Space-Time as a Primitive for Space and Motion

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Abstract. *This paper deals with the issue of the representation of space and motion, and argues that motion can be taken as a primitive notion on which a theory of space can be built, in which every object is an occurrent and has temporal parts. There has been a lot of discussion around the continuants/ occurrents opposition; while some authors have advocated the use of occurrents only for theories of parts and the geometry of common-sense, the few detailed or convincing work that has been devoted to solving the inherent problems of such an approach has made it easy for its detractors to claim it is a dead-end street. We present here a theory of spatio-temporal entities and show how this theory can be used to define a theory of motion. Thus we define a notion of continuity that is more appropriate than mathematical continuity for characterizing motion, and argue that we have here a basis for a theory of spatio-temporal objects.*

1 Introduction

This paper deals with the issue of the ontology and the representation of space and motion, and argues that motion can be taken as a primitive notion on which a theory of space can be built, in which every object is an occurrent and has temporal parts, all within a mereo-topological framework.

Space and motion are critical concepts in a number of fields, and their nature has been the center of much debate in philosophy since Aristotle until now. The debate has been strongly influenced by the work of physicists (from Newton to Einstein), and only recently by researchers from the artificial intelligence (AI) community, who have tried to come up with theories of space more suited to their needs, that is coping with uncertainty, impreciseness, and reasoning about space and time in a computational perspective. This has lead to question the ontological assumptions made in physics about space and time and renew the discussions of a cognitively acceptable characterization of them.

Dealing with spatial representations is very often dealing with changing representations and that is where time enters the picture. Motion has then often be assumed to be a construction of space and time, separately, combining difficulties from both concepts. This view has thus given rise to a number of problems, ranging from Zeno's paradoxes (how can one tell at a given instant if a body is in motion or at rest) to the problem of identity of spatial objects though time. Objects are indeed considered as persistent through time (*continuants*) as opposed to events and processes (*occurrents*, which have temporal parts) but since some of their properties are changing there has been much debate about some criteria that would determine whether two objects at different times are the "same" object. The problem of the *continuity* of continuants still lacks some convincing treatment. There is a number of possibilities in the literature to cope with the problems mentioned above. Some solutions involve refining the temporal structure [15], while others focus on the reasoning process about spatial entities (e.g. [37]) or on a

revised theory of parts ([38]). A few authors have proposed to deal with this by considering all objects to be occurrents[1] and to be considered as whole space-time histories. These authors have not really exploited much the idea however: Hayes's theory of space-time is somewhat coarse, as shown in [43], in which there is, to the best of our knowledge, the closest attempt to a mereological spatio-temporal theory; still its properties are not investigated and it has not been used in further work by this author. All in all, there is still strong oppositions to this solution for an ontology of objects (see below, and [38] for a general discussion). We think on the contrary that there are good reasons to commit oneself to such an ontology of spatial entities in general and that it is the lack of precise work to address the undeniable problems of this approach that has left it in the limbo. We present here an axiomatic theory of spatio-temporal regions that arguably addresses and solves some of the issues mentioned before. This theory enables us to define a topology of extended spatial and temporal entities and to propose an ontology for (topological) motion. We place ourself in a mereo-topological framework (see [38] and [42] for a general presentation of available theories along this line). We think this theory then gives a good basis for a theory of objects having spatio-temporal referents.
In the following we present in section 2 a survey of different approaches to motion representation. Section 3 defines the issues related to the occurrents/continuants distinction. Section 4 presents a spatio-temporal topology based on a theory similar to that of [9]. Section 5 discusses a possible definition of continuity that characterizes common-sense motion. Section 6 points to some properties of our theory that make it a sound basis for a further extension to actual objects.

2 Classical Representations of Motion

Motion is differently approached from various disciplines in which it is a key notion such as philosophy, physics, psychology, linguistics or artificial intelligence. The Newtonian framework in physics has of course widely influenced most of those approaches, until its shortcomings became evident when it came to account for the way motion is perceived by humans, as shown in language for instance, and the way it could be treated by machines. It is still prevalent in a number of domains, but with some changes according to the field that make use of the concept. Whether in qualitative reasoning about physical systems, representation of lexical knowledge, cognitive modeling, geographical information systems, (artificial) vision and perception, theories have more or less departed from the classical view that motion is a continuous function from a time isomorphic to the real line into a space isomorphic to Euclidean space. We have tried to classify these approaches in order to show how and by how much our model differs from related work[2].
There are a few key choices that distinguish the various approaches, listed below, concerning space in general and thus, motion:

1. The choice of an absolute space (existing separately from objects in it) vs. a relative space (where only physical objects exist and are located with respect to each other).

