A Simple Framework for Cognitive Planning

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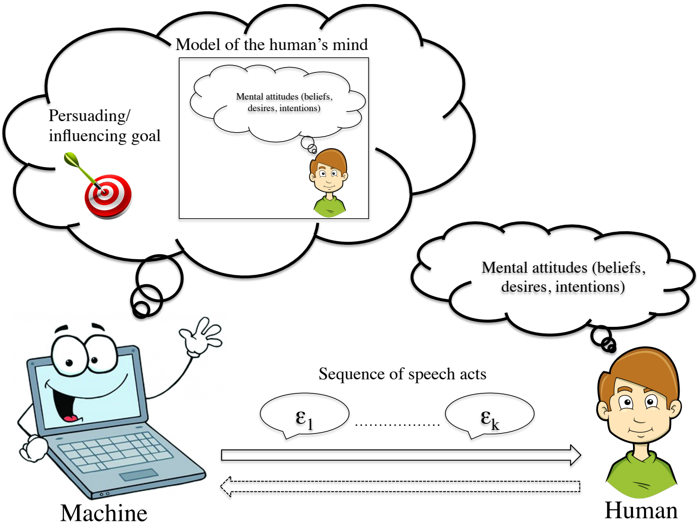
AAAI 2021

Cognitive planning



ANR project CoPains: `https://www.irit.fr/CoPains/`

Our contribution

- We encode the cognitive planning problem in an epistemic logic with a semantics exploiting belief bases (Lorini 2018, 2019, 2020)
    - We study a NP-fragment of the logic whose satisfiability problem is reduced to SAT
    - We provide complexity results for the cognitive planning problem
- We illustrate its potential for applications in HMI: persuasive artificial agent

Language

- A countably infinite set of atomic propositions $Atm = \{p, q, \ldots\}$
- A finite set of agents $Agt = \{1, \ldots, n\}$

Language:

$$\begin{array}{llll} \mathcal{L}_0 : & \alpha & ::= & p \mid \neg\alpha \mid \alpha_1 \wedge \alpha_2 \mid \alpha_1 \vee \alpha_2 \mid \triangle_i \alpha, \\ \mathcal{L} : & \varphi & ::= & \alpha \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid \Box_i \varphi \mid \Diamond_i \varphi, \end{array}$$

with $p \in Atm$ and $i \in Agt$

$\triangle_i \alpha$: agent i explicitly believes that α

$\Box_i \varphi$: agent i implicitly believes that φ

Semantics

Definition (State)

A state is a tuple $B = (B_1, \ldots, B_n, V)$ where:

- for every $i \in Agt$, $B_i \subseteq \mathcal{L}_0$ is agent i's belief base,
- $V \subseteq Atm$ is the actual environment.

The set of all states is denoted by $\mathbf{S}$.

Definition (Satisfaction relation)

Let $B = (B_1, \ldots, B_n, V) \in \mathbf{S}$. Then:

$$
\begin{aligned}
B \models p &\iff p \in V \\
B \models \neg\alpha &\iff B \not\models \alpha \\
B \models \alpha_1 \wedge \alpha_2 &\iff B \models \alpha_1 \text{ and } B \models \alpha_2 \\
B \models \triangle_i \alpha &\iff \alpha \in B_i
\end{aligned}
$$

Semantics (cont.)

Definition (Multi-agent belief model)

A multi-agent belief model (MAB) is a pair (B, Cxt), where $B \in \mathbf{S}$ and $Cxt \subseteq \mathbf{S}$. The class of MABs is denoted by $\mathbf{M}$.

Definition (Epistemic alternatives)

Let $B = (B_1, \ldots, B_n, V), B' = (B'_1, \ldots, B'_n, V') \in \mathbf{S}$. Then,

$$B\mathcal{R}_i B' \text{ if and only if } \forall \alpha \in B_i : B' \models \alpha.$$

Semantics (cont.)

Definition (Satisfaction relation (cont.))

