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# Property-Based Timing Analysis and Optimization for Complex Cyber-Physical Real-Time Systems

Jian-Jia Chen

**TU Dortmund**

13.06.2023

# Outline

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Motivations of Property-Based Analysis and Optimizations

Self-Suspension Properties

Symmetric Multicore Timing Analysis

Gang Scheduling

Further Properties and Obstacles

Conclusion and Summary

# Contents

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Motivations of Property-Based Analysis and Optimizations

Self-Suspension Properties

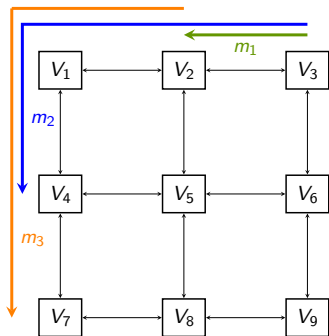
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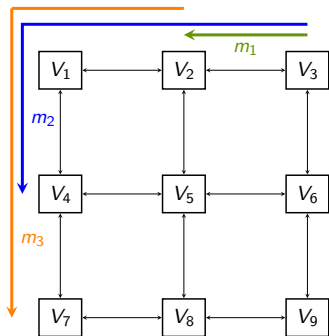
# An Example of Classical Timing Analysis Flow



- Suppose to analyze lowest-priority message  $m_3$
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3 periodic messages  
fixed path, preemptive  
priority:  $m_1 > m_2 > m_3$

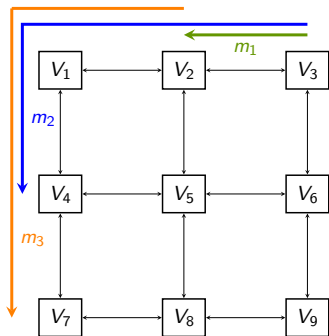
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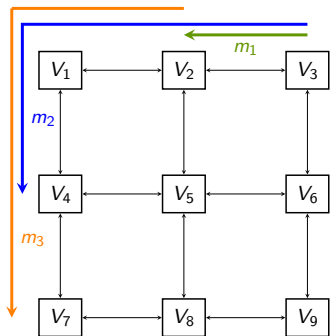
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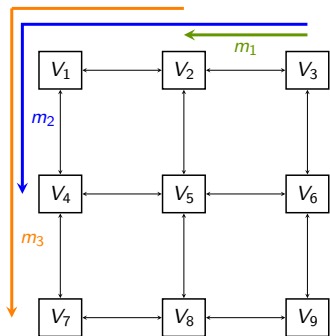
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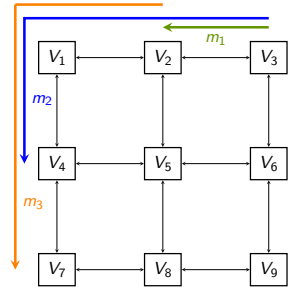
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- At least 8 papers introduced unsafe analyses



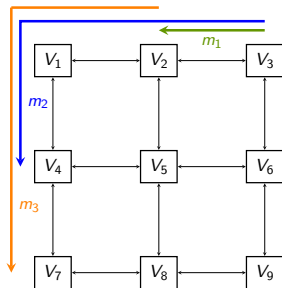
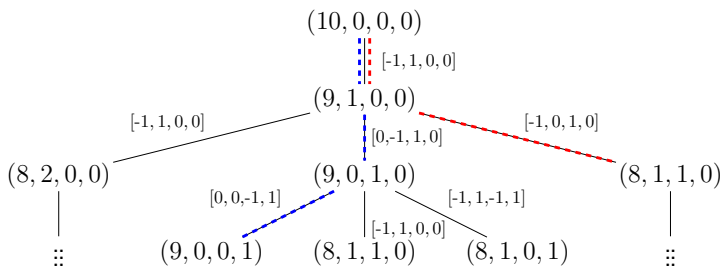
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- Imagine that the path of  $m_3$  is equivalent to a uniprocessor
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- Ad-hoc solutions for one dedicated problem
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- Misuse of critical instant theorem in the first four analyses of previous example

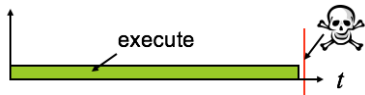


Radically new  
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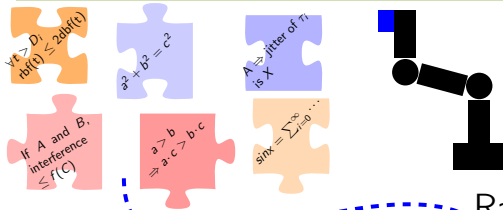
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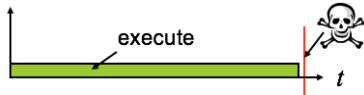
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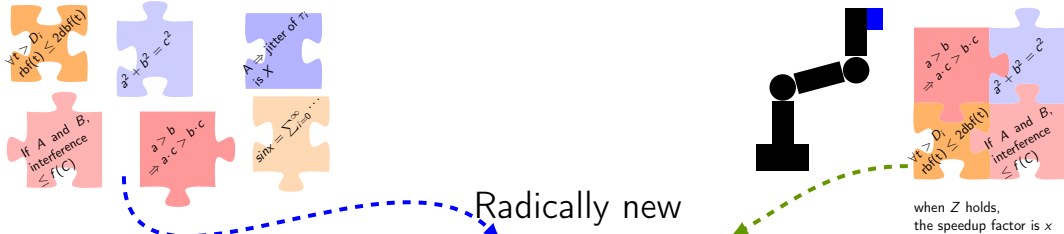


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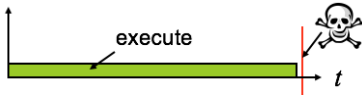




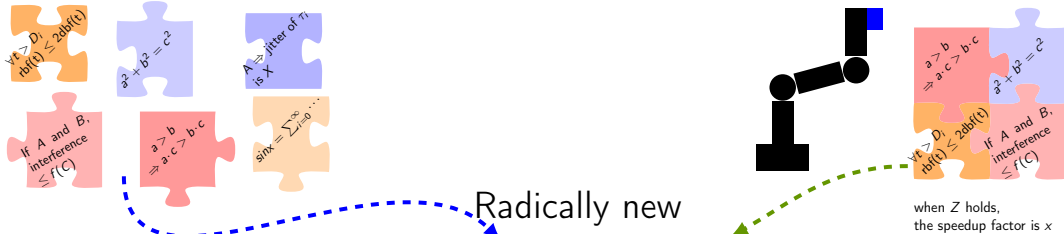
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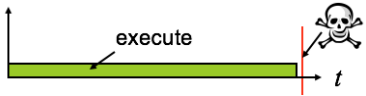
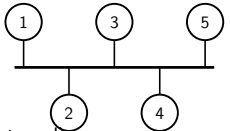
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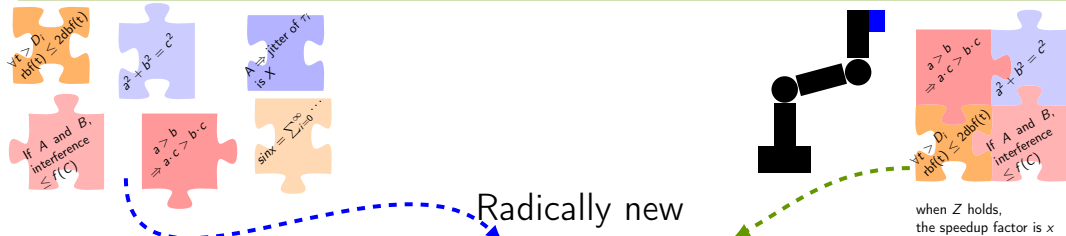
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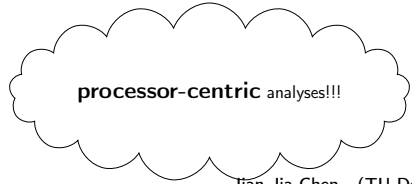
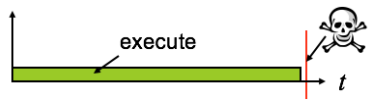
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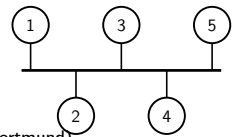


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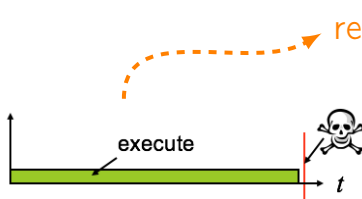
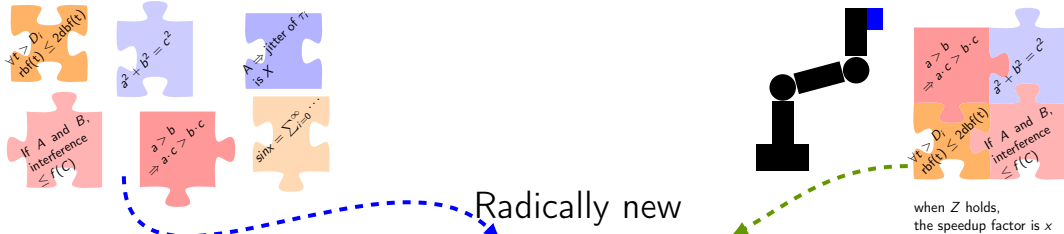


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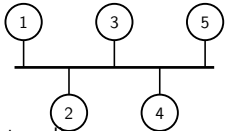
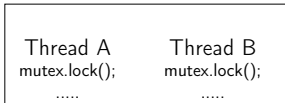
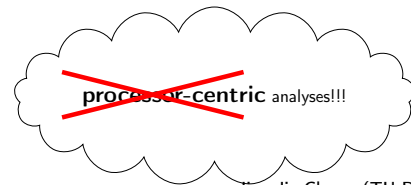
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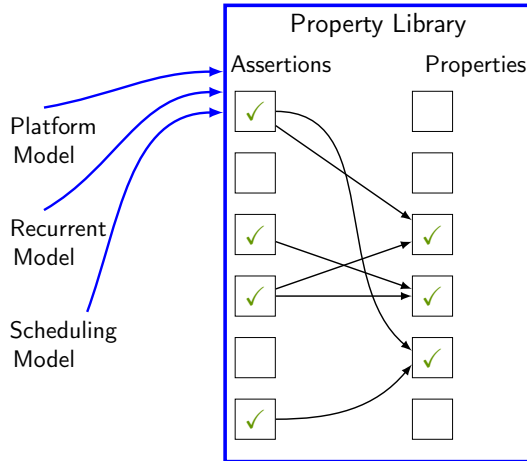