2. The choice of extended objects (or regions) vs. points as spatial primitives and the corresponding choice for time (not always in accordance when combined).

3. Considering relative or absolute motion, independently of the choice made about space.

[1] Among others: [36], [6], [33] and more recently [19, 43].

[2] Most of the references here emerge from linguistics, AI, or formal ontology, without aiming at any completeness. We focused on representational issues, thus ignoring most of the discussion in philosophy and psychology about the perception of motion, for instance, e.g. [7], [30].

4. The assumption that space is discrete or dense (hence motion can be continuous or not).
5. Modeling change as constraints on state/situation transitions or constraints on relations (in axiomatic theories).
6. The choice of a primitive space-time as opposed to an approach where time and space are separate dimension.

It turns out most of these choices can be made independently from each other.

The choice of an absolute space combined with points as primitive for time and space is the basis of modern pre-relativist physics, along with the assumption they are both dense (but see [13] for a physicist's dissident opinion). It is moreover used in robotics, and in a qualitative way in most work done under the label "qualitative reasoning" ([10], [12], [11], [34]), mostly qualitative kinematics. It has also been used for a representation of motion verbs properties in [20], although the cognitive adequacy of such primitives is doubtful.

Relative space is indeed advocated in most linguistically and/or cognitively oriented work ([39], [2], [22]), but it does not entail motion is considered relatively; for instance in [2], a motion path is represented as a series of preexisting connected locations, whereas a purely relative theory would talk about changing relations between entities[3].

The relative approach is often combined with the choice of regions as primitive entities, and has been a recent trend in AI in what is called qualitative spatial reasoning, [35], [3], inspired by [9] and pioneer work in mereology and mereo-topology ([27], [26]). It has led to the investigation of motion in a combined region-based space and a mixed interval/point-based time by Galton [14, 15]. This was a departure from the traditional state-based approach of qualitative kinematics and other AI-oriented authors (such as [37, 17]) (motion constraining possible transitions between physical states) to an approach where motion is a change in *relations* bearing on individuals objects.

As for the last representational choice (space-time vs. space and time), an overwhelming majority of authors take space and time as two separate dimensions, and the exceptions are few, although many philosophers have advocated their contrary view. Carnap had defined languages in which primitive entities are spatio-temporal [6], but stopped short of any characterization of their properties. Hayes in [18] has made use of space-time histories as representations for objects, but his theory has trouble departing from the standard (space$\otimes$time) product, as shown by [43], who gave hints as what such a theory should be. Recently, [5], has shown the equivalence between a theory of primitive space-time lines and a theory of spatial locations and time instants, but it is mostly a translation of Euclidean geometry that is far from being cognitively justified or usable (and going from points to regions is then far from obvious). It moreover makes the strong assumption that points all have a constant speed.

We will see how we can overcome the flaws of these approaches to provide an alternate model for the representation of motion, where all entities are spatio-temporal. First we will have a look at some reasons for this ontological choice.

3 The Occurrent/Continuant Opposition

The classical ontological distinction, in most axiomatic theories of objects and their parts, between continuants (objects of the everyday world, persisting through time and which cannot have temporal parts) and occurrents (states or events, bounded in time and which can have

[3]The former approach is thus closer to the absolutist view where motion is a change of value of a location function, usually assumed to be continuous, see also [24, 29].

temporal parts) lead to some problems when considering *change*. It is indeed difficult to define criteria to identify objects whose parts or properties are changing, and it can even entail paradoxes (see the Flux argument in [38], pp117-127).

The problem disappear when considering all material objects are occurrents since properties become relative to temporal parts of these objects (and there is no need for tensing logical predicates, thus avoiding the aforementioned paradoxes). This ontology of objects allows for a simple reformulation of spatio-temporal facts in a not-so-unintuitive way, as it does seem natural to consider objects are bounded in time and are similar to events in that respect.

The main criticism against this approach (again see [38]) is that it is not easily understandable and that it is difficult to think of the world in terms of four-dimensioned objects [4]. It is true that few people have investigated this angle and that almost nobody has made significant work on building common-sense theories of space-time (with the few exceptions already mentioned —Hayes, Vieu, but without much success). But most of the arguments presented in [38] eventually boil down to the same and only one: the "alien nature [of a process ontology]", since it gets rid of objects and the concept of change altogether. The theory we present here is an attempt at making it more understandable, and adding the time dimension *into* "spatial" objects, showing objects then have different interpretations, and that change is still a valid concept, although it has a somewhat different manifestation. For that purpose, we have focused on spatio-temporal regions of space.

