

# QUERYING AND REPAIRING INCONSISTENT PRIORITIZED KNOWLEDGE BASES:

*Complexity Analysis & Links with Abstract Argumentation*

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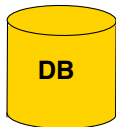
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Paper at KR'2020, long version on arXiv

# ONTOLOGY-MEDIATED QUERY ANSWERING

dataset



Prof(ann)  
Teaches(ann, cs100)

ontology



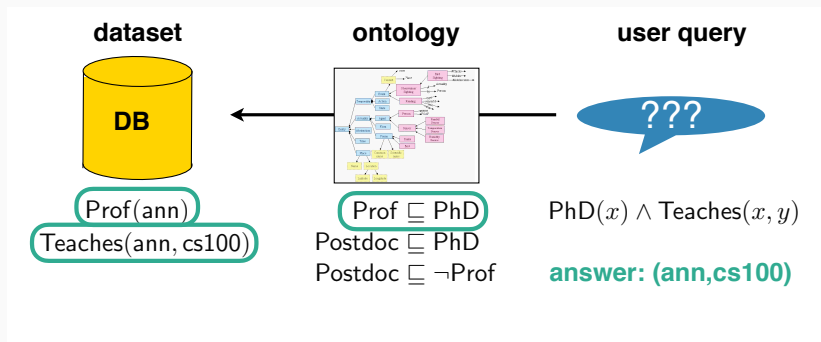
$\text{Prof} \sqsubseteq \text{PhD}$   
 $\text{Postdoc} \sqsubseteq \text{PhD}$   
 $\text{Postdoc} \sqsubseteq \neg\text{Prof}$

user query

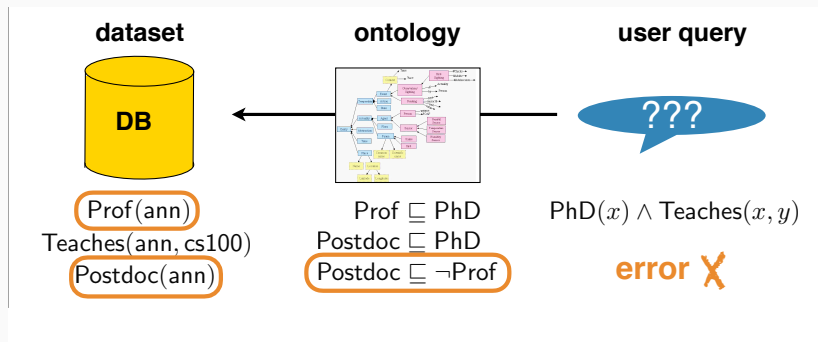


$\text{PhD}(x) \wedge \text{Teaches}(x, y)$

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Handling errors in data: [restore consistency](#) / [alternative semantics](#)

Repairs = maximal subsets of data consistent with the ontology

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likely answers

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- tuple is answer  $\Leftrightarrow$  holds w.r.t. all “possible worlds”

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IAR-answers  $\subseteq$  AR-answers  $\subseteq$  brave-answers

## EXAMPLE: INCONSISTENCY-TOLERANT SEMANTICS

Consider the following knowledge base (KB):

$$\begin{aligned}\mathcal{O} &= \text{Prof} \sqsubseteq \text{PhD}, \text{Postdoc} \sqsubseteq \text{PhD}, \text{Postdoc} \sqsubseteq \neg\text{Prof}\} \\ \mathcal{D} &= \{\text{Postdoc}(a), \text{Prof}(a), \text{Teaches}(a, c)\}\end{aligned}$$

**Inconsistent KB** with two repairs:

$$\mathcal{R}_1 = \{\text{Postdoc}(a), \text{Teaches}(a, c)\} \quad \mathcal{R}_2 = \{\text{Prof}(a), \text{Teaches}(a, c)\}$$

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Get the following results:

- **IAR semantics:**  $\text{Teaches}(a, c)$
- **AR semantics:**  $\text{PhD}(a), \text{Teaches}(a, c)$
- **brave semantics:**  $\text{Postdoc}(a), \text{Prof}(a), \text{PhD}(a), \text{Teaches}(a, c)$

Want to exploit information about **relative reliability of facts**

In this work: focus on **fact-level preferences**

- **priority relation**  $\succ$  over set of facts
- $\alpha \succ \beta$ :  $\alpha$  **more trusted** than  $\beta$

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**Prioritized knowledge base**  $\mathcal{K}_\succ$  consists of:

- a TBox (ontology)  $\mathcal{T}$
- a ABox (dataset)  $\mathcal{A}$
- a **priority relation**  $\succ$  over  $\mathcal{A}$

Question: how to **select 'best' repairs of a prioritized KB?**

Adapt three notions of optimal repair from database setting

(Staworko et al. 2012)

- **Pareto-optimal repair (P)**: cannot 'improve'  $\mathcal{R}$  by adding  $\alpha \in \mathcal{R} \setminus \mathcal{A}$ , then removing  $\beta_1, \dots, \beta_n$ , with  $\alpha \succ \beta_i$  ( $1 \leq i \leq n$ )

