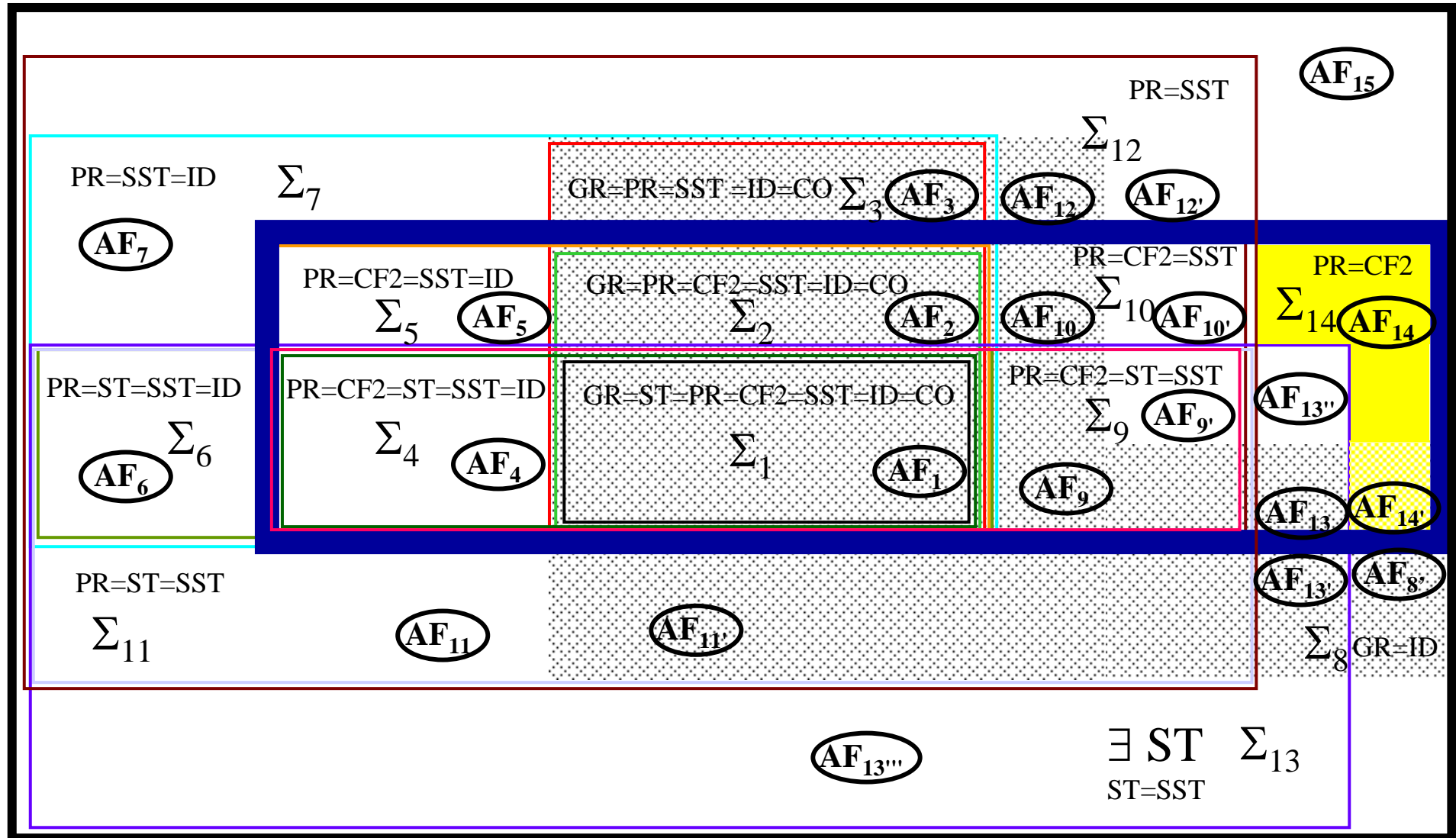
## The $\Sigma_{13}$ class

---

$\Sigma_{13}$  is the class where ***ST*** is defined (and hence agrees with ***SST***) and may differ from any other

