Retailer-group supplier assignment: comparison of three collaborative decision making models

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Abstract. In this paper, we consider a supply chain management problem where a group of small capacity retailers need to mutually select a unique large capacity supplier considering the efficiency of the assignment at a group level and their individual preferences. The problem is modeled as a multi-agent system where retailers are represented by agents. We present a modified Vickrey auction with regret minimization and compare it experimentally with aggregation of preferences through voting and standard Vickrey auction. The solutions are evaluated in terms of egalitarian, utilitarian, elitist, and Nash social welfare values.

1 Introduction

Companies form alliances by combining their individual resources with the objective of advancing their mutual interests. Alliance formation is an important part of business strategic decision-making in many areas, such as: industrial procurement, manufacturing, crisis management, and logistics. These environments are typically characterized by companies with different and contrary goals and preferences. It is crucial, for preserving the alliance, to maintain a fine balance between the objectives of the alliance as a group and the objectives of the alliance members as individuals.

The alignment of individual and mutual (group) objectives is a non-trivial problem. Agents can be competitive, \(e.g.,\) self-interested with conflicting goals, or collaborative, \(i.e.,\) working together, sharing harmonious goals. Even when agents are collaborative, the utilities of the individual may not necessarily be aligned with the utilities of the group. Therefore, reaching an optimal solution for the heterogeneous group decision problem is a complex task.

Assuming that a group performs a unified action, a globally optimal action is the one that minimizes the cost and/or maximizes the profit of the group as a whole. The quality of the group action for an individual group member can be assessed in local terms, namely through fairness, \(i.e.,\) individual agent satisfaction with the group behavior. Since a globally optimal action is not necessarily optimal individually, there might be agents individually unsatisfied with the one. The increase in the number of unsatisfied agents can influence the stability of the group and, therefore, its performance.

In this paper we focus on the specific problem of selecting a unique large supplier for a group of multiple small retailers. The problem can be similarly
formulated as the assignment of a group of multiple small suppliers to a large-sized retailer. Relating to the former problem formulation, we assume retailers are grouped based on the similarity of their utility functions. Furthermore, we assume the existence of more large size suppliers, each endowed with complementary or substitutable products, and for delivery of products, they deliver only large size shipments which demands can be formed only by a larger retailer group. Furthermore, we assume that self-interested low capacity retailers are already grouped based on the utility, i.e., a cost function for a supplier assignment which corresponds at most to the retailer group’s cost function. In this context, a group-optimal, but unequally fair supplier assignment can serve as an incentive for unsatisfied retailers to break-up from their current group and join a competitor one.

Assuming that self-interested group members have no insight into the preference orders of each other, the question we consider is: is it necessary for an individual to reveal a sensitive (cardinal) information or ordinal preferences suffice to achieve a fair and efficient group assignment? A key difficulty in collective choice is that agents generally have conflicting preferences over the outcomes. To resolve this issue, classical coalition formation and matching algorithms might be inappropriate due to the competitive problem context and their focus on socially optimal solutions, while ignoring the issue of fairness at the individual level, see, e.g., [8,17]. However, some solutions in collective choice settings which respond to these challenges can be found in, i.e., the election of a president through citizens voting.

Both elections (voting) and auctions require a central authority, an elicitor and an auctioneer, respectively, to determine winners. The auctioneer in combinatorial auctions can disinsensitivize misreporting by applying mechanism design (MD) techniques while the elicitor can not do the same in voting, since MD applied in voting does not give relevant results. Therefore, other methods should be explored to prevent misreporting, e.g., making it computationally hard to misreport beneficially [5].

For the presented retailer group assignment problem, we apply auction, which is a market-based choice method, and compare it with voting, which is a classical social choice method. Namely, for the computation of the acceptable group assignment solution, we propose a modified Vickrey auction with regret minimization. We use regret minimisation to achieve a fair solution. In voting, the preferences of each retailer over the available suppliers are aggregated using the Borda method. Both of these approaches are seen in this context as profit sharing mechanisms showing favoritism towards the least happy group members, thus resulting in the favorable conditions for group stability.