Among the problems such an approach has to address are the following:

- What should be the links between spatial, spatio-temporal and temporal relations ?
- What kinds of temporal parts of entities can be legitimately defined and how ?
- What does change mean in that context ?

We will address these questions in turn in the following sections.

4 A Mereo-Topological and Spatio-Temporal Theory

The basis of our representation is a mereo-topology in the spirit of [9]; we have taken over the axiomatization given in [3] whose models have been clearly characterized. The primitive entities of this theory are supposed to be spatial extents (regions of space), but will be interpreted here as "regions" of a primitive space-time; the only primitive relation is that of connection (C), symmetric, reflexive and extensional from which the notion of parthood (P) and topological concepts (open/closed regions) can be defined and axiomatized in an appropriate manner. The principle of extensionality of classical mereology has been often criticized (for good reasons) when applied to physical objects, since many people consider that two objects can be said to have the same parts and still be different; when considering mereo-topology, the extensionality of C means that two objects occupying the same region of space are identical. This is perfectly acceptable here since we consider only the spatial extent of objects. Section 6 gives some elements as to how these issues can be addressed in our framework if it is to become a theory of objects. A more serious criticism of this version of mereo-topology is the absence of boundary entity, which has been considered a serious flaw by [38] and [42]; we will see that

[4] It has been argued that physics of the relativity theory would justify such an approach as being closer to the "real" universe, but we think the debate has nothing to do with this since relativity is still far from being part of the layman's common-sense knowledge and brings no intuition on our perception of the everyday world. We claim that the "space-time" approach is an interesting track anyway, regardless of modern physics theories, for solving ontological problems of space and time.

is does not entail anything uncomfortable in our theory and preserves a certain homogeneity of the formulations of problems tackled here.

We make use of other classical definitions: partial overlap (PO), external connection (EC), sum of objects ($x{+}y$), intersection of two overlapping objects ($x{\cdot}y$), closure and interior (c and i), complement ($-x$), all defined in [3]. An illustration of a spatio-temporal interpretation of the relations is given figure 1, where space here is one-dimensional.

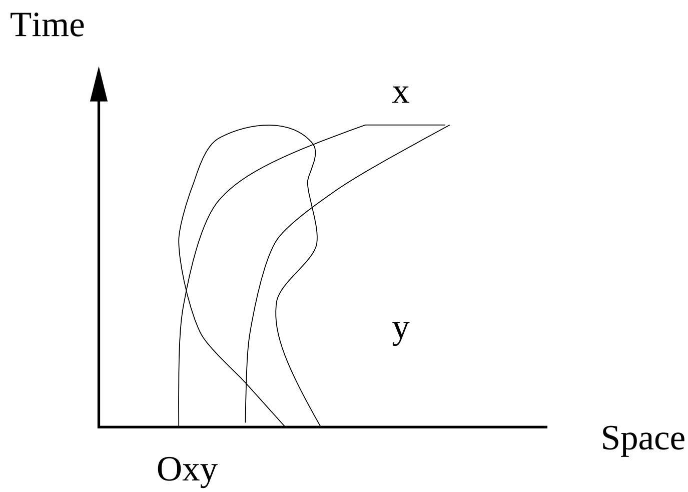

Figure 1: A spatio-temporal interpretation of O(verlap)

4.1 Temporal Relations

The theory we mentioned in the previous section is generally thought to provide a topological account of space seen as made up of extended regions. In order to be regarded as a spatio-temporal theory, we must introduce the structural properties of time into it. The most intrinsic and uncontroversial property of time is that of *order*, but is not sufficient to express relations between extended entities. Indeed, since our entities are spatio-temporally extended we have to make use of temporal relations similar to what exists in the literature for extended periods of time, as opposed to point-based theories (cf. [41, 25]). The main difference will be that temporal equivalence will not entail equality (since entities are not temporal only). Entities are extended in a primitive space-time so that temporal relations actually bear on the spatio-temporal regions *themselves*, and not on time extents.