$\Sigma_{13}$  has articulated intersections with  $\Sigma_{14}$  to be examined later

# Agreement classes: multiple-status behavior



## The $\Sigma_{14}$ class – Regions of interest

---

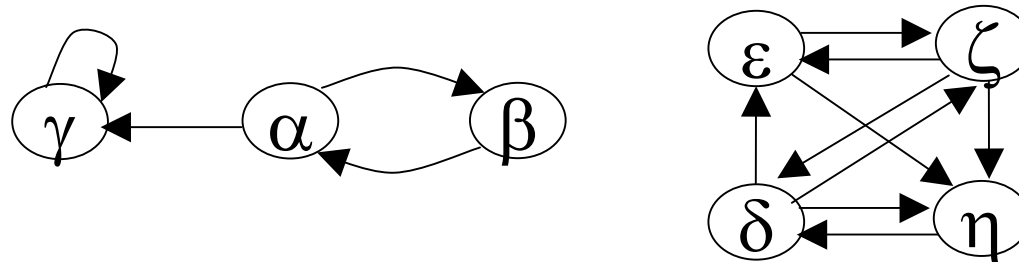
$\Sigma_{14}$  is the class corresponding to agreement between the last pair to be considered: **PR** and **CF2**

Let us now examine the distinct regions related to  $\Sigma_{13}$  and  $\Sigma_{14}$  in the Venn diagram

$(\Sigma_{13} \setminus \Sigma_{12}) \cap \Sigma_8 \cap \Sigma_{14} \neq \emptyset$  as it includes  $AF_{13}$



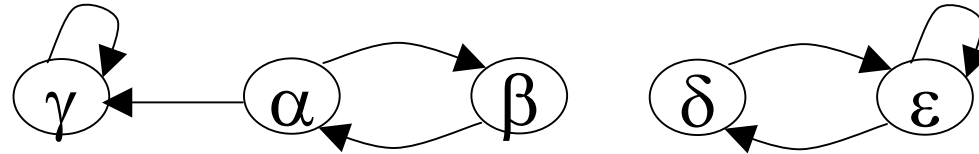
$(\Sigma_{13} \setminus \Sigma_{12}) \cap (\Sigma_8 \setminus \Sigma_{14}) \neq \emptyset$  as it includes  $AF_{13}'$



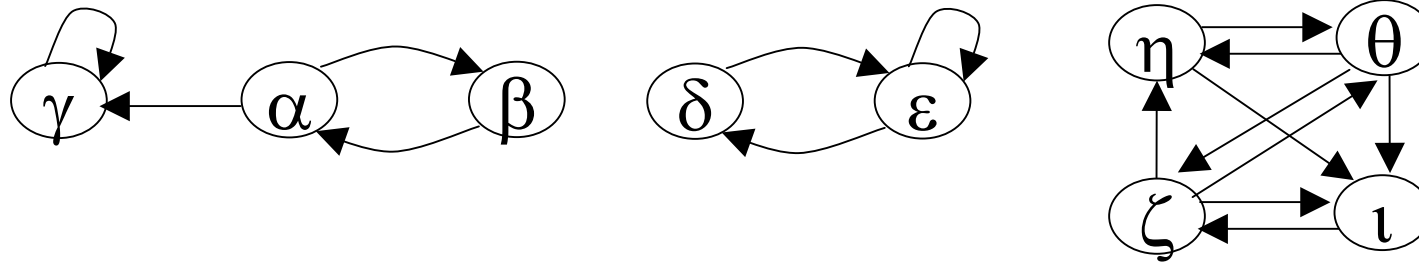
# The $\Sigma_{14}$ class – Regions of interest

---

$(\Sigma_{13} \setminus \Sigma_{12}) \cap (\Sigma_{14} \setminus \Sigma_8) \neq \emptyset$  as it includes  $AF_{13}''$



