Effective Types for C formalized in Coq
(work in progress)

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Aliasing

Aliasing: different pointers pointing to the same object

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int f(int *p, int *q) {
  int x = *p; *q = 314; return x;
}
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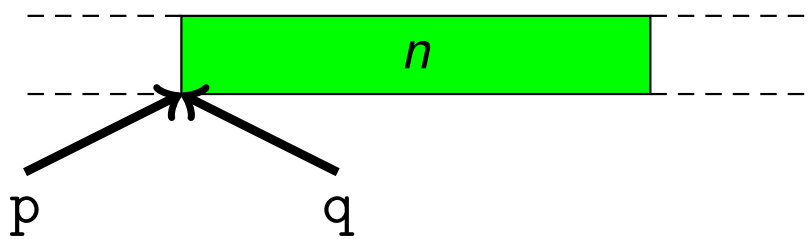
When called with aliased pointers, the original value *n* of `*p` is returned



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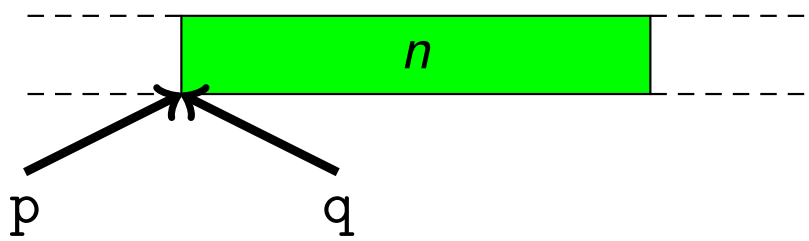

Optimizing `x` away is unsound, as 314 would always be returned

Aliasing with different types

The function

```
int h(int *p, float *q) {
  int x = *p; *q = 3.14; return x;
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can still be called with aliased pointers

```
union { int x; float y; } u;
u.x = 271;
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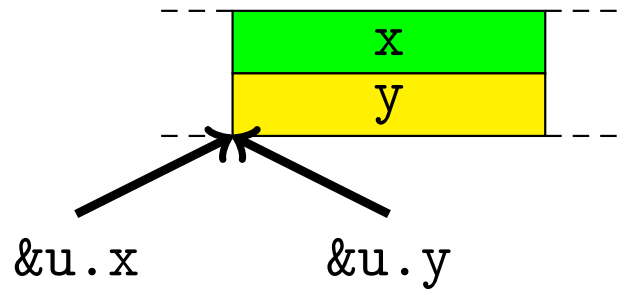

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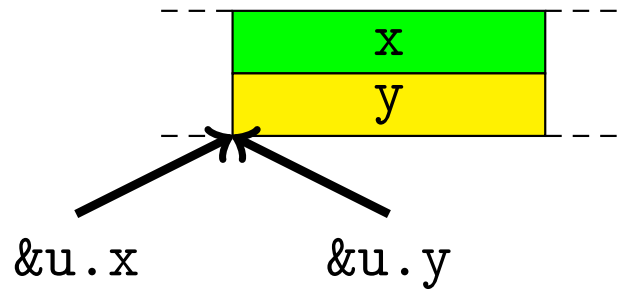

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In C11, types can be used for non-aliasing hypotheses:

- Reads/writes "using the wrong type" are "not allowed"
- A compiler can use that to assume that `p` and `q` do not alias

Undefined behavior according to the C standard

- The C standard classifies these "not allowed" programs as *undefined behavior*
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A formal C semantics should thus account for undefined behavior

Other difficulties of the C11 memory model

Interplay between low- and high-level

- Each object *should be* represented as a sequence of bytes
  . . . which can be inspected and manipulated *in C*
- Each object can be accessed using typed expressions
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The standard is unclear on many of such difficulties. Opportunities for a formal semantics to resolve this unclarity!