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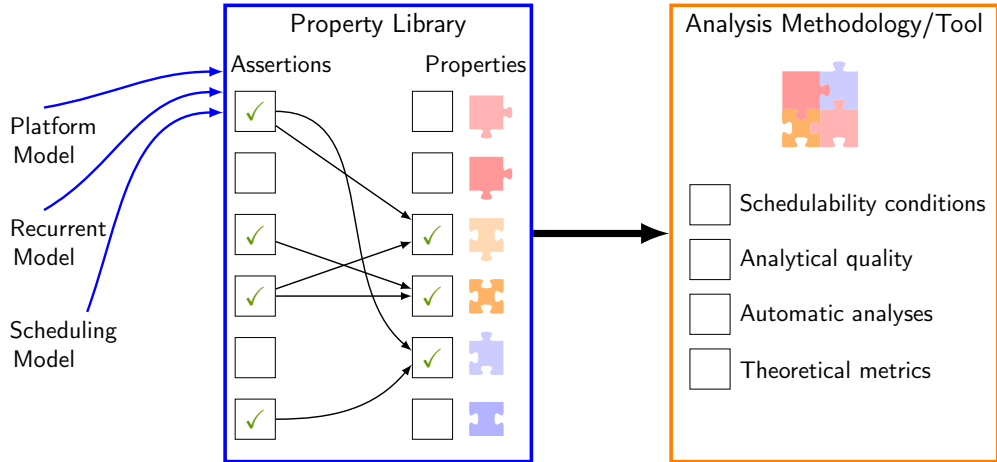


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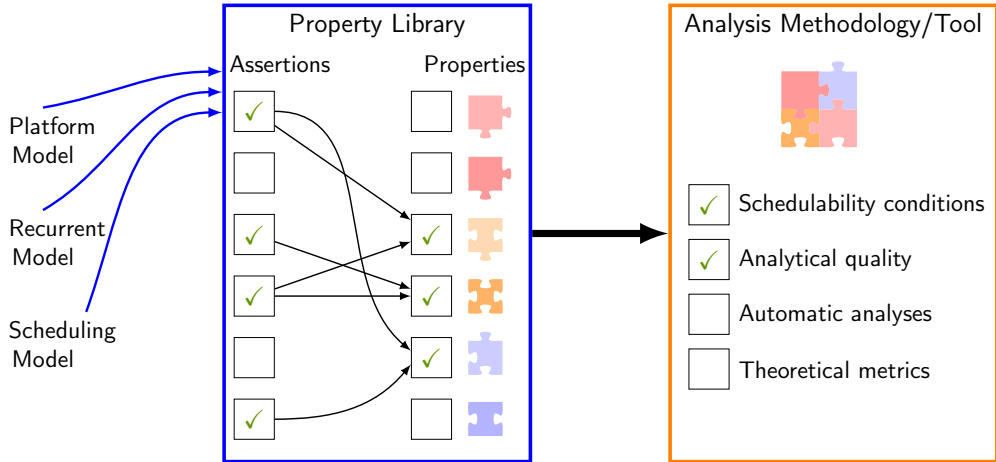
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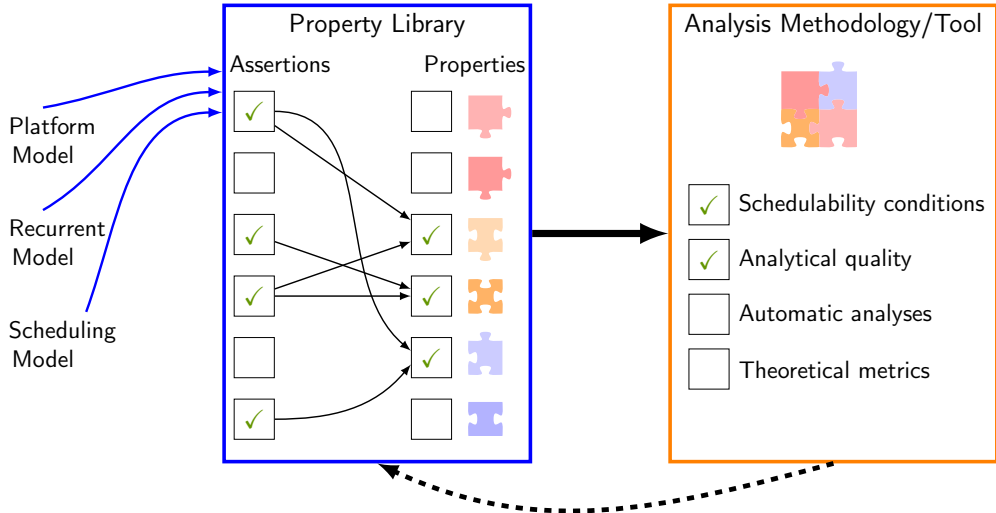
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# Goals of PropRT

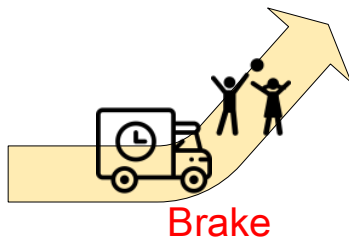
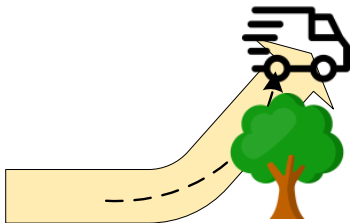
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- Formal properties to modularly compose real-time embedded systems
- Methodologies for generating/verifying properties and modular compositions
- Predictable interplay of computation, communication, and synchronization for complex real-time embedded systems

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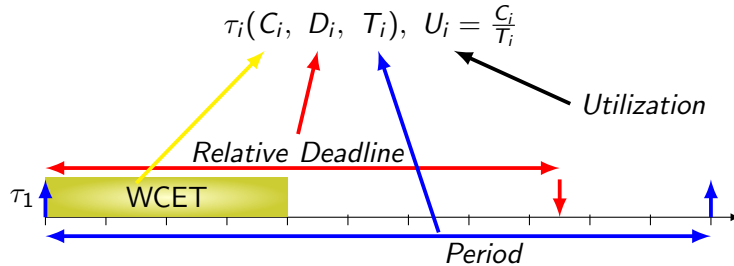
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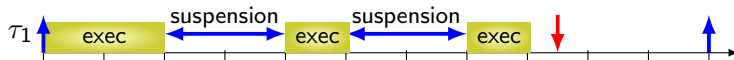
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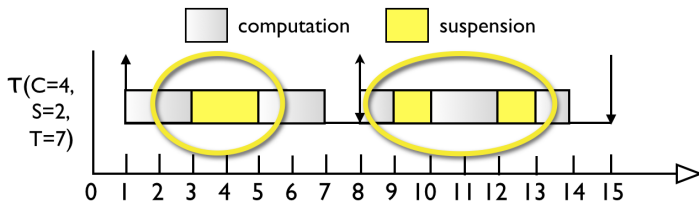
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Suspension time:  $S_i$

# Possible Self Suspensions

Implicit-deadline sporadic suspending task:  $\tau(C, S, T)$



Jobs may alternate between **computation** and **suspension** phases **without any restriction** on how they **interleave**

- 1-Segmented self-suspension: 2 computation segments separated by a suspension interval
- Segmented self-suspension:  $f$  computation segments separated by  $f - 1$  suspension intervals
- Dynamic self-suspension: the suspension pattern is unknown and can be arbitrary

# Reasons for Suspension: Computation Offloading

## Pseudo-code for this system

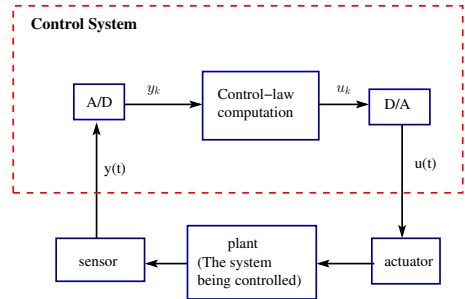
set timer to interrupt periodically with period  $T$ ;

at each timer interrupt

do

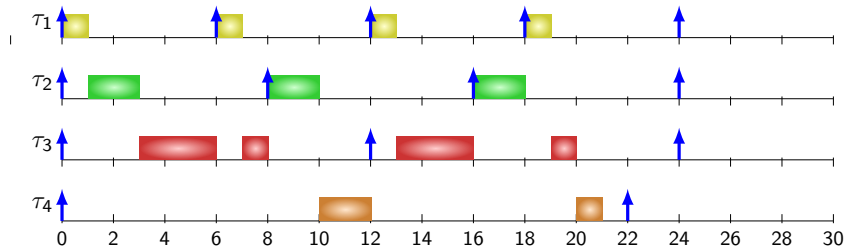
- perform analog-to-digital conversion to get  $y$ ;
- compute control output  $u$  by using accelerators;
- output  $u$  and do digital-to-analog conversion;

od





# The Golden Critical Instant Theorem (without Suspension)



- Release the higher-priority tasks at the same time as task (i.e.,  $\tau_k$ ) under analysis
- The following jobs of a higher-priority task should be released then by following the period constraint

$$\exists t | 0 < t \leq D_k \text{ s.t. } C_k + \sum_{\tau_j \in \text{higher\_priority}(\tau_k)} \left\lceil \frac{t}{T_j} \right\rceil C_j \leq t.$$

# Suspension Induces Jitter under Fixed-Priority

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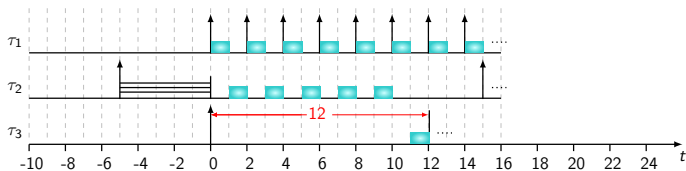
Schedulability test of task  $\tau_k$ :

