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## **Proof Mining for the Fixed Point Theory of Nonexpansive Semigroups**

Angeliki Koutsoukou-Argyraki

Research Group Logic

Department of Mathematics

Technische Universität Darmstadt

Proof mining is a research program in applied proof theory aiming at the extraction of constructive information (in particular computable bounds) even from nonconstructive proofs in mathematics. The extractability of the bounds for statements of a certain logical form (and within an appropriate formal framework) is a priori guaranteed by certain logical metatheorems that have been shown by employing variations of Gödel's functional Dialectica interpretation. The bounds are obtained via a study of the underlying logical structure of a proof and are explicit, computable, highly uniform and of low complexity. Proof mining finds its early origins in the ideas of Kreisel and has been developed in recent years by Kohlenbach and his collaborators. The program has been fruitfully applied in many different mathematical disciplines mostly within analysis. We will present here an instance of a recent application in nonlinear operator theory and fixed point theory in which we extract computable information for the approximate common fixed points of one-parameter nonexpansive semigroups on a subset of a Banach space. The results are included in [3] and [2] and obtained via proof mining on a proof in [4]. (A completely different bound had been obtained in [1] (also see [2]) via proof mining on a completely different proof in [5] of a generalization of the statement in [4].) We will also present an example of a metatheorem to illustrate how the extractability of the bound is guaranteed in a specific mathematical context.

### REFERENCES

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