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Related as follows:

$$CRep(\mathcal{K}_\succ) \subseteq GRep(\mathcal{K}_\succ) \subseteq PRep(\mathcal{K}_\succ) \subseteq Rep(\mathcal{K})$$

For  $X \in \{P, G, C\}$ , can consider  $X$ -AR,  $X$ -IAR,  $X$ -brave semantics

- same definitions, but restrict to repairs from  $XRep(\mathcal{K}_\succ)$

Some natural reasoning tasks for optimal repairs  $X \in \{P, G, C\}$

**ISREP** Decide if set of facts is X-optimal repair of  $\mathcal{K}_{\succ}$

**AR, IAR, BRAVE** Decide whether  $\mathcal{K}_{\succ}$  entails a given Boolean query under X-AR (resp. X-IAR, X-brave) semantics

**UNIQUE** Determine if there is a unique X-optimal repair

**ENUM** Enumerate all elements of  $XRep(\mathcal{K}_{\succ})$

Study the **data complexity** to understand difficulty of these tasks

## DATA COMPLEXITY RESULTS

Results below hold for **typical OMQA settings**:

- **DL-Lite<sub>R</sub> (OWL 2 QL) ontologies** (+ other common DL-Lite dialects)
- **conjunctive queries** (lower bounds for atomic queries)

	Standard	Pareto	Global	Completion
ISREP	in P	in P	coNP-c	in P
AR	coNP-c	coNP-c	$\Pi_2^P$ -c	coNP-c
IAR	in AC <sup>0</sup>	coNP-c	$\Pi_2^P$ -c	coNP-c
BRAVE	in AC <sup>0</sup>	NP-c	$\Sigma_2^P$ -c	NP-c
UNIQUE	in P	coNP-c	$\Pi_2^P$ -c	in P
ENUM	DELAYP	not TOTALP	not TOTALP	DELAYP

**Adding preferences increases complexity** of most tasks

Two **important open questions** (for OMQA and DB settings):

- Which notion of **optimal repair is 'most natural'**?
- How to obtain **lower complexity while exploiting preferences**?

To help answer these questions:

**develop connections with argumentation**

**Argumentation framework (AF)** is a pair  $(Args, \rightsquigarrow)$  where:

- $Args$  is a finite set of **arguments**
- $\rightsquigarrow$  is an **attack relation** between arguments
  - $\alpha \rightsquigarrow \beta$ :  $\alpha$  attacks  $\beta$

Key notion: **extension**  $\sim$  'coherent' position (subset of arguments)

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Several different notions of extension, in particular:

- **preferred extension**:  $\subseteq$ -maximal conflict-free self-defending set
- **stable extension**: conflict-free, attacks all non-included arguments

Stable extensions are also preferred extensions

- **Coherent AF**: stable and preferred extension coincide

Many variants of AFs have been studied

Set-based AFs (SETAFs):

- allow **collective attacks**  $S \rightsquigarrow \alpha$  ( $S$  finite set of arguments)

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- prove **new results for PAFs, SETAFs, and PSETAFs**



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**Theorem:** Every strongly symmetric PSETAF with a transitive preference relation is coherent.

Translation of a prioritized KB  $\mathcal{K}_\succ = (\mathcal{T}, \mathcal{A}, \succ)$  into a PSETAF  $F_{\mathcal{K}, \succ}$ :

- use  $\mathcal{A}$  as the arguments
- use  $\succ$  as the preference
- the attacks are of the form  $C \setminus \{\alpha\} \rightsquigarrow \alpha$ ,  
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Globally- and completion-optimal repairs:

- no corresponding notion of extension

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Inspires **new grounded semantics:**

- **query set of facts in grounded extension of  $F_{\mathcal{K}, \succ}$**

Desirable properties:

- **tractable (PTIME-complete) data complexity**, for DL-Lite KBs
- **computable via logic programming** (well-founded semantics)
- amenable to **preprocessing**
- **more productive than recent Elect semantics** (Belabbes et al. 2019)

## Contributions

- **complexity analysis of optimal repairs** DL-Lite KBs
- clarified **relationship with argumentation** general KBs & DBs
- **new tractable grounded semantics** DL-Lite KBs, DBs

## Topics for future work

- design **SAT-based procedures** for (co)NP reasoning tasks
- further study **properties of PSETAFs**
- explore how to **specify / elicit priority relations**

QUESTIONS ?



### Cited papers:

Staworko, Chomicki, and Marcinkowski. Prioritized repairing and consistent query answering in relational databases. *Ann. Math. Artif. Intell.* 64(2-3):209–246, 2012.

Belabbes, Benferhat, and Chomicki. Elect: An inconsistency handling approach for partially preordered lightweight ontologies. *Proc. of LPNMR*, 2019.

### Surveys of inconsistency-tolerant OMQA:

Bienvenu and Bourgaux. Inconsistency-tolerant querying of description logic knowledge bases. *Reasoning Web*, 2016.

Bienvenu. A Short Survey on Inconsistency Handling in Ontology-Mediated Query Answering. *KI*, 2020.