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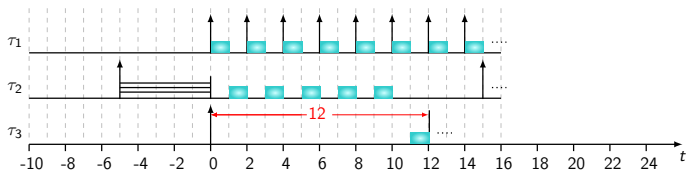


$\tau_i$	$C_i$	$S_i$	$T_i$
$\tau_1$	1	0	2
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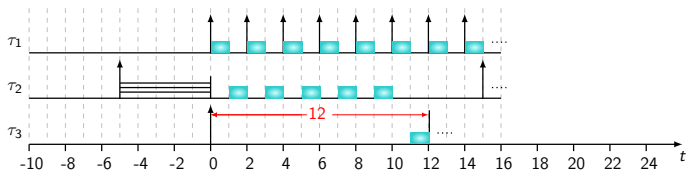
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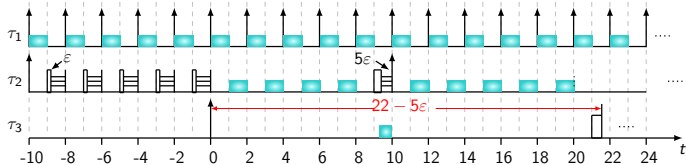
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Worst Case



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- Ridouard et al. in RTSS 2004
  - Uniprocessor scheduling of self-suspending tasks is NP-hard in the strong sense
- Bletsas and Audsley in RTAS 2004 (**flawed**), ECRTS 2004 (**flawed**), RTCSA 2005 (**flawed**)
- Lakshmanan et al. in RTAS 2010 (**flawed**)
- Kim et al. in RTSS 2013 (**flawed**)
- Ding et al. 2009 (**flawed**)
- Meng RTCSA 1994 (**flawed**)
- Kim et al. in RTCSA 1995 (**flawed**)
- Rajkumar IBM report 1991 (**inconclusive**)



# My Contribution for Systems with Self Suspensions

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- A summary of misconceptions in the literature (Real-Time Systems Journal 2019)
- 15+ technical papers
  - Publications detailing how to safely analyze the timing properties for different self-suspension models
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- Extension for resource-centric analyses
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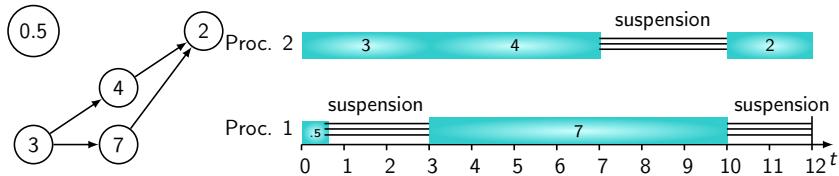
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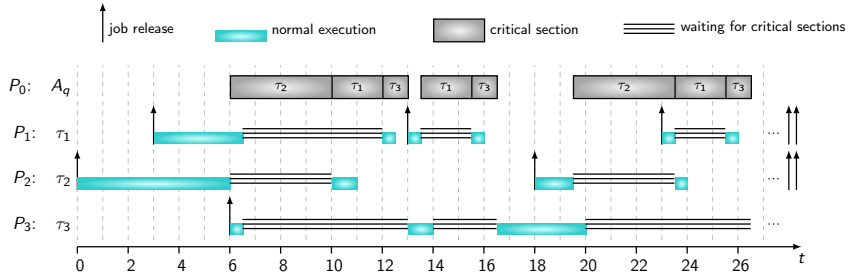
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  - Type-aware scheduling (IEEE TC 2023)
- Extension in multiprocessor locking protocols
  - Resource-oriented partitioned scheduling (RTSS)
  - Dependency-graph approach (RTSS 2018, RTAS 2019)

# Reasons for Suspension: DAG Structure



- A task may be parallelized such that it can be executed simultaneously on some processors to perform independent computation
- To this end, we can use a *directed acyclic graph (DAG)* to model the dependency of the subtasks in a sporadic task
- Each vertex in the DAG represents a subtask

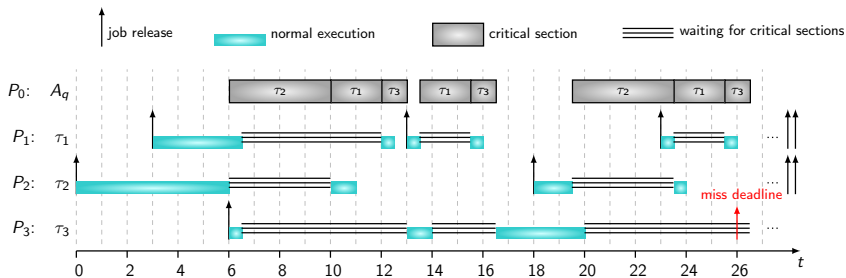
# Reasons for Self-Suspensions: Locking Protocols



- Distributed PCP in the above example
- Semaphores in multiprocessor systems: remote blocking due to mutual exclusion

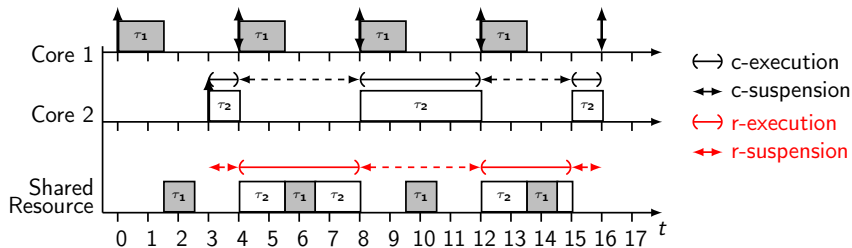
# Does Spinning Avoid Self-Suspension?

$\tau_i$	$Proc(\tau_i)$	$C_i$	$T_i (= D_i)$	$N_k$	$L_i$
$\tau_1$	$Proc_1$	6	10	1	2
$\tau_2$	$Proc_2$	11	18	1	4
$\tau_3$	$Proc_3$	8	20	3	1



A job of task  $\tau_3$ : run 0.5 time unit on  $Proc_3$ , critical section 1 time unit, run 1 time unit on  $Proc_3$ , access the critical section for 1 time unit, run 3.5 time units on  $Proc_3$ , and access the critical section for 1 time unit

# Reasons for Self-Suspensions: Physical Resource Sharing



- Multiple cores may share a bus
- The contention on the bus can be considered as a suspension problem (with respect to the bus access)



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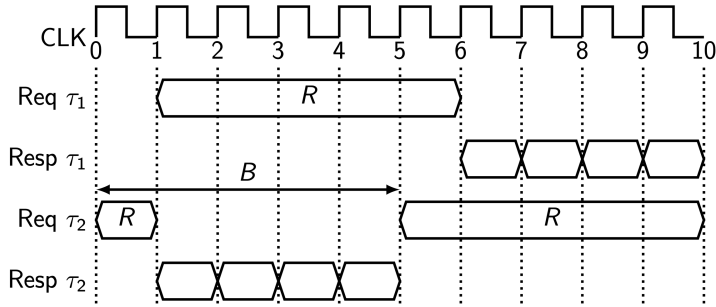
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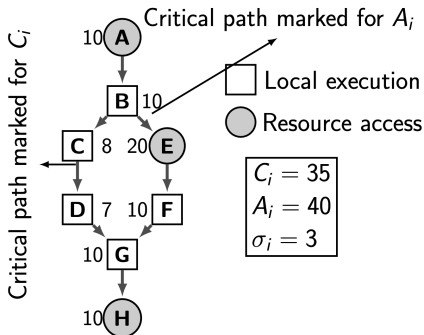
# Platform Model

- Multicore with a share resource
- For example, atomic (non-split-transaction) bus
  - Bus sits idle while memory processes the request and sends the response
- Fixed-priority arbitration



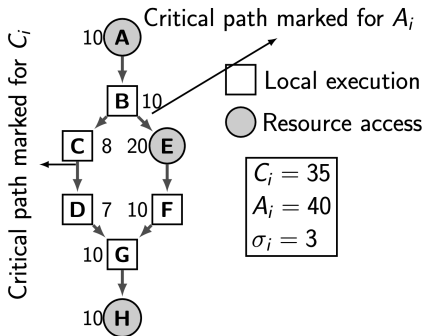
# Task and Scheduling Model

- Resource access task  $\tau_i$   
( $C_i, A_i, T_i, D_i, \sigma_i$ )
  - $C_i$ : upper bound on local computation
  - $A_i$ : upper bound on resource accesses
  - $T_i$ : minimum inter-arrival time
  - $D_i$ : relative deadline ( $D_i \leq T_i$ )
  - $\sigma_i$ : the maximum number segments of consecutive resource accesses
- Path analysis
- Fixed-priority scheduling (we use deadline-monotonic scheduling)



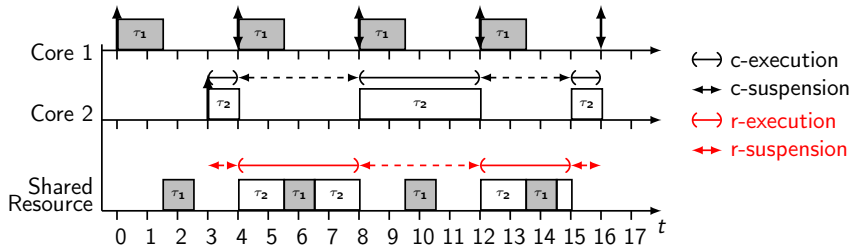
# Task and Scheduling Model

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Assume compositional properties: 75 is a safe upper bound.