Let $(B, Cxt) \in \mathbf{M}$. Then:

$$(B, Cxt) \models \alpha \iff B \models \alpha$$
$$(B, Cxt) \models \Box_i \varphi \iff \forall B' \in Cxt : \text{if } B \mathcal{R}_i B' \text{ then } (B', Cxt) \models \varphi$$

Theorem

Checking satisfiability of $\mathcal{L}(Atm, Agt)$ formulas in the class $\mathbf{M}$ *is a PSPACE-hard problem.*

NP-fragment

Single-reasoner fragment:

$$\mathcal{L}_{\mathsf{Frag}} : \quad \varphi \quad ::= \quad \alpha \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid \Box_{\mathfrak{m}}\alpha \mid \Diamond_{\mathfrak{m}}\alpha$$

where α ranges over $\mathcal{L}_0$ and $\mathfrak{m}$ a special agent in *Agt* called ‘machine’

Recall:

$$\mathcal{L}_0 : \quad \alpha \quad ::= \quad p \mid \neg\alpha \mid \alpha_1 \wedge \alpha_2 \mid \alpha_1 \vee \alpha_2 \mid \triangle_i\alpha$$

Polynomial reduction to SAT

$$\mathcal{L}_{\mathsf{Frag}} \overset{nnf}{\dashrightarrow} \mathcal{L}^{\mathsf{NNF}}_{\mathsf{Frag}} \overset{tr_1}{\dashrightarrow} \mathcal{L}_{\mathsf{Mod}} \overset{tr_2}{\dashrightarrow} \mathcal{L}_{\mathsf{Prop}}$$

Figure: Summary of reduction process

- $\mathcal{L}^{\mathsf{NNF}}_{\mathsf{Frag}}$: NNF variant of $\mathcal{L}_{\mathsf{Frag}}$
- $\mathcal{L}_{\mathsf{Mod}}$: mono-modal language with no nested modalities, $\triangle_i\alpha$ are treated as atoms (fresh atoms $p_{\triangle_i\alpha}$)
- $\mathcal{L}_{\mathsf{Prop}}$: propositional language, atoms have 'world'-indexes+atoms for 'simulating' accessibility relations

Theorem

Checking satisfiability of formulas in $\mathcal{L}_{\mathsf{Frag}}$ in the class **M** *is an NP-complete problem.*

Dynamic extension

Dynamic language:

$$\mathcal{L}^{+}_{\mathsf{Frag}}: \quad \varphi \quad ::= \quad \alpha \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid \Box_{\mathfrak{m}}\alpha \mid \Diamond_{\mathfrak{m}}\alpha \mid [+_i\alpha]\varphi$$

$[+_i\alpha]\varphi$: φ holds after agent i has privately expanded her belief base with α

Definition (Satisfaction relation, cont.)

Let $B = (B_1, \ldots, B_n, V) \in \mathbf{S}$ and let $(B, Cxt) \in \mathbf{M}$. Then:

$$(B, Cxt) \models [+_i\alpha]\varphi \iff (B^{+_i\alpha}, Cxt) \models \varphi$$

with $V^{+_i\alpha} = V$, $B_i^{+_i\alpha} = B_i \cup \{\alpha\}$ and $B_j^{+_i\alpha} = B_j$ for all $j \neq i$.

Dynamic extension (cont.)

The following equivalences are valid in the class $\in \mathbf{M}$:

$$[+_i\alpha]\alpha' \leftrightarrow \begin{cases} \top, & \text{if } \alpha' = \triangle_i\alpha, \\ \alpha', & \text{otherwise;} \end{cases}$$

$$[+_i\alpha]\neg\varphi \leftrightarrow \neg[+_i\alpha]\varphi;$$

$$[+_i\alpha](\varphi_1 \wedge \varphi_2) \leftrightarrow [+_i\alpha]\varphi_1 \wedge [+_i\alpha]\varphi_2;$$

$$[+_i\alpha](\varphi_1 \vee \varphi_2) \leftrightarrow [+_i\alpha]\varphi_1 \vee [+_i\alpha]\varphi_2;$$

$$[+_i\alpha]\Box_{\mathfrak{m}}\alpha' \leftrightarrow \begin{cases} \Box_{\mathfrak{m}}(\alpha \rightarrow \alpha'), & \text{if } i = \mathfrak{m}, \\ \Box_{\mathfrak{m}}\alpha', & \text{otherwise;} \end{cases}$$

$$[+_i\alpha]\Diamond_{\mathfrak{m}}\alpha' \leftrightarrow \begin{cases} \Diamond_{\mathfrak{m}}(\alpha \wedge \alpha'), & \text{if } i = \mathfrak{m}, \\ \Diamond_{\mathfrak{m}}\alpha', & \text{otherwise.} \end{cases}$$

Polynomial reduction of satisfiability for $\mathcal{L}^+_{\mathsf{Frag}}$ to satisfiability for $\mathcal{L}_{\mathsf{Frag}}$ via the previous reduction axioms

Theorem

Checking satisfiability of formulas in $\mathcal{L}^+_{\mathsf{Frag}}$ in the class $\mathbf{M}$ is an NP-complete problem.

