Discussion on

Uncertainty handling in Logic Programming

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Uncertainty / Fuzziness

- **uncertainty**
  
  due to incomplete information or randomness on Boolean events
  
  truth-degrees $\in \{0, 1\}$
  
  can be evaluated in a quantitative / qualitative way
  
  uncertainty measures on possible worlds
  
  uncertainty degrees $\in [0, 1]$ (usually)
  
  various models: probabilistic, possibilistic, belief functions, etc.

- **fuzziness**
  
  partial satisfaction of gradual properties
  
  truth-degrees $\in [0, 1]$ (usually)
  
  full compositional laws for compound formulas
Logic Programming and Uncertainty

A variety of logic programming languages handling different uncertainty and fuzzy models. One can classify them by:

- Uncertainty / fuzzy model chosen:
  - probabilistic l.p.
  - possibilistic l.p.
  - belief l.p.
  - fuzzy (choices of aggregation operations)

- Annotation-based / implication-based rules

  annotated rule: \[ A : \mu \leftarrow B_1 : \mu_1 \land \ldots \land B_n : \mu_n \]

  (a interpretation makes true or false each basic annotated fact)

  weighted implication: \[ (A \leftarrow B_1 \land \ldots \land B_n, \mu) \]

  (mv-valued interpretation of facts / rules)
Logic Programming and Uncertainty

- **definite programs**: no negation involved
  fix point semantics (minimal models)

- **normal programs**: negation by failure in the body of the rules
  links to non-monotonic reasoning: \( \text{not } A = A \) is not believed, \( \neg A \) is consistent
  answer set semantics (stable models): minimal models of program reducts (Gelfond-Lifschitz reduction)

- **extended programs**: negation by failure + classical negation
  answer set semantics: coherent stable models

- **disjunctive programs**
  disjunctions in the head of rules
  qualitative form of uncertainty
Annotated logic programming languages

- Generalized Annotated Programs GAP (Kifer-Subrahmanian, 89)

- Probabilistic logic programs PLP (Ng-Subrahmanian, 92)
  Hybrid Probabilistic logic programs (Dekhtyar-Subrahmanian, 97)
  (Saad 06)

- Action probabilistic programs (Khuller et al., 07), (Simari et al., SUM 2010)

- Extended fuzzy logic programs (Saad, SUM 2009)
  Disjunctive Extended fuzzy logic programs (Saad, SUM 2010)
Conditional / Implication -based approaches

- **Conditional probability-based logic programs** (Lukasiewicz, 2001)
  
  rules: \((A \leftarrow B, [\alpha, \beta])\)

  interpretations: \(Pr : 2^{HB} \rightarrow [0, 1]\) probability function

  \(Pr \models (A \leftarrow B, \alpha) \iff Pr(A \mid B) \in [\alpha, \beta]\)

  inference: linear optimization techniques

- **Possibilistic logic programs** (Dubios-Lang-Prade, 1991)
  
  rules: \((A \leftarrow B, \alpha)\)

  interpretations: \(N : 2^{HB} \rightarrow [0, 1]\) necessity function

  \(N \models (A \leftarrow B, \alpha) \iff N(\neg B \lor A) \geq \alpha\)

  Immediate Consequence operator based on weighted modus ponens:
  from \((A \leftarrow B, \alpha)\) and \((B, \beta)\) derive \((A, \min(\alpha, \beta))\)
Conditional / Implication -based approaches

• Fuzzy / many-valued logic programs
  rules: \((A \leftarrow B, \alpha)\)

\[ I : At \rightarrow [0, 1] \text{ extends to rules by } I(A \leftarrow B) = I(A) \Rightarrow I(B), \text{ where } \Rightarrow \text{ is the residuum of a conjunctive aggregation operator (t-norm)} \]

\[ I \models (A \leftarrow B, \alpha) \text{ iff } I(A) \Rightarrow I(B) \geq \alpha \text{ iff } I(B) \geq I(A) \ast \alpha \]

Immediate Consequence operator based on fuzzy modus ponens:
from \((A \leftarrow B, \alpha)\) and \((B, \beta)\) derive \((A, \alpha \ast \beta)\)
Implication-based logic programming languages

• Answer set semantics for possibilistic logic programs
  - (Nicolás et al., 2005, 2006)
  - (Bauters-Schockaert-De Cock-Vermeir, 2010)
  - (Nieves-Osorio, 2007)

• Residuated Logic programs (Damasio-Pereira, 2001)
  truth-values domain: abstract residuated lattice

• Normal logic programs over lattices and bilattices (Straccia, 2005)

• Answer set semantics for fuzzy L.P.s
  - (Madrid-Ojeda, 2009)
  - (Janssen, Schockaert, Vermeir, De Cock, 2009)
Discussion

- Annotated versus implication based approaches:
  - extendability?
  - expressiveness?
  - applicability? (Simari et al, SUM 2010)

- Fuzzy logic programming languages:
  - weak link to well-established systems of formal fuzzy logic (e.g. Łukasiewicz, Gödel, product logics)
  - answer set semantics: introducing non-monotonicity into fuzzy logics (fuzzy equilibrium logic - Schockaert et al.)

- Integration of uncertainty and fuzziness handling
  - disjunctive Fuzzy LP (Saad, SUM 2010)

- Scalability