# Key Observations: Symmetric Property



- From the core perspectives for  $\tau_2$ 
  - accessing or waiting: [3,4), [8,12), [15, 16)
  - suspension: [4,8), [12, 15)
- From the shared resource perspectives for  $\tau_2$ 
  - executing or waiting: [4,8), [12, 15)
  - suspension: [3,4), [8,12), [15, 16)

# Schedulability Test for Task $\tau_k$

---

- WCRT is upper bounded by the minimum  $t | 0 < t \leq D_k$

$$(C_k + \text{exec\_core}(t)) + (A_k + \text{exec\_resource}(t)) \leq t$$

# Schedulability Test for Task $\tau_k$

---

- WCRT is upper bounded by the minimum  $t | 0 < t \leq D_k$

$$\left( C_k + \sum_{\tau_i \in hp(\tau_k, c)} \left\lceil \frac{t + T_i}{T_i} \right\rceil C_i \right) + \sigma_k B + \left( A_k + \sum_{\tau_i \in hp(\tau_k, r)} \left\lceil \frac{t + T_i}{T_i} \right\rceil A_i \right) \leq t$$

- $\sigma_k B$ : the maximum blocking time by the lower priority tasks on the shared resource
- $hp(\tau_k, c)$ : higher-priority tasks than  $\tau_k$  on the same core
- $hp(\tau_k, r)$ : higher-priority tasks than  $\tau_k$  on shared resource

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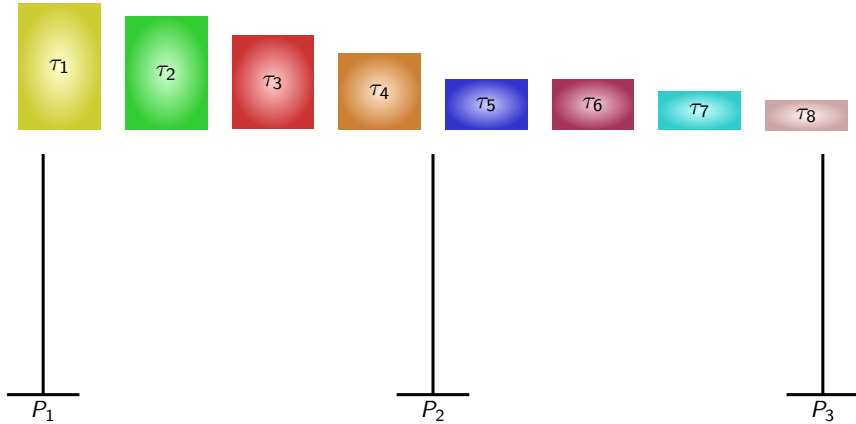
$$\left( C_k + \sum_{\tau_i \in hp(\tau_k, c)} \left\lceil \frac{t + T_i}{T_i} \right\rceil C_i \right) + \sigma_k B + \left( A_k + \sum_{\tau_i \in hp(\tau_k, r)} \left\lceil \frac{t + T_i}{T_i} \right\rceil A_i \right) \leq t$$

- $\sigma_k B$ : the maximum blocking time by the lower priority tasks on the shared resource
- $hp(\tau_k, c)$ : higher-priority tasks than  $\tau_k$  on the same core
- $hp(\tau_k, r)$ : higher-priority tasks than  $\tau_k$  on shared resource
- Pessimism of the above response time analysis: number of resource access segments was not exploited
- In our paper, we explain how to [calculate and utilize](#) the information  $\sigma_k$  in a symmetric and more precise manner



# Task Assignment (Partition)

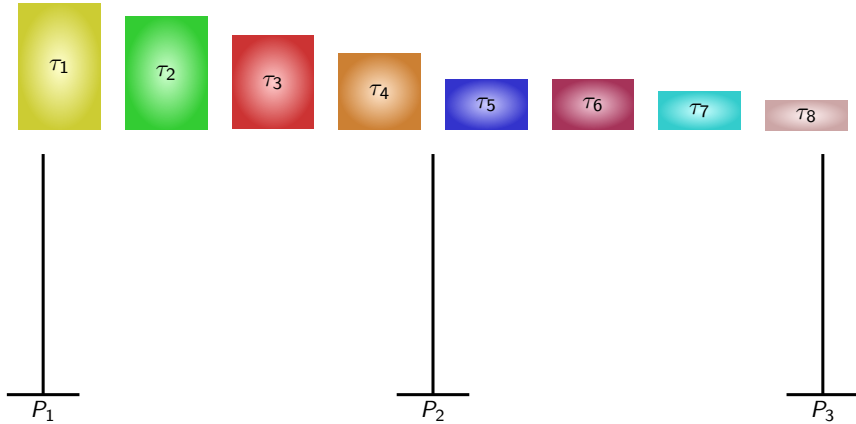
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- Schedulability tests are based on the previous slide.
- Fitting can be First-Fit (FF), Worst-Fit (WF), Best-Fit (BF)

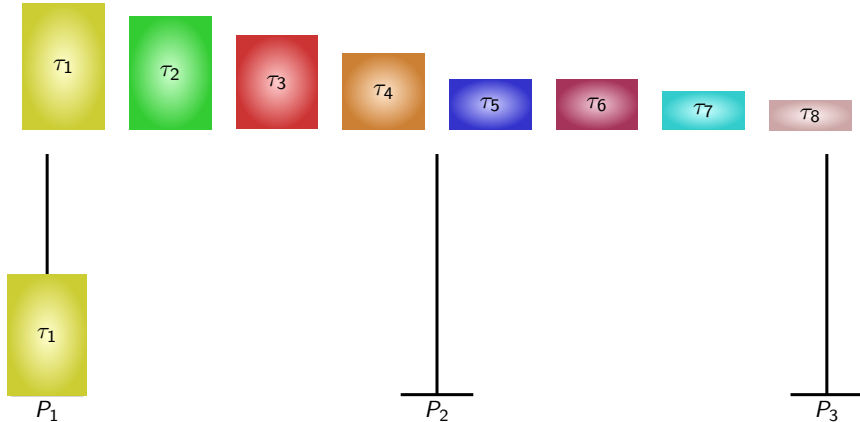
# Task Assignment (Partition)

---



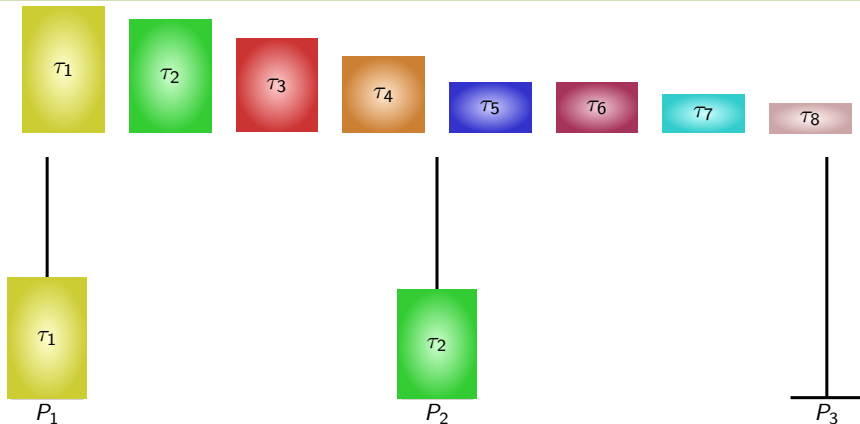
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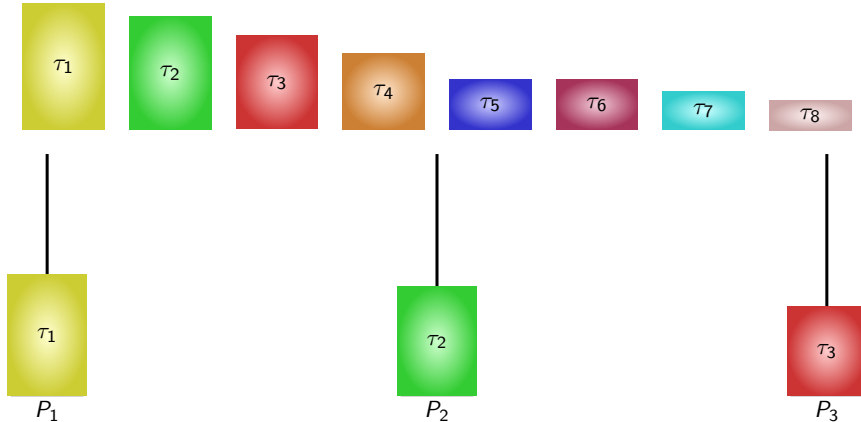
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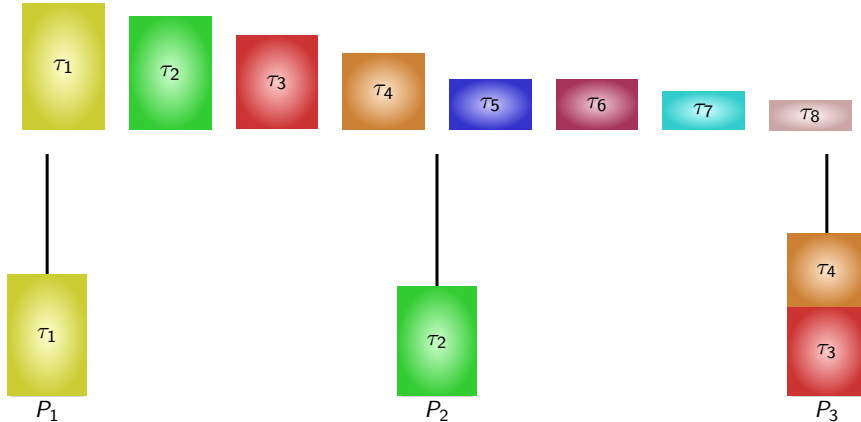
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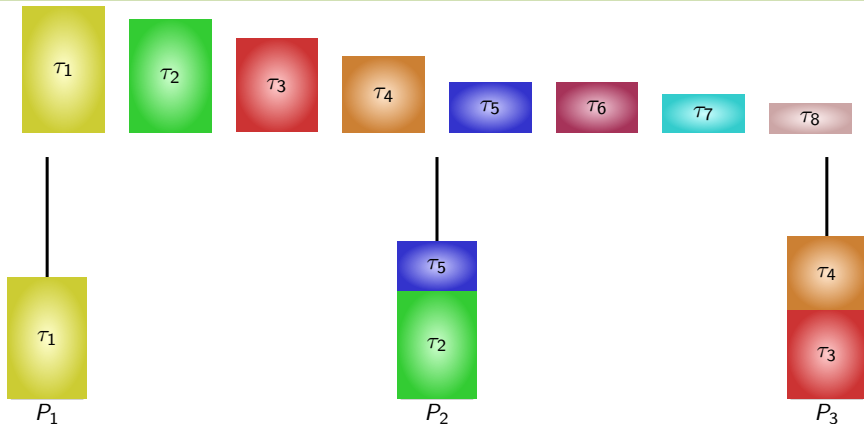
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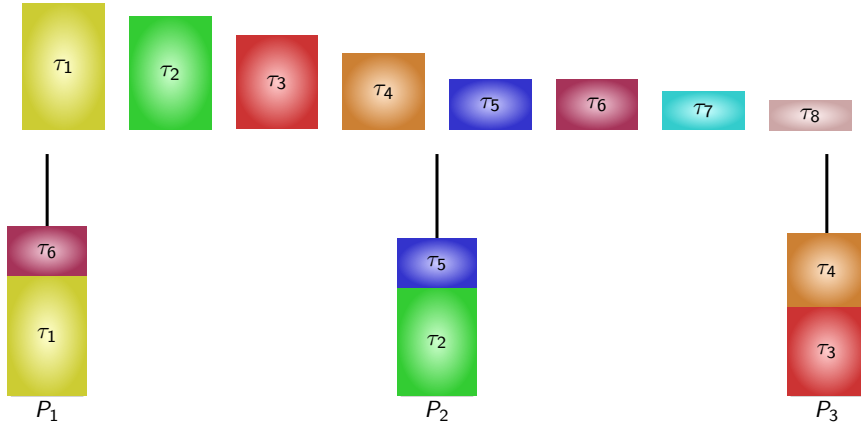
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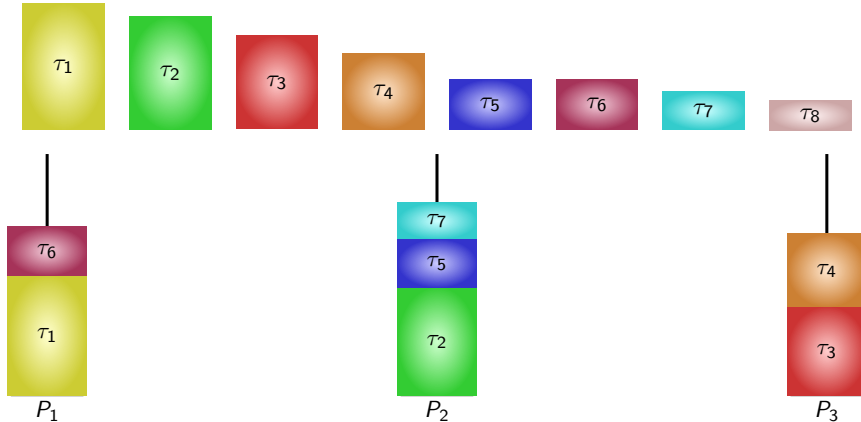
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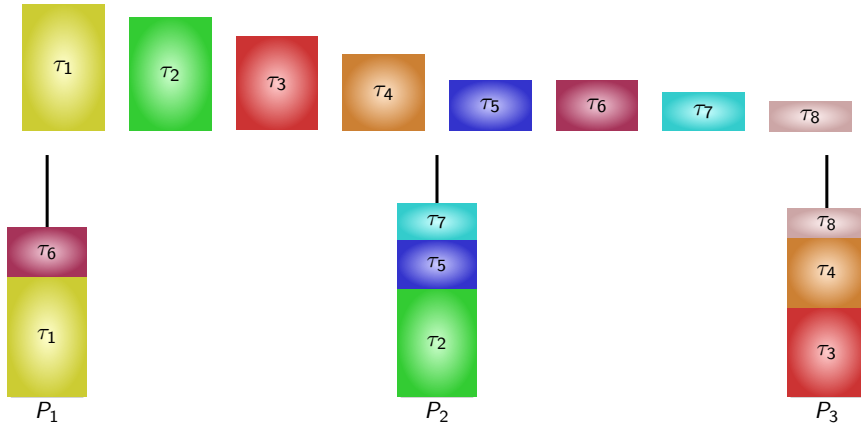


