The J-Curve and Transaction Taxes: Insights from an Artificial Stock Market

Lina Kalimullina and Rainer Schöbel

Abstract We investigate the distribution of relative returns (RR) among agents who possess varying levels of information in an artificial stock market (ASM). We demonstrate the existence of the J-curve in this market. In contrast to previous studies, the agents in possession of the least information are statistically not different net losers. Moreover, we find that the J-curve is not valid if the relative number of random traders is high. We introduce Tobin-like transaction taxes and show that they destroy liquidity and harm market efficiency. With high taxes, the inequality between agents possessing varying levels of information decreases. However, tax levels dealt with in recent studies influence the market parameters and the J-curve only marginally.

1 Introduction

A significant underperformance of the majority of actively managed funds in comparison with the market average raises the question whether additional information can be beneficial to agents. Studies with informed agents and non-informed agents agree that knowing more is better than knowing less [7]. Following the argumentation in [6], only in efficient financial markets does obtaining additional information not result in increased returns, since all information is already reflected in market prices. However, ‘strong’ market efficiency is hardly possible in reality. Therefore information may be beneficial to market participants. According to [21], economic intelligence is believed to have an increasing function: the more information one has, the better results one can obtain. Nevertheless, when there are several information levels present, a more complex relation can be observed [10, 23]. Various
studies of experimental financial markets have discovered J-shaped relative returns among information levels: the best informed agents outperform the market, but only averagely informed agents perform worse than the least informed or completely uninformed agents [10]. These results suggest that partial information may be harmful. In this paper, we examine the J-curve existence in a simulated financial stock market. In addition, we explore a case in which the RR’s function has a different shape. The discussion concerning the pros and cons of implementing transaction taxes in financial markets is still viable. Supporters claim that taxes reduce market volatility. Opponents argue that taxes result in a decreased market efficiency. Apart from checking the effects of a tax levy on these market parameters, we also investigate whether this levy causes any changes to the J-curve.

In order to achieve our goals, we reproduce from scratch an agent-based ASM, as described in [22, 23]; it models heterogeneously behaving agents who possess heterogeneous levels of information. This approach allows us to incorporate and replicate stylized properties of real financial markets [25] as well as market imperfections [13].

The rest of the paper is organized as follows. In Section 2, we briefly describe the ASM implemented in Matlab. In Section 3, we discuss the J-structure of final returns among the varying information levels resulting from extensive simulations, and explore a case when this shape is not valid. Section 4 extends the discussion by introducing transaction taxes and examines how this affects the market parameters and the J-curve. Section 5 offers a conclusion.

2 Market Structure

In this section, we describe the continuous double auction (CDA) trading system, the economic environment and the market process sequence. We focus on two sources of agent’s heterogeneity: the amount of information they possess (‘informational endowment’) and their trading strategies.

2.1 Trading System

The marketplace mechanism, where the selling and buying of stocks occurs, is modeled as an order-driven CDA. This trading mechanism is common to many stock exchanges [1]. In such a market, agents trade directly with each other and the market maker does not participate in transactions. As in [4], traders arrive at the market sequentially and place either marketable orders, leading immediately to a transaction, or limit orders, which are written down in the order book. By allowing traders to send limit orders, liquidity is secured. If an agent wants to buy the stock, he submits an order to buy - a bid order; if he wants to sell the stock, he submits an order to sell - an ask order. Trade is facilitated by the order book that represents a queue of
limit orders which, once sent to the market, stay there until they are satisfied or until the end of a period of time. In the limit order book, the price is set in multiple steps when a newly placed market order matches any limit order previously sent in by another agent. As in [1], the best limit order serves as the transaction price if orders are matched. Furthermore, transactions follow a time priority rule for orders with identical prices, and a price priority rule for orders across all price levels [5].

2.2 Economic Environment

The economic environment of our ASM is similar to those presented in [10, 23]. We designed a multi-period market where heterogeneously informed artificial agents (computer algorithms) trade a risky asset, and possess a portfolio consisting of this risky asset (stock) and a risk-free asset (cash). In our simplified case, only one risky asset is available for trading. Going short in cash or stock is not allowed. At the end of each period, interest on the cash account and dividends on stocks are paid out. Dividends are simulated as a random walk process before each trading session:

$$D_t = |D_{t-1} + 0.1 \times N(0,1)| \forall t = 0...T,$$

where $T$ is the number of simulated periods and $N(0,1)$ denotes a standard normally distributed random variable. The starting value of dividend is $D_0 = 0.2$. The dividend process is assumed to be non-negative, meaning that whenever $D_t < 0$, the absolute value is taken for the purpose of the dividend process [22, 23]. As in [21], we define our market as a pure circulation market: no new stocks are issued. The market is populated with ten heterogeneous agents; each of them possibly has a different information endowment and uses a different trading strategy. Information about orders and past prices is public; it is available to all agents free of charge at all times. The final endowments of stock and cash resulting from one period are carried over to the next period.