Cognitive planning problem

⇒ Agent $\mathfrak{m}$'s set of informative actions:

$$Act_{\mathfrak{m}} = \{+_{\mathfrak{m}}\alpha : \alpha \in \mathcal{L}_0\}$$

Elements of $Act_{\mathfrak{m}}$ are noted $\epsilon, \epsilon', \ldots$

⇒ Executability precondition function:

$$\mathcal{P} : Act_{\mathfrak{m}} \longrightarrow \mathcal{L}_{\mathsf{Frag}}$$

⇒ Successful occurrence of an informative action:

$$\langle\!\langle \epsilon \rangle\!\rangle \varphi \stackrel{\text{def}}{=} \mathcal{P}(\epsilon) \wedge [\epsilon]\varphi$$

$\langle\!\langle \epsilon \rangle\!\rangle \varphi$: agent $\mathfrak{m}$'s informative action ϵ can take place and φ holds after its occurence

Cognitive planning problem (cont.)

Definition ($\mathcal{L}^{+}_{\mathsf{Frag}}$-planning problem)

A $\mathcal{L}^{+}_{\mathsf{Frag}}$-planning problem is a tuple $\langle \Sigma, Op, \alpha_G \rangle$ where:

- $\Sigma \subset \mathcal{L}_0$ is a finite set of agent $\mathfrak{m}$'s available information,
- $Op \subset Act_{\mathfrak{m}}$ is a finite set of agent $\mathfrak{m}$'s operators,
- $\alpha_G \in \mathcal{L}_0$ is agent $\mathfrak{m}$'s goal.

A solution plan to a $\mathcal{L}^{+}_{\mathsf{Frag}}$-planning problem $\langle \Sigma, Op, \alpha_G \rangle$ is a sequence of operators $\epsilon_1, \ldots, \epsilon_k$ from Op such that $\Sigma \models_{\mathbf{M}} \langle\!\langle \epsilon_1 \rangle\!\rangle \ldots \langle\!\langle \epsilon_k \rangle\!\rangle \Box_{\mathfrak{m}} \alpha_G$

Theorem

Checking plan existence for a $\mathcal{L}^{+}_{\mathsf{Frag}}$-planning problem is in $\mathsf{NP}^{\mathsf{NP}} = \Sigma^{\mathsf{P}}_2$.

Example

- Agent $\mathfrak{h}$: human user who has to choose a sport to practice
- Agent $\mathfrak{m}$: artificial assistant
- Agent $\mathfrak{m}$'s goal: agent $\mathfrak{h}$ forms the intention to practice a sport
- Solution plan: sequence of speech acts by $\mathfrak{m}$

	env	**loc**	**soc**	**cost**	**danger**	**intens**
sw	*water*	*mixed*	*single*	*med*	*low*	*high*
ru	*land*	*outdoor*	*single*	*low*	*med*	*high*
hr	*land*	*outdoor*	*single*	*high*	*high*	*low*
te	*land*	*mixed*	*mixed*	*high*	*med*	*med*
so	*land*	*mixed*	*team*	*med*	*med*	*med*
yo	*land*	*mixed*	*single*	*med*	*low*	*low*
di	*water*	*mixed*	*single*	*high*	*high*	*low*
sq	*land*	*indoor*	*mixed*	*high*	*med*	*med*

Table: Properties of sports

Example (cont.)

Model of the human's mind:

$$\alpha_1 \stackrel{\text{def}}{=} \bigwedge_{\substack{o \in Opt \\ x \in Var \\ v, v' \in Val_x : v \neq v'}} (\triangle_{\mathfrak{h}} \mathsf{val}(o, x \mapsto v) \rightarrow \triangle_{\mathfrak{h}} \neg \mathsf{val}(o, x \mapsto v'))$$

$$\alpha_2 \stackrel{\text{def}}{=} \bigwedge_{\Gamma, \Gamma' \in 2^{Des*} : \Gamma \neq \Gamma'} (\mathsf{des}(\mathfrak{h}, \Gamma) \rightarrow \neg \mathsf{des}(\mathfrak{h}, \Gamma'))$$

$$\alpha_3 \stackrel{\text{def}}{=} \bigvee_{\Gamma \in 2^{Des*}} \mathsf{des}(\mathfrak{h}, \Gamma),$$