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# Experiments

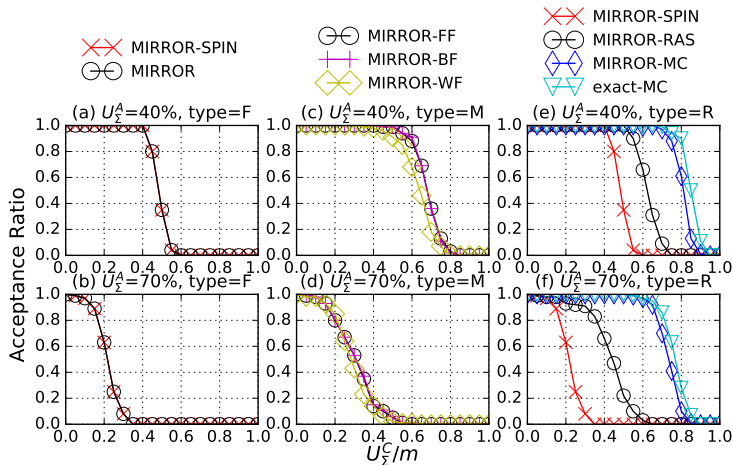
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- Configuration
  - 4-core platform ( $m=4$ )
  - 20 tasks
  - Periods [10-1000ms]
  - Each utilization level: 100 task sets
- Comparison:
  - Exact-MC (Bonifaci et al. in RTNS 2015): do memory access and then do execution
  - MIRROR-SPIN (This resembles the test from Altmeyer et al. in RTNS 2015)
- Evaluation Metrics:
  - The acceptance ratio of a level: the number of task sets that are schedulable by the test divided by the number of task sets.

# Experiments

Resource access segments  $\sigma_i$ :

- 1 (rare access, type=R),
- 2 (moderate access, type=M),
- 10 (frequent access, type=F).



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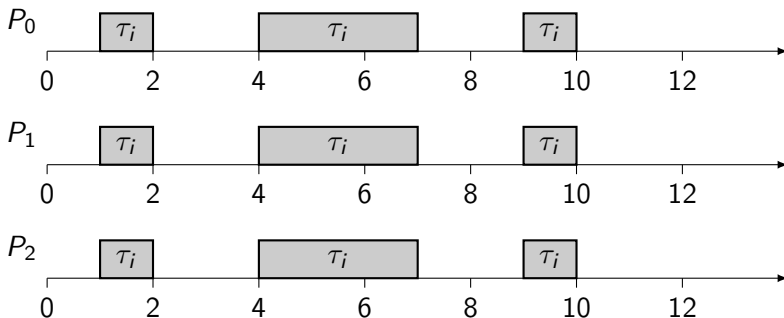
**Gang Scheduling**

Further Properties and Obstacles

Conclusion and Summary

# Gang Scheduling

A set of threads is grouped together into a *gang* s.t. they must be *co-scheduled at the same time*

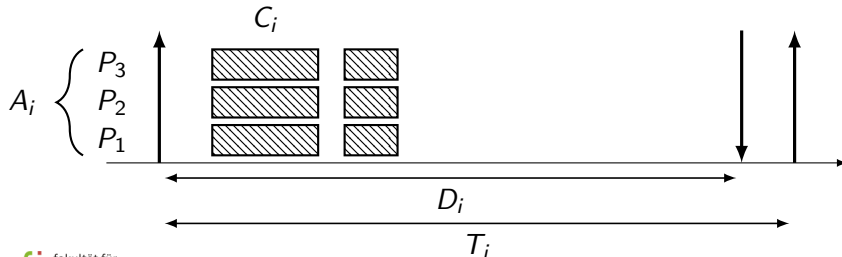


# Gang Task Model

## Definition

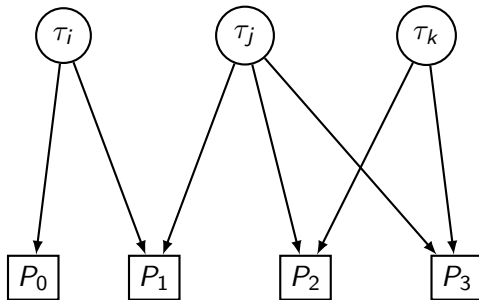
[A *sporadic constrained-deadline gang task*  $\tau_i$ ]

- WCET:  $C_i$
- Gang size:  $E_i$
- Relative deadline:  $D_i \leq T_i$
- Minimal inter-arrival time:  $T_i$



# Exemplary Stationary Gang Assignment

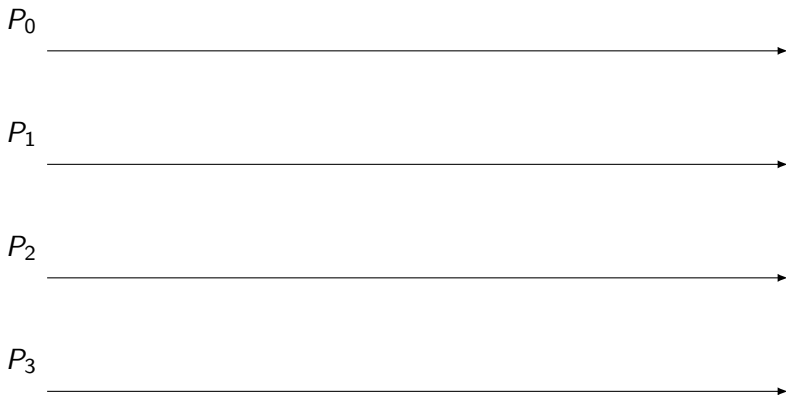
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# Fixed-Priority Stationary Gang Schedule

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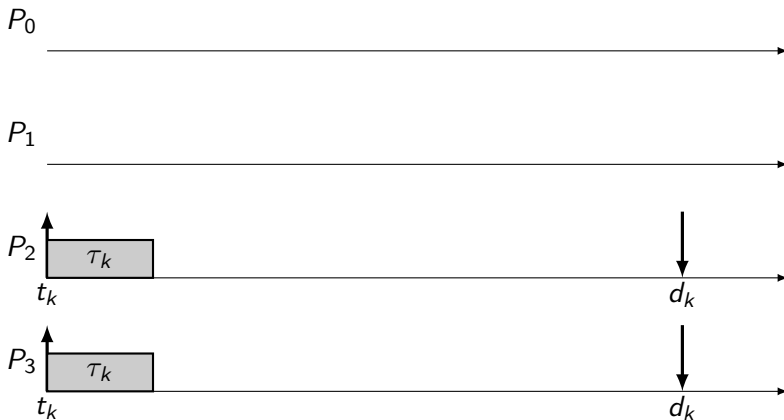
# Fixed-Priority Stationary Gang Schedule