Contribution

An abstract formal memory for C supporting

- Types (arrays, structs, unions, . . .)
- Strict aliasing restrictions (effective types)
- Indeterminate values
- Pointers one past
- Byte-level operations
- Parametrized by an interface for machine integers
- Multiple object representations
- Alignment
- Formalized in Coq

How others treat pointers

Many existing formal C semantics (*e.g.* CompCert) treat the memory as containing just bytes:

- The memory is a finite map of cells, each cell consisting of a array of bytes
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Too little information to capture strict aliasing restrictions

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Gives a semantics for strict aliasing restrictions

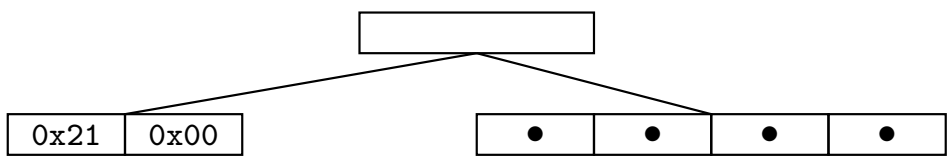
Overview of our memory

Our memory consists of three layers. For

```
struct { short x, *p; } s = { 33; &s.x }
```

consider:

- *Memory values* with arrays of bytes as leafs:



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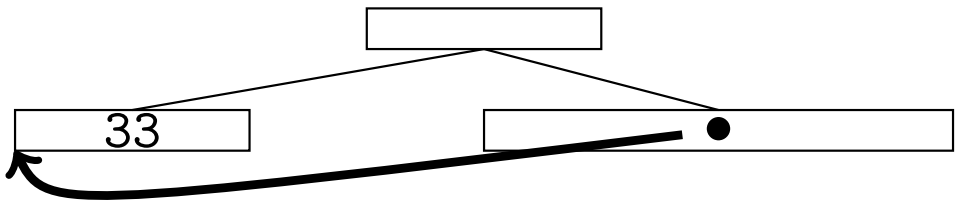

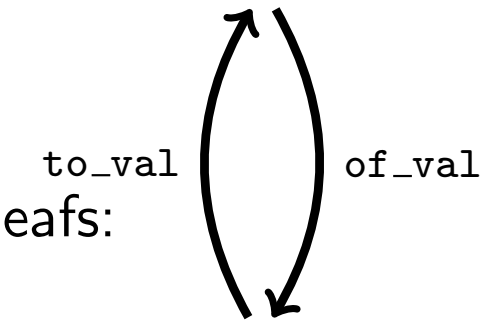

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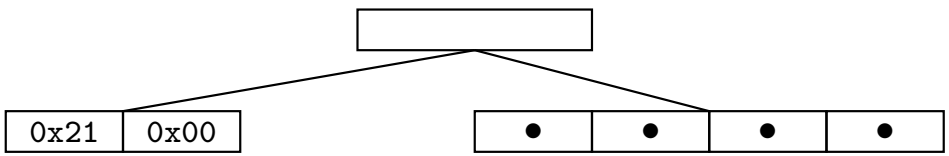

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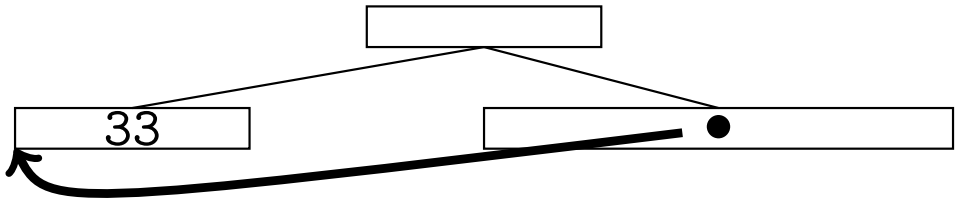

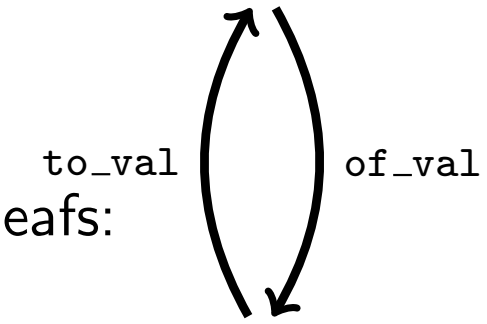

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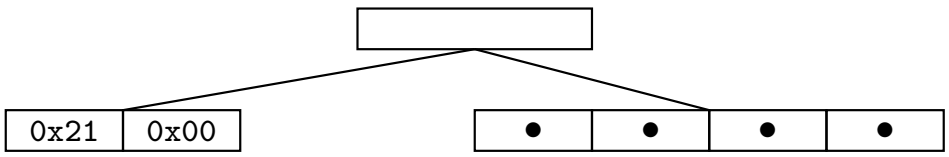