$$\alpha_4 \stackrel{\text{def}}{=} \bigwedge_{o \in Opt} \Big(\mathsf{ideal}(\mathfrak{h}, o) \leftrightarrow \bigvee_{\Gamma \in 2^{Des*}} (\mathsf{des}(\mathfrak{h}, \Gamma) \wedge \bigwedge_{\gamma \in \Gamma} f_{comp}(o, \gamma)) \Big)$$

$$\alpha_5 \stackrel{\text{def}}{=} \bigwedge_{o \in Opt} \Big(\mathsf{justif}(\mathfrak{h}, o) \leftrightarrow \bigvee_{\Gamma \in 2^{Des*}} (\mathsf{des}(\mathfrak{h}, \Gamma) \wedge \bigwedge_{\gamma \in \Gamma} f^{\mathfrak{h}}_{comp}(o, \gamma)) \Big)$$

$$\alpha_6 \stackrel{\text{def}}{=} \mathsf{des}(\mathfrak{h}, \Gamma_{\mathfrak{h}})$$

with

$$\Gamma_{\mathfrak{h}} = \{\mathbf{env} \mapsto \mathit{land}, \mathbf{intens} \mapsto \mathit{med}, \sim\mathbf{loc} \mapsto \mathit{indoor}, [\mathbf{cost} \mapsto \mathit{high}] \rightsquigarrow \mathbf{soc} \mapsto \mathit{mixed}\}$$

Example (cont.)

Information about properties of sports:

$$\alpha_7 \stackrel{\text{def}}{=} \bigwedge_{\substack{o \in Opt \\ x \in Var \\ v, v' \in Val_x : v \neq v'}} \big(\mathsf{val}(o, x \mapsto v) \to \neg \mathsf{val}(o, x \mapsto v')\big)$$

$$\alpha_8 \stackrel{\text{def}}{=} \bigwedge_{o \in Opt, x \in Var} \mathsf{val}(o, x \mapsto v_{o,x})$$

Potential intention: $\mathsf{potIntend}(\mathfrak{h}, o) \stackrel{\text{def}}{=} \triangle_{\mathfrak{h}} \mathsf{ideal}(\mathfrak{h}, o) \wedge \mathsf{justif}(\mathfrak{h}, o)$

Example (cont.)

Operators:

$$Op = \big\{ \mathit{inform}\big(\mathfrak{m},\mathfrak{h},\text{val}(o,a)\big) : o \in \mathit{Opt} \text{ and } a \in \mathit{Assign} \big\} \cup \big\{ \mathit{inform}\big(\mathfrak{m},\mathfrak{h},\text{ideal}(\mathfrak{h},o)\big) : o \in \mathit{Opt} \big\}$$

Executability preconditions: $\mathcal{P}\big(\mathit{inform}(\mathfrak{m},\mathfrak{h},p)\big) = \Box_{\mathfrak{m}} p$

Planning goal: $\alpha_G \stackrel{\text{def}}{=} \bigvee_{o \in \mathit{Opt}} \text{potIntend}(\mathfrak{h}, o)$

$\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5$ is a solution to the planning problem $\langle \Sigma, Op, \alpha_G \rangle$ with $\Sigma = \{\alpha_1, \ldots, \alpha_8\}$ and

$\epsilon_1 \stackrel{\text{def}}{=} \mathit{inform}\big(\mathfrak{m},\mathfrak{h},\text{ideal}(\mathfrak{h},\mathrm{te})\big)$

$\epsilon_2 \stackrel{\text{def}}{=} \mathit{inform}\big(\mathfrak{m},\mathfrak{h},\text{val}(\mathrm{te}, \textbf{env} \mapsto \mathit{land})\big)$

$\epsilon_3 \stackrel{\text{def}}{=} \mathit{inform}\big(\mathfrak{m},\mathfrak{h},\text{val}(\mathrm{te}, \textbf{intens} \mapsto \mathit{med})\big)$

$\epsilon_4 \stackrel{\text{def}}{=} \mathit{inform}\big(\mathfrak{m},\mathfrak{h},\text{val}(\mathrm{te}, \textbf{loc} \mapsto \mathit{mixed})\big)$

$\epsilon_5 \stackrel{\text{def}}{=} \mathit{inform}\big(\mathfrak{m},\mathfrak{h},\text{val}(\mathrm{te}, \textbf{soc} \mapsto \mathit{mixed})\big)$

Perspectives

- Implementation of a cognitive planning algorithm using a SAT-solver
- Extension by belief revision: feedback from human to machine
- Extension by 'yes-no' questions: extension of language $\mathcal{L}^{+}_{\mathsf{Frag}}$ by program constructions of propositional dynamic logic (PDL)