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# Fixed-Priority Stationary Gang Schedule

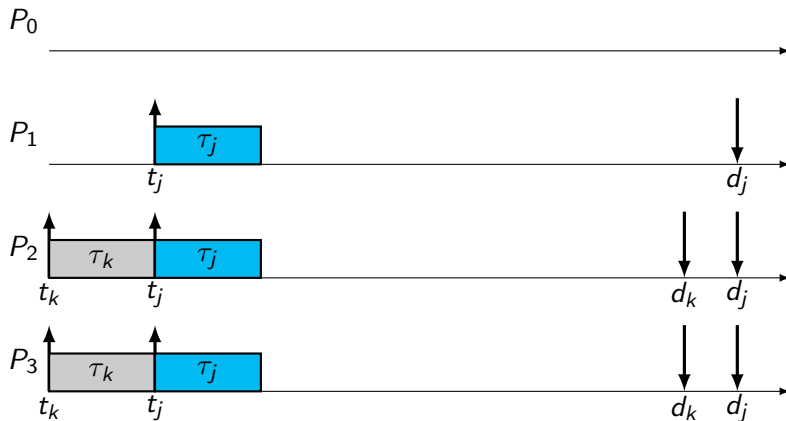
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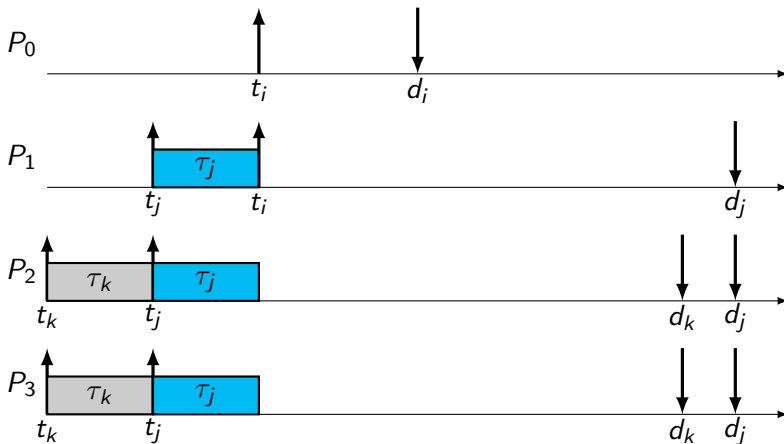
# Fixed-Priority Stationary Gang Schedule



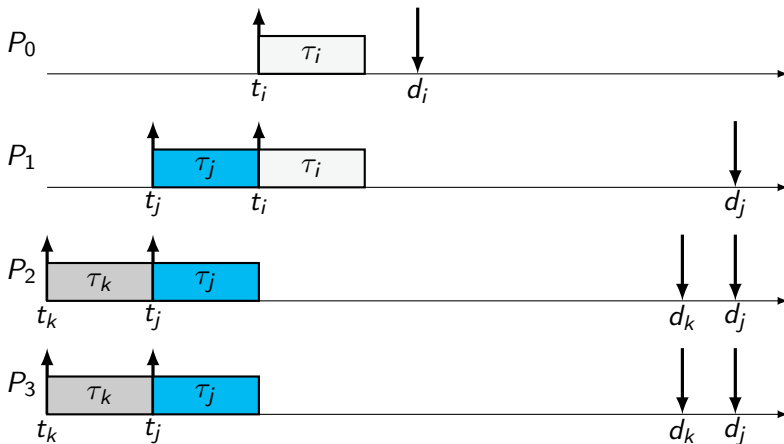
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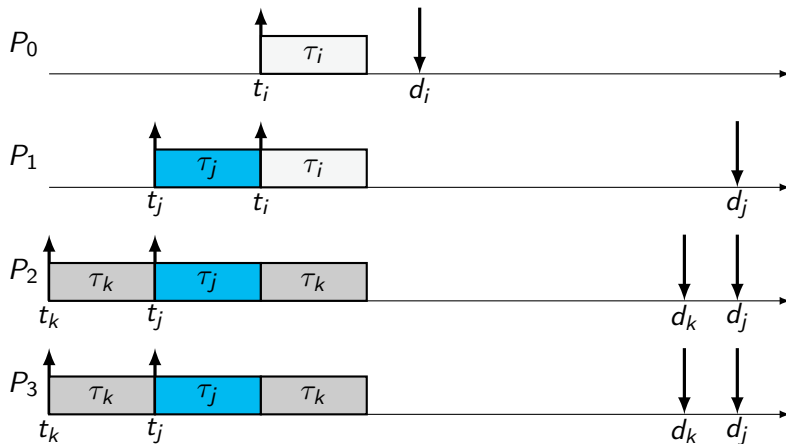
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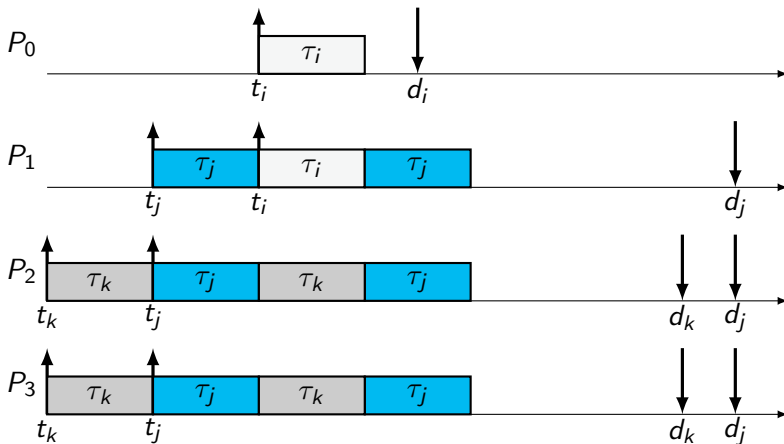


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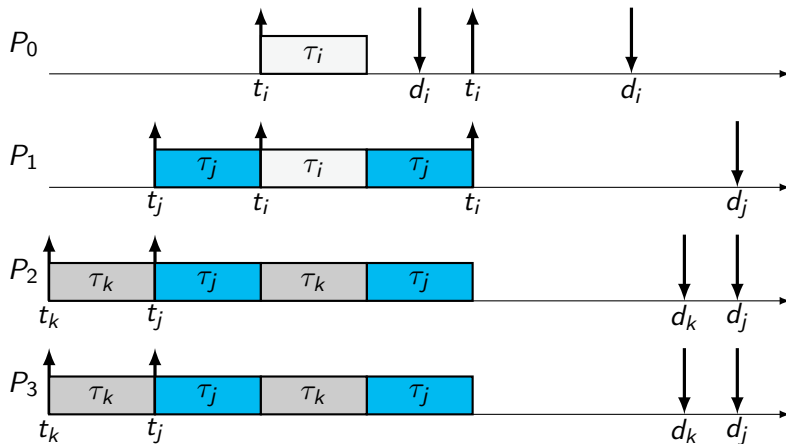