We investigate the connection between fairness and the efficiency of those methods and compare the resulting solutions in terms of utilitarian, egalitarian, elitist and Nash social welfare. Furthermore, we draw conclusions on their relative beneficial contexts of application and limitations in group assignment.

This paper is organized as follows. Section 2 treats related work. In Section 3 we formulate the retailer-group supplier assignment problem and give basic defi-
nitions regarding social welfare. Section 4 describes briefly the modified Vickrey auction method with regret minimization. The voting method is described in Section 5. Section 6 contains simulation results comparing the proposed approaches. We discuss our results and draw conclusions in Section 7.

2 Related work

Coalition formation is one of the ways in which self-interested agents with similar or complementary goals seek to cooperate. Coalition formation is a subject of different fields, e.g., game theory, where aspects like stability and payoff distribution are important elements [1], and economics, which is concerned with defining and analyzing criteria for distinguishing among strategic and social factors which propel many firms into coalition formation [6]. One of the most prominent criteria for coalition formation are the efficiency and the equity of assignments [19]. Assuming a preexistence of a lasting coalition, in this paper we are not concerned with the formation of coalitions per se but with the influence of coalition collective decisions on the coalitional stability.

Methods for reaching globally optimal or efficient solutions have been in the research focus of economists [14,21,22], while notions of equality (local optimality) have been somewhat neglected [7]. However, in a self-interested agents setting, where each group member is crucial for the success of the group, a feasible locally acceptable solution is preferable to a globally optimal one. Coalition stability has been considered extensively in the literature. Assuming that there is substantial variability of preferences across states of nature, Pycia in [13] shows that there exists a core stable coalition structure in every state if and only if agents preferences are pairwise-aligned in every state. Furthermore, core-stability was considered within various models of cooperative games with structure in [4].

In competitive contexts, it is difficult to incentivize agents to collaborate and unconditionally share information, especially if the collaboration might give a more costly result for an agent than its non-collaborative behavior. In economics, a common resource assignment approach for self-interested agents is auction. Common auction forms are English, Dutch, first-price sealed-bid, and Vickrey auction (VA). Different incentive models can be integrated in auctions’ rules so that a desirable fair outcome is chosen [15].

Assuming that collusion is not allowed and that agents have quasilinear utility functions, Vickrey auction (second-price sealed-bid auction) is strategy-proof or truthful, i.e., it gives bidders an incentive to bid their true values [20]. In this auction, bidders place their bid in a sealed envelope and simultaneously submit them to the auctioneer without knowledge of other bids. The highest bidder wins, paying a price equal to the second-highest bid. If everyone bids their true value, the bidder with the highest valuation will receive the good. Since bidders have an incentive to bid truthfully in this type of auction, it is the quickest and most likely to achieve Pareto efficiency and profit maximisation as a result. Furthermore, in the jargon of game theory, bidding truthfully is a dominant
strategy. If all players bid truthfully, then the VA produces an outcome that maximizes the social surplus. Finally, the VA is computationally tractable and it can be implemented in polynomial time.

As pointed out in [7], the quality of a group action can be assessed using tools from formal economic sciences. Pareto optimality and utilitarianism are two of the most frequently used efficiency and equity criteria from economical theories that have found use in multiagent systems. Furthermore, in [7] authors argue that in addition to efficiency characteristics of a solution, egalitarian characteristics and negotiation methods that produce egalitarian solutions are needed.

Regret, or loss of opportunity, is a concept in decision theory that calculates the difference between the utilities of two choices or outcomes. Regret theory [11] models how choices can be made under uncertainty by minimising the maximal possible regret that can be incurred by a choice. A minimax regret is a decision criterion first suggested by [16] in the realm of statistics. However, ever since, economists have been interested in it as well [18]. Minimax regret concept was used in multi-agent task-assignment in [10] while in [12] the minimization of voter’s regret for a possible voting outcome was applied to reduce the elicitation requirements.

3 Problem formulation

The retailer-group supplier selection problem we focus on in this paper consists of selecting a unique supplier for a group (coalition) of retailers out of a set of all suppliers. We assume the existence of multiple suppliers, and every supplier can service those retailer groups that satisfy his minimal requirements, e.g., pay his asking price and/or request a minimum number of products to be supplied. Furthermore, for simplicity but without loss of generality, we assume that the suppliers differ only in the costs of their services for the group, while the revenues for the group members of every supplier servicing the group are equal. In this way we simplify the group profit maximization problem concentrating only on the minimization of group assignment costs.