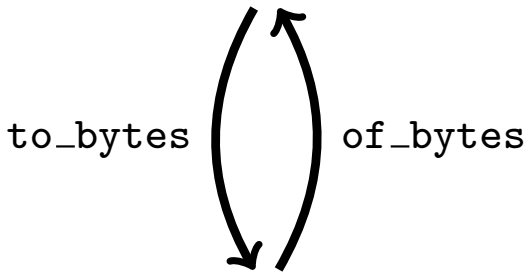

- Arrays of bytes:

Bytes and memory values

- *Bytes* are represented symbolically (à la CompCert):

$$b ::= \text{“}\textit{bit sequence}\text{”} \mid (\texttt{ptr}\ p)_i \mid \texttt{undef}$$

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- *Memory values* are defined as:

$$\begin{aligned} w ::= {} & \texttt{base}_{\tau_\mathsf{b}}\ \vec{b} \mid \texttt{array}_\tau\ \vec{w} \\ & \mid \texttt{struct}_s\ \vec{w} \mid \texttt{union}_s\ (i, w) \mid \overline{\texttt{union}_s}\ \vec{b} \end{aligned}$$

- With corresponding typing judgments

Example

Consider:

```
struct T {
  union U { short x; int y; } u;
  short *p;
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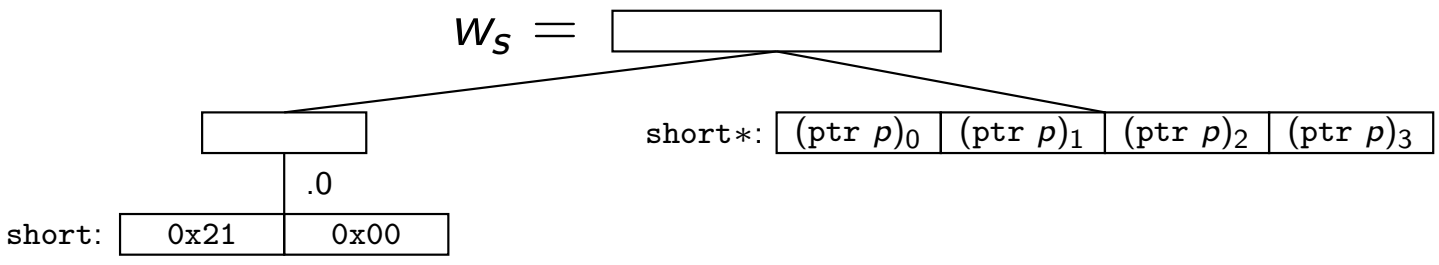

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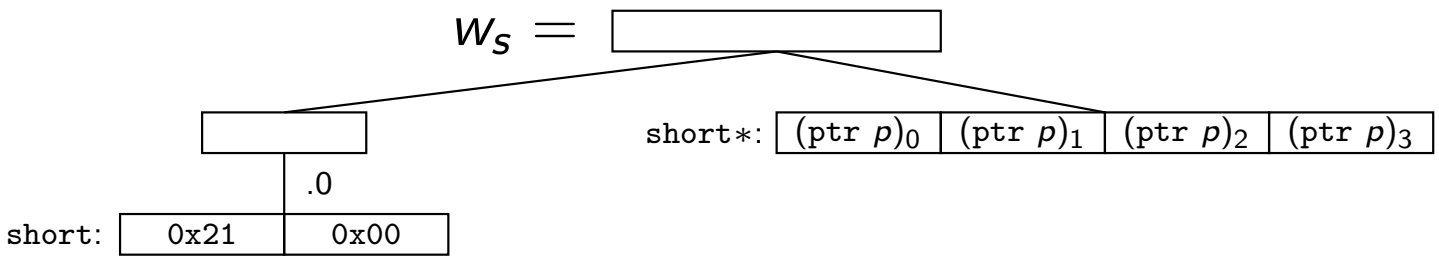

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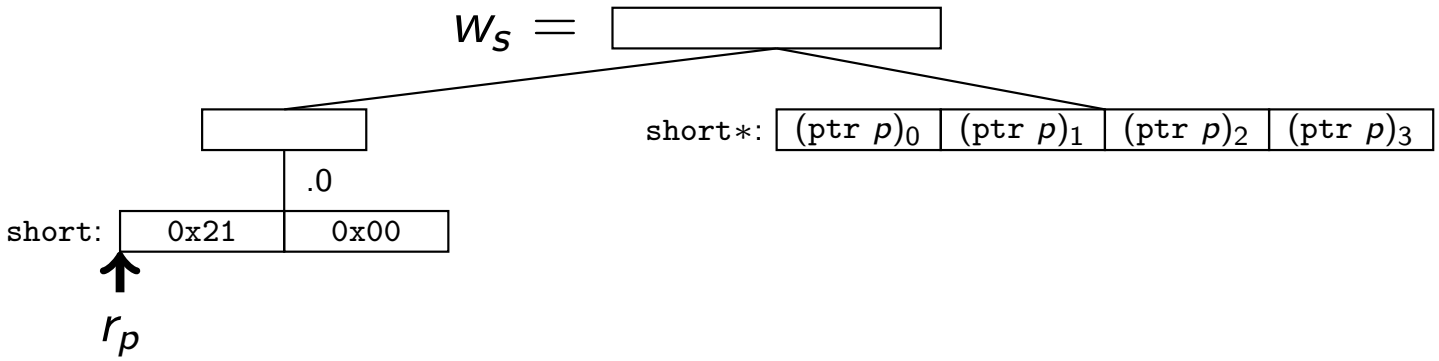

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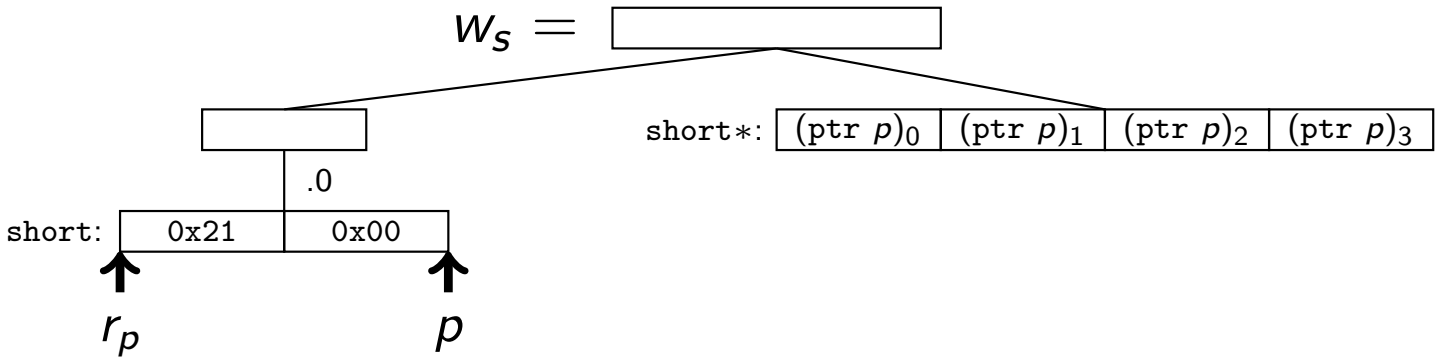

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Hard part: dealing with this choice in *abstract values* and the various operations

Formalization in Coq

Type theory is ideal for the combination programming/proving

- *The devil is in the details*, Coq is extremely useful for debugging of definitions
- Useful to prove meta-theoretical properties
  e.g. memory embeddings and commuting diagrams
- Use of type classes for parametrization by machine integers
- Use of type classes for overloading of notations
- Light use of dependent types to get better properties on pseudo terms *e.g.* ranges of pointers
- 7.500 lines of code for the memory model out of 28.000

Future research

- Floating point numbers
- Variable length arrays
- The `const`, `volatile` and `restrict` qualifier
- Integration into our operational semantics
  ... and make it (reasonably efficiently) executable
- Memory injections à la CompCert
- Correspondence with CompCert
- Integration into our axiomatic semantics
- Verification Condition generator in Coq

Questions

Sources: see `http://robbertkrebbers.nl/research/ch2o/`

Aliasing optimizations

Is the optimization `h` $\rightarrow$ `h_opt` correct?

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int h(int *p, float *q) {
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  Four cases:
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  4. $m(z, r) = \overline{\texttt{union}_s}\ \vec{b}$:
     Now `*p` changes the memory to $m(z, r) = \texttt{union}_s\ (i, v)$
     After that, `*q = 3.14` fails