2.3 Information Structure

Each trader is endowed with some information about future dividends depending on his information level. The mechanism of information allocation takes the form of a moving window: the best informed agent has the most comprehensive information, the second best informed trader obtains the same information set but after one period, while the least informed agent has access to the same information set after the time lag of eight periods. At the beginning of each new period, an agent receives additional information about one further dividend realization. Agent $I_0$ (information level of zero) does not possess any information. Such an agent is also called a completely uninformed agent; he trades randomly on the market.
Agent $I_1$ has information about dividend payments at the end of the current period; he is called the least informed agent. In general, an agent $I_j$ receives information about $j$ future dividends. The best informed agent is $I_9$. The better informed agents have an informational advantage since they gain access to desirable information earlier than the worse informed agents [10]. It is assumed that information is exact, and dividend values do not suffer from any noise or inaccuracy.

Once the information about future dividends is obtained by traders, it is modified in the subsequent prediction process. We assume that all agents use the same prediction mechanism. With the help of Gordon’s formula, agents calculate the expected present value of stock (EPV) conditioned on their forecasting horizon [23]:

$$EPV_{j,k} = \frac{D_{k+j-1}}{r_e(1+r_e)^{j-2}} + \sum_{i=k}^{k+j-2} \frac{D_i}{(1+r_e)^{i-k}}.$$ 

Here $EPV_{j,k}$ denotes the conditional EPV of a stock for a trader $I_j$ with information level $j$ in a period $k$, and $r_e$ is a risk-adjusted interest rate.

In contrast to the experimental market with human agents in [10], in our simulation set-up there is no feedback on the information level: an agent does not realize that the other agents might be better informed than himself.

### 2.4 Trading Strategies of Market Participants

Artificial agents use certain algorithms to decide on a market transaction. Three types of agents are modeled to interact in the market. We model random traders, fundamental traders, and chartists, as described in [23].

A random trader constructs a new limit order as a random deviation from the previous transaction price. We assume that random traders believe that they are trading based on information which they think they possess; or that they simply like to trade [3].

The main assumption about fundamentalists is their belief that information about future changes of fundamental value is not reflected in the price process yet but that the price will converge to its fundamental value in the future [20]. Their trading decisions are based on EPVs. If the EPV is larger (smaller) than the best ask (bid), it is treated as a new marketable bid (ask) order.

A chartist (or technical trader) ignores the information about the future dividends and follows a trend (or momentum) strategy. He analyzes only price changes and does not conduct any economic analysis. His decisions are based on the presence of uptrends or downtrends or on the absence of any type of clear trends in the price evolution process. Including a technical trader in the simulated market is a natural choice, in light of the conclusions from the experimental results in [13]: human agents, even after being told that the RR are randomly drawn variables, tend to attach considerable importance to past performance; the majority of traders believe that the future will be either like the past or contrary to the past.
It is assumed that agents cannot change their strategy, they lack social interaction and the ability to evaluate success or failure. Learning or adaptive mechanisms are not incorporated.

### 2.5 Market Session Sequence

The following steps summarize the sequence of main activities of the simulated ASM. After the dividend process is randomly simulated and the agents are assigned exogenously with information levels, each of them receives a corresponding amount of information about future dividends. The market session begins with the ‘opening’ procedure, which sets the starting price through a Warlasian auction by determining the equilibrium price of supply and demand [2]. In the following market session, agents are chosen randomly to act sequentially in the market. If a transaction is executed, the wealth bookkeeping mechanism changes the current endowments of the agents. At the end of each period, the order book is emptied: all the unmatched limit orders are removed from the order book. Moreover, dividend and interest payments are added to the final endowments.

