Institut de Recherche en Informatique de Toulouse
CNRS - INP - UT3 - UT1 - UT2J

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TouIST tutorial #2: Modeling and solving problems

1 Prerequisites

You must have done the previous tutorial. You should be familiar with the simple use of touist (writing a set of formulas, checking its satisfiability, extracting (counter) models).

2 Goal of the tutorial

At the end of this session, you should be able to transform a combinatorial type problem (of which Sudoku is a typical example) into a satisfiability problem of a set of logical formulas. Then, you must be able, from the models obtained (or from their absence) to conclude as for the solutions of the concrete problem from which you will have left.

3 Knowledge representation and reasoning

We have seen that a problem can be encoded in the form of a set of formulas representing its hypotheses and the negation of the desired conclusion. If the conclusion is a logical consequence of all the hypotheses, then the system obtained must be unsatisfiable.

In what follows, we use the following example to show how we can model a problem using propositional logic and find a solution using TouIST.

Exercise 1 *Show the validity of the following reasoning: After a horse race between black and white horses, the winning horse was the one who had a black dress and a white mane. They were PetitGris, Qristal or Rikita. However PetitGris has a black mane and a white tail, Qristal has a black dress and a white tail, and finally Rikita has a black dress. Finally the winner had its mane and tail opposite color. Consequence: Rikita is the winner.*

To model this example, you have to give yourself a language (a list of propositional variables). It is necessary to choose a language which makes it possible to preserve the direction of each propositional variable whatever the sentence in which it replaces a proposition (a piece of sentence without connector and having a value of truth).

Here is an example of language:

- The conclusion being "Rikita is the winner", we will model it by the propositional variable R.

- Likewise, P and Q will represent "PetitGris is the winner" and "Qristal is the winner" respectively.
  It is important to emphasize that "Qristal" for instance, cannot be represented by a propositional variable, since this word in itself has no truth value.

- The beginning of the first hypothesis tells us that the only colors are black and white. It is about tail, dress, and mane. We must therefore choose a propositional variable describing the state of each element *of the winner* (because we are trying to determine who is the *winner*). Thus, we can define:
  - Wt as "The winner has a white tail";
  - Wd as "The winner has a white dress";
  - Wm as "The winner has a white mane".

  The same propositions relating to the color black will be given by the negation of these variables. For example, "The winner has a black dress" will be represented by $\neg Wd$.

Now, we can model the sentence "the winning horse was the one who had a black dress and a white mane" by something like: "The winner has a black dress and the winner has a white name", that is:

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not Wd and Wm
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In the same way, "They were PetitGris, Qristal or Rikita." can be modeled by "PetitGris is the winner or Qristal is the winner or Rikita is the winner".

Question 1.1 *What formula must be added to the* TouIST *program for modeling this sentence?*

And so on for "PetitGris has a black mane and a white tail, Qristal has a black dress and a white tail, and finally Rikita has a black dress" that can be reformulated as:

- "if PetitGris is the winner, then the winner has a black mane and the winner has a white tail";
- "if Qristal is the winner, then the winner has a black dress and the winner has a white tail";
- "if Rikita is the winner, then the winner has a black dress".

Question 1.2 *What formulas model the above three sentences? Add them to your* TouIST *program.*

Question 1.3 *What is the formula that models the sentence "the winner had its mane and tail opposite color"? Please, add this formula to your* TouIST *program.*

Question 1.4 *What formula represents "Rikita is the winner" ? And what formula must be added to your program?*

Question 1.5 *You should be able to solve the exercise now. What happens when you click on* Solve *button? What does the result mean?*

Exercise 2 *Consider the following reasoning: A theft was committed, traces of size 40 shoes were found and the thief was seen: he is small. Among the 3 suspects (A, B and C), A is large and has a size 40, B is small and has a size 45, finally C is small and has a size 40. The thief is one of the three suspects. So C is the thief.*

We give you the following propositional variables:

- `A` *for "A is the thief";*
- `B` *for "B is the thief";*
- `C` *for "C is the thief";*
- `T40` *for "The traces left by the thief were 40";*
- `S40` *for "The thief had a size 40";*
- `S45` *for "The thief had a size 45";*
- `S` *for "The thief was small".*

Building on the previous exercise:

1. *use this language to model the problem in* TouIST*;*
2. *show that this reasoning is not valid (we cannot deduce the guilt from C);*
3. *show that from the hypotheses, we can still conclude that A is not guilty;*
4. *add a hypothesis allowing the reasoning to be valid and to prove the guilt of C (note: you cannot reasonably wear 40 if you have a size 45).*