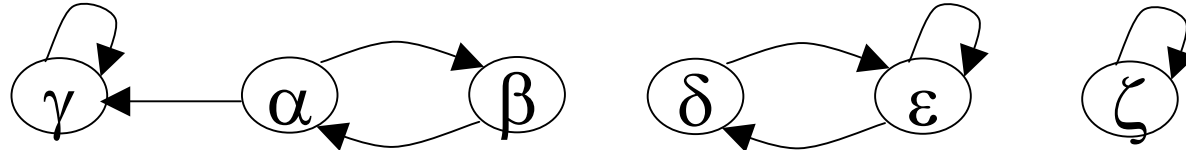
$\Sigma_{13} \setminus (\Sigma_8 \cup \Sigma_{12} \cup \Sigma_{14}) \neq \emptyset$  as it includes  $AF_{13}'''$



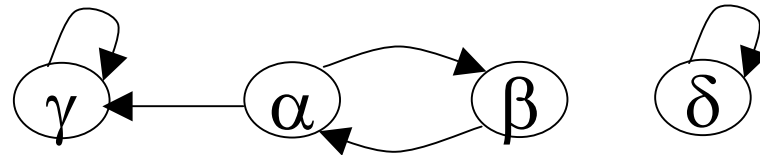
# The $\Sigma_{14}$ class – Regions of interest

---

$\Sigma_{14} \setminus (\Sigma_8 \cup \Sigma_{12} \cup \Sigma_{13}) \neq \emptyset$  as it includes  $AF_{14}$