Let \( A = \{a_1, \ldots, a_r\} \) be a group of \( r \) retailer agents. Moreover, let \( \Theta = \{\theta_1, \ldots, \theta_s\} \) be a set of all available suppliers where \( s \) is the cardinality of set \( \Theta \).

Each retailer \( a_i \in A \) associates a cost \( c_{a_i,\theta_j} > 0 \) for each of suppliers \( \theta_j \in \Theta \). The cost \( c_{a_i,\theta_j} \) is the price retailer \( a_i \) pays for services when supplier \( \theta_j \) is selected by the group \( A \) to which \( a_i \) belongs. The profile of the costs that agents \( a_i \in A \) attribute to suppliers \( \theta_j \in \Theta \) can be represented through a group cost matrix \( C_A = [c_{a_i,\theta_j}]_{r \times s}, C_A \in \mathbb{R}_{>0}^{r \times s} \).

Let supplier \( \theta_j \) be a qualifying supplier if for every agent \( a_i \in A \), the cost \( c_{a_i,\theta_j} \) of having \( \theta_j \) as a supplier of group \( A \) is such that

\[
    c_{a_i,\theta_j} \leq c_{a_i}^q,
\]

where \( c_{a_i}^q = \beta_i \cdot c_{a_i,\theta_{j \min}} \) is a qualifying supplier cost for agent \( a_i \) and \( c_{a_i,\theta_{j \min}} = \min_j c_{a_i,\theta_j} \), is the cost of individually optimal supplier \( \theta_{j \min} \) for agent \( a_i \in A \), and
\( \beta_i \geq 1 \) is a tolerance factor of agent \( a_i \) representing the maximum acceptable cost ratio between group's assigned and individual agent's locally optimal supplier. Therefore, \( \theta_j \) is a qualifying supplier for group \( A \) if for each group member \( a_i \in A \), Formula (1) holds. For every agent for which the latter does not hold, we will call him \( \beta \)-unsatisfied. \( \beta \) should be agreed within the group based on different individual agents characteristics.

Regarding the information available to each agent, we assume that each retailer agent \( a_i \in A \) has at its disposal the information vector regarding its cost \( c_{a_i, \theta_j} \), for the procurement of every of suppliers \( \theta_j \in \Theta \), and that agents have no insight into each other's information regarding supplier cost values.

In this setting, the objective is to assign group \( A \) to a unique supplier \( \theta_j \in \Theta \), such that the group assignment cost \( \sum_{a_i \in A} c_{a_i, \theta_j} \) is minimal and that Formula (1) holds for all \( a_i \in A \).

Furthermore, regarding the solution equity, we consider the following four social welfare functions:

- **Utilitarian social welfare**:
  \[
  u(\alpha, C) = \sum_{a_i \in A, \theta_j} c_{a_i, \theta_j},
  \]

- **Egalitarian social welfare**:
  \[
  e(\alpha, C) = \max_{a_i \in A, \theta_j} c_{a_i, \theta_j},
  \]

- **Elitist social welfare**:
  \[
  el(\alpha, C) = \min_{a_i \in A, \theta_j} c_{a_i, \theta_j}, \text{ and}
  \]

- **Nash social welfare**:
  \[
  n(\alpha, C) = \prod_{a_i \in A, \theta_j} c_{a_i, \theta_j}.
  \]

For group \( A \), supplier \( \theta \in \Theta \) is better than supplier \( \theta' \in \Theta \) if and only if a social welfare function \( SWF(\theta, C_A) < SCW(\theta', C_A) \), since the lower the cost a retailer group has to pay, the better off the group is.

While the utilitarian, egalitarian and elitist welfare functions are well known, the Nash social welfare is perhaps less familiar. A low Nash value, when it is defined in terms of costs, is an indication of both good utilitarian and egalitarian value, *i.e.*, assignment solutions with a low Nash value are both locally and globally good solutions. They can be seen as a kind of a compromise between the collective and individually optimal assignment.