Most first-order temporal logics take as a primitive relation (besides a relation of partial order) a temporal inclusion or a temporal overlap [40, 23, 4]. But for the same reasons that mereology alone cannot account for topological concepts (as is shown in [42]), we cannot rely on those notions alone to distinguish between open and closed entities at the temporal level, a distinction that needs to be kept if "connection" and "overlap" must remain separate relations. The common-sense theory of time in [1] uses a "meet" relation as a temporal primitive between intervals but it can only account for relations between self-connected intervals (a very strong restriction for most theories of space, time or space-time), and other relations can only be derived by assuming the existence of many other intervals than the ones initially introduced; it seems as if the author assumes the existence of an underlying absolute space, which goes against the whole enterprise of only introducing a minimal number of entities. But the main problem with this popular theory is the absence of the open/closed distinction, which can be considered as central to any theory of change (at least at the temporal level), as shown by

Galton. Galton [15] uses a hybrid ontology mixing intervals and instants to keep that expressive power, but we believe this hypothesis is not necessary and keep a more general (where entities can be disconnected) and ontologically more parsimonious (assuming only extended entities) model which turns out to be quite practical. That is why we introduce a "temporal connection" relation, noted $\asymp$, added to the obvious partial order $<$ [5] ; much in the same way as spatio-temporal connection, this relation is the most economical way of defining temporal parts, overlap and other desirable relations.

Our theory is a first order theory with equality, where logical and, or, negation and implication are denoted $\wedge$, $\vee$,$\neg$,$\rightarrow$; a definition is introduced by $\triangleq$. In the following, universal quantifiers scoping over whole formulas are omitted. Upper case symbols stand for predicates, lower case ones for variables or constants. Thus, $\asymp$ is symmetric and reflexive:

A 4.1 $x \asymp y \rightarrow y \asymp x$

A 4.2 $x \asymp x$

$\asymp$ and $<$ are incompatible:

A 4.3 $x \asymp y \rightarrow \neg x < y$

Obviously, $<$ is antisymmetric:

A 4.4 $x < y \rightarrow \neg y < x$

The composition of $<$ and $\asymp$ behaves as follows (this kind of property is sometimes called "transfer" in temporal axiomatic theories):

A 4.5 $(x < y \ \wedge \ y \asymp z \ \wedge \ z < t) \rightarrow x < t$

And the following more intuitive relations can now be defined:

D 4.1 $x \subseteq_t y \triangleq \forall z (z \asymp x \rightarrow z \asymp y)$ (temporal inclusion)

D 4.2 $x \sigma y \triangleq \exists z (z \subseteq_t y \ \wedge \ z \subseteq_t x)$ (temporal overlap)

D 4.3 $(x \equiv_t y) \triangleq x \subset_t y \wedge y \subset_t x$ (temporal equivalence)

The transitivity of $<$ can be derived from A4.2 and A4.5 and its irreflexivity from A4.3 and A4.2. The relation $<$ is therefore a strict (partial) order. We can derive all the axioms of other systems based on $<$ and $\subseteq_t$ or σ ([41, 4, 23]) from the previous axioms, which have been mostly inspired by [4], thus giving the intended properties for the temporal relations:

Th 4.1 $x \subseteq_t x$

Th 4.2 $x \sigma y \rightarrow y \sigma x$

Th 4.3 $x < y \rightarrow \neg x \sigma y$

Th 4.4 $(x < y \wedge y \sigma z \wedge z < t) \rightarrow x < t$

Th 4.5 $(x < y \wedge y \subseteq_t z \wedge z < t) \rightarrow x < t$

Th 4.6 $(x \subseteq_t y \ \wedge \ y \subseteq_t z) \rightarrow x \subseteq_t z$

Th 4.7 $x \subseteq_t y \rightarrow \forall z \ (z \sigma x \rightarrow z \sigma y)$

Th 4.8 $x \subseteq_t y \rightarrow \forall z \ (z < y \rightarrow z < x) \wedge (y < z \rightarrow x < z)$

