Positive Logic Is **2-Exptime** Hard

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University of Warsaw

TYPES, Toulouse, April 26, 2013

Work in Progress...

Positive Logic Is **2-UExptime** Hard

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Positive Logic Is **2-co-NExptime** Hard

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Motivation

Foundational research on:

- Expressive power of “weak” intuitionistic logics.
- High-level properties of proof tactics.

Prenex normal form

In classical logic:

Every formula is classically equivalent to one of the form:

$$Q_1 x_1 Q_2 x_2 \ldots Q_k x_k . \mathit{Body}(x_, x_2, \ldots, x_k),$$

where *Body* has no quantifiers.

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Formulas may be classified according to the quantifier prefix, e.g. universal ($\forall^*$) formulas are in Π_1, and Π_2 is $\forall^* \exists^*$.

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In intuitionistic logic:

The prenex fragment is decidable in Pspace.

The language we study

To make things simpler, we consider first-order formulas

- with universal quantifiers and implications;
- without function symbols

This fragment is known to be undecidable.

Mints Hierarchy

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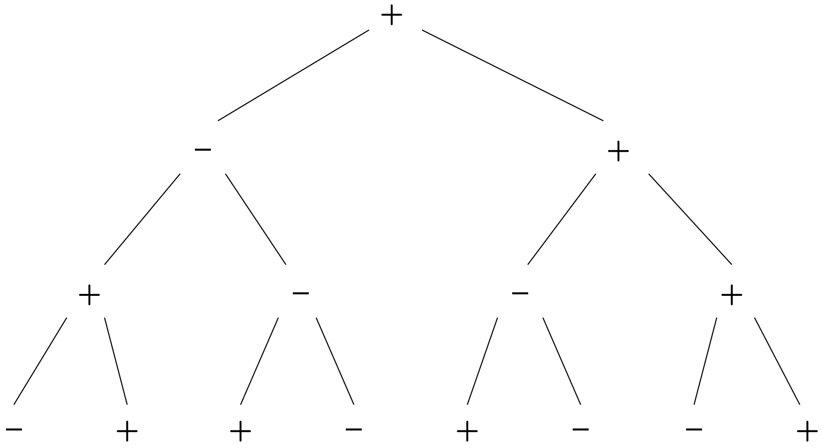
a formula *would get* if classically normalized.

For example, $\forall$ quantifiers occurring at *positive* positions
will remain $\forall$ in the prefix.

Positive and Negative



Mints Hierarchy

Π_1 – All quantifiers at positive positions.

Σ_1 – All quantifiers at negative positions.

Π_2 – One alternation: some negative quantifiers in scope of some positive ones.

Σ_2 – One alternation: some positive quantifiers in scope of some negative ones.

And so on.

Lower bounds for Mints Hierarchy

Π_1 – **2-UExptime**-hard

Σ_1 – At least **Exptime**-hard

Π_2 – Undecidable

Σ_2 – Undecidable

Work in progress: with function symbols

- Class Σ_1 becomes undecidable.
- Class Π_1 is of the same complexity as before.

A positive example

Let $\varphi = (I \to Tv) \to Ap(x), \quad \psi = I \to (D \to Tv) \to Ap(x)$ and $\vartheta = D \to Ap(x)$, and prove the formula

$$(\forall x(\varphi \to \psi \to \vartheta \to Ap(x)) \to Tv) \to Tv.$$

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Example continued

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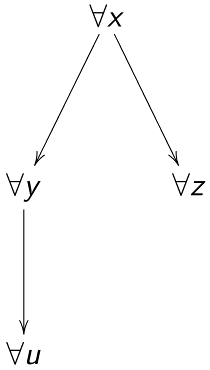
Nested quantifiers