4 **Modified Vickrey auction method with regret minimization**

Since the retailer-group choice of a group supplier depends on the individual group members' costs, the Vickrey auction is a straightforward method of resolving this problem in a distributed way for self-interested agents.
Vickrey auction is performed in two stages: bidding and assignment. Bidders are individual retailers within the group whose objective is to minimize their total individual costs.

- In a bidding phase, bidders \( a_i \in A \) submit to the auctioneer in a sealed bid their full list of costs \( c_{a_i, \theta_j} \) for a set of suppliers \( \theta_j \in \Theta \).
- In the assignment phase, auctioneer calculates group total cost \( C_{(A, \theta_j)} \) for every supplier \( \theta_j \in \Theta \), and assigns the group to the supplier with the least total cost. Group total cost \( C_{(A, \theta_j)} \) is measured as the sum of the individual bidder costs for each \( \theta \in \Theta \).

**Vickrey rules** For the services of the assigned supplier to the group, each member of the group \( a_i \in A \) pays an individual price \( p_i \) which is calculated based on the Vickrey rules. The auctioneer computes price \( p_i \) in the following way:

- Let \( C \) denote the total cost from the efficient allocation \( (C = \sum_{a_i \in A} c_{a_i, \theta_j}) \).
- Let \( C^{-i} \) denote the total cost that could be generated if \( a_i \) did not participate, and the auctioneer allocated (not necessarily the same) supplier \( \theta_j \in \Theta \) to the rest of the bidders to minimize total group assignment cost.

Then, \( a_i \)'s payment in the regular Vickrey auction is

\[
p_i = c_{a_i, \theta_j} + (C - C^{-i}). \tag{2}
\]

The first part of Formula (2), \( c_{a_i, \theta_j} \), represents the individual cost of agent \( a_i \) for being serviced by \( \theta_j \), while its second part \( (C - C^{-i}) \) represents the cost of agent \( a_i \)'s group participation. However, the regular Vickrey auction does not consider the fairness issues and there might be group members who have to accept the group assignment even though they are \( \beta \)-unsatisfied. If all group members are considered necessary for contracting with a supplier, it is in the interest of a group to maintain every group member within the group. The latter is stable as long as all the members are \( \beta \)-satisfied with the assignment and the group profit is strictly positive. To consider the fairness in the group, we propose modified Vickrey rules in the following.

**Modified Vickrey rules** We propose the modification of Vickrey rules by the integration of regret which is seen here as an opportunistic cost of \( \beta \)-unsatisfied agents for the group assignment. In this context, the regret of agent \( a_i \in A \) with respect to supplier \( \theta_A \) assigned to a group, is:

\[
r(a_i, \theta_A) = c_{a_i, \theta_A} + (C - C^{-i}) - c_{a_i}^\beta, \forall a_i \in A. \tag{3}
\]

Let \( \Psi \subset A \) be a set of all \( \beta \)-unsatisfied agents for which \( r(a_i, \theta_A) > 0 \) and \( \Phi = A \setminus \Psi \) be a set of \( \beta \)-satisfied agents. Payment \( p(a_i) \) for each \( \beta \)-unsatisfied agent \( a_i \in \Psi \) is lowered by the value sufficient to reach its minimally acceptable group assignment solution,

\[
p(a_i) = c_{a_i, \theta_A} + (C - C^{-i}) - r(a_i, \theta_A), \forall a_i \in \Psi. \tag{4}
\]
After all the individual $\beta$-unsatisfied agents’ payments are transferred, their total regret $r(\Psi, \theta_A) = \sum_{a_i \in \Psi} r(a_i, \theta_A)$ is then distributively paid by bidders $a_i \in \Phi$ as an additional cost $\delta(a_i)$ to their Vickrey payment $p(a_i)$. The latter is calculated as $\delta(a_i) = r(\Psi, \theta_A)/|\Phi|$.

For the distribution of additional cost $\delta(a_i)$ over agents $a_i \in \Phi$, we apply a heuristic approach of ordering agents $a_i \in \Phi$ in a non-increasing order of their individual payments $p(a_i)$, Formula (2), and iteratively charge each one of them in rounds additional unitary payment $\eta_a(t)$ until total regret $r(\Psi, \theta_A)$ isn’t distributed over all $\beta$-satisfied agents.