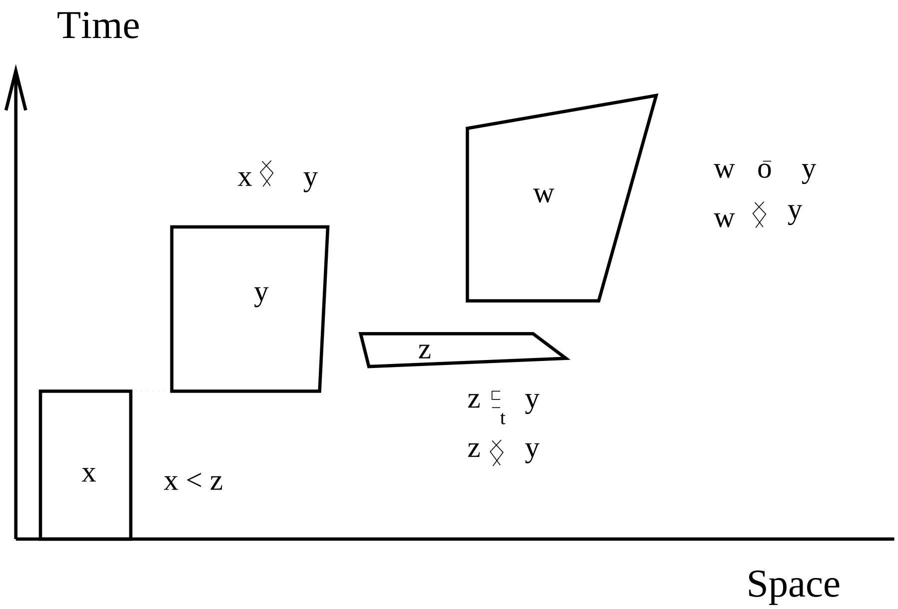

Figure 2: Temporal relations

Figure 2 shows examples of the temporal relations thus defined over spatio-temporal regions. We can also define a notion of temporal self-connectedness:

D 4.4 $\text{CON}_t x \triangleq \forall x_1 \forall x_2 (x = x_1 + x_2 \rightarrow (cx_1 \bowtie cx_2))$

We can moreover impose the linearity of the underlying temporal order by stating that there must be a relation ($<$ or $\bowtie$) between two temporally self-connected entities (and only in that case since the theory allows for the sum of arbitrary entities, therefore not necessarily self-connected). The following axiom indeed eliminates branching time models.

A 4.6 $(\text{CON}_t x \wedge \text{CON}_t y) \rightarrow (x < y \ \vee \ x \bowtie y \ \vee \ y < x)$

4.2 Spatio-Temporal Interactions

Since our theory is more than just space "and" time, we must define how time interacts with the spatio-temporal relations. We refine here the axioms given in [43], which uses a "temporal overlap"relation and a partial order relation, and add more constraints that we believe necessary to characterize a truly spatio-temporal theory. The original axioms for this relation had undesirable consequences for external connection and overlap. Besides they did not guarantee that temporal and similar spatio-temporal relations were really distinct.
First, two connected entities are also time-connected:

A 4.7 $\text{C}xy \rightarrow x \bowtie y$

The models must not be temporal only, so C and $\bowtie$ are different

A 4.8 $\exists x \exists y \ x \bowtie y \ \wedge \ \neg\text{C}xy$

A 4.9 $\exists x \exists y \ x < y$

[5]For readability's sake, and to distinguish them from the spatio-temporal relations, temporal relations are infixed.

The interaction between $+$ and $<$ and σ need to be specified:

A 4.10 $(x < y \wedge z < y) \leftrightarrow (x + z) < y$

A 4.11 $(x + y) \lessgtr z \leftrightarrow x \lessgtr z \vee y \lessgtr z$

So that we have the following properties (axioms of [43]):

Th 4.9 $\mathrm{O}xy \rightarrow x\sigma y$

Th 4.10 $(x < y \wedge \mathrm{P}zx \wedge \mathrm{P}ty) \rightarrow z < t$

Th 4.11 $(x\sigma y \wedge \mathrm{P}xz \wedge \mathrm{P}yt) \rightarrow z\sigma t$