### 2.6 Simulation Set-Ups

We conducted two extensive simulations with slightly changed parameters. The first set of parameters corresponds to the simulation for Section 3. The market is populated with ten agents; only one of them is a random trader while the others are fundamentalists. There is only one fundamental trader per information level. Chartists are not included in the market population at this stage as they do not use the fundamental information (later on we describe how including the chartists influences the simulation results). The market session lasts 100 periods, each consisting of 100 steps. Initially the agents are endowed with high amounts of cash and stock, so that they are very unlikely to face any budget constraints during a market session: the initial cash endowment is 10,000 units of currency and the initial stock endowment is 1,000 units. The initial price of the stock is 40 units of currency. For the purpose of statistical reliability, we conduct 100,000 market iterations (100 × 100 simulation): 100 sessions are completed with different random dividend processes and 100 further runs are repeated within each session under the same dividend process to account for randomness in the sequence of agents’ draw and market actions. The second set of parameters corresponds to the simulation for Section 4. In comparison to the previous set-up, the market session lasts 75 periods. The initial cash endowment is 1,000 units of currency and the initial stock endowment is 100 units. The simulation repeats 2,800 market iterations for each unique set of changing mar-

---

1 Learning mechanisms in a CDA are considered in [17, 18].
ket parameters (70 × 40 simulation): there are 70 sessions with different random dividend processes and 40 further runs within each session.

3 Evaluation of the Agents’ Success: the J-Curve

The J-curve of RRs, thoroughly investigated in [21] for the first time, describes the inequality of RRs obtained by the heterogeneously informed agents. In this section, we investigate the J-curve resulting from the simulation of our artificial market model.

We assess the final wealth allocation with the overall relative return (ORR). ORR is the RR estimated from the market opening until its closing, minus the market return. The market return is calculated as the average individual return and is taken as a benchmark for the comparison of returns among the agents.

The Mann-Whitney U-test confirmed the J-shape for our market setting: not all informed traders perform better than the random trader, see Table 1 and Figure 1.

Table 1 Mann-Whitney U-test for pairs of ORRs

<table>
<thead>
<tr>
<th>Agent</th>
<th>I₀</th>
<th>I₁</th>
<th>I₂</th>
<th>I₃</th>
<th>I₄</th>
<th>I₅</th>
<th>I₆</th>
<th>I₇</th>
<th>I₈</th>
<th>I₉</th>
</tr>
</thead>
<tbody>
<tr>
<td>I₀</td>
<td>-</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>I₁</td>
<td>1</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>I₂</td>
<td>1</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>I₃</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>I₄</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>I₅</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>I₆</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>I₇</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>I₈</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>I₉</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-</td>
</tr>
</tbody>
</table>

Note: The mean ORRs are compared pairwise (I₀ with I₁, I₁ with I₂, and so forth). The null hypothesis \( H₀ \) states that the two samples are drawn from the distributions with equal medians or, in other words, that two information levels have equal average ORRs. In the table the logical value \( h \) is presented: it represents the test result. \( h = 1 \) means that \( H₀ \) is rejected at 5% significance level (two-sided test): the ORRs of the two compared groups are statistically different. \( h = 0 \) means that \( H₀ \) could not be rejected at 5% significance level.

The ORRs of the uninformed I₀ and medium informed I₆ agents are very close to zero. The random trader ends up with the market average return, which is expected to be true under the market efficiency hypothesis\(^2\). Moreover, his return is not statistically different from the return of the averagely informed agent I₆ whose information level serves as the border line between the ‘public information’ and the ‘insider

\(^2\) We thank the anonymous referee for pointing this out. This is true if the random trader does not have any influence on the market price, or if he has an equal probability to beat the market and to be beaten by the market.
information’ and as the break-even level between information levels providing negative and positive ORRs. The ORRs of the informed agents up to $I_5$ are negative. The information of all these agents is already included in the prices of the current period. The ORRs of the agents $I_1 - I_6$ are not proved to be significantly different to one another. All these agents lose in comparison with the $I_0$ or $I_6$. At the expense of these badly informed agents, the best informed agents $I_7 - I_9$ (or ‘insiders’) beat the market. For these agents, the higher the information level is, the significantly higher the ORR will be.

![Mean ORRs](image)

**Fig. 1 Mean ORRs**

The resulting J-curve has minor deviations from the curve presented in [10] which is based on the outcomes of an experimental market with human agents. The J-shape produced there suggests lower returns for the averagely- rather than for the low-informed agents. However, our results show that the averagely informed agents do not receive a return smaller than that of the least informed agent. Our results are in line with [23].

We also run the simulation under a changed composition of the market population. If chartists are included, they become net losers. In addition, this change influences the position of the break-even point among the fundamental traders: the more chartists there are, the more informed fundamentalists manage to beat the market.

Furthermore, we discovered a market setup in which the J-curve is not valid. We change the share of random traders in the population (or the probability of them being drawn in the market). With an extremely high random trader population, the J-structure is not observed, see Figure 2. The random traders experience negative ORRs because they influence the price if their population is high, and they have a greater risk of being beaten by the informed fundamentalists. At the expense of the random traders, all of the informed traders utilize the information profitably; it corroborates with the predictions from [3]. On the other hand, when the number of random traders is relatively small, the benefits of the best informed traders are bolstered additionally by the losses of the least- and averagely-informed agents. Therefore, in Figure 3 the mean ORRs of the best informed agent remain approximately unchanged, the ORRs of the random trader change slightly from positive to
negative values, whereas the ORRs of the least informed trader increase drastically from negative to positive values when the share of random traders increases.