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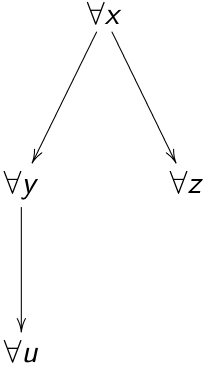
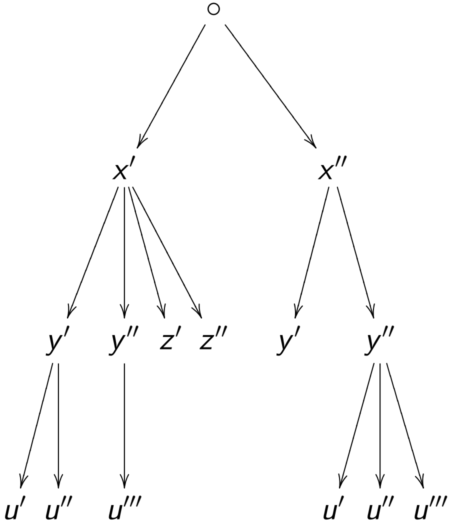
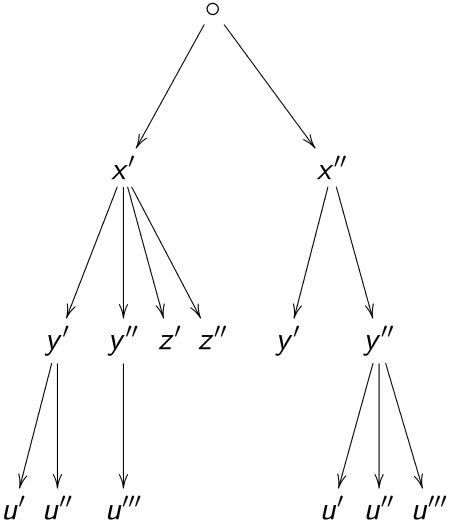




The tree of knowledge



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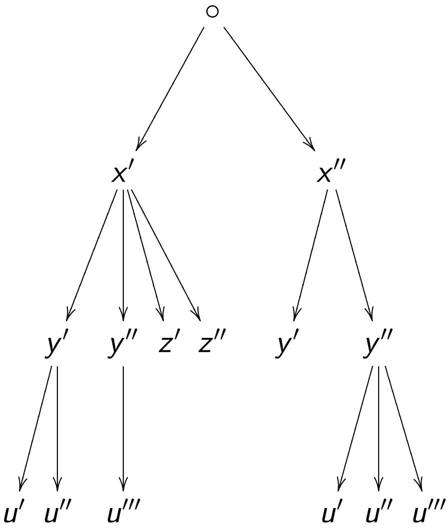
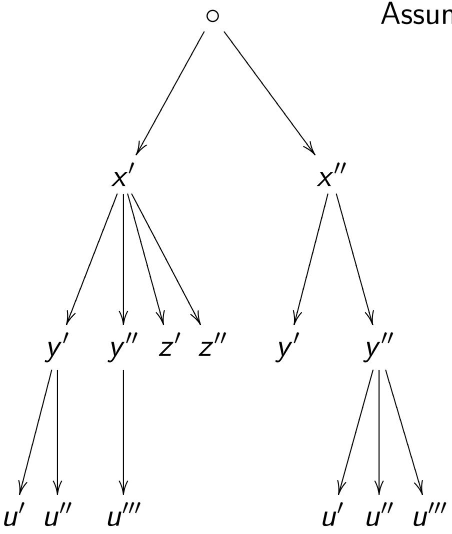


There may be different assumptions about each of the variables.

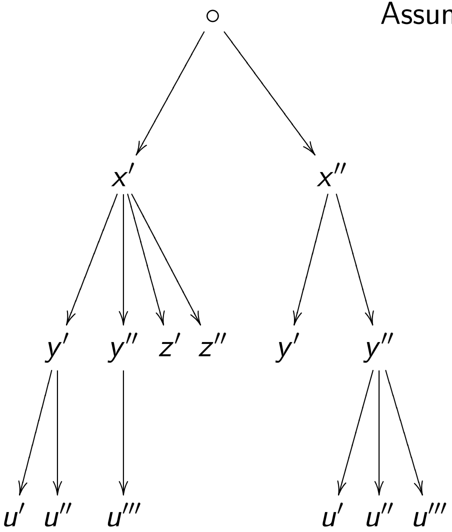
In other words, every node in the tree has a different "knowledge".