Th 4.12 $(x + y)\sigma z \leftrightarrow x\sigma z \vee y\sigma z$

4.3 Temporal Parts

In order to define relations between regions that can vary through time, we now define a notion of "temporal slice", i.e. the maximal part corresponding to a certain time extent. We thus assume that any entity can have temporal parts. This notion is present in classical approaches where time and space are separate: for instance [38] define "temporal parts" ($<_T$) as follows (the notation is Simons'):
$e' <_T e \equiv e' < e \wedge \forall f([f < e \wedge spl[f] < spl[e']] \rightarrow f < e')$
where $<$ is a part-whole relation between objects and spl (spell) is a function giving the stretch of time corresponding to the objects it is applied to. Simons makes a distinction between objects and the space they occupy but still applies the same relations to these entities. He then goes on defining a "phase" as a temporally connected temporal part, without giving a formal definition of connection, which is a notion not definable in a purely mereological framework. Finally, a slice is defined (informally) as a phase of zero duration, which has no explicit meaning in his theory (the intended model being here implicitly the real line for time, which is somewhat surprising in a mereological framework). This is the reason why we need topology to define such objects and this is why we don't want to talk about durations, which entails a metric. We confine ourselves to a theory where the shortest "durations" are the smallest events or objects you can talk about and nothing exterior to the theory.
Carnap [6] had also proposed to consider objects as having a spatio-temporal extension and built a few theories on that basis, but making each time different assumptions about objects of that theory: he had obviously in mind the classical models of topology (points and set of points) but axiomatized it in many different ways. Another attempt in the direction of logical modeling of physical objects was made in much the same spirit by Woodger to apply it to biological systems [44], and Carnap took it as a basis for another theory extending the language of space-time objects. He combined a theory of part-whole relations with a temporal order and sketched what the links between them should be (most incompletely though). He moreover defined a notion of "momentary" objects close to what would be now called an atomic temporal part, and assumed that every object has a momentary part. A "slice" is then a maximal momentary part of an object. Although his axioms are not really consistent with his definition of a temporal order, we have here the germ for the idea of temporal slice. However, in our theory, we make no assumption as to the nature of time with respect to atomicity.
These somewhat different approaches have in common that they assume the existence of a spatio-temporal continuum (Carnap assumes space-time is dense, unbounded at both ends,

and mathematically continuous), which seems to us too strong and unpractical a theory for defining common-sense notions. Besides they jeopardize any attempt at consistency or completeness by taking aboard notions that are not definable in their own framework. The theory we propose avoids such pitfalls by staying within a topological theory enriched with a temporal orientation, and by making no unnecessary assumption about the nature of space-time; thus we avoid a proliferation of objects that have no intuitive meaning or relevance for common sense. The definition of a temporal slice then is:

D 4.5 $\text{TS}xy \triangleq \text{P}xy \wedge \forall z\ ((\text{P}zy \wedge z \subseteq_t x) \rightarrow \text{P}zx)$

It has the following properties:

Th 4.13 $\text{TS}xx$

Th 4.14 $(\text{TS}xy \wedge \text{TS}yx) \rightarrow x = y$

Th 4.15 $(\text{TS}xy \wedge \text{TS}yz) \rightarrow \text{TS}xz$

Th 4.16 $(\text{TS}xy \wedge \text{TS}zy\ \wedge\ x \subset_t z) \rightarrow \text{TS}xz$

In order to prevent the proliferation of unnecessary entities, we only want to impose the existence of a slice of an object when it temporally intersects another (σ), so that we state only the existence of *meaningful* temporal parts, corresponding to interactions between different terms. For that purpose, it is enough to state the following axiom:

A 4.12 $y \subseteq_t x \rightarrow \exists u\ (\text{TS}ux\ \wedge\ u \equiv_t y)$
(any object x has a slice u t-equivalent to any object y temporally included in x)

and we then have the following properties:

Th 4.17 $\forall x, y (x\sigma y \rightarrow \exists u (\text{TS}ux\ \wedge\ u \subseteq_t y))$

Th 4.18 $\text{P}xy \rightarrow \exists z (\text{TS}zy\ \wedge\ z \equiv_t x)$
(there is a temporally equivalent slice for all parts of an entity)

Th 4.19 $(\text{TS}xy \wedge \text{TS}zy\ \wedge\ x \equiv_t z) \rightarrow x = z$
(this t-equivalent slice is unique)

We will note $x_{/y}$ this corresponding slice; $x_{/y}$ is the part of x corresponding to the “lifetime” of y, when $y \subseteq_t x$. This is the way all temporal parts can be defined.

This allows for the definition of complex relations corresponding to the trajectories of regions with respect to one another; for instance, a region x “leaving” another one y “during” a third one z can be expressed as[6]:

D 4.6
$$POx_{/z}y_{/z}\ \wedge\ \forall u (u < (x \cdot y)_{/z} \rightarrow u < z)\ \wedge\ \exists v_1 \exists v_2 (TSv_1x\ \wedge\ TSv_2x\ \wedge\ Pv_1y_{/z}\ \wedge\ \neg Cv_2y_{/z})$$

[6] A systematic exploration of motion classes and of valid inferences on them can be found in [31].