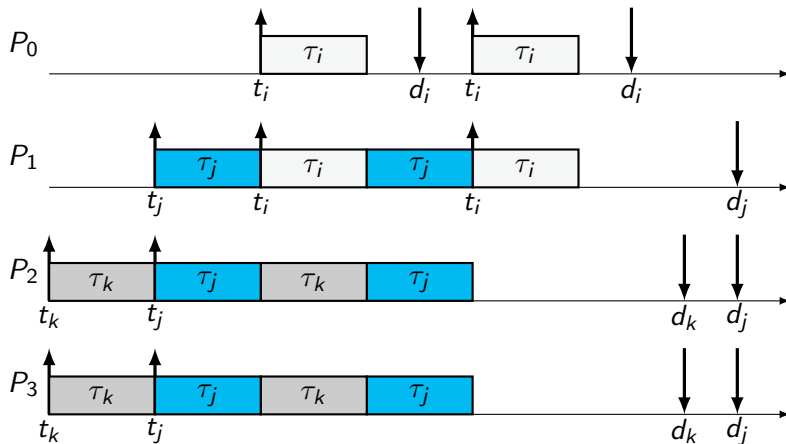
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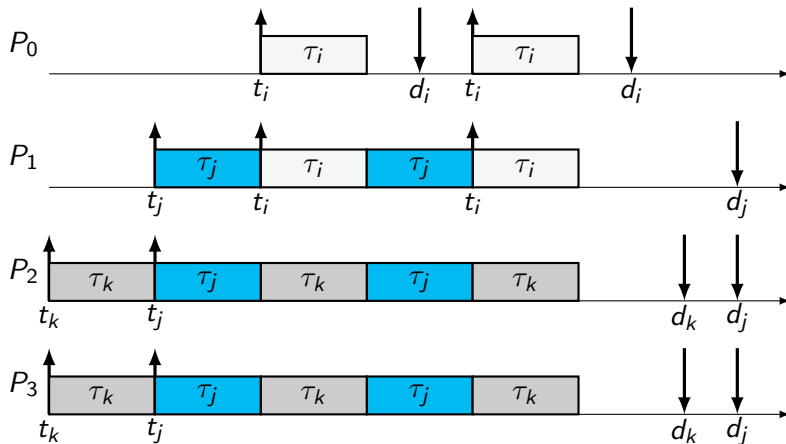
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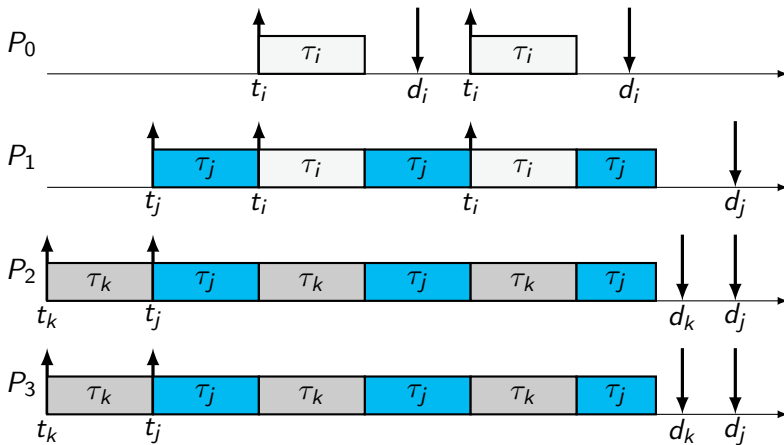
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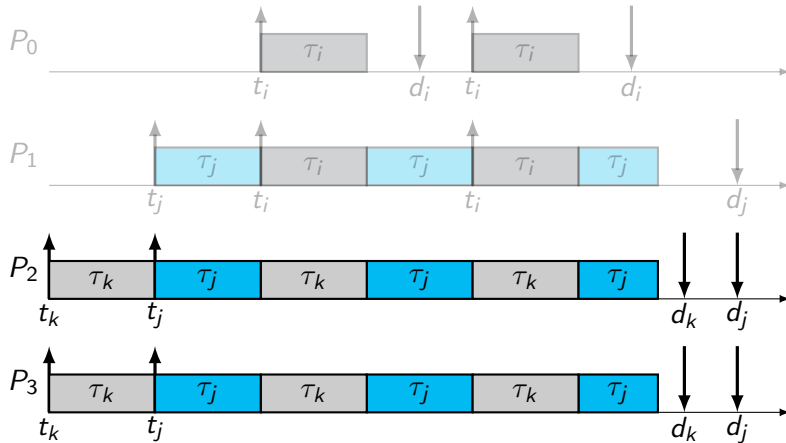
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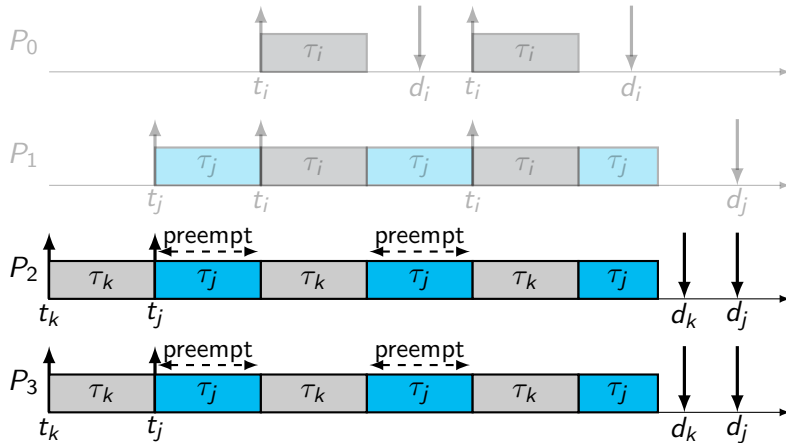
# Fixed-Priority Stationary Gang Schedule



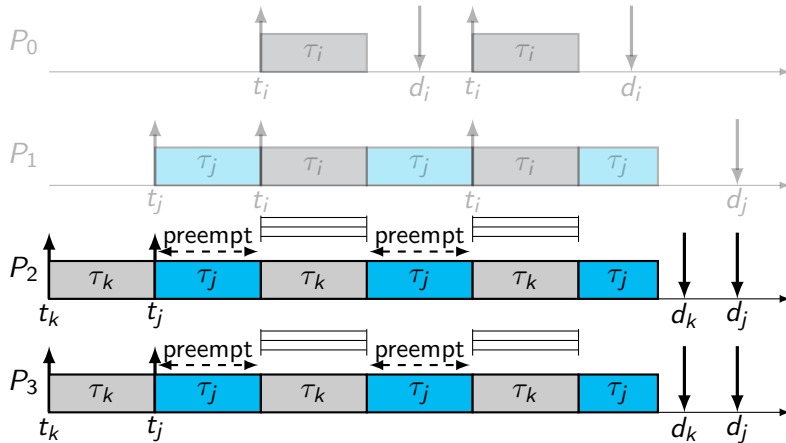
# Response-Time Analysis



# Response-Time Analysis



# Response-Time Analysis

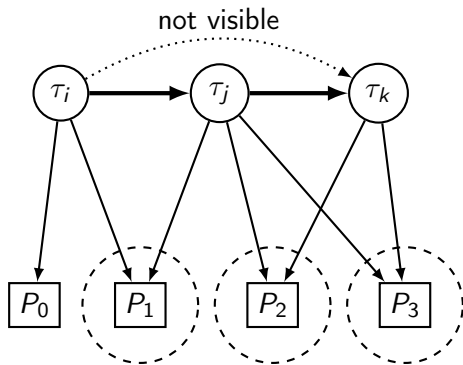




# Exemplary Stationary Gang Assignment

## Definition

[Self-Suspension] Higher-priority tasks that **do not interfere** with the job under analysis may cause **self-suspension** like behaviour of interfering tasks



# Transformation and Schedulability Analysis

For each task in priority order **do**:

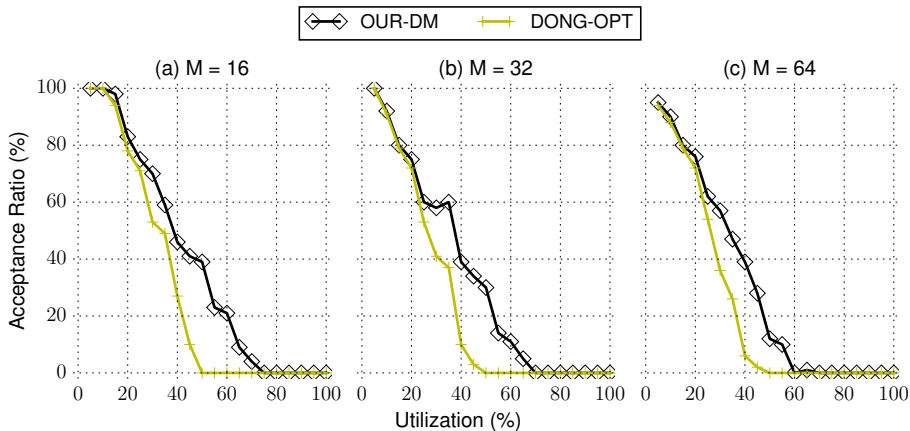
- 1 Transform higher-priority task set
- 2 Analyze worst-case response-time based on uniprocessor self-suspension analysis

## Definition

[Transformation] Let a sporadic gang task  $\tau_i$  be transformed to the corresponding self-suspending task  $(C_i, D_i, T_i, S_{i,k})$  with the same  $C_i$ ,  $D_i$ , and  $T_i$  as for  $\tau_i$ , where

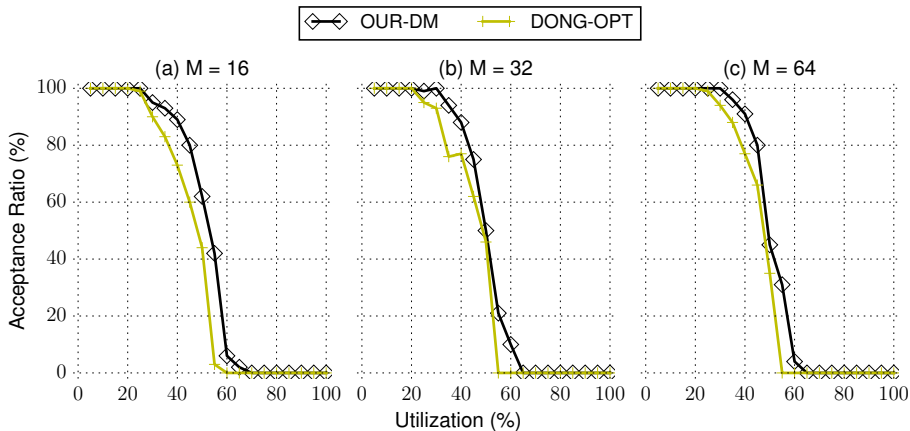
$$\begin{cases} S_{i,k} = \min \left\{ R_i - C_i, \sum_{\tau_j \in V_{i,k}} \left( 1 + \left\lceil \frac{R_i}{T_j} \right\rceil \right) \cdot C_j \right\} & \text{if it has suspension behaviour} \\ S_{i,k} = 0 & \text{otherwise} \end{cases}$$

# Evaluation: Gang Size [1, M/4]



- OUR-DM: Ueter et al. ECRTS 2021
- Dong-OPT: Dong and Liu, RTSS 2017

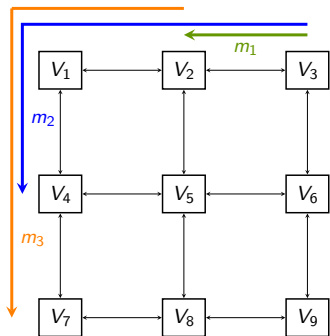
# Evaluation: Gang Size [M/8, M/2]



- OUR-DM: Ueter et al. ECRTS 2021
- Dong-OPT: Dong and Liu, RTSS 2017

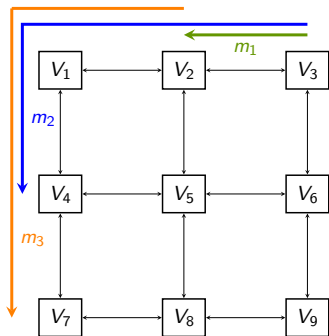
# Real-Time Networks on Chip (Revisited)

- We can map this to gang scheduling, using each link as a processor



3 periodic messages  
fixed path, preemptive  
priority:  $m_1 > m_2 > m_3$

# Real-Time Networks on Chip (Revisited)



3 periodic messages  
fixed path, preemptive  
priority:  $m_1 > m_2 > m_3$

- We can map this to gang scheduling, using each link as a processor
  - Assuming that the switching is reserved for one stream completely along the path
  - Ueter et al. RTCSA 2020

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## Worst Case Response Time Exceedance Probability (WCRTEP)