To avoid agents $a_i \in \Phi$ becoming $\beta$-unsatisfied, we limit additional unitary payment $\eta_a(t)$ of agent $a_i$ in round $t$ as follows:

$$\eta_a(t) = \min(p(a_i)(t) + \delta(a_i) - c_{a_i}, 0), \forall a_i \in \Phi.$$  

We assume that the assignment information is known only within the same group and any time the group structure changes, the group members recalculate the assignment. In the next section we present the voting-based approach.

5 Voting with Borda Count

Voting is a general group option-choosing method for societies of self-interested agents [3] and is also used for the purpose of fair distribution of desired items. Formally, a voting problem is specified by a non-empty set of social options $O$ and a set $A = \{a_1, \ldots, a_n\}$ of at least two agents. Each agent $a_i \in A$ reports his/her preferences over elements in $O$, which are represented by a complete, transitive preference relation $\succ_i$. A profile $P = \{\succ_i\mid a_i \in A\}$ is the set of the preference orders of the agents $A$. A voting rule is a function $F$, that assigns to each tuple of $n$ preference orders a non-empty sub-set of options from $O$. The choice of voting rule is determined by the nature of the problem.

The problem we are considering, finding a common supplier for a coalition of retailers, can be naturally represented as a voting problem, by setting $O = \Theta$. Each retailer in the coalition $a_i \in A$ constructs a preference order over the available supplier in the following manner: $\theta_j \succ_i \theta_k$ if and only if $c_{i,j} < c_{i,k}$ and $\theta_j \sim_i \theta_k$ if and only if $c_{i,j} = c_{i,k}$. Thus a retailer $a_i$ prefers a supplier $\theta_j$ over supplier $\theta_k$ if and only if the cost of the supplier $\theta_k$ is lower than that of $\theta_j$.

We use the Borda count rule, which is a scoring rule and the easiest to implement from the rules which consider not only who the top ranked candidate is, like the plurality rule and the fallback bargaining rule [2], but also how strongly a candidate is preferred to other candidates. An additional advantage of the Borda count rule is the low computational complexity of the determining the winner, which is not necessarily the case with other voting rules [9].

According to the Borda count rule, each supplier $\theta_j \in \Theta$ is given a score based on its position in the individual preference orders in $P$. The scores for the $\theta \in \Theta$ are defined as

$$sb(\theta) = \sum_{a_i \in A} \#\{(\theta')\mid \theta' \in \Theta \text{ and } \theta \succ_i \theta'\}.$$
The number \( \#\{(\theta')|\theta' \in \Theta \text{ and } \theta \succ_i \theta'\} \) is effectively the position of the option \( \theta \) in the retailer \( i \)'s preference order. For example, in the order \( \theta_1 \succ_i \theta_2 \succ_i \theta_3 \), the top ranked option \( \theta_1 \) is assigned a value 3 because it at least as good as 3 other options including itself. The Borda count rule \( F_B(P) \) returns the option with the highest score as a winner of the election \( F_B(P) = \arg \max_{\theta \in \Theta} sb(\theta) \).

It is directly observable that the Borda score for each option can be calculated in linear time of the size of the profile of preferences and consequently the respective linear order over the options can be generated in time \( O(m^2 \times n) \), where \( m \) is the number of options and \( n \) is the number of agents. As other voting rules, Borda can sometimes produce tied alternatives. We use the lexicographic order over suppliers to break ties.

We present the detailed description of our supplier-selection algorithm based on the Borda count voting rule.

We assume that \( \succ \) default order over \( \Theta \) is used in case of a tie. Then each retailer \( a_i \in A \) does the following steps:

1. it keeps in its memory a set of suppliers \( \theta_j \in \Theta \) and associates for each of the suppliers, the cost \( c_{a_i, \theta_j} \).
2. constructs the preference order over \( \Theta \) as described previously.
3. it receives preference orders from other retailer agents and constructs a preference profile \( P \) to which he applies the \( F_B \) rule. If \( F_B(P) \) produces a tie, the supplier who is ranked the highest according to \( \succ \) is chosen.

