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## What is a Derivation?

It seems to be simple to answer the question, what a derivation in the calculus of Natural Deduction is. Logicians could refer in their answer to Gentzen's famous article "*Investigations into Logical Deduction*" and to the recursive definition of derivations found in this paper.

One crucial aspect of these derivations is the possibility of hypothetical reasoning represented by the discharge of assumptions. Most logicians are able to discharge previously assumed assumptions in a derivation, when it is advisable to do so. But it is not so clear, how to represent formally such a discharge.

Usually, the discharge of assumptions is described as a relation (or a function) relating occurrences of assumptions somehow with the inference step in which they are discharged. But we find only few details about the notion of occurrences. We mention a symptomatic quote by Prawitz: "I take for granted the notion of an *occurrence of a formula* or (synonymously) a *formula occurrence* in a formula-tree." (Dag Prawitz, "*Natural Deduction*", p. 25.)

Taking the notion of occurrences granted means to have only an intuitive and informal notion of assumptions and inference steps. No adequate definition of the discharge of assumptions can be given; consequently, the derivations are not defined adequately. This cannot be satisfactory for a logician.

The main goal of our talk is to illustrate how to change this situation and to provide a formally adequate definition of derivations.

In a first step, we introduce a formal theory of occurrences with focus on the occurrences of subtrees of formula trees. In the light of this theory, we analyse the involved informal notions in order to determine the suitable representation of assumptions, of inference steps and of the discharge of assumptions.

We choose a specific account to these informal notions, and present subsequently the necessary formal methods and concepts. Finally, we are able to provide an adequate definition of a discharge function (mapping assumptions to inference steps) and of derivation which is, essentially, an ordered pair of a formula tree and a suitable discharge function. Depending on the time left, we may conclude our talk with some more elaborate details about the adequate treatment of the composition of derivations and of proof conversions.