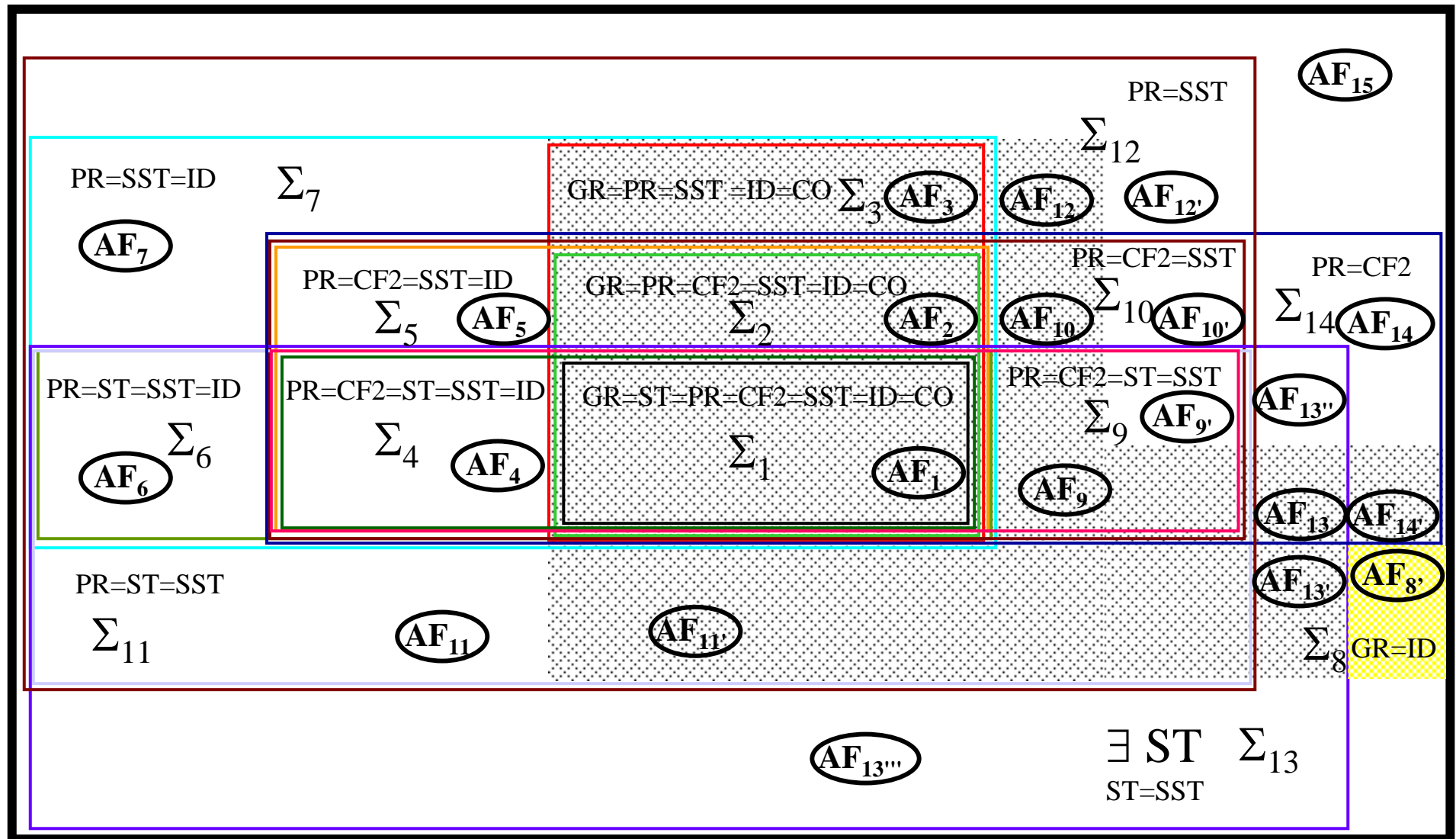


$(\Sigma_{14} \cap \Sigma_8) \setminus (\Sigma_{12} \cup \Sigma_{13}) \neq \emptyset$  as it includes  $AF_{14}''$





# Agreement classes: multiple-status behavior



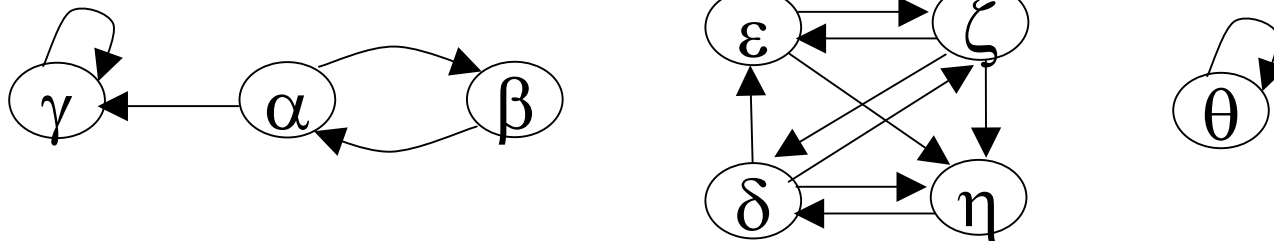
Σ<sub>8</sub> GR=ID

## $\Sigma_8$ and nothing else

---

There are argumentation frameworks where **GR** and **ID** agree while all other semantics disagree (and **ST** is undefined)

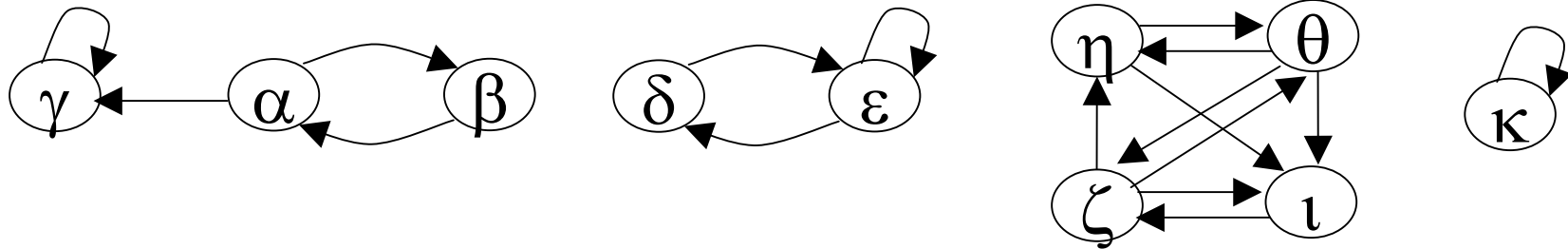
$\Sigma_8 \setminus (\Sigma_{12} \cup \Sigma_{13} \cup \Sigma_{14}) \neq \emptyset$  as it includes  $AF_8$