5 The Problem of Continuity

Common-sense motion is something more than just a change of location. It is often assumed motion is a *continuous* change of location. Continuity can be intuitively seen, independently of any framework, as the following property: between two consecutive states of the "world" (or situations), there must be a state whose characteristics are intermediary between those of the two states. When situations are arbitrary and one can make any distinction about the space one consider, continuity of motion is usually thought to be something like a continuous function from the real line (to which time is assimilated) to the domain of locations. It is questionable that such a concept is of any use in a mereo-topological framework, since this kind of continuity implies a metrics on time and space. It moreover gives rise to famous paradoxes which conclude that motion must be impossible. Galton's work have investigated that kind of problems related to the classical representation of motion and has come to the conclusion that mathematical continuity was not necessary for common-sense representations of motion: since mereo-topology tells us only about connections between entities and parts of entities, it is enough to characterize continuity as a set of constraints on transitions between these relations through time: an object cannot be a proper part of another and then suddenly be disconnected from it. We see at this point how this connects with the identity problem of continuants: two objects have often be said to be the same if one is a continuous evolution of the other. Thus a definition of continuity would constitute a criterion for identity of objects.
Galton does not give us a generic characterization of spatial continuity though, he merely states what transitions should be acceptable in a qualitative framework and over what kinds of temporal entities (open or closed ones); moreover this was done only for closed spatial entities. In contrast, our framework allows for a simple characterization of continuity. A necessary condition is temporal self-connectedness (CON_t); the other has to do with mereology: for an object to be continuous, it must not suddenly lose or gain parts. In our case it means a spatio-temporal object cannot have time-slices disconnected from a temporally connected part of itself (see figure 3). Consider a spatio-temporal object w, a time slice x of w, and u,

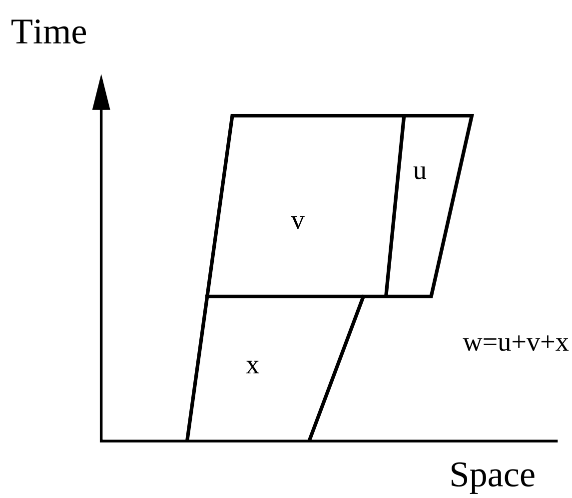

Figure 3: A non continuous region

a part of w such that $u \gtrless x$. As x is a time-slice it must be connected to any time-connected part of w for w to be said continuous, hence we want $\mathrm{C}xu$; otherwise one can have what is shown figure 3, where a discontinuity corresponds to an horizontal leap. Formally, we define a continuous spatio-temporal object:

D 5.1 CONTINU$w \triangleq$CON$_t w \wedge \forall x \forall u\ ((\text{TS}xw\ \wedge\ x \asymp u \wedge \text{P}uw) \rightarrow \text{C}xu)$

This rather intuitive definition has proved to be quite powerful; we have shown in [31] that the conceptual neighborhoods of RCC8 [35] can be recovered from it under certain conditions. It also shows what was left more or less implicit in other work on these conceptual neighborhoods: that the objects considered must be closed and non-atomic (or have a non-atomic interior) for this notion of continuity to be valid. Open entities allow for different continuous transitions (e.g. from ¬C to PO), and they cannot simply be ignored by a common-sense theory of space (for an idea of their importance, see [8]). This is why we use Asher and Vieu's topological theory instead of RCC, which does not distinguish open and closed entities.

6 Towards a Theory of Objects

The theory presented here is only concerned with the spatio-temporal extents of what could be physical objects or events. In this framework, because of the extensionality of C, any two co-located regions of space-time are equal, which is not the case of objects or events. This theory could however be applied to objects if extensionality was modified to hold only of objects of the same type (two co-located objects of the same type are equals); in that case spatio-temporal equality would correspond to an equivalence relation on spatio-temporal objects. We could consider an ontology of objects such as [16, 43], in which one distinguishes (among others) physical objects, substances, locations; this way, for instance, objects and substances could be differentiated from the region of space they occupy, and an object would be different from the substance that it is made of (although occupying the same region of space-time). In the same way, we might want to distinguish a forest from the collection of its trees, and this could be taken care of by a separate ontological component dealing with plural terms (in the spirit of [28] for example). We do not insist on what should be the actual ontological distinctions of the resulting system, as it is not uncontroversial, but we claim that our theory is a sound base for that purpose. The case of events is still a bit different, since there is no accepted way of individuating events and, a fortiori, this should not be done on the spatio-temporal level; one thing that should be expressed at least is that the spatio-temporal extent of an event should include the spatio-temporal parts of the objects involved in that event corresponding to the time extension of the event.