![3D representation of ORRs and share of random traders](image1)

**Fig. 2** ORRs and share of random traders: 3D representation

![2D representation per information level](image2)

**Fig. 3** ORRs and share of random traders: 2D representation per information level

## 4 Transaction Tax: Market Parameters and the J-Curve

Recently, the discussion about the necessity of transaction taxes has become topical once more. Transaction taxes are usually considered a typical regulation mechanism for stock markets, but the policy makers do not agree as to whether they have a positive or negative effect, and, consequently, upon the necessity of their implementation. The proposed Tobin tax varies between 0.005% and 0.5% depending on the country and security. Supporters believe that taxes may help to attain market stability. As a result, a share of the traders simply refrains from trading [14]. The argument against taxes is that it causes a decrease in trading volume. The impact on market efficiency and volatility is still being discussed [8]. We examine whether our model supports the results of the previous research and explore the ORR structure change along the tax level growth.

Our results show that transaction taxes negatively influence the trading volume as
well as the acceptance ratio (parameters are defined as in [12]): see Figure 4. However, small tax levels do not significantly affect these parameters. We examined normalized and mean relative absolute deviations as well as normalized returns and found no observable influence, which supports the results in [12] and [24] but contradicts the investigations in [14] and [15]. [16] proposes that in a CDA, the stabilizing impact of transaction taxes is diminished and the volatility is not decreased, since a market liquidity reduction leads to a higher price impact of each order. On the contrary, the dealership market benefits from tax, since taxes deter speculative traders but the liquidity is still provided by specialists (dealers).

The liquidity decrease leads to another negative impact of transaction taxes, namely to a reduction in market efficiency: this supports the results from [19]. The market efficiency is measured by the Pearson correlation between the average prices per period and the EPVs. Figure 5 shows that the average price process has the highest correlation with the EPV of the agent with the median information level, which is in agreement with [10]. It implies that the market prices do not reveal all available information. Only ‘public information’, or the information of the least and averagely informed traders, is contained in the prices. The insiders’ information is not reflected in the current prices, thus the market cannot be defined as one with ‘strong’ efficiency. According to the classification of Fama [6], our market shows ‘semistrong form’ efficiency. Figure 5 also illustrates that high transaction tax rates lessen the degree to which prices reflect the amount of available information: the Pearson correlation becomes weaker. However, for very small transaction tax levels, the market efficiency is not critically affected, which confirms results in [12]. Moreover, we find that an increasing tax level influences the structure of tax revenues received by the collecting institute from agents of different information lev-

---

3 The results are based on the treatment of the real stock market in Sweden.  
4 According to their results, the volatility is reduced through the introduction of a higher tax level.  
5 They found that tax increases volatility and decreases trading volume.
Fig. 5 Transaction taxes and the market efficiency

Fig. 6 Transaction taxes and tax revenues (the directions of tax axis and information level axis have been inversed for the purpose of better demonstration)

Fig. 7 Transaction taxes and ORRs (the directions of tax axis and information level axis have been inversed for the purpose of better demonstration)
els: see Figure 6. For a low level of transaction tax, the $I_9$-investor pays the minimal amount of tax, while the maximum amount is contributed by the $I_1$-investor. As tax increases, the relative tax revenues from the $I_1$ and $I_9$ increase, while those of averagely informed agents remain nearly unchanged.

Finally, with increasing tax rates, the inequality between information levels decreases: see Figure 7. The J-curve is more pronounced if there is a small or zero transaction tax level.

Overall, in our simulated model, the market liquidity and efficiency decrease with tax growth, while the J-curve becomes less pronounced. However, at the level of transaction tax discussed in the literature, all of these changes are small.

5 Conclusion

This paper has provided new insights into the topic of relative wealth distribution amongst agents with heterogeneous information levels. For this purpose we reconstructed a computational ASM model, as presented in [23]. The J-shape of the ORRs, mentioned in the previous studies, was confirmed with some modifications. We found that none of the low informed agents outperformed the least informed agent. We explored a counterexample in which the J-shape was not observed.

We extended the discussion by analyzing the J-curve reaction to the introduction of a transaction tax. The inequality between information levels became less critical with the introduction of taxes, while the market liquidity and efficiency were negatively influenced. However, the transaction tax levels considered in recent discussions had only marginal effects on the parameters discussed here.

References