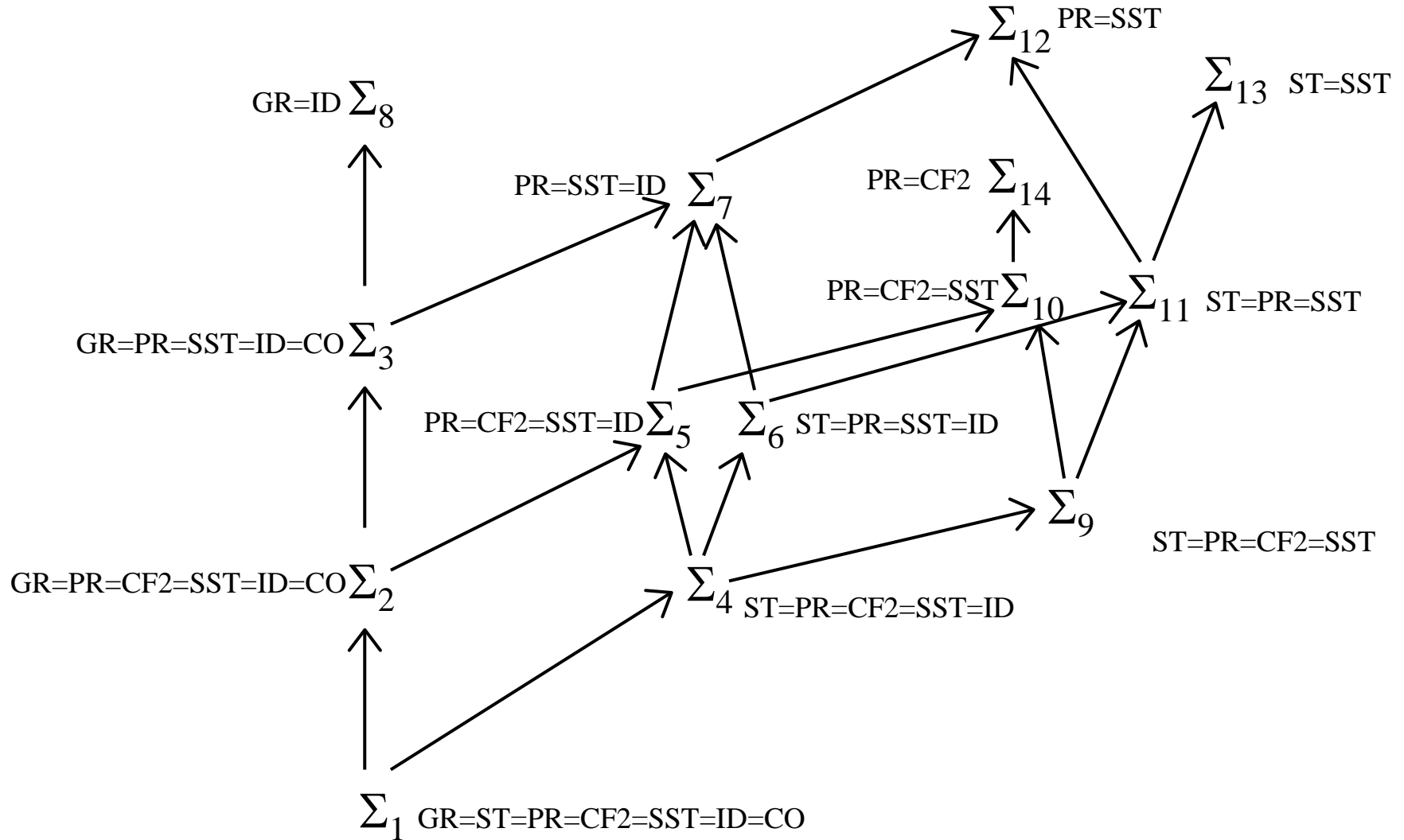
# Universal disagreement is possible

---

There are argumentation frameworks where no two semantics agree (while **ST** is undefined) like  $AF_{15}$