Some properties of our model are already quite close to what we might expect from a theory of temporal parts. For instance, parts of a region can persist after its "death", and the sum of these parts after it no longer exists is thus different from it; this is simply expressed in our formalism as shown below, and is illustrated Figure 4. To illustrate this, let's consider a molecule that was part of Julius Caesar and that now belongs to my body[7]. Can I be said to overlap with Julius Caesar ? The answer in our framework is no:
let 'j' denote Julius Caesar, 'i' denote myself, living in the present and m a molecule. m was part of JC can be expressed as: $\text{P}(m/_j)j$ (m was part of j during j's lifespan). Likewise, "m is now a part of myself" can be expressed as: $\text{P}(m/_i)i$ (considering i will probably not survive the molecule either). In our framework the previous two pieces of information are quite consistent with the fact that $j < i$, which implies $\neg\text{O}ji$.

Another thing to consider is the place of atoms in a theory of space-time. Most mereological theories are undecided about the existence of atoms. In case atoms are allowed for spatial models, topological change concerning them cannot be continuous in the traditional sense, as

[7]This example was suggested to us by an anonymous reviewer; one has to keep in mind when considering this example that we have introduced a theory of spatio-temporal regions, which can be interpreted as referents of physical objects, but which are *not* physical objects.

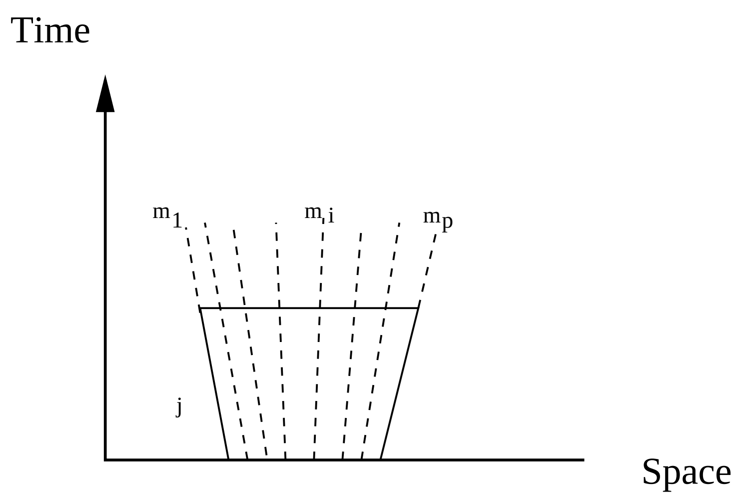

Figure 4: Parts m_i surviving a whole j

we have seen section 4. In our framework, it means entities having atomic slices are the limit of what can be distinguished in the framework. Entities can “gain” or “lose” atoms and still be continuous in the sense defined above. This is a satisfactory behavior if atoms of a space-time theory (or a mereological theory of objects) are considered to be the finest distinctions needed for this particular theory. As soon as an object gains or loses a non-atomic part, it ceases to be continuous.

We thus believe the theory we have presented can lay the foundations for an intuitive and expressive theory of four-dimensional objects, characterized by a notion of continuity that stays within a mereo-topological framework.

7 Conclusion

We have presented here a mereo-topological theory of spatio-temporal regions and studied the properties of such a theory with respect to a few issues that are common in mereological and topological theories: the definition of temporal parts, the problem of continuity, the meaning of atoms. We have used this theory for reasoning on motion in [31] and shown it was a interesting way of dealing with underspecified representations for qualitative reasoning. We have moreover applied it to the representation of the semantics of some motion verbs in [32]. We believe this work shows the interest and exploitability of a theory of space-time in a variety of tasks, ranging from cognitive modeling to qualitative reasoning in AI, and that the ontological model is a practical and rigorous tool for the reformulation of many existing problems in AI and philosophical ontology. A natural extension to this work would be now to deal with objects of our everyday world (and not just spatio-temporal regions), as sketched in Section 6.

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