6 Simulation setup and results

We simulate a multi-agent system with retailer and supplier agents applying the modified Vickrey auction method with regret minimization and the voting method in MatLab.

In the following, the results of 10 different instances are presented for the problem with 100 retailer agents and 100 suppliers. We tested different scenarios with up to 100 agents in a discrete simulation setting where the initial retailer agent costs are based on the Euclidian distances from their to the suppliers positions, and the positions are generated uniformly randomly in the range \([0, 100]^2 \in \mathbb{R}^2 \). The experiments with less agents have similar result dynamics but are not presented here due to the lack of space.

We concentrate on the minimization of the group assignment cost, and, therefore, measure the sum of differences between agents \( a_i \in A \) and \( \theta \in \Theta \) and calculate utilitarian, egalitarian, elitist, and Nash social welfare values for the best group supplier as negative values of the total, maximum, minimum, and product of distances in the multi-agent system. Moreover, the cost of assignment is proportional to the measured distance. Therefore, the elitist welfare is measured as the utility of the agent that is currently best off as negative distance cost \( -\min_{i} \sum_{t=1}^{T} dist_{ai}(t) \). The utilitarian social welfare is the sum of individual utilities \( -\sum_{i=1}^{n} \sum_{t=1}^{T} dist_{ai}(t) \), while the egalitarian social welfare is given by the utility of the agent that is currently worst off \( -\max_{i} \sum_{t=1}^{T} dist_{ai}(t) \).
From Figure 1 it can be seen that the utilitarian welfare is the highest for the Vickrey auction algorithm without considering fairness issues even though the proposed two solutions follow quite well the best Vickrey solution. However, the egalitarian (Figure 2) and the utilitarian welfare in both modified auction and modified voting cases are very close in all the experimented instances to the regular Vickrey algorithm, except that for Egalitarian welfare, the modified auction algorithm performs in average better than the other two. However, due to the inclusion of regret in the bid calculation, the egalitarian welfare of the auction algorithm with regret is in average (7 out of 10 instances) better than the one of the regular Vickrey auction algorithm. The elitist welfare of the regular Vickrey auction algorithm (Figure 4) is the best in 4 and second best in 2 out of 10 instances, followed by the auction algorithm with regret which only in 2 instances has a lower result than the voting mechanism.

Furthermore, we measured also Nash welfare (Figure 3) which shows the superiority of the regular Vickrey auction algorithm, followed by the modified Vickrey auction algorithm and the modified voting mechanism. Please note, that, since the objective is to lower the total cost, lower values show better group performance.

A bit inferior behavior of the modified voting mechanism can be explained by the ordinal and not cardinal order of the options which doesn’t give a mathematical basis based on which fine-tuning and the mathematical optimization can be made. The individual assignment process of the voting mechanism does not take into consideration individual agent bias and is performed taking into considera-
Fig. 2: Egalitarian welfare.

Fig. 3: Nash welfare, logarithmic scale.

tion all the information at disposal with the same weight. Furthermore, since a random factor is introduced through lexicographic ordering of equivalently valued suppliers, the distribution of the quality of the solution on the best-case and worst-case retailer agent cannot be achieved.
7 Conclusions

In this work we proposed a new approach for the retailer group supplier assignment problem: modified Vickrey auction with regret minimization, and compared it experimentally with the standard Vickrey auction and the voting method where the preferences of each retailer over the available suppliers are aggregated using the Borda rule. The proposed regret-based Vickrey auction distributes the burden of group assignment costs which might put at stake the coalition, among the most stable agents whose costs are the least.

The proposed modified Vickrey auction is applicable to competitive and to collaborative environments, due to the intrinsic strategy-proofness of the original Vickrey auction. Our results show that the proposed auction method on average reaches solutions that are cost optimal for the system as a whole and sufficiently good for the agents individually.

The research in this paper is also related to the importance of the cardinal vs. ordinal information exchange in the efficiency and the fairness of the group assignment. The experimented voting, compared to the auction methods, demonstrated quite good results in respect to both of them. However, a great disadvantage of the voting approach stems from the ordinal preference values which influences its inferiority to auction assignment results.

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