Report on a 3-Year Project on Efficient and Natural Proof Systems
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I propose to present generalised normalisation methods and to use example proof systems to illustrate the main points.

I will argue that normalisation in Gentzen is a conflation of two mechanisms that operate on two distinct composition methods:

1) sharing/contraction: this is the composition method that, when normalised upon, generates complexity;
2) linear cut: this composition method, when normalised upon, reduces complexity.

As is well known and intuitive, normalising sharing/contraction means, computationally, applying beta reduction to several instances of a variable (at the propositional level) or generating Herbrand expansions (in a predicate calculus).

Since the two mechanisms pull in opposite directions from the point of view of complexity, separating them would allow us to obtain finer computational control. Such separation is impossible in Gentzen formalisms, due to their limitations in proof composition, but it is natural in deep inference.

An additional, natural composition method could be considered in deep inference (and not in Gentzen), namely

3) substitution of proofs inside other proofs.

This is a sound generalisation of Frege substitution, which in turn is conjectured to provide a superpolynomial speed-up in the size of proofs.

I will argue that those three composition methods and their associated normalisation mechanisms can be separated and made independent to a very large extent in deep inference. I will illustrate the impact of these ideas on the design of a proof system addressing some computational requirements. This proof system, called KV, is a multiplicative linear logic with a self-dual non-commutative connective 'seq' and a self-dual modality 'star' providing contraction over seq. Given this characteristics, KV could form the core of a proof system for a Kleene algebra. KV can also be seen as an extension of BV with the star modality. Remarkably, under certain natural and logic-independent constraints, system KV seems to be canonical, in the sense that there is no room to manoeuvre in the design of the rules.