Exercise 3 *To join the* No SKRuB club*, it is necessary and sufficient to fulfill ALL of the following conditions:*

1. *if he plays the bagpipes and if he does not wear a kilt then he is not Scottish;*
2. *he plays rugby or he wears a kilt;*
3. *if he plays the bagpipes and if he does not play rugby then he is not Scottish;*
4. *he is Scottish or he plays the bagpipes;*
5. *if he is Scottish and if he wears a kilt then he plays the bagpipes;*

6. *if he is not Scottish then he does not play the bagpipes;*
7. *if he is Scottish and if he plays rugby then he plays the bagpipes;*
8. *if he plays rugby and if he wears a kilt then he does not play the bagpipes.*

After having modeled this example under TouIST*:*

- *check that all the conditions are unsatisfactory. What do you deduce from this in relation to the number of members of this club, and why?*
- *show that it is enough to delete at least one of these conditions for all the remaining conditions to be satisfied;*
- *say which one should be removed to obtain the largest number of models?*
- *find out why the club is called that.*

4 For further...

4.1 The 3×3 Latin square

We consider a Latin square grid 3×3. Each of the 9 boxes must contain a value from 1 to 3, and only one. The boxes on the same line all contain different values. The boxes in the same column all contain different values.

Exercise 4 *Suggest a modeling of the Latin square 3×3 in order to calculate all the possible solutions of the following Latin square:*

2		*1*
3		

As this is the first transformation of a combinatorial problem into a problem of satisfying constraints, you will be guided step by step.

For each of the cells, you can define three propositional variables corresponding to the three possible values $\{1, 2, 3\}$. Thus, $C(i, j, k)$ will represent the proposition "The coordinate cell (i, j) contains the value k", and $\neg C(i, j, k)$ will represent the proposition "The coordinate cell (i, j) does not contain the value k".

Question 4.1 *Using formulas of type $C(i, j, n)$, describe the initial state of the grid.*

To impose that the cell $(2, 1)$ contains one of the three possible values, you can use the formula:

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C(2,1,1) or C(2,1,2) or C(2,1,3)
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Question 4.2 *Write the set of formulas modeling the fact that each cell contains at least one of the values between 1 and 3. How many formulas must be written to process all the cells in the grid?*

Question 4.3 *Now write all the formulas to force each cell to have at most one value (between 1 and 3). How many formulas do you get?*

Note: if a cell has a certain value, then it does not have the other two possible values.

Question 4.4 *Write the constraint (set of formula) indicating that each value between 1 and 3 only appears once and only once on each line.*

(Note: this means that if a cell has a certain value, the other two cells on the same row cannot have this value.)

Question 4.5 *Finally, write the constraint (set of formula) indicating that each value between 1 and 3 only appears once and only once on each column.*

Question 4.6 *Press* Solve*: you should obtain 1 model. Write the corresponding square.*

4.2 Reasoning with a set of rules

We have a set of rules in botany that allow us to classify plants:

$$BR = \left\{ \begin{array}{l} flower \wedge seed \rightarrow phanerogame, \\ phanerogame \wedge nakedSeed \rightarrow firTree, \\ phanerogame \wedge 1\_cotyledon \rightarrow monocotyledon, \\ phanerogame \wedge 2\_cotyledon \rightarrow dicotyledon, \\ monocotyledon \wedge rhizome \rightarrow thrush, \\ dicotyledon \rightarrow windFlower, \\ monocotyledon \wedge \neg rhizome \rightarrow lilac, \\ leaf \wedge flower \rightarrow cryptogam, \\ cryptogam \wedge \neg root \rightarrow moss, \\ cryptogam \wedge root \rightarrow fern, \\ \neg leaf \wedge plant \rightarrow thallophyte, \\ thallophyte \wedge chlorophyll \rightarrow seeWeed, \\ thallophyte \wedge \neg chlorophyll \rightarrow mushroom, \\ \neg leaf \wedge \neg flower \wedge \neg plant \rightarrow colibacillus \end{array} \right\}$$

We add a set of facts BF which describes an example to classify. We will assume here that BF contains the following facts: $\{rhizome, flower, seed, 1\_cotyledon\}$.

Exercise 5 *Check that both BR is satisfiable and $BR \cup BF$ is satisfiable. Check that we can deduce thrush from the union of the two bases (that is, $BR \cup BF \models thrush$).*

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