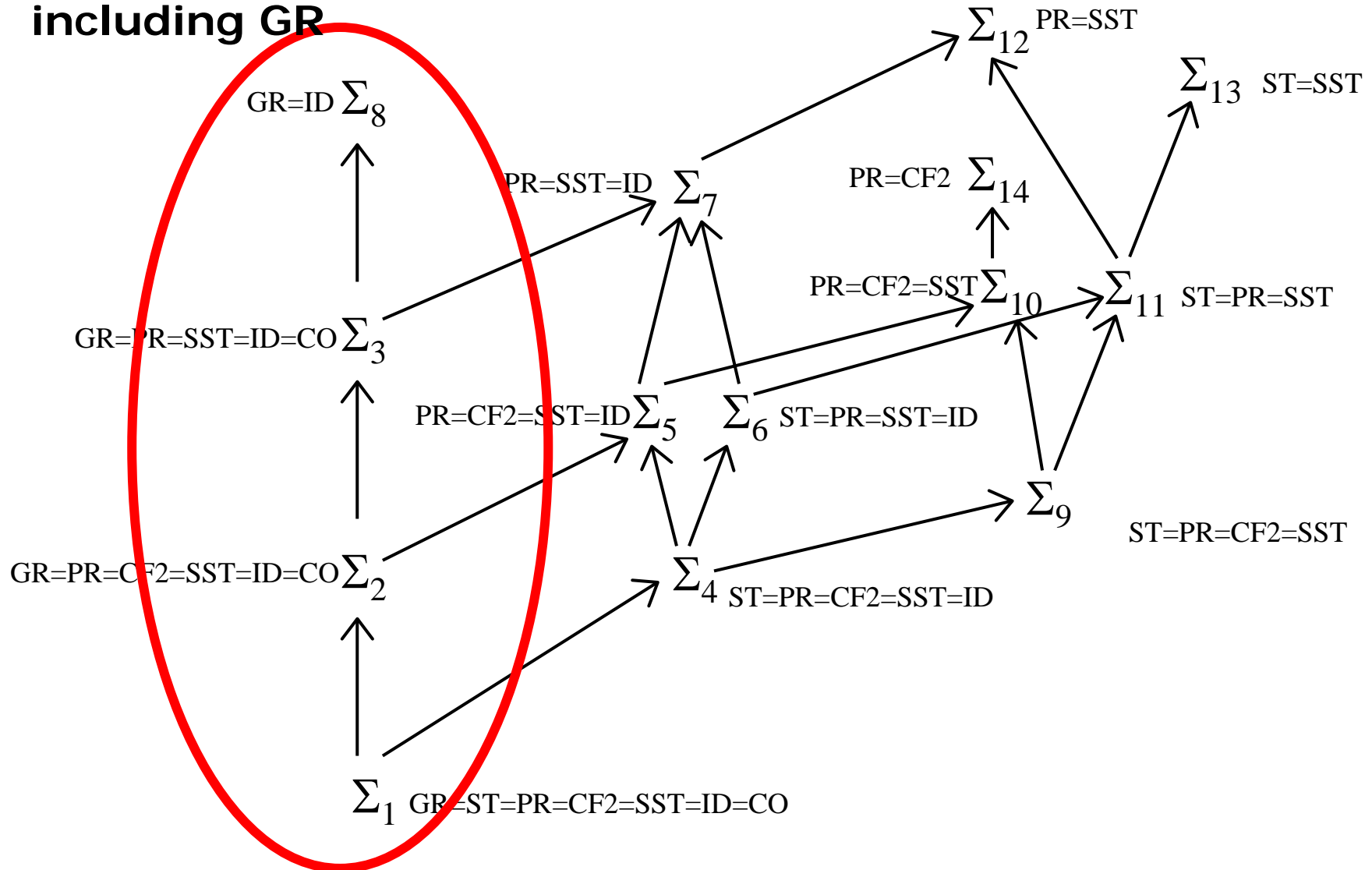


# A synthetic view



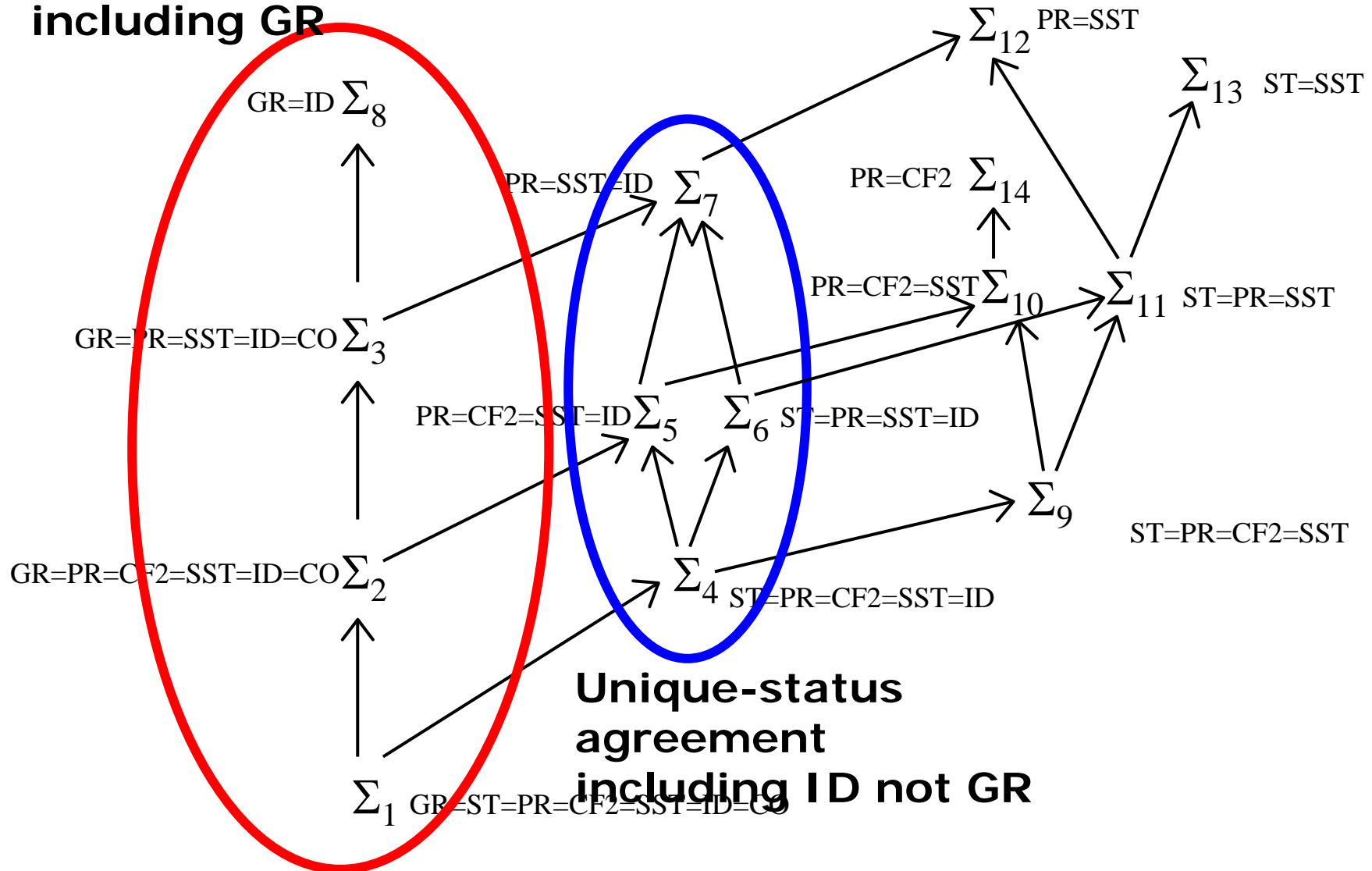
# A synthetic view

## Unique-status agreement including GR



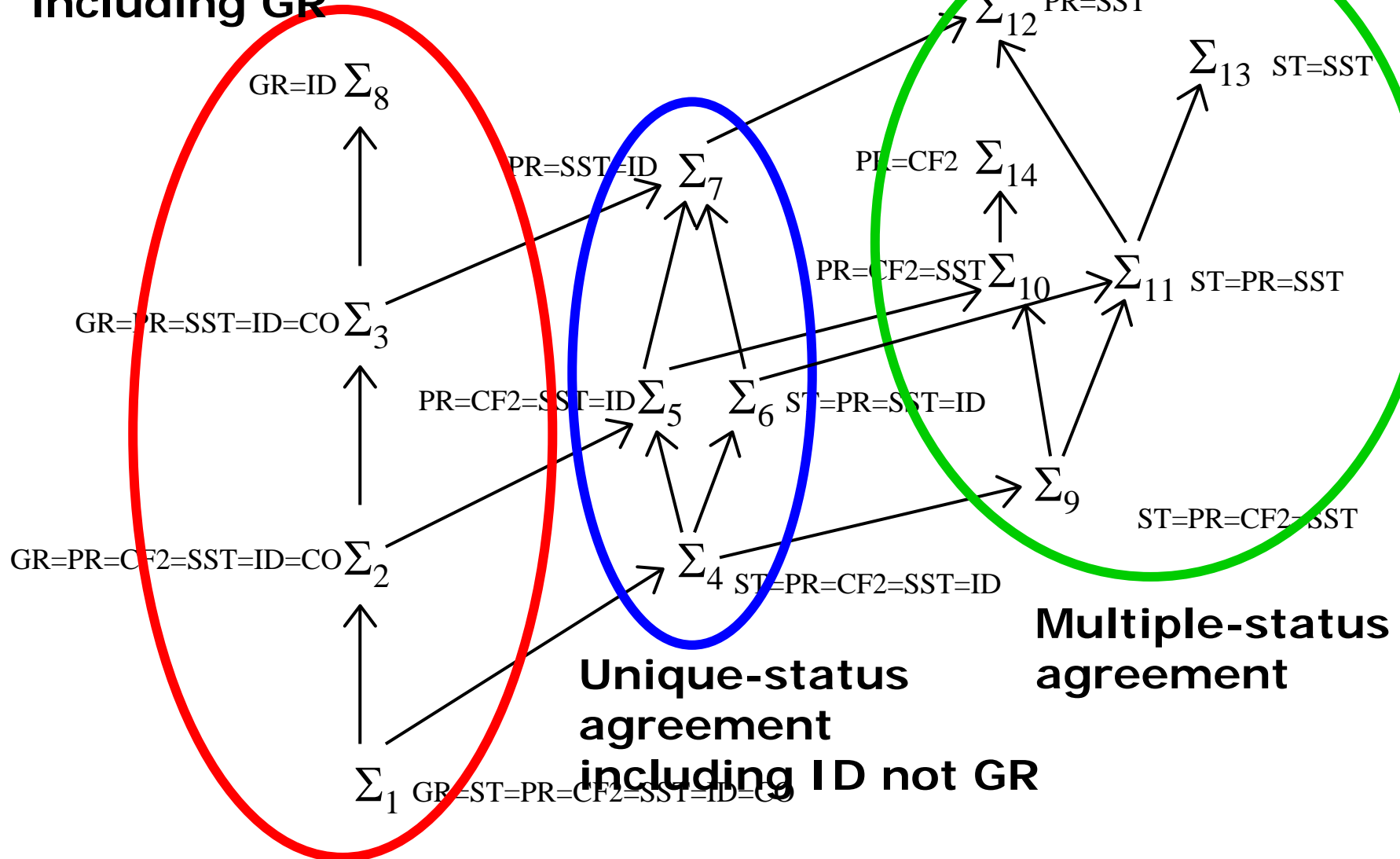
# A synthetic view

## Unique-status agreement including GR



# A synthetic view

**Unique-status agreement including GR**



# Conclusions

---

A systematic analysis of agreement providing a reference framework independent of any topological characterization

## **Next steps:**

analyzing relationships between agreement classes and topological families of argumentation frameworks

considering other argumentation semantics