The WCRTEP of task  $\tau_k$  is an upper bound on the probability that the response time of a job of  $\tau_k$  is greater than  $t$ , i.e.,

$$\sup_{j \in \mathbb{N}} \{\mathbb{P}(R_{k,j} > t)\}, \quad (1)$$

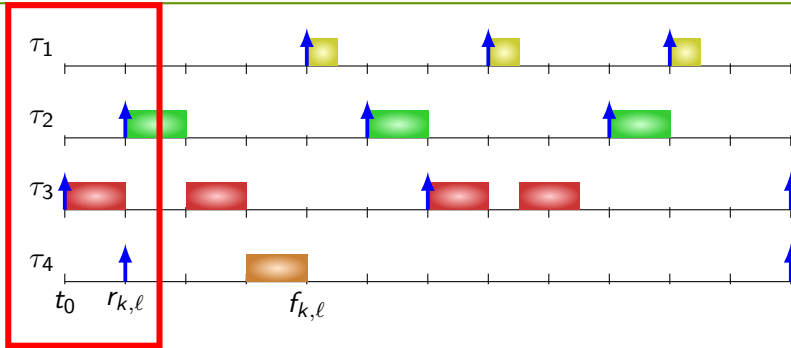
## Worst Case Deadline Failure Probability (WCDFP)

The WCDFP of task  $\tau_k$  is an upper bound on the probability that a job of  $\tau_k$  misses its relative deadline  $D_k$ :

$$\sup_{j \in \mathbb{N}} \{\mathbb{P}(R_{k,j} > D_k)\} \quad (2)$$

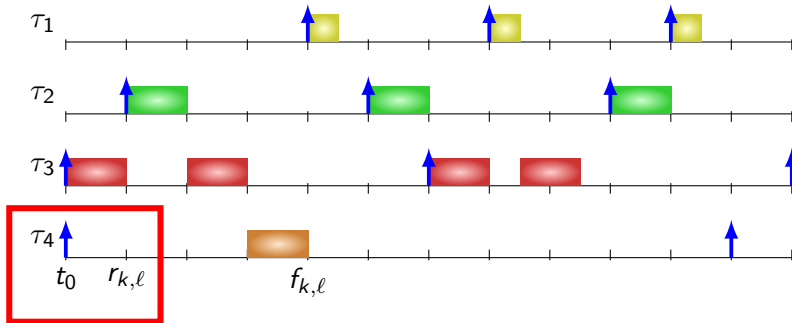


# Critical Instant



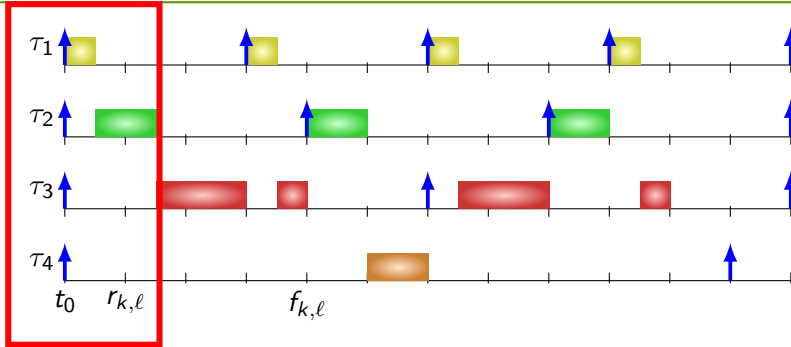
- 1) Interval extension

# Critical Instant



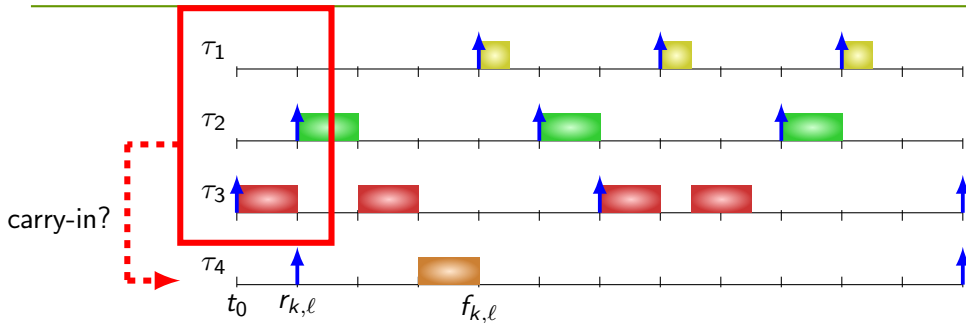
- 1) Interval extension 2) release time modification

# Critical Instant



- 1) Interval extension 2) release time modification 3) simultaneous release

# Critical Instant for Probabilistic Setup? **Refuted**

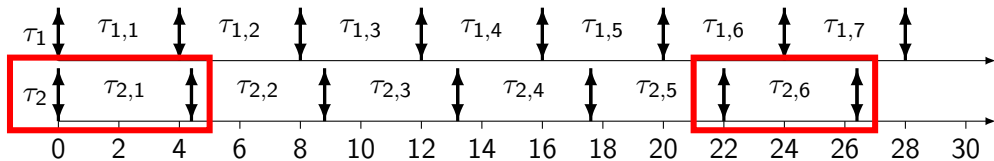


- 1) Interval extension **is not deterministic**: a probabilistic distribution function!
- Detailed in Chen et al. RTSS 2022

## Counterexample: $\mathbb{P}(R_{2,6} > t)$ is higher than $\mathbb{P}(R_{2,1} > t)$

Periodic task  $\tau_1$  and  $\tau_2$ , simultaneously released at time 0, for all  $j \in \mathbb{N}$ :

- $T_1 = 4$ ,  $\mathbb{P}(C_{1,j} = 1) = 0.9$ ,  $\mathbb{P}(C_{1,j} = 2.5) = 0.1$
- $T_2 = 4.4$ ,  $\mathbb{P}(C_{1,j} = 3) = 1.0$

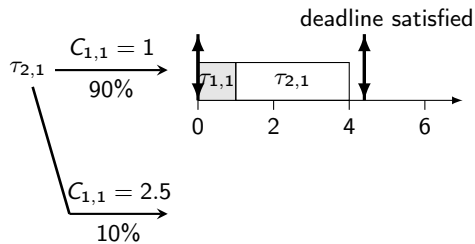


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Consider  $t = 4.4$ , we obtain:

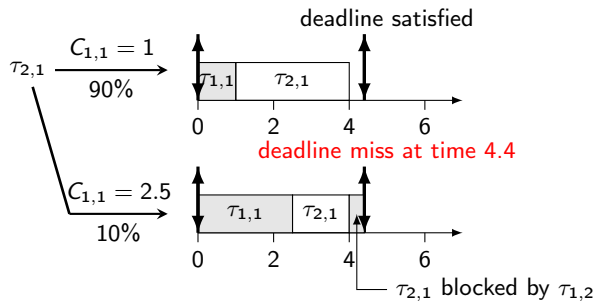
$$\mathbb{P}(R_{2,1} > 4.4) = \mathbb{P}(C_{1,1} = 2.5) = \mathbf{0.1}$$



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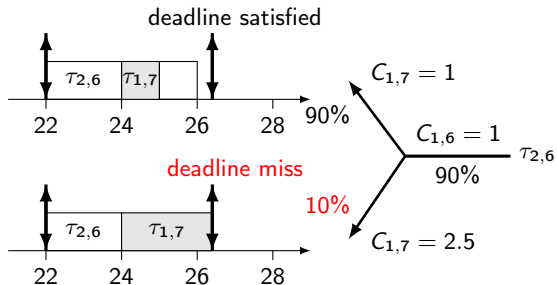
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Consider  $t = 4.4$ , we obtain:

$$\mathbb{P}(R_{2,6} > 4.4) = \mathbb{P}(C_{1,6} = 1) \cdot \mathbb{P}(C_{1,7} = 2.5)$$

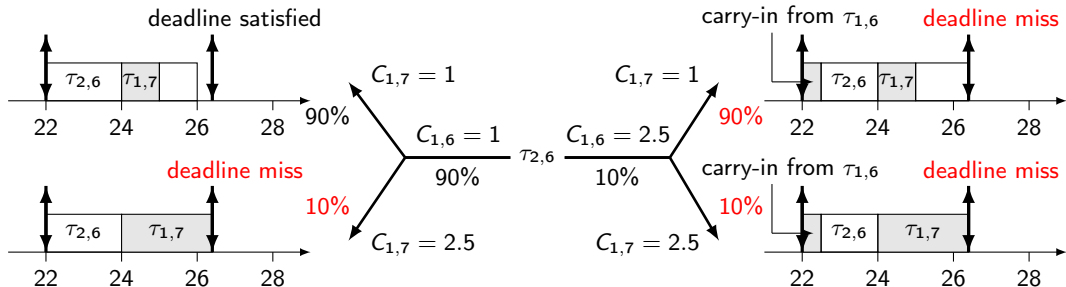




## Counterexample: $\mathbb{P}(R_{2,6} > t)$ is higher than $\mathbb{P}(R_{2,1} > t)$

Consider  $t = 4.4$ , we obtain:

$$\mathbb{P}(R_{2,6} > 4.4) = \mathbb{P}(C_{1,6} = 1) \cdot \mathbb{P}(C_{1,7} = 2.5) + \mathbb{P}(C_{1,6} = 2.5) = 0.9 \cdot 0.1 + 0.1 = \mathbf{0.19}$$



Higher priority tasks like  $\tau_1$  may still provide carry-in!

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- To effectively analyze real-time properties in multicore systems, we need
  - *formal properties* to modularly compose real-time embedded systems
  - *methodologies* for generating/verifying properties and modular compositions
  - *predictable interplay* of complex real-time embedded systems

# Conclusion

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  - *formal properties* to modularly compose real-time embedded systems
  - *methodologies* for generating/verifying properties and modular compositions
  - *predictable interplay* of complex real-time embedded systems
- Classical computation-centric view is limited and may be prone to error
- The focus should be shifted to
  - communication,
  - synchronization, and
  - parallelization.

# Conclusion

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  - *formal properties* to modularly compose real-time embedded systems
  - *methodologies* for generating/verifying properties and modular compositions
  - *predictable interplay* of complex real-time embedded systems
- Classical computation-centric view is limited and may be prone to error
- The focus should be shifted to
  - communication,
  - synchronization, and
  - parallelization.

*It is the worst of time and also the best of time.*