How many of them?



Assume n unary predicates.

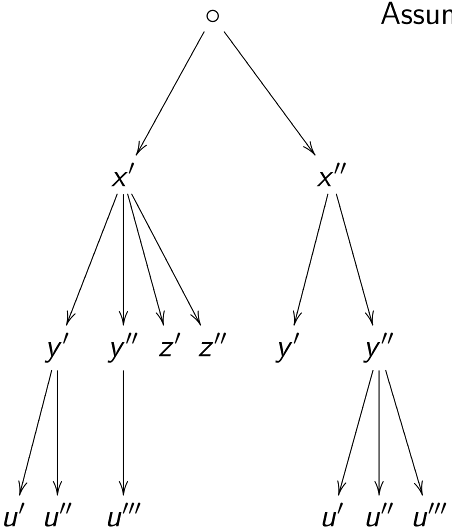
How many of them?



Assume n unary predicates.

$\Longleftarrow$ 2^n possibilities.

How many of them?



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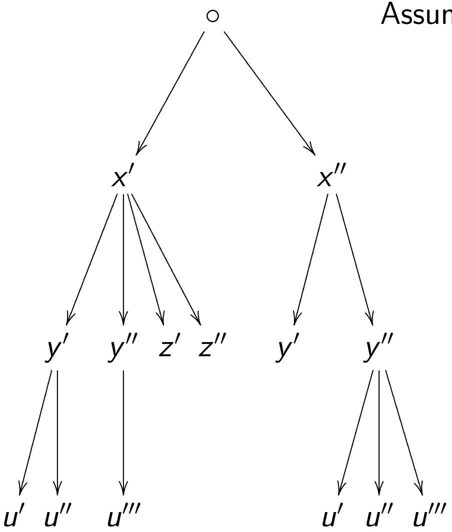

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$\Longleftarrow$ $2^{2^{2^n}}$ etc. etc.

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The state of the automaton corresponds to the proof goal.

The available assumptions on a variable y'' constitute the
„knowledge" of node y'' of the tree. This can be modeled
by memory registers associated to every node.

Proof search as computation

Assumptions about y'' = data written in registers at node y''

Present goal $P(y'')$ = machine at node y'' in state P.

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$R(x') \to Q(x') \to P(y'')$ = action possible only if register R at node x' is "1".

$\forall z(T \to Q_0(z)) \to P(y'')$ = create a new child z' of y''; enter node z' in initial state Q_0.

The tree of knowledge of good and...

Restricted access to data:

In intuitionistic logic one cannot reason from non-existence of assumptions, or delete assumptions.

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In this tree there is only good!

Alternation

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Existential choice because there may be more than one usable assumption.

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Universal choice because an assumption may have more than one premise.

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Universal choice because an assumption may have more than one premise.

(To derive $Ap(x)$ from $\varphi \to \psi \to Ap(x)$ one has to prove both φ and ψ.)

Eden automaton (simplified)

An ID is a triple $\langle q, T, w \rangle$, where q is a state, T is a tree of knowledge, and w is the *current apple* (a node of T).

There is a fixed number of binary registers at every node.

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- Check if a selected flag is up at a given ancestor of w;

Eden automaton (simplified)

An ID is a triple $\langle q, T, w\rangle$, where q is a state, T is a tree of knowledge, and w is the *current apple* (a node of T).

There is a fixed number of binary registers at every node.

Possible actions:

- Move the apple can up to the father of w or down to a nondeterministically chosen child of w;
- Raise a selected flag at a given ancestor node of w;
- Check if a selected flag is up at a given ancestor of w;
- Create a new child of w and move the apple there.

The main result (1)

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From a universal Turing Machine M working in time $2^{2^{\mathcal{O}(n)}}$ and an input word x of length n

we construct (in Logspace) an Eden automaton A such that

M accepts x iff A terminates.

The main result (2)

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From an Eden automaton A we define (in Logspace) a positive first-order formula Φ such that

A